Cross-referencing the documentation

When reading this manual, you will find references to other Stata manuals, for example, [U] 27 Overview of Stata estimation commands; [R] regress; and [D] reshape. The first example is a reference to chapter 27, Overview of Stata estimation commands, in the User’s Guide; the second is a reference to the regress entry in the Base Reference Manual; and the third is a reference to the reshape entry in the Data Management Reference Manual.

All the manuals in the Stata Documentation have a shorthand notation:

- [GSM] Getting Started with Stata for Mac
- [GSU] Getting Started with Stata for Unix
- [GSW] Getting Started with Stata for Windows
- [U] Stata User's Guide
- [R] Stata Base Reference Manual
- [BAYES] Stata Bayesian Analysis Reference Manual
- [BMA] Stata Bayesian Model Averaging Reference Manual
- [D] Stata Data Management Reference Manual
- [FN] Stata Functions Reference Manual
- [LASSO] Stata Lasso Reference Manual
- [XT] Stata Longitudinal-Data/Panel-Data Reference Manual
- [MI] Stata Multiple-Imputation Reference Manual
- [RPT] Stata Reporting Reference Manual
- [SVY] Stata Survey Data Reference Manual
- [TABLES] Stata Customizable Tables and Collected Results Reference Manual
- [I] Stata Index

Intro — Introduction to causal inference and treatment-effects estimation

Description

This entry provides an introduction to causal inference and treatment-effects estimation. It presents concepts, frameworks, and assumptions that researchers consider when they wish to draw causal inferences in their analyses.

For information on Stata commands that estimate treatment effects and that are specifically designed for causal inference, see \texttt{Causal inference commands}. For more in-depth introductions to causal inference, see Imbens and Rubin (2015), Robins and Greenland (1992), Hernán and Robins (2020), and Pearl (2009).

Remarks and examples

Remarks are presented under the following headings:

- Motivation: Causation versus association
- Causal inference workflow
- Potential-outcomes framework
  - Treatment-effect estimands
  - Assumptions required in potential-outcomes framework
  - Relaxing causal assumptions
- Causal diagrams
- Importance of identification before estimation

Motivation: Causation versus association

Research may be driven by the desire to evaluate causation or association. Causal questions explore changes in the outcome when we change a variable under our control or examine what would happen to the outcome if a variable of interest had not changed. For example:

- Does receiving a treatment cure the illness?
- What would have happened to the inflation rate if the Federal Reserve had not increased interest rates?
- Does smoking reduce fetal growth?
- Does raising the minimum wage decrease unemployment?

In contrast, associative questions observe patterns in data. For example:

- How does the cure rate between patients who received treatment and those who did not receive treatment compare?
- Is there a correlation between interest rates and inflation?
- Is there a difference in the mean birthweight of infants born to mothers who smoke versus those born to mothers who do not smoke?
- What is the difference in the unemployment rate between states that have implemented an increase in minimum wage and those that have not?
The first set of questions asks what happens when there is an intervention or imagine a scenario where a variable changes versus does not change. The second set of questions observe only the pattern without intervention.

To examine some of the considerations we must face when performing causal inference, we consider a hypothetical study, conducted by a software company called Statanium, that examines the relationship between the number of breaks taken by software developers and their productivity. The company wants to find out whether the number of breaks impacts productivity. As a user of Statanium and an expert in causality, you are hired to advise the company whether it should encourage developers to take additional breaks during their workday. The question the company is interested in is causal because it aims to compare the productivity when the number of breaks is increased versus the productivity when the number of breaks (and other possible factors) remains unchanged. If the two outcomes of productivity are different, then the action of increasing the number of breaks has a causal effect. In causal inference literature, the action of increasing the number of breaks is referred to as a treatment or an intervention.

You could estimate an association between increased breaks and productivity via correlation or many other statistical methods that estimate dependence. However, the interest is in estimation of causal effect. The well-known expression “association is not causation” suggests that for any given amount of association, only some part or none of it is causal. Thus, a challenge in causal inference is to identify and eliminate relationships that are only associative. To perform causal inference in our case, we want to create a hypothetical scenario where the number of breaks is increased and all other factors that may influence productivity remain fixed. Then we can determine the causal effect of the additional break.

When it is possible, randomized experiments are a popular method to estimate causal effects because the treatment (number of breaks) is controlled by the experimenter. Therefore, the treatment assignment is guaranteed to be random and unrelated to all other factors that may determine the outcome. This can be represented using causal diagrams, as in figure 1, where $T$ represents the treatment, $Y$ represents the productivity or outcome, and $X$ represents all other factors. The absence of an arrow between $X$ and $T$ indicates that $X$ does not affect $T$.

![Figure 1](image)

However, randomized experiments are not always feasible, and causal effects need to be inferred from observational data. In the observational data case, the experimenter does not have control over the treatment assignment, and the assumption that all other factors are held constant in both the observed and hypothetical worlds may not hold. This is due to the presence of confounding factors (for example, job satisfaction) that affect both the treatment and outcome. This is reflected in the causal diagram in figure 2 by arrows from $X$ to both $T$ and $Y$. 
Here the measured total association between $T$ and $Y$ contains both causal path $T \rightarrow Y$ and association path $T \leftarrow X \rightarrow Y$, highlighted in red. To identify the causal effect and make further causal inference, you need to eliminate the association represented by the red path by accounting or adjusting for other confounding factors.

The following highlights the significance of adjusting for confounding variables. Suppose that in figure 3 you plot the productivity of software developers as a function of the average number of breaks.

The plot shows a positive association between the number of breaks and productivity, implying that taking more breaks leads to higher productivity. The questions of interest are whether this association can be considered causal and whether Statanium should motivate its hardworking developers to take more breaks during the workday. The conclusion that more breaks are beneficial would be valid if the model assumed is as shown in figure 1. However, as an expert, you believe that there are confounding factors, such as workload or job satisfaction, that need to be considered, as shown in figure 2. To account for the effect of job satisfaction, in figure 4, you plot the productivity of software developers against the average number of breaks they take, considering different levels of job satisfaction—dissatisfied (orange), satisfied (red), and highly satisfied (yellow).
The plot shows that when job satisfaction is accounted for, the effect of the number of breaks on productivity changes from positive to negative. This phenomenon is known as Simpson’s paradox (Blyth 1972), where the overall effect appears to be positive but, when it is adjusted for a confounder, the effect sign changes direction.

The above example illustrates the fundamental difference between causation and association. For example, if for some reason the researcher does not account for the job satisfaction or it is not observed, then the estimated association cannot be interpreted as causal.

Below, we introduce the workflow, popular causal inference frameworks, and assumptions that allow researchers to draw causal inferences rather than merely find associations.

Causal inference workflow

The causal inference literature recommends the following three-phase workflow (Pearl 2009; Imbens 2020; and Heckman and Pinto 2022) when a research question is causal in nature:

1. Hypothetical modeling: Researchers make assumptions about relationships among variables based on their understanding and expertise. These assumptions are related not only to the treatment variable and the outcome of interest but also to any variables that might be related to the treatment or the outcome. The assumptions regarding these relationships cannot be tested from data; therefore, the validity of these assumptions must come from previous theory or the researcher’s own expertise.

2. Causal effect identification: Based on the assumptions made in the first phase, the researcher tries to determine whether the causal effect can be identified.

3. Parameter estimation: If the answer to the second phase is positive, the researcher can then use various estimation techniques, such as those provided by the commands discussed in [CAUSAL] Causal inference commands, to estimate the causal effect.

Potential-outcomes framework

The potential-outcomes framework is one of the most commonly used theories for understanding and evaluating causal inference. The foundation of this framework is motivated by the idea of a natural experiment (Imbens 2020), which focuses on finding settings and identification strategies under which the assignment of a treatment can be considered as good as random even though the data are observational rather than from a randomized experiment. The roots of the potential-outcomes framework trace back to a seminal paper by Splawa-Neyman (1923), and the framework was formally
introduced in Rubin (1974). Fisher (1925) built upon Neyman’s ideas and introduced the idea of physical randomization, which formally defines the concept of a treatment-assignment mechanism. For further reading, see Imbens and Rubin (2015).

As discussed earlier, the goal of causal inference is to estimate the change in an outcome as the treatment varies. In the Statanium example, we now assume that the treatment (number of breaks) and the outcome (productivity) are binary. That is, \( T_i = 1 \) means the Statanium developer takes additional breaks, and \( T_i = 0 \) otherwise. Similarly, \( Y_i = 1 \) if a developer’s productivity increases, and \( Y_i = 0 \) otherwise. Thus, we are interested in estimating the change in productivity if the developer takes additional breaks, \( T_i = 1 \), versus if the developer does not take additional breaks, \( T_i = 0 \). However, for a given developer, we can observe only one outcome. This is known as the fundamental problem of causal inference (Holland 1986). The potential-outcomes framework provides tools and assumptions to solve this problem.

The important concepts in potential outcomes are the unit, treatment, and outcome. A unit is the research object to which treatment is assigned. It can be, for example, a person, a company, a school, or a county. Treatment is the action that we apply to a unit. In this entry, treatment is denoted as \( T \). For binary treatment, the units to which treatment is applied (\( T = 1 \)) are called the treated group, and the units to which the treatment is not applied (\( T = 0 \)) are called the control group. The potential outcome of treatment with value \( t \) for unit \( i \) is denoted by \( Y_i(t) \) or \( Y_i(0) \). In our running example, the unit is the developer, treatment is taking additional breaks, and the outcome is whether productivity improves.

The observed outcome is related to the potential outcome through

\[
Y_i = T_i Y_i(1) + (1 - T_i) Y_i(0).
\]

That is, if the unit receives the treatment \( T_i = 1 \), then \( Y_i = Y_i(1) \) and \( Y_i(0) \) otherwise.

A potential outcome that is not observed is called a counterfactual outcome. For example, for the developer who took additional breaks, \( Y_i(1) \) is the observed outcome and \( Y_i(0) \) is the counterfactual outcome.

### Treatment-effect estimands

**Individual treatment effect (ITE).** For each developer or unit \( i \), the ITE is defined as

\[
Y_i(T_i = 1) - Y_i(T_i = 0)
\]

Because only one of the potential outcomes is observed, we cannot identify this quantity directly from the data without making assumptions about the unobserved counterfactuals and the assignment to treatment. However, estimates of individual effects can be useful for providing insight into how a treatment may affect an individual. For instance, in epidemiology, estimated ITEs could help determine whether a treatment is likely to be helpful for a particular patient.

We can also define and estimate effects that allow us to draw interesting causal inferences for the population instead of for each individual.

**Average treatment effect (ATE).** The ATE is also known as the average causal effect. The ATE at the population level can be defined as the mean difference in potential outcomes when units received a treatment versus when units did not receive any treatment,

\[
\text{ATE} = E[Y(1) - Y(0)] = E[Y(1)] - E[Y(0)]
\]

where \( Y(t) = Y(T = t) \), for \( t = \{0, 1\} \). We say that the ATE of treatment \( T \) on outcome \( Y \) exists if \( E[Y(1)] \neq E[Y(0)] \).
The ATE provides an estimate of the expected average effect in the population and can therefore be used to answer many interesting questions that could help policymakers make decisions. In our Statanium example, the ATE is the expected difference in productivity if all developers took extra breaks versus none taking extra breaks. Statanium could determine whether extra breaks are beneficial and set break policies based on the estimated ATE. Similarly, we interpret the ATE for a couple of our motivating causal questions. When investigating whether a treatment cures an illness, we could interpret the ATE as the expected difference in the proportion of individuals who were cured from an illness when everyone received the treatment versus when no one received the treatment. When evaluating a raise in minimum wage, we could interpret the ATE as the expected change in unemployment rate when the minimum wage is raised for everyone versus when the minimum wage stays the same.

When the ATE is of interest, we must use an appropriate method that leads to an estimate of this quantity. We might be tempted to use association difference to estimate ATE, where association difference is the conditional mean difference of outcomes between treatment and not treated units. The treatment $T$ and $Y$ are associated if $E[Y|T = 1] \neq E[Y|T = 0]$. Thus, one might try to estimate the causal quantity in (1) with the statistical quantity $E[Y|T = 1] - E[Y|T = 0]$. For the Statanium example, this quantity could be estimated from the data by contrasting the sample average of the developers that took the treatment with the ones that did not. Mathematically,

$$\frac{\sum_{i=1}^{N} Y_i T_i}{\sum_{i=1}^{N} T_i} - \frac{\sum_{i=1}^{N} Y_i (1 - T_i)}{\sum_{i=1}^{N} (1 - T_i)}$$

However, in general, ATE and association difference are different; otherwise, association would be causation. Figure 5 highlights the causation–association difference.

![Figure 5](image.png)

To infer causation, we imagine that each treatment or intervention is applied to the entire population, and the difference between the red and white circles is observed in the same population. In contrast, to infer association, we condition on $T = t$ and estimate the difference between subsets of populations.

**Average treatment effect on the treated (ATET).** For the treated group, the treatment-effect estimand is the ATET:

$$\text{ATET} = E[Y(1)|T = 1] - E[Y(0)|T = 1]$$
The ATET is useful when researchers are interested in the effect on those who received the treatment. This effect may be of particular interest when the goal is to understand how a treatment performs for the subpopulation at which the treatment was targeted. The effect is a comparison with what would have happened in this subpopulation if they had not received the treatment. In our Statanium example, we estimate the expected difference in productivity for developers who took extra breaks compared with the productivity of these developers if they had not taken extra breaks. When we investigate whether a treatment cures an illness, the ATET is focused only on those who received the treatment. For this group, what is the expected difference in the proportion of individuals who were cured when given the treatment versus the proportion cured if they had not received the treatment? When evaluating a raise in minimum wage, we might be interested in the effect for states that enacted a minimum wage increase. The ATET is the expected difference in unemployment rate for these states compared with a situation where these states did not raise minimum wage.

Additional estimands. In specific situations, there are several other treatment-effect estimands that may be of interest and that can be defined in the potential-outcomes framework. We briefly mention a few here.

Sometimes, it is assumed that the effect of $T$ on $Y$ may involve both a direct effect and an indirect effect such that $T$ has an effect on another variable $M$, known as a mediator, and that $M$ in turn has an effect on $Y$. In the Statanium example, we might believe that increased breaks could improve the developers’ focus while working and that improved focus leads to increased productivity. In such a situation, comparisons of average direct treatment effects and average indirect treatment effects may be of interest. See Robins and Greenland (1992), VanderWeele (2015), and Pearl and MacKenzie (2018) for discussions of causal mediation analysis and definitions of applicable direct and indirect effect estimands.

Recently, there has been a surge of interest in estimating the treatment effect when it differs between subgroups, also known as the heterogeneous treatment effect (Athey and Imbens 2016; Künzel et al. 2019; and Nie and Wager 2021). At the subgroup level, the treatment-effect estimand is called conditional average treatment effect (CATE),

$$\text{CATE} = E[Y(1)|X = x] - E[Y(0)|X = x]$$

where $Y(t)|X = x$ for $t = \{0, 1\}$ are the potential outcomes of the subgroup $X = x$.

Assumptions required in potential-outcomes framework

At this point, it is natural to ask under which conditions treatment effects can be estimated from observational data. For illustration purposes, our focus will be on ATE. We are interested in assumptions for which

$$\text{ATE} = E[Y(1)] - E[Y(0)] = E[Y|T = 1] - E[Y|T = 0]$$

A causal quantity, that is, ATE, is “identifiable” if it can be computed from a statistical quantity $E[Y|T = t]$. By statistical quantity, we mean an object that can be estimated from data.

In the potential-outcome framework, commonly used assumptions are the stable unit treatment value assumption (SUTVA), unconfoundedness assumption, and overlap assumption.

SUTVA. The SUTVA, along with consistency, states that for a given unit, the treatment of other units does not affect the outcome of the treatment received by that unit. Consequently, there are two different sources in which SUTVA could be violated. The first source is a violation of the consistency condition, which might not hold in some studies (Cole and Frangakis 2009; and Schwartz, Gatto,
Usually, the problem arises from the vagueness of the assigned treatment. For example, for the Statanium example, if the treatment is additional breaks, this may be one additional break or three additional breaks. If we observe only whether a treatment has been assigned, the counterfactual \(Y(T = t)\) is not well defined because different numbers of breaks have different causal effects. The second source of violation arises if some units are influenced by the assignment of the treatment of other units. For example, if some developers in the control group noticed that the developers in the treatment group are less productive, they might change their lifestyle and start taking fewer breaks, which can lead to an increase in productivity. Typically, interference can occur because of spillover effects or noncompliance or because the units are members of a social network. For details, see Hernán and Robins (2020, chap. 3).

**Unconfoundedness assumption.** The unconfoundedness assumption goes by many names, including the conditional-independence assumption, the ignorability assumption, and the exchangeability assumption. The impetus behind this assumption is to make the treatment and control group comparable within strata defined by \(X\). It states that the probability of a positive outcome in the control group (white group in the figure above) would be the same as the probability of a positive outcome in the treatment group (red group) had units in the control group received the treatment given to those in the treatment group. In other words, under unconfoundedness, if by accident the treatment were given to the white group instead of the red group, then the ATE would remain the same. Mathematically, this is represented by \((Y(1), Y(0)) \perp \perp T|X\), where \(\perp \perp\) denotes (conditional) independence and \(X\) are potential confounders. It is important to differentiate between \(Y(t) \perp \perp T|X\), which utilizes potential outcomes, and \(Y \perp \perp T|X\). The unconfoundedness assumption does not imply that \(Y \perp \perp T|X\). On the contrary, if the ATE is not zero, then \(Y\) and \(T\) are associated. In the context of the Statanium example, the underlying intuition is that for two developers \(i\) and \(j\), their potential outcomes should be independent of the treatment assignment \(P(Y_i(0), Y_i(1)|T = t_i, X) = P(Y_j(0), Y_j(1)|T = t_j, X)\), for \(t_i, t_j \in \{0, 1\}\).

**Overlap assumption.** Finally, the overlap or positivity assumption \(P(T = t|X = x) > 0\) implies that the treatment assignment should be stochastic. For example, if the developers who are highly satisfied are always assigned treatment \(T = 1\), then there is no meaning in studying the treatment \(T = 0\). In contrast to unconfoundedness, the overlap assumption can sometimes be verified from the data. For details, see Hernán and Robins (2020, chap. 12).

When these three main assumptions in the potential-outcome framework are satisfied, then the ATE is identified, and estimation methods available in the \texttt{teffects} suite of commands can be used.

### Relaxing causal assumptions

One of the crucial questions in causal inference is whether all confounders have been accounted for in the study. Unfortunately, the unconfoundedness is not testable from data. There are observable and unobservable confounders that, unaccounted for, will lead to incorrect conclusions. This problem of unobserved confounders or endogeneity is usually addressed using estimators that account for an endogenous treatment or instrumental-variable method.

Discussions of these issues can be found in Imbens and Rubin (2015). For a general treatment, see Wooldridge (2010) and Angrist and Pischke (2009).

The potential-outcome framework is not restricted only to models in which identification relies on conditional unconfoundedness, which is sometimes referred to as selection on observables. It also allows us to recover the ATET by controlling for individual and time-varying unobservables, without the need to control for covariates. One example of this is the difference-in-differences method, which can be used to estimate ATET by comparing the change in outcomes between the treatment and control groups over time.
Causal diagrams

In Motivation: Causation versus association, we used causal diagrams, specifically directed acyclic graphs (DAGs), to represent our assumptions about causal relationships. DAGs are helpful in phases 1 and 2 described in Causal inference workflow. The intellectual predecessor of causal diagrams, or more generally, the structural causal models (SCMs) framework, goes back to the pioneering work of geneticist Wright (1921, 1934) and econometricians Frisch and Waugh (1933) and Haavelmo (1944) and the references therein. Early econometricians were attempting to conceptualize the fact that, unlike correlation, the regression of a variable $Y$ on $T$ has a natural direction and is different from the regression of $T$ on $Y$. This distinction led to the development of causal frameworks by Pearl (2009) and Heckman and Pinto (2022). Furthermore, this differentiation is linked to the concept known as “the ladder of causality” (Pearl and MacKenzie 2018), where the lowest rung signifies association and higher rungs address causal queries. Here our focus is on causal diagrams. We provide only a brief introduction and explore the usefulness of causal diagrams in determining causal-effect identification before estimating treatment effects using one of Stata’s estimation commands. We will mention some of the common terminology used in SCM, but for more detailed descriptions of these terms and additional details on SCM, we refer you to Pearl (2009), Peters, Janzing, and Schölkopf (2017), Bareinboim et al. (2022), and the references therein.

DAGs are characterized by nodes and directed edges between the nodes (shown as circles and arrows, respectively, in diagrams below), which represent causal relationships. The absence of edges between nodes indicates that there is no direct causal effect between those nodes. Unlike path diagrams in SEM, independent unobserved error terms are not depicted in DAGs.

For the Statanium example, we capture our assumptions using the DAG in the left panel of figure 6.

![DAG example](image)

Figure 6. (left) Unconfoundedness and (right) Instrumental variable

Here $X$ is a confounder in the causal relationship for $T$ and $Y$, and because $X$ is observed, we can control or adjust for it by including it as a covariate in our model when we estimate the causal effect. With this adjustment, we eliminate associative paths and identify the causal relationship between $T$ and $Y$. In the causal diagram literature, the path $T ← X → Y$ is known as a backdoor path because it contains an arrow that goes to the back of node $T$ (hence, the name “backdoor”). If we do not condition on $X$, we say that we are leaving the path open, and we cannot estimate the causal effect. In this situation, we meet what is known as the backdoor path criterion because, by adjusting for $X$, we close all paths entering $T$.

In comparison, in the DAG shown on the right of figure 6, $X$ is an unobserved confounder, which is indicated by shading the node for this variable. In this case, we cannot condition on $X$ by including it as a covariate when we estimate the causal effect. Thus, the backdoor path criterion is not satisfied, and the causal effect of $T$ on $Y$ cannot be identified in this way. Fortunately, in this case, the assumed DAG still allows us to estimate the causal effect because $Z$ can be used as an instrumental variable. A proper instrumental variable can be included as a predictor of the treatment to eliminate unobserved sources of confounding such as that from $X$ here. $Z$ can be used as a proper instrumental variable because there is no direct effect of $Z$ on the outcome $Y$ and there are no unobserved confounders between $Z$ and $X$ or $Z$ and $Y$.

For our simple DAGs, we were able to evaluate identification easily by considering the relationships among the few variables. For more complex situations, there exist several algorithms and criteria that are designed to check and achieve causal effect identification using graphical approach. We do not
discuss those algorithms here; we instead refer the interested reader to Shpitser and Pearl (2008), Pearl (2009), Maathuis and Colombo (2015), Perković et al. (2015), and van der Zander, Liškiewicz, and Textor (2019). These algorithms can select variables that form a valid adjustment set that can be used for treatment-effect estimation. Once the valid adjustment set is selected, researchers can use their preferred treatment-effect estimation method from the list of commands in [CAUSAL] Causal inference commands.

Below, we demonstrate how a DAG can be used in the phases discussed in the Causal inference workflow. Consider a study that examines the effect on five-year mortality of polycystic kidney disease (PKD) among patients undergoing peritoneal dialysis (PD). Recall that in phase 1 of the workflow, we construct a hypothetical model that represents researchers’ assumptions. We begin our causal analysis by drawing a DAG that represents our assumptions about the causal effects we are interested in estimating and the relationship among all variables, observed and unobserved, that may affect the variables of interest. Here we borrow the hypothetical model from Evans et al. (2012), illustrated in figure 6.

The goal of phase 2 is to check whether the causal effect of the treatment variable PKD on mortality M is identifiable. For the variable definitions, see table 2. The gray node in the graph indicates that the variable AT is not observed.

<table>
<thead>
<tr>
<th>PKD - polycystic kidney disease; treatment variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>M - mortality; outcome variable</td>
</tr>
<tr>
<td>A - patient age</td>
</tr>
<tr>
<td>G - patient gender</td>
</tr>
<tr>
<td>C - patient’s comorbidities summary index</td>
</tr>
<tr>
<td>PD - indicator for patients undergoing peritoneal dialysis</td>
</tr>
<tr>
<td>AT - indicator for patients undergoing peritoneal dialysis</td>
</tr>
<tr>
<td>DT - type of medical assistance</td>
</tr>
<tr>
<td>DT - type of peritoneal dialysis</td>
</tr>
</tbody>
</table>

Note that if all variables in the causal graph are observed, then the causal effect can be identified using backdoor path criteria (Pearl 2009).
However, in our example, \( AT \) is not observed, and there are unblocked noncausal (backdoor) paths that hinder identification:

\[
\begin{align*}
P1: & \quad PKD \leftarrow CI \rightarrow AT \rightarrow M \\
P2: & \quad PKD \leftarrow A \rightarrow AT \rightarrow M \\
P3: & \quad PKD \leftarrow CI \leftarrow G \rightarrow AT \rightarrow M
\end{align*}
\]

Note that, despite a common misconception, conditioning on all observed variables does not mitigate the identification issue. For example, consider the path \( PKD \rightarrow DT \leftarrow AT \). This type of path is known as a v-structure, and DT is called a collider. PKD and AT are unrelated because the collider closes or blocks the path. However, we would not want to include DT in the model because conditioning on the collider DT introduces a selection bias through \( PKD \rightarrow DT \leftarrow AT \rightarrow M \). For more examples, see \textit{Importance of identification before estimation}. Interestingly, the above noncausal paths P1, P2, and P3 can be blocked if we condition on variables CI and A. In fact, CI and A are the necessary minimal sufficient set that makes the causal effect of PKD to M identifiable. Now that we know that the causal effect is identified, we are ready to move to phase 3, estimation of the treatment effect. In this case, CI and A can be used as covariates in, for instance, the inverse-probability weighting estimator. For details, see [CAUSAL] \texttt{teffects ipw}.

\[
. \texttt{teffects ipw (M) (PKD CI A)}
\]

\texttt{(output omitted)}

**Importance of identification before estimation**

In this section, we provide numerical examples that reemphasize the importance of the identification step. If a causal effect is not identified, the results of any treatment-effect estimation method are unreliable. We illustrate this point with two simple examples, which also aim to clear up the common misconception that it is harmless to control for more variables in treatment-effect estimation. For more examples, the reader is referred to Cinelli, Forney, and Pearl (forthcoming) and Hünermund, Louw, and Caspi (2021).

\> \textbf{Example 1}

The presumed relationship among variables in figure 7 is well known for introducing collider bias, sometimes called selection bias, in the estimated effect of \( T \) on \( Y \). In the causal diagram literature, the represented DAG is known as a v-structure, and the variable \( X \) as a collider. A v-structure \( T \rightarrow X \leftarrow Y \) has an interesting characteristic that the causal effect of \( T \) on \( Y \) is 0 because the collider \( X \) blocks the path. If we adjust for \( X \), we introduce a selection bias because the adjustment results in an induced association between \( T \) and \( Y \).

![Figure 7.](https://example.com/figure7.png)
Suppose the data come from the data-generation process as

\[
T := \varepsilon_T \\
Y := \varepsilon_Y \\
X := 2 \times T - 0.5 \times Y + \varepsilon_X
\]

where \(\varepsilon_T, \varepsilon_Y, \varepsilon_X \sim N(0, 1)\). The regression of \(Y\) on \(T\) and \(X\) shows that \(T\) and \(X\) are for prediction purposes if we were to determine this on the basis of their \(p\)-values. However, the \(X\) is a bad control in terms of answering a causal question about the effect of \(T\) on \(Y\).

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 1,000</th>
<th>F(2, 997) = 111.51</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>187.350778</td>
<td>2</td>
<td>93.675389</td>
<td>Prob &gt; F = 0.0000</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>837.504999</td>
<td>997</td>
<td>.840025074</td>
<td>R-squared = 0.1828</td>
<td>Adj R-squared = 0.1812</td>
</tr>
<tr>
<td>Total</td>
<td>1024.85578</td>
<td>999</td>
<td>1.02588166</td>
<td>Root MSE = .91653</td>
<td></td>
</tr>
</tbody>
</table>

| \(Y\) Coefficient Std. err. | \(t\)  | P>|t| | [95% conf. interval] |
|-----------------------------|-------|-------|---------------------|
| \(T\)                       | .8088171 | .0607918 | 13.30 | 0.000 | .6895225 .9281117 |
| \(X\)                       | -.3942266 | .0263994 | -14.93 | 0.000 | -.4460314 -.3424219 |
| _cons                        | .0345212 | .0289833 | 1.19 | 0.234 | -.0223541 .0913965 |

Fortunately, the true causal effect 0 is recovered if we do not control for \(X\).

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 1,000</th>
<th>F(1, 998) = 0.02</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>.025504326</td>
<td>1</td>
<td>.025504326</td>
<td>Prob &gt; F = 0.8748</td>
<td></td>
</tr>
<tr>
<td>Residual</td>
<td>1024.83027</td>
<td>998</td>
<td>1.02688404</td>
<td>R-squared = 0.0000</td>
<td>Adj R-squared = -0.0010</td>
</tr>
<tr>
<td>Total</td>
<td>1024.85578</td>
<td>999</td>
<td>1.02588166</td>
<td>Root MSE = 1.0134</td>
<td></td>
</tr>
</tbody>
</table>

| \(Y\) Coefficient Std. err. | \(t\)  | P>|t| | [95% conf. interval] |
|-----------------------------|-------|-------|---------------------|
| \(T\)                       | .0049212 | .0312266 | 0.16 | 0.875 | -.0563562 .0661986 |
| _cons                        | .0347662 | .0320452 | 1.08 | 0.278 | -.0281175 .0976498 |

Example 2

Here we demonstrate another type of bias that can be induced in the estimated effect of \(T\) on \(Y\).
Compared with the previous example, the harmfulness for controlling $X$ is not explicit. Recall that in the previous section, we mentioned that the nodes for unobserved error terms ($\varepsilon_T, \varepsilon_Y$) are omitted from the causal graph, but they are there. If we imagine that the error term for $Y$ is there, then $Y$ would be a collider because $T \rightarrow Y \leftarrow \varepsilon_Y$. We also see $Y \rightarrow X$, so we can say that $X$ is a descendant of the collider $Y$. It turns out that by controlling on the descendant $X$ of collider $Y$, we induce an association between $T$ and $\varepsilon_Y$ (Pearl 2009, sec. 11.3). This conditioning opens a backdoor path between $T$ and $Y$, which makes the causal effect of $T$ on $Y$ unidentified.

Suppose the data arise from the following data-generation process:

\[
T := \varepsilon_T \\
Y := 2 \times T + \varepsilon_Y \\
X := -0.5 \times Y + \varepsilon_X
\]

Then the causal effect of $T$ on $Y$ is 2. Similar to the previous example, even though $X$ is good for prediction purposes, it is a bad control for causality.

\[
\text{regress } Y \ T \ X
\]

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4420.49975</td>
<td>2</td>
<td>2210.24987</td>
<td>F(2, 997) = 2631.17</td>
</tr>
<tr>
<td>Residual</td>
<td>837.504998</td>
<td>997</td>
<td>.840025073</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>5258.00474</td>
<td>999</td>
<td>5.26326801</td>
<td>R-squared = 0.8407</td>
</tr>
</tbody>
</table>

| Y       | Coefficient | Std. err. | t    | P>|t| | [95% conf. interval] |
|---------|-------------|-----------|------|-------|-----------------------|
| T       | 1.626137    | .0379613  | 42.84| 0.000 | 1.551644 - 1.700631   |
| X       | -.3942266   | .0263994  | -14.93| 0.000 | -.4460314 -- -.3424219 |
| _cons   | .0345212    | .0289833  | 1.19 | 0.234 | -.0223541 -- .0913965 |

Again, the true causal effect is recovered if we do not control for $X$.

\[
\text{regress } Y \ T
\]

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th>Number of obs = 1,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>4233.17448</td>
<td>1</td>
<td>4233.17448</td>
<td>F(1, 998) = 4122.35</td>
</tr>
<tr>
<td>Residual</td>
<td>1024.83027</td>
<td>998</td>
<td>1.02688404</td>
<td>Prob &gt; F = 0.0000</td>
</tr>
<tr>
<td>Total</td>
<td>5258.00474</td>
<td>999</td>
<td>5.26326801</td>
<td>R-squared = 0.8051</td>
</tr>
</tbody>
</table>

| Y       | Coefficient | Std. err. | t    | P>|t| | [95% conf. interval] |
|---------|-------------|-----------|------|-------|-----------------------|
| T       | 2.004921    | .0312266  | 64.21| 0.000 | 1.943644 -- 2.066199  |
| _cons   | .0347662    | .0320452  | 1.08 | 0.278 | -.0281175 -- .0976498 |

These examples demonstrate that, before estimating a treatment effect, we must determine whether and how the effect can be identified based on our assumed relationships among variables. Once we have evaluated identification, we can select an appropriate method for estimating the causal effect. In these examples, we used linear regression. Actually, many of Stata’s regression commands can be used for estimating treatment effects, provided that identification assumptions hold. Stata also offers estimation commands that are specifically designed for estimating treatment effects in various situations. For more information on these specialized commands, see [CAUSAL] Causal inference commands.
References


Description

In this entry, we give you an overview of the estimation commands in Stata that are designed for causal inference. We provide important details about each command so that readers can select the one that best fits their data and research needs.

Here we assume that you are familiar with causal inference and the most common assumptions. For an introduction to these concepts, see [CAUSAL] Intro.

Remarks and examples

Below, we introduce Stata commands that are specifically designed for causal inference. For each command, we provide information on the type of data required and the necessary assumptions. In addition, we outline the type of statistics that can be estimated—typically one or more of the average treatment effect (ATE), the average treatment effect on the treated (ATET), or the potential-outcome means (POM). We also indicate the type of outcome variable (continuous, binary, count, fractional, or nonnegative) and the type of treatment (binary, multivalued, or continuous) that each command supports. Finally, we note which models must be specified: a model for the outcome, a model for the treatment, both, or none.

Remarks are presented under the following headings:

teffects
stteffects
telasso
Difference in differences
Endogenous treatment
Causal mediation
Extended regression models
margins

teffects

The teffects suite of commands is useful for estimating treatment effects from cross-sectional data. These commands rely on the stable unit treatment value assumption (SUTVA), unconfoundedness (conditional-independence) assumption, and overlap assumption.

The commands in the teffects suite and the type of estimator provided by each are as follows:

<table>
<thead>
<tr>
<th>Command</th>
<th>Type of Estimator</th>
</tr>
</thead>
<tbody>
<tr>
<td>teffects ra</td>
<td>Regression adjustment</td>
</tr>
<tr>
<td>teffects ipw</td>
<td>Inverse-probability weighting</td>
</tr>
<tr>
<td>teffects ipwra</td>
<td>Inverse-probability-weighted regression adjustment</td>
</tr>
<tr>
<td>teffects aipw</td>
<td>Augmented inverse-probability weighting</td>
</tr>
<tr>
<td>teffects nnmatch</td>
<td>Nearest-neighbor matching</td>
</tr>
<tr>
<td>teffects psmatch</td>
<td>Propensity-score matching</td>
</tr>
</tbody>
</table>
Details on the available estimands, types of outcomes and treatments supported, and the models to be specified are given below:

<table>
<thead>
<tr>
<th>Command</th>
<th>Estimand</th>
<th>Outcome types</th>
<th>Treatment types</th>
<th>Models specified</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>teffects ra</code></td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>outcome</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>binary</td>
<td>multivalued</td>
<td></td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td>count</td>
<td>nonnegative</td>
<td></td>
</tr>
<tr>
<td><code>teffects ipw</code></td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>treatment</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>binary</td>
<td>multivalued</td>
<td></td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td>count</td>
<td>nonnegative</td>
<td></td>
</tr>
<tr>
<td><code>teffects ipwra</code></td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>outcome</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>binary</td>
<td>multivalued</td>
<td>treatment</td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td>count</td>
<td>nonnegative</td>
<td></td>
</tr>
<tr>
<td><code>teffects aipw</code></td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>outcome</td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td>binary</td>
<td>multivalued</td>
<td>treatment</td>
</tr>
<tr>
<td><code>teffects psmatch</code></td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>treatment</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>binary</td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>teffects nnmatch</code></td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>outcome*</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>binary</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*`nnmatch` includes covariates for modeling the outcome but does not require specification of a functional form for the outcome model.

For further information on these commands and the properties of the estimators that they implement, see `[CAUSAL] teffects intro`. 
**stteffects**

The `stteffects` suite of commands is useful for estimating treatment effects from survival-time data. These commands rely on the SUTVA, unconfoundedness (conditional-independence) assumption, and overlap assumption. They also rely on an assumption that the correct adjustment is made for censoring.

The commands in the `stteffects` suite and the type of estimator provided by each are as follows:

<table>
<thead>
<tr>
<th>Command</th>
<th>Type</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>stteffects ra</code></td>
<td>Survival-time regression adjustment</td>
</tr>
<tr>
<td><code>stteffects wra</code></td>
<td>Survival-time weighted regression adjustment</td>
</tr>
<tr>
<td><code>stteffects ipw</code></td>
<td>Survival-time inverse-probability weighting</td>
</tr>
<tr>
<td><code>stteffects ipwra</code></td>
<td>Survival-time inverse-probability-weighted regression adjustment</td>
</tr>
</tbody>
</table>

Details on the available estimands, types and treatments supported, and the models to be specified are given below:

<table>
<thead>
<tr>
<th>Command</th>
<th>Estimand</th>
<th>Treatment types</th>
<th>Models specified</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>stteffects ra</code></td>
<td>ATE</td>
<td>binary</td>
<td>outcome</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>multivalued</td>
<td></td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>stteffects wra</code></td>
<td>ATE</td>
<td>binary</td>
<td>outcome</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>multivalued</td>
<td>censoring</td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>stteffects ipw</code></td>
<td>ATE</td>
<td>binary</td>
<td>treatment</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>multivalued</td>
<td>censoring</td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td></td>
<td></td>
</tr>
<tr>
<td><code>stteffects ipwra</code></td>
<td>ATE</td>
<td>binary</td>
<td>outcome</td>
</tr>
<tr>
<td></td>
<td>ATET</td>
<td>multivalued</td>
<td>treatment</td>
</tr>
<tr>
<td></td>
<td>POM</td>
<td></td>
<td>censoring (optional)</td>
</tr>
</tbody>
</table>

For further information on these commands and the properties of the estimators that they implement, see [CAUSAL] `stteffects intro`.

**telasso**

The `telasso` command is useful for estimating treatment effects from cross-sectional data and using lasso to select from among many potential control variables to be included in the model. This estimator relies on the SUTVA, unconfoundedness (conditional-independence) assumption, and overlap assumption.

`telasso` allows a continuous, binary, count, or nonnegative outcome and requires a binary treatment variable. Models are specified for both the outcome and the treatment. The ATE, ATET, or POM may be requested.

For further information on this command and the properties of the estimator that it implements, see [CAUSAL] `telasso`. 
The difference-in-differences suite of commands is useful for estimating treatment effects from data in which some of the units are observed both before and after a treatment and some units remain untreated. The difference-in-differences suite comprises the following commands:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>didregress</td>
<td>Difference in differences</td>
</tr>
<tr>
<td>xtdidregress</td>
<td>Difference in differences for panel data</td>
</tr>
<tr>
<td>hdidregress</td>
<td>Heterogeneous difference in differences</td>
</tr>
<tr>
<td>xthdidregress</td>
<td>Heterogeneous difference in differences for panel data</td>
</tr>
</tbody>
</table>

The didregress and hdidregress commands estimate treatment effects for repeated cross-sectional data, while xtdidregress and xthdidregress estimate treatment effects for panel data. The didregress and xtdidregress commands estimate a single ATET. The hdidregress and xthdidregress commands allow for heterogeneous treatment effects and report separate ATETs for each time and treatment cohort.

These estimators rely on the SUTVA, unconfoundedness (conditional-independence) assumption, and overlap assumption. In addition, they rely on an assumption of parallel trends in the treatment and control groups.

For further information on these commands and the properties of the estimators that they implement, see [CAUSAL] DID intro.

Endogenous treatment

The et commands are useful for estimating treatment effects from cross-sectional data in cases where the unconfoundedness (conditional-independence) assumption is violated because treatment assignment is not independent of the potential outcomes. The et commands comprise the following:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>eteffects</td>
<td>Endogenous treatment-effects estimation</td>
</tr>
<tr>
<td>etpoisson</td>
<td>Poisson regression with endogenous treatment effects</td>
</tr>
<tr>
<td>etregress</td>
<td>Linear regression with endogenous treatment effects</td>
</tr>
</tbody>
</table>

Details on the estimands, types of outcomes and treatments supported, and the models to be specified are given below:

<table>
<thead>
<tr>
<th>Command</th>
<th>Estimand</th>
<th>Outcome types</th>
<th>Treatment types</th>
<th>Models specified</th>
</tr>
</thead>
<tbody>
<tr>
<td>eteffects</td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>outcome treatment</td>
</tr>
<tr>
<td>eteffects</td>
<td>ATET</td>
<td>binary</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eteffects</td>
<td>POM</td>
<td>count</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eteffects</td>
<td></td>
<td>fractional</td>
<td></td>
<td></td>
</tr>
<tr>
<td>eteffects</td>
<td></td>
<td>nonnegative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>etpoisson</td>
<td>ATE</td>
<td>count</td>
<td>binary</td>
<td>outcome treatment</td>
</tr>
<tr>
<td>etpoisson</td>
<td>ATET</td>
<td>nonnegative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>etpoisson</td>
<td>POM</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>etregress</td>
<td>ATE</td>
<td>continuous</td>
<td>binary</td>
<td>outcome treatment</td>
</tr>
<tr>
<td>etregress</td>
<td>ATET</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>etregress</td>
<td>POM</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Note that `eteffects` provides the ATE, ATET, and POM directly. `etregress` estimates the ATE directly, while the ATET and POM can be obtained from `margins` after estimation. For `etpoisson`, ATE, ATET, and POM can all be obtained from `margins` after estimation.

For further information on these commands and the properties of the estimators that they implement, see [CAUSAL] `eteffects`, [CAUSAL] `etpoisson`, and [CAUSAL] `etregress`.

**Causal mediation**

The `mediate` command is useful for estimating direct, indirect, and total treatment effects from cross-sectional data in some cases where the treatment may affect an outcome both directly and indirectly. An indirect effect is one in which the treatment affects another variable, called a mediator, and the mediator in turn affects the outcome.

The `mediate` command allows both outcome and mediator variables to be continuous, binary, count, and nonnegative. The treatment may be binary, multivalued, or continuous. Models may be specified for the treatment and the mediator.

This estimator relies on the SUTVA, unconfoundedness (conditional-independence) assumption, and overlap assumption.

`mediate` provides estimates of the following statistics:

<table>
<thead>
<tr>
<th>Estimand</th>
<th>Synonym</th>
</tr>
</thead>
<tbody>
<tr>
<td>average indirect treatment effect (AITE)</td>
<td>natural indirect effect (NIE)</td>
</tr>
<tr>
<td>average direct treatment effect (ADTE)</td>
<td>natural direct effect (NDE)</td>
</tr>
<tr>
<td>total average treatment effect (ATE)</td>
<td>marginal total effect (MTE)</td>
</tr>
<tr>
<td>average indirect treatment effect with respect to controls (AITEC)</td>
<td>pure natural indirect effect (PNIE)</td>
</tr>
<tr>
<td>average direct treatment effect with respect to the treated (ADTET)</td>
<td>total natural direct effect (TNDE)</td>
</tr>
</tbody>
</table>

For further information on this command and the properties of the estimator that it implements, see [CAUSAL] `mediate`.

**Extended regression models**

The extended regression model (ERM) suite of commands is designed to account for treatment (exogenous or endogenous), endogenous covariates, and nonrandom sample selection one at a time or in combination. Commands are available for both cross-sectional and panel data. The following commands are comprised in the ERM suite:

- `ereg recess`  
  - Extended linear regression
- `eintreg`     
  - Extended interval regression
- `eprobit`     
  - Extended probit regression
- `eoprobit`    
  - Extended ordered probit regression
- `xteregession` 
  - Extended linear regression for panel data
- `xteintreg`   
  - Extended interval regression for panel data
- `xtprobit`    
  - Extended probit regression for panel data
- `xteoprobit`  
  - Extended ordered probit regression for panel data
eregress and xteregress fit models for continuous outcomes. eintreg and xteintreg fit models for interval-censored outcomes. eprobit and xteprobit fit models for binary outcomes. eoprobit and xteoprobit fit models for ordinal outcomes. All commands allow binary and multivalued treatments.

After fitting a model that accounts for endogenous or exogenous treatment with one of the ERM commands, you can use estat teffects to estimate the ATE, ATET, or POM.

For further information on these commands and the properties of the estimators that they implement, see [ERM] Intro 1.

Other commands in Stata provide some of the features found in the ERM commands. For instance, when you account only for endogenous covariates, eregress and ivregress provide equivalent parameter estimates. Instrumental-variable commands—ivregress, ivprobit, ivpoisson, and ivtobit—are designed to account for endogeneity (unobserved confounding) and provide consistent parameter estimates in this situation. Thus, these commands are used when the goal is causal inference. In some cases, a parameter estimated by these commands can be directly interpreted as the causal effect of interest, and in other cases, postestimation commands can be used to obtain the ATE, ATET, and POM.

margins

The margins command is available after many estimation commands in Stata. When a researcher has determined that appropriate assumptions have been satisfied for performing causal inference, many estimation commands can be used in combination with margins to estimate the ATE, ATET, and POM. As a simple example, you may type

```
. regress y c.x##i.trt, vce(robust)
```

to fit a linear regression of y on treatment trt and adjusted for covariate x. To estimate the POM, you could type

```
. margins trt, vce(unconditional)
```

The ATE is a contrast of the POM, and margins uses the r. operator to request such a contrast:

```
. margins r.trt, vce(unconditional)
```

The margins command can be used similarly after other estimation commands, and the results can be interpreted causally when proper assumptions for causal inference have been met.

For more information on margins, see [R] margins.

Also see

[CAUSAL] Intro — Introduction to causal inference and treatment-effects estimation
[CAUSAL] Glossary
**Description**

Difference in differences (DID) is a method to estimate the average effect of a treatment on those who receive the treatment. The method can be applied to two types of observational data: repeated cross-sections, in which different individuals are observed at different time points, and panel data, in which the same individuals are observed over time. We provide commands that estimate the average treatment effect on the treated (ATET). The effect may be the same for all (homogeneous) or may differ across time and across groups (heterogeneous). *didregress* and *xtdidregress* are for homogeneous treatment effects; *hdidregress* and *xthdidregress* are for heterogeneous treatment effects.

*didregress* and *xtdidregress* estimate the ATET of a binary or continuous treatment on a continuous outcome by fitting a linear model with time fixed effects and group or panel fixed effects. These commands also estimate difference in difference in differences (DDD), in which we augment the DID framework to include additional control groups to obtain the ATET.

*hdidregress* and *xthdidregress* estimate ATETs that vary over time and over treatment cohorts. Treatment cohorts are groups that are subject to intervention at different points in time. As in the homogeneous case, we can fit a linear model with time fixed effects and group or panel fixed effects, but we also incorporate interactions of the treatment with time and treatment cohorts. We fit these models using the two-way fixed-effects (TWFE) estimator. *hdidregress* and *xthdidregress* additionally allow for regression adjustment (RA), inverse-probability weighting (IPW), and augmented inverse-probability weighting (AIPW) to estimate the ATETs. See [CAUSAL] teffects intro for a discussion of RA, AIPW, and IPW estimators.

**Remarks and examples**

Remarks are presented under the following headings:

- Introduction
- Intuition for estimating effects
- DID with heterogeneous treatment effects
- Standard error considerations
- Different types of data and specification
  - Specifying groups and time as binary indicators
  - Excluding group and time effects
  - Exploring treatment-effect heterogeneity
- Conclusion

This entry presents the intuition and some of the technical details for the estimators in *didregress*, *xtdidregress*, *hdidregress*, and *xthdidregress* and the diagnostics available after estimation. See [CAUSAL] didregress, [CAUSAL] didregress postestimation, [CAUSAL] hdidregress, [CAUSAL] xthdidregress, and [CAUSAL] hdidregress postestimation for details on the syntax and worked examples. For a more complete discussion and references on homogeneous DID, see Angrist and Pischke (2009), Blundell and Dias (2009), Imbens and Wooldridge (2009), Lechner (2011), Angrist and Pischke (2015), Abadie and Cattaneo (2018), and Wing, Simon, and Bello-Gomez (2018). For more details on heterogeneous DID, see Roth et al. (2022) and de Chaisemartin and D’Haultfœuille (forthcoming) and the references therein.
Introduction

DID is one of the most venerable causal inference methods used by researchers. DID provides estimates of treatment effects on those that receive a treatment. Examples of such treatment effects are the effect of a minimum-wage increase on employment for those that see their minimum wage increase or the effect of water pollution on health outcomes for those that were exposed to water pollution.

Unlike the treatment-effects estimators described in [CAUSAL] `teffects`, which are applied to data from one cross-section, DID allows us to consider variation over time. For example, DID allows for panel data, for which we observe the same unit over time. It also works for repeated cross-sections, for which individuals sampled within groups differ across time periods but the groups sampled are the same. An appealing feature of DID is that our model specification does not require us to control for individual characteristics to identify treatment effects, something that is fundamental to the estimators described in [CAUSAL] `teffects`.

When thinking about ATETs in a DID framework, we compare a group before and after the treatment, perhaps by looking at a graph such as the one below:

![Graph showing outcome of interest over time](image)

Figure 1.

A treatment occurred in the year 2010. This might be a government policy, a change in medicine dosage, or any other intervention of interest. We would like to know if the treatment had a causal effect. It is clear from the graph that the outcome of interest changed after 2010. Is this due to the treatment or is something else occurring? Perhaps there are unobserved time effects that affect the treatment group after the treatment. For instance, there could have been a change in weather conditions or an economic downfall that affected the treatment group but was not captured in the model or the data. If this is the case, it does not suffice to look at the treatment group before and after the policy. DID addresses this by finding a control group, that is, a group that was subject to the same unobserved time effects but was not exposed to the treatment. Comparing the treatment group with the control group before and after the treatment might give us a better understanding of whether the treatment made a difference. A graph looking at a treatment and control group might look like this:
For both the treatment and the control group, we see that there was a decrease in the mean of the outcome after 2010. Therefore, the decrease we saw in the treatment group cannot be attributable entirely to the treatment. (In fact, these are simulated data, and we know the treatment has no effect.)

In a DID setup, if the treated group had not received the treatment, we would expect the treated and control groups to experience the same trends. A treatment effect implies a systematic deviation from a common trend that can be observed graphically. This is what we observe:

Here both groups experienced a decrease after 2010, but the treatment-group decrease was more substantial. The difference in the decreases across groups may indicate the effect of the treatment.

Researchers may motivate their analysis with such graphs. However, graphical evidence is not enough. We need statistical validation, so we fit a model.
The ATET is consistently estimated by differencing the average outcome for the treatment and control groups over time to eliminate time-invariant unobserved characteristics and also differencing the average outcome of these groups to eliminate time-varying unobserved effects common to both groups. These two differences give the method its name and highlight its intuitive appeal. More appealing is the fact that you can get the effect of interest, the ATET, from one parameter in a linear regression.

When talking about DID, people cite Snow (1849) and Snow (1855) as the first known applications. Snow claimed that cholera was not transmitted by contaminated air or contaminated blood, as was thought by some academics of his time. Snow hypothesized the disease was communicated via water that had been polluted with sewage. Below, he describes how he came up with an idea for a natural experiment to validate his hypothesis:

In Thomas Street, Horsleydown, there are two courts close together, consisting of a number of small houses or cottages, inhabited by poor people. The houses occupy one side of each court or alley—the south side of Trusscott’s Court, and the north side of the other, which is called Surrey Buildings, being placed back to back, with an intervening space, divided into small back areas, in which are situated the privies of both the courts, communicating with the same drain, and there is an open sewer which passes the further end of both courts. Now, in Surrey’s buildings the cholera has committed fearful devastation, whilst in the adjoining court there has been but one fatal case, and another case that ended in recovery. In the former court the slops of dirty water poured down by the inhabitants into a channel in front of the houses got into the well from which they obtained their water, this being the only difference . . . .

In the first edition (1849) of the text, Snow reports the deaths from cholera from September 23, 1848, to August 25, 1849, for five London districts. The number of deaths is higher in the South and East districts relative to the other three districts, arising from the source of their water supply. Snow obtains a clear motivation for his theory. In the second edition (1855), he collects data before and after a pump with contaminated water in Broadstreet, London, is closed. It is then that he can compare a treated with a control group before and after a treatment to establish a treatment effect.

Intuition for estimating effects

We can build our intuition about the causal inference implied by the DID setup by using the potential-outcomes framework described in [CAUSAL] Intro, [CAUSAL] teffects intro, and [CAUSAL] teffects intro advanced. We consider individual-level data for which we sample different individuals at different points in time, that is, a repeated cross-section. The treatment occurs at the group level. For example, the treatment may occur at the state, county, or hospital level. All individuals in a given state, county, or hospital either are treated or are controls at a given point in time. We index individuals by \( i \), groups by \( g \), and time by \( t \). We are interested in the effect of a treatment, \( D_{igt} \in \{0, 1\} \), on an outcome, \( Y_{igt} \). Suppose the potential-outcome mean of an individual in group \( g \) at time \( t \) that does not receive the treatment is given by the following:

\[
E \{Y_{igt}(0) \mid g, t\} = \gamma_g + \gamma_t
\]

Above, \( \gamma_g \) denotes the group effects, and \( \gamma_t \) denotes the time effects. Also suppose the potential-outcome mean for someone who receives the treatment is given by the following:

\[
E \{Y_{igt}(1) \mid g, t\} = \gamma_g + \gamma_t + \delta
\]

The potential outcomes described above allow us to think of the regression model

\[
Y_{igt} = \gamma_g + \gamma_t + D_{igt}\delta + \varepsilon_{igt}
\]
A regression estimate of $\delta$, the coefficient on the indicator of treatment, consistently estimates the ATET in this simplified framework, if we meet the overlap assumption, the stable unit treatment value assumption, and conditional independence (described in [CAUSAL] Intro and [CAUSAL] teffects intro advanced), plus one additional assumption.

To introduce this additional assumption, we think it is sometimes more intuitive to look at a two-period, two-group example. In this case, $g \in \{0, 1\}$, where 0 is the control group and 1 is the treatment group, and $t \in \{0, 1\}$, where 0 is the period before the treatment and 1 occurs after the treatment. To guarantee a consistent estimate of the ATET, we need to make the parallel-trends assumption:

$$E(Y_{igt}(0) \mid D_{gt} = 1) - E(Y_{igt}(0) \mid D_{gt} = 0) = E(Y_{igt}(1) \mid D_{gt} = 1) - E(Y_{igt}(1) \mid D_{gt} = 0)$$

The parallel-trends assumption is stated in terms of the potential outcomes of not being treated, $Y_{igt}(0)$, conditional on treatment, $D_{gt}$. It implies that if the treated had not received the treatment, the groups defined by $D_{gt} = 1$ and $D_{gt} = 0$ would have the same trends. For this to be true, we need group effects to be time invariant and time effects to be group invariant. The simple framework described in (1) and (2) satisfies the parallel-trends assumption.

The parallel-trends assumption has a graphical representation. Let’s think again about the case with multiple time periods. The parallel-trends assumption implies the paths of the outcome variable over time are parallel between the control and treatment groups prior to the date of the treatment. We can visually check this assumption by plotting the means of the outcome over time for both groups or by visualizing the results of the linear trends model. For instance, we might use a graph like the one in figure 2, where we plotted the means over time. After fitting a model using didregress and xtdidregress, you can get both the mean outcome plot and the trends plot by typing

```
. estat trendplots
```

Another way to think about the parallel-trends assumption in the pretreatment period is that treatment and control groups do not change their behavior in anticipation of the treatment. We can think of the parallel-trends assumption as implying that there should be no treatment effect in anticipation of the treatment. To test this assumption, we could fit a Granger-type causality model where we augment our model with dummies for each pretreatment period for the treated observations. A joint test of the coefficients on these dummies against 0 can be used as a test of the null hypothesis that no anticipatory effects have taken place. We can perform this test by typing

```
. estat granger
```

**DID with heterogeneous treatment effects**

When we introduced DID estimation above, we built our intuition for a case in which the treatment effect is the same for every group and in which the treatment effect does not change over time. We imposed this when we wrote

$$E \{Y_{igt}(0) \mid g, t\} = \gamma_g + \gamma_t$$

$$E \{Y_{igt}(1) \mid g, t\} = \gamma_g + \gamma_t + \delta$$

Also, we discussed the model using two groups and two time periods, a $2 \times 2$ model. But nothing precludes us from thinking of a set of treatment effects, say, $\delta_{gt}$, that varies over multiple groups, $g$, and time periods, $t$. More importantly, when we assume that the effect is homogeneous but the true model is heterogeneous, our treatment-effect estimates are going to be inconsistent, as was shown by de Chaisemartin and D’Haultfœuille (2020), Borusyak, Jaravel, and Spiess (2021), Sun and Abraham (2021), and Goodman-Bacon (2021).
Goodman-Bacon (2021) characterizes the ATET estimate obtained from a DID of the form

\[ Y_{igt} = \gamma_g + \gamma_t + D_{gt}\delta + \varepsilon_{igt} \]

when there are multiple time periods and treatment occurs at different points in time as a deviation from the 2 × 2 framework. He shows the estimated coefficient, \( \hat{\delta} \), is a weighted average of contrasts between groups treated at different points in time and groups that are not treated. When contrasts occur over time between treated groups and groups that are never treated or not yet treated, the comparisons are informative. When contrasts occur between groups that are both already treated, this component of the weighted average is not a treatment effect and thus introduces bias. The bias can be significant and change not only the magnitude but also the sign of the coefficient.

Goodman-Bacon (2021) decomposes \( \hat{\delta} \) and estimates all contrasts and weights that lead to the homogeneous estimate. For balanced-panel datasets, after `xtdidregress`, you can get the decomposition of the ATET by typing

```
. estat bdecomp
```

The result will let you know if the homogeneity assumption is warranted. This would be the case if the coefficients of the different contrasts are similar to the ATET estimate. If the contrasts differ substantially from the ATET estimate, our estimates are misleading.

If the decomposition suggests the homogeneous treatment effect is unwarranted or you do not want to impose homogeneity in treatment, you may use `xthdidregress`. If you are concerned about heterogeneity, you may choose from two workflows. With balanced panels, you might choose to fit the model for homogeneous treatment effects first and use `estat bdecomp` to determine whether effects are heterogeneous. With any panel or repeated cross-sectional data, you can fit a heterogeneous treatment-effects model and then use `estat aggregation` to ascertain whether effects are heterogeneous.

The treatment effects you are estimating using `hdidregress` and `xthdidregress` are of the form

\( \text{ATET}(c, t) \)

where \( c \) corresponds to the moment in time when a group receives the treatment, a cohort, and \( t \) corresponds to time. For example, a policy might be administered at the state level starting in 1995 for some states and in 1998 for other states. In this case, we would have treatment effects of the form \( \text{ATET}(1995, t) \) and \( \text{ATET}(1998, t) \). We are saying that the effect of the policy is different for states treated in 1995 than it is for those treated in 1998. Also, we are saying that the effect changes over time. Thus, if our sample goes from 1993 to 2000, for the 1995 cohort, we have treatment effects of the form \( \text{ATET}(1995, 1993), \text{ATET}(1995, 1994), \ldots, \text{ATET}(1995, 2000) \).

As is the case for homogeneous treatment, we need to satisfy a parallel-trends assumption and no anticipation of treatment. Both of these assumptions now need to hold for each cohort. As with the homogeneous treatment effects, we provide graphical diagnostics and tests. For a test of parallel trends, we have the command

```
. estat ptrends
```

For graphical inspection, you may type

```
. estat atetplot
```

which results in a graph for each cohort before and after treatment that lets you see whether there is no treatment effect, before treatment, for each cohort.

For a discussion of these assumptions and the requirements needed for consistency of the heterogeneous treatment implemented in `hdidregress` and `xthdidregress`, see Wooldridge (2021), Callaway and Sant’Anna (2021), and Roth et al. (2022).
Standard error considerations

While a standard linear regression model can be used to estimate the ATET in the homogeneous case, when assuming homogeneous treatment, the best estimate of the standard error requires some consideration. Many standard-error estimates have been proposed, and each one performs differently depending on the type of DID model being fit and the structure of the data. Below, we provide a discussion of some of the issues centered on the available standard-error estimates for `hdidregress`, `didregress`, `xthdidregress`, and `xtdidregress`. For a more complete discussion of inference for the homogeneous treatment-effect estimators, see Cameron and Miller (2015) and MacKinnon (2019) and the references therein. For a more complete discussion of the heterogeneous case, see Wooldridge (2021), Callaway and Sant’Anna (2021), and Roth et al. (2022). We begin our discussion with `didregress` and `xtdidregress`.

Bertrand, Duflo, and Mullainathan (2004) show that the standard errors for DID estimates are inconsistent if they do not account for the serial correlation of the outcome of interest. Because the outcomes studied usually vary at the group and time levels, it makes sense to correct for serial correlation. The authors show that using cluster–robust standard errors at the group level where treatment occurs provides correct coverage in the presence of serial correlation when the number of groups is not too small. Bester, Conley, and Hansen (2011) further show that using cluster–robust standard errors and using critical values of a $t$ distribution with $G - 1$ degrees of freedom, where $G$ is the number of groups, is asymptotically valid for a fixed number of groups and a growing sample size. In other words, you do not need the number of groups to be arbitrarily large, that is, to grow asymptotically. Cluster–robust standard errors with $G - 1$ degrees of freedom are the default standard errors of `didregress` and `xtdidregress`.

The results of Bertrand, Duflo, and Mullainathan (2004) and Bester, Conley, and Hansen (2011) demonstrate that we could still obtain reliable standard errors even when the number of groups was not large. But what about data with a very small number of groups? Several simulation and theoretical results suggest that cluster–robust standard errors may still have poor coverage when the number of groups is very small or when the number of treated groups is small relative to the number of control groups. These scenarios with small group sizes are common in DID studies, and another method of standard error estimation should be considered in these situations.

When the number of groups is small, we provide three alternatives. The first alternative is to use the wild cluster bootstrap that imposes the null hypothesis that the ATET is 0. Cameron, Gelbach, and Miller (2008) and MacKinnon and Webb (2018) show that the wild cluster bootstrap provides better inference than using cluster–robust standard errors with $t(G - 1)$ critical values. The second alternative comes from Imbens and Kolesář (2016), who show that with a small number of groups, you may use bias-corrected standard errors with the degrees-of-freedom adjustment proposed by Bell and McCaffrey (2002). For the third alternative, you may use aggregation-type methods like those proposed by Donald and Lang (2007); they show that their method works well when the number of groups is small but the number of individuals in each group is large.

When the disparity between treatment and control groups is large, for example, because there is only one treated group or because the group sizes vary greatly, cluster–robust standard errors and the other methods mentioned above underperform. Yet the bias-corrected and cluster–bootstrap methods provide an improvement over the cluster–robust standard errors.

What we said above for `didregress` and `xtdidregress` applies to `hdidregress` and `xthdidregress` when the underlying estimator is a linear regression. Considerations are different when we talk about the RA, AIPW, and IPW estimators implemented by the heterogeneous DID commands. These last three estimators can be understood as estimates from a multiple-equation model, in which each ATET($c, t$) could have been estimated separately. The standard errors of the estimates are, by default, equivalent to what you would obtain using a method of moments estimator and clustering at the group level where the treatment is administered; see [R] `gmm`.
Because heterogeneous DID has multiple equations and parameters, you may want to consider inference of all the ATET(c,t) estimates simultaneously: uniform inference. For such a scenario, you may use the bootstrap procedure suggested by Callaway and Sant’Anna (2021) by typing

```
.estat sci
```
after hdidregress and xthdidregress to obtain simultaneous confidence intervals.

**Different types of data and specification**

We allow for DID estimation of ATETs with two types of data: repeated cross-sections and panel data. The type of estimator to use depends on the assumptions you would like to make about the model and what type of data you have. Below, we walk you through the different types of models you may want to fit and how to fit them with didregress, hdidregress, xtdidregress, and xthdidregress.

Suppose you have a repeated cross-section of individuals i over a period of time t. The treatment is implemented at the state level, s, and it is denoted by \( D_{st} \), where \( D_{st} = 1 \) for all observations that are subject to the treatment in state s at time t. The indexing of the treatment makes clear that all individuals in the state at a given time are either treated or untreated. The model to be fit is given by

\[
y_{ist} = \gamma_s + \gamma_t + z_{ist}\beta + D_{st}\delta + \varepsilon_{ist} \tag{3}
\]

In the expression above, \( y_{ist} \) is the outcome, \( y \) in our data; \( z_{ist} \) are individual-level characteristics, given by \( z_1 \) and \( z_2 \); and \( D_{st} \) is given by the variable \( d \). To obtain the ATET, we type

```
.didregress (y z1 z2) (d), group(state) time(year)
```

Within the first set of parentheses, we list the outcome and covariates \( z_{ist} \) from (3). In the second set of parentheses, we specify the treatment variable. Group and time effects are included in the model through the `group()` and `time()` options, respectively. Also, by default, cluster-robust standard errors are computed at the state level. The command above is equivalent to typing

```
.regress y z1 z2 i.year i.state d, vce(cluster state)
```

The model we fit above assumes the treatment effect is homogeneous. Yet we may want to allow the treatment effect to vary over treatment-time cohort and over time. If this is the case, we may type

```
.hdidregress twfe (y) (d), group(state) time(year)
```

or, instead of \texttt{twfe}, we could specify \texttt{ra}, \texttt{ipw}, or \texttt{aipw}. All of these estimators will provide multiple cohort-time treatment effects instead of one ATET parameter. Notice that we excluded \( z_1 \) and \( z_2 \) in the specification above. Depending on the estimator we select, we could include them in either of the sets of parentheses. If we include them only in the first parentheses, we are modeling the outcome. If we include them in the second parentheses, we are modeling the treatment. If we include regressors in both parentheses, we are modeling the treatment and the outcome. This is equivalent to what we specify when we use the estimators discussed in \texttt{[CAUSAL] teffects ra}, \texttt{[CAUSAL] teffects ipw}, and \texttt{[CAUSAL] teffects aipw}.

\texttt{didregress} also allows us to specify a DDD model for situations in which we would like to augment the DID framework to include another control group. For instance, starting from (3), let’s assume the treatment occurs at the state level but also varies for people older than 65 versus people who are younger, defined by a binary variable \( g \). The DDD model is now given by

\[
y_{isgt} = \gamma_s + \gamma_g + \gamma_t + \gamma_s\gamma_t + \gamma_g\gamma_t + \gamma_s\gamma_g + z_{isgt}\beta + D_{sgt}\delta + \varepsilon_{isgt}
\]
To fit this model, we would type

```
. didregress (y z1 z2) (d), group(state g) time(year)
```

Although the model has a large set of interactions and looks much more complex than the DID model, the only difference in what we type is the addition of `g` to the set of group variables. The cluster–robust standard errors are computed at the highest level of clustering, in this case, `state`.

In some cases, data do not include a time component but rather multiple grouping variables across which differences may be taken. You could fit DDD models for three groups or DID for two groups. For DDD with `state`, `g1`, and `g2`, you could type

```
. didregress (y z1 z2) (d), group(state g1 g2)
```

Or you could fit a DID by typing

```
. didregress (y z1 z2) (d), group(g1 g2)
```

With these last two specifications, diagnostics that rely on time variables are not available.

With panel data, the basic model is given by

\[
y_{ist} = \gamma_i + \gamma_t + z_{ist}\beta + D_{st}\delta + \varepsilon_{ist}
\]  
(4)

Again, `i` denotes the individual (panel), `t` is the time period, and `s` is a group-level index. Individuals are assumed to be nested within the group, and treatment occurs at the group and time levels.

The specification is analogous to the one in (3), except that instead of \( \gamma_s \) we have \( \gamma_i \). Suppose that our data now include a panel identifier variable, `id`, that corresponds to `i` in (4). To fit the model above, we would type

```
. xtset id
. xtdidregress (y z1 z2) (d), group(state) time(year)
```

In this model, the group variable is different from the `xtset` identifying variable. The group variable defines the level of clustering, whereas the `xtset` variable defines the panel identifier. In other words, we are fitting a fixed-effects model with individual fixed effects, not one with state fixed effects. Of course, the group and `xtset` variables could be the same, but they do not need to be.

If we want to assume heterogeneous treatment, we must `xtset` our data with respect to the panel variable and with respect to time. The heterogeneous estimates rely on first differences of the outcome variable. For the RA estimator, you would type

```
. xtset id time
. xthdidregress ra (y z1 z2) (d), group(state)
```

Again, the panel variable and the group variable need not be the same, but the group and panel variable need to be related.

**Specifying groups and time as binary indicators**

The cross-sectional and panel-data models discussed above for homogeneous treatment are referred to as generalized DID models by Wing, Simon, and Bello-Gomez (2018). They are a generalization of the two-period, two-group specification that is usually discussed in the literature. This is also the way Angrist and Pischke (2009) write the model.
It is not uncommon, however, to write the DID model in terms of binary indicators. Instead of having group effects and time effects in the model for all groups and times in the data, you could instead include only a posttreatment effect and a treatment-group effect. To do this, you would create and include an indicator in your model that takes the value of 1 if the observation belongs to a treated group and 0 otherwise as well as an indicator that is 1 if the period considered occurs after the treatment. You can specify this model by using the `nogteffects` option and including your own indicators.

Say, for example, you create a time-constant indicator, `gtreated`, that is 1 if a group is treated and 0 otherwise, as well as an indicator, `post`, that is 1 after the policy was implemented and 0 before. You would type

```stata
.didregress (y z1 z2 i.post i.gtreated) (d), group(state) time(year) nogteffects
```

The syntax, in which we include indicators and use `nogteffects`, is equivalent for `xtdidregress`.

The model you specify in this case is given by

\[
y_{ist} = 1 \{s = \text{treated}\} + 1 \{t = \text{post}\} + z_{ist}\beta + D_{st}\delta + \varepsilon_{ist}
\]

where \(1 \{s = \text{treated}\}\) is an indicator function that is 1 if group \(s\) receives the treatment and 0 otherwise. Similarly, \(1 \{t = \text{post}\}\) indicates that we are in the period for which the treatment is active. In the expression above, we still specify the `group()` and `time()` options to obtain the correct standard errors, to validate the assumptions of the DID specification, and to obtain group- and time-level statistics.

### Excluding group and time effects

`didregress` and `xtdidregress` allow you to forgo the group and time effects that are generated by default. In this case, you would have to add your own group and time effects or omit them entirely by using the `nogteffects` option, which excludes group and time effects from your specification. For DDD models, you may also use the `nointeract` option, which will exclude group and time interactions from your model. Combining both options excludes group and time effects altogether. You may type

```stata
.didregress (y z) (d), group(s g) time(t) nogteffects nointeract
```

You would then fit

\[
y_{isgt} = \beta_0 + z_{isgt}\beta + D_{sgt}\delta + \varepsilon_{ist}
\]

You could still include group and time interactions by adding them in the first set of parentheses.

### Exploring treatment-effect heterogeneity

The heterogeneous treatment-effect estimators assume that the treatment effect changes over cohorts and over time. Yet we may want to see how the treatment effects vary only in one of those dimensions. For instance, we may want to look at the average of the treatment effects for the 1995 cohort \(\text{ATET}(1995, t)\) within all time periods. We could also average within cohorts at a given point in time to get a treatment effect for a particular year. Or even more, we might average over cohort and over time to get back one single \(\text{ATET}\). After estimation, we can do this using the `estat aggregation` command; see [CAUSAL] `hididregress postestimation`. 
For the \texttt{twfe} estimator, we can additionally decide at the estimation stage what level of heterogeneity we want to impose to our model. We can decide whether we want to allow for time heterogeneity or cohort-level heterogeneity using the \texttt{hettype()} option. For instance, if we wanted to disregard time heterogeneity but model cohort heterogeneity, we would type

\begin{verbatim}
  . hdidregress ..., ... hettype(cohort)
\end{verbatim}

\section*{Conclusion}

didregress and xtdidregress compute the ATET using DID and DDD for panel data and repeated cross-sections. Additionally, hdidregress and xthdidregress estimate ATETs that vary over time and over treatment cohorts. All four commands offer standard error computations that address some of the issues that researchers face, such as a small number of groups. After you fit models with these commands, diagnostics and tests to validate the assumptions and internal validity of the DID and DDD results are available.

\section*{References}


Also see

[CAUSAL] `teffects` — Treatment-effects estimation for observational data

[CAUSAL] `teffects intro` — Introduction to treatment effects for observational data

[CAUSAL] `teffects intro advanced` — Advanced introduction to treatment effects for observational data
**didregress** estimates the average treatment effect on the treated (ATET) from observational data by difference-in-differences (DID) or difference-in-difference-in-differences (DDD). The ATET of a binary or continuous treatment on a continuous outcome is estimated by fitting a linear model with time and group fixed effects. The DID and DDD estimation performed by **didregress** can be applied to data comprising repeated cross-sections in which different groups of individuals are observed at each time period.

**xtdidregress** estimates the ATET from observational data by DID or DDD for panel data. The ATET of a binary or continuous treatment on a continuous outcome is estimated by fitting a linear model with time and panel fixed effects.

**Quick start**

DID estimate of the ATET of **treat1** on outcome **y1** modeled using covariates **x1** and **x2**, and **grpvar1** and **tvar** fixed effects, with the treatment occurring at the **grpvar1** and **tvar** levels

```
didregress (y1 x1 x2) (treat1), group(grpvar1) time(tvar)
```

Same as above, but compute wild cluster-bootstrap *p*-values and confidence intervals with **grpvar1** as the clustering variable

```
didregress (y1 x1 x2) (treat1), group(grpvar1) time(tvar) ///
wildbootstrap
```

Aggregate data at the **grpvar1** and **tvar** levels to estimate the ATET

```
didregress (y1 x1 x2) (treat1), group(grpvar1) time(tvar) ///
aggregate(standard)
```

Same as above, but use the Donald and Lang (2007) method to compute the ATET and standard errors

```
didregress (y1 x1 x2) (treat1), group(grpvar1) time(tvar) ///
aggregate(dlang)
```

DDD estimate of the ATET of **treat2** on outcome **y2** modeled using covariates **x1** and **x2** and fixed effects defined by two-way interactions for **grpvar1**, **grpvar2**, and **tvar**, with the treatment occurring at the **grpvar1**, **grpvar2**, and **tvar** levels

```
didregress (y2 x1 x2) (treat2), group(grpvar1 grpvar2) time(tvar)
```

DID estimate of ATET of **treat3** on outcome **y3** using xtsset data; **y3** modeled using covariates **x1** and **x2**, and individual (panel) and **tvar** fixed effects, with the treatment occurring at the **grpvar1** and **tvar** levels

```
xtdidregress (y3 x1 x2) (treat3), group(grpvar1) time(tvar)
```
**Menu**

didregress

Statistics > Causal inference/treatment effects > Continuous outcomes > Difference in differences (DID) > DID

xtdidregress

Statistics > Causal inference/treatment effects > Continuous outcomes > Difference in differences (DID) > Panel-data DID (FE)

**Syntax**

*DID for repeated cross-sectional data*

```
didregress (ovar omvarlist) (tvar [, continuous]) [if] [in] [weight],
    group(groupvars) [time(timevar) options]
```

*DID for panel data*

```
xtdidregress (ovar omvarlist) (tvar [, continuous]) [if] [in] [weight],
    group(groupvars) [time(timevar) options]
```

*ovar* is the outcome of interest.

*omvarlist* specifies the covariates in the outcome model and may contain factor variables; see [U] 11.4.3 Factor variables.

*tvar* must be a binary variable indicating observations subject to treatment or a continuous variable measuring treatment intensity.

*groupvars* are categorical variables that indicate the group level at which the treatment occurs. At least one group variable must be specified. If *timevar* is specified, at most two group variables may be specified. If *timevar* is not specified, at most three group variables may be specified.

*timevar* is a time variable. It must be specified if *groupvar* has only one variable.
### Options

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>group(groupvars)</em></td>
<td>specify group variables</td>
</tr>
<tr>
<td>†time(timevar)</td>
<td>specify time variable</td>
</tr>
<tr>
<td>nointeract</td>
<td>exclude group() and time() interactions</td>
</tr>
<tr>
<td>nogteffects</td>
<td>do not include group and time effects in the model</td>
</tr>
<tr>
<td>aggregate(aggmethod)</td>
<td>aggregate to the levels defined by interacting groupvars and timevar</td>
</tr>
<tr>
<td>wildbootstrap[ (wildopts) ]</td>
<td>compute confidence intervals and p-values with the wild bootstrap</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SE/Robust</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>vce(vcetype)</td>
<td>vcetype may be cluster clustvar, robust, hc2, or bootstrap</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reporting</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>level(#)</td>
<td>set confidence level; default is level(95)</td>
</tr>
<tr>
<td>aequations</td>
<td>display auxiliary-equation results</td>
</tr>
<tr>
<td>display_options</td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td>coeflegend</td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>

*group(groupvars) is required.  
†time(timevar) is required when only one group is specified.  
For xtdidregress, a panel variable must be specified using xtset; see [XT] xtset.  
by, collect, and statsby are allowed; see [U] 11.1.10 Prefix commands.  
fweights, aweights, and pweights are allowed; see [U] 11.1.6 weight.  
coeflegend does not appear in the dialog box.  
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

### aggmethod

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>standard</td>
</tr>
<tr>
<td>dlang[, dlopt]</td>
</tr>
</tbody>
</table>

### wildopts

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>errorweight(edtype)</td>
</tr>
<tr>
<td>reps(#)</td>
</tr>
<tr>
<td>rseed(#)</td>
</tr>
<tr>
<td>blocksize(#)</td>
</tr>
</tbody>
</table>

### Options

- **group(groupvars)** specifies group variables. It indicates the group level at which the treatment occurs.  
- **groupvars** may be, for example, states, counties, or hospitals. **groupvars** define the group levels for which group effects are included in the model used to perform DID estimation and for which group
interactions are included in the model used to perform DDD estimation. `group()` also defines the level clustering for the default cluster–robust standard errors. `group()` is required.

You may specify at most two group variables if you also specify a time variable in the `time()` option or at most three group variables if no time variable is specified.

`time(timevar)` specifies the time variable. You may specify `time()` when one or two group variables are specified in the `group()` option. `time()` is required when only one group variable is specified in the `group()` option.

`nointeract` excludes `groupvars` and `timevar` interactions from being included in the model. By default, `didregress` and `xtdidregress` include group and time interactions for your specification if there is more than one group variable.

`nogteffects` specifies to not include group and time effects. By default, `didregress` adds group and time dummies to the regression specification. By default, `xtdidregress` adds time dummies to the fixed-effects specification.

`aggregate(aggmethod)` fits the model by aggregating data at the `groupvars` and `timevar` levels. `aggmethod` may be either `standard` or `dlang[ , dlopt]`.

- `standard` specifies that aggregation is performed using the standard aggregation method. In this case, a regression model is fit of the original outcome on covariates that vary within levels of the `groupvars` and `timevar` interaction. The estimates of the group-time level effects from this regression are used to construct a new dependent variable. These effects along with the remaining covariates are then aggregated to the level of the of the `groupvars` and `timevar` interaction. The final results are obtained by regressing the estimated group-time level effects on the remaining covariates with this aggregated dataset and estimating group-level cluster–robust standard errors.

- `dlang[ , dlopt]` aggregates data using methods proposed by Donald and Lang (2007). `dlopt` may be either `constant` or `varying`.
  - `constant` requests that standard errors be estimated using the standard ordinary least-squares method, as suggested by Donald and Lang (2007). With this method, as with the `standard` aggregation method, a single regression model is fit in the first step, so the coefficient estimates are constant across levels of the `groupvars` and `timevar` interaction.
  - `varying` specifies that the aggregation method allow for varying coefficients on the covariates. Specifically, in this case, separate regression models of the original outcome on covariates that vary within levels of `groupvars` and `timevar` are fit for each level of the `groupvars` and `timevar` interaction. Thus, this method allows the coefficients on these variables to vary. The constant from each of these regressions forms the new dependent variable. The final regression and standard error computations are equivalent to those used by the `constant` method.

`wildbootstrap[ (wildopts) ]` computes confidence intervals and p-values with the wild bootstrap. The wild bootstrap is constructed imposing the null hypothesis that the ATET is 0; that is, it is a restricted wild bootstrap. Confidence intervals are computed separately from the p-values. The bounds of the confidence interval are computed using a bisection optimization algorithm described in Methods and formulas. `wildopts` are `errorweight(edtype)`, `reps(#)` , `rseed(#)` , and `blocksize(#)`.

- `errorweight(edtype)` defines the error weight used to draw residuals from the wild bootstrap. `edtype` is one of `rademacher` (the default), `mammen`, `webb`, `normal`, or `gamma`.
  - `rademacher` multiplies the residuals at each bootstrap replication with a randomly generated variable that takes the value of 1 with probability 0.5 and the value of $-1$ with probability 0.5. `errorweight(rademacher)` is the default.
mammen multiplies the residuals at each bootstrap replication with a randomly generated variable that takes the value of $1 - \phi$ with probability $\phi/\sqrt{5}$ and $\phi$ otherwise, where $\phi = (1 + \sqrt{5})/2$. webb multiplies the residuals at each bootstrap replication with a randomly generated variable that takes the values $-\sqrt{3}/2$, $-\sqrt{2}/2$, $-\sqrt{1}/2$, $\sqrt{1}/2$, $\sqrt{2}/2$, and $\sqrt{3}/2$, each with probability $1/6$.

normal multiplies the residuals at each bootstrap replication with a randomly generated normal distribution variable with the first four moments given by 0, 1, 0, and 3.

gamma multiplies the residuals at each bootstrap replication with a randomly generated gamma distribution variable with shape parameter 4 and scale parameter $1/2$.

reps(#) performs # wild bootstrap replications. The default is reps(1000).
rseed(#) sets the random-number seed to #.

blocksize(#) specifies that the wild bootstrap be performed in blocks, with # replications per block. The wild bootstrap computation requires a matrix with dimensions (# groups) × (# replications). If this is too large, you can reduce the matrix to (# groups) × (# block size) and loop (# replications)/(# block size) times. When the same random seed is set, using a different block size does not change the numerical results; it only modifies the computation method. The block size must be less than or equal to the number of bootstrap replications.

**vce(vcetype)** specifies the type of standard error reported, which includes types that allow for intragroup correlation (cluster clustvar), that are robust to some kinds of misspecification (robust), that are bias-corrected cluster–robust using the degrees-of-freedom adjustment proposed by Bell and McCaffrey (2002) (hc2), and that use bootstrap sampling done at the group level (bootstrap); see [R] vce option.

vce(cluster clustvar), the default, uses the first variable specified in the group(groupvars) option.

vce(hc2) specifies bias-corrected cluster–robust standard errors with the degrees-of-freedom adjustment proposed by Bell and McCaffrey (2002). As with vce(hc2) in [R] regress, the residuals are rescaled by the projection matrix to improve the small-sample properties of the variance estimates. For more details, see Methods and formulas.

Specifying vce(robust) is equivalent to specifying vce(cluster clustvar), where clustvar is the first variable specified in the group(groupvars) option.

level(#); see [R] Estimation options.

eaqeuations specifies that the results for the outcome-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

The following option is available with didregress and xtdidregress but is not shown in the dialog box:

coefflegend; see [R] Estimation options.
Remarks and examples

Remarks are presented under the following headings:

- Introduction
- DID estimation
- Graphical diagnostics and tests
- Specifying a 2-by-2 DID
- Standard error considerations
- Default cluster–robust standard errors

Introduction

DID is one of the most venerable causal inference methods used by researchers. DID estimates the average treatment effect on the treated group (ATET).

To obtain the ATET using DID, one must compute the difference of the mean outcome for the treatment and the control groups before and after the treatment. This eliminates time-invariant unobservable group characteristics that confound the ATET; however, this is not enough to identify an effect. There may be time-varying unobservable confounders with an effect on the treatment group even after we control for time-invariant unobservable group characteristics. DID eliminates time-varying confounders by including a control group that is subject to the same time-varying confounders as the treatment group.

The ATET is then consistently estimated, differencing the mean outcome for the treatment and control groups over time to eliminate time-invariant unobservable characteristics and also differencing the mean outcome of these groups to eliminate time-varying unobservable effects common to both groups. These two differences give the DID method its name and highlight its intuitive appeal. More appealing is the fact that you can get the effect of interest, the ATET, from one parameter in a linear regression.

Below, we illustrate how to use didregress and xtdidregress. For more information about the methods used below, see [CAUSAL] DID intro. For general discussions of the DID methodology, see Angrist and Pischke (2009, 2015), Blundell and Dias (2009), Imbens and Wooldridge (2009), Lechner (2011), Abadie and Cattaneo (2018), and Wing, Simon, and Bello-Gomez (2018) and the references therein.

DID estimation

Example 1: Fitting a DID model

A health provider is interested in studying the effect of a new hospital admissions procedure on the satisfaction of patients. The provider has monthly data on patients from January to July. The new admissions procedure was implemented in April by hospitals that were under new management. Of the 46 hospitals in the study, 18 implemented the new procedure.

The health provider will use a DID regression to analyze the effect of the new admissions procedure on the hospitals that participated in the program. The outcome of interest is patient satisfaction, satis, which is recorded as an average of the responses to a set of four questions asked to patients. satis may take values between 0 and 10, where 10 is the greatest possible level of satisfaction and 0 is utter disappointment. The procedure variable marks the treated observations; it is 1 if a surveyed individual was admitted to the hospital using the new procedure after March and 0 otherwise. To get the ATET on the outcome satis, we type

```
.didregress (satis) (procedure), group(hospital) time(month)
```
The first set of parentheses is used to specify the outcome of interest followed by the covariates in the model. In this case, there are no covariates, just the outcome, `satis`. The second set of parentheses is used to specify the binary variable that indicates the treated observations, `procedure`. The `group()` and `time()` options are used to construct group and time fixed effects that are included in the model. The variable specified in `group()` is also important because it defines the level of clustering for the default cluster–robust standard errors; in this case, we cluster at the hospital level. The results from this command are as follows:

```
use https://www.stata-press.com/data/r18/hospdd
(Artificial hospital admission procedure data)
didregress (satis) (procedure), group(hospital) time(month)
```

**Treatment and time information**

<table>
<thead>
<tr>
<th></th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group (hospital)</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>Time</td>
<td>Minimum</td>
<td>1</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>1</td>
</tr>
</tbody>
</table>

**Difference-in-differences regression**

Data type: Repeated cross-sectional  
(Std. err. adjusted for 46 clusters in hospital)

|          | Coefficient | Robust std. err. | t    | P>|t|  | [95% conf. interval] |
|----------|-------------|------------------|------|------|----------------------|
| ATET     |             |                  |      |      |                      |
| procedure (New vs Old) | .8479879 | .0321121          | 26.41| .000 | [.7833108 .912665]   |

Note: ATET estimate adjusted for group effects and time effects.

The first table gives information about the treated and control groups and about treatment timing. The first segment with the title `Group` tells us the number of treated and control hospitals: 28 hospitals were using the old procedure and 18 hospitals were using the new one. The second segment of the table gives information about the first time we observe hospitals in the control group and the first time we observe the treatment (the new admission procedure) for hospitals in the treatment group. In this example, all hospitals that adopted the new procedure did so in April, time period 4. If some hospitals had adopted the policy later, the minimum and maximum time of first treatment would differ.

The ATET is 0.85, almost a 1-point increase in satisfaction relative to the case where none of the treated hospitals enacted the new procedure. In other words, if the hospitals that implemented the new admission procedure had not done so, their satisfaction ratings would be lower by almost one point on average.

We now explore whether the trajectories of `satis` are parallel for the control and treatment groups prior to the date when the new procedure was implemented. We are checking what is known as the parallel-trends or common-trends assumption, an important assumption of the DID model, as discussed in [CAUSAL] DID intro. A visual diagnostic of this assumption can be obtained by plotting the means
of the outcome over time for both groups or by visualizing the results of the linear-trends model. We can perform both of these diagnostic checks by using `estat trendplots`. To obtain figure 1 below, we type

```
. estat trendplots
```

![Graphical diagnostics for parallel trends](image)

Figure 1.

The graph seems to indicate that the parallel-trends assumption is satisfied. Prior to the policy implementation, treated and control hospitals followed a parallel path.

We could also perform a test to see if the trajectories are parallel by augmenting our original model to include variables representing time trends before and after the treatment for both groups of hospitals. The linear-trends model estimates a coefficient for the differences in linear trends prior to treatment, and if that coefficient is 0, the linear pretreatment trends are parallel. Otherwise, identification of the ATET may become questionable.

We can perform this test by using `estat ptrends`:

```
. estat ptrends
```

```
  Parallel-trends test (pretreatment time period)
  H0: Linear trends are parallel
  F(1, 45) = 0.55
  Prob > F = 0.4615
```

We do not have evidence to reject the null hypothesis of parallel trends in this case. Both the test and the graphical analysis support the parallel-trends assumption and, therefore, our ATET estimate.

Example 2: Fitting a DDD model

The results in example 1 could come into question if they could be the consequence of other unobserved variables rather than the consequence of the new hospital admissions procedure. The health provider administrators believe that responses to the survey are related to the frequency of individuals’ hospital visits. The patients may have unobserved characteristics that affect both how frequently they visit the hospital and how they feel about the admissions procedure. In other words, there might be unobserved characteristics that confound the effect of the new hospital admissions procedure. The administrators decide to obtain the ATET by using a DDD model. They want to estimate the average treatment effect on patients who visit the hospital with high or very high frequencies.
To do this, we will first create a new variable hightrt to be our new treatment identifier. Observations are now marked as treated (hightrt = 1) if hospital visit frequency by an individual is high or very high (frequency = 3 or 4) and if the hospital implemented the new admissions procedure in April.

```
. generate hightrt = procedure==1 & (frequency==3 | frequency==4)
. label define trt 0 "Untreated" 1 "Treated"
. label values hightrt trt
```

The DDD model will incorporate both hospital and frequency of usage effects as well as their interaction with time effects. To fit the model, we incorporate a new group variable, frequency:

```
. didregress (satis) (hightrt), group(hospital frequency) time(month)
```

Treatment and time information

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>hospital</td>
<td>28</td>
<td>18</td>
</tr>
<tr>
<td>frequency</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Maximum</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Triple-differences regression

| satis          | Robust Coefficient | std. err. | t       | P>|t|  | [95% conf. interval] |
|----------------|---------------------|-----------|---------|------|---------------------|
| ATET           | hightrt             | .764154   | .0402603| 18.98| 0.000              |
| (Treated vs Untreated) |                  |           |         |      | .6830655 .8452425   |

Note: ATET estimate adjusted for group effects, time effects, and group- and time-effects interactions.

The omitted output after the command corresponds to the factor-variable interactions that include the base categories. This is common when you fit DDD models that by default include group interactions and group and time interactions.

The first table above has information on the second group variable, frequency, for which low and medium frequencies are controls and high and very high frequencies are treated. The second table shows that the ATET is now smaller, but the policies still increase satisfaction.
Example 3: DID for panel-data model

Moser and Voena (2012) look at the effect of compulsory licensing on domestic inventions. Compulsory licensing allows firms in developing countries to produce foreign inventions without the consent of foreign patent owners. Having access to foreign technology may discourage domestic inventions, but it could also enhance local production.

Moser and Voena consider legislation that occurred during World War I called Trading With the Enemy Act (TWEA). By 1919, German-owned patents were systematically licensed to U.S. companies. In Moser and Voena (2012), the treated observations correspond to a subclass in the chemical industry that was granted at least one of the TWEA patents after 1918, as reported by United States Patent and Trademark Office (USPTO). A subclass is a group of firms in an industry that employ similar technologies, as defined by the USPTO.

The outcome of interest is the number of patents granted to inventors from the U.S. in that subclass, uspatents. This measures domestic innovation. Moser and Voena also include the number of non-TWEA patents granted to the subclass that were from foreign inventors, fpatents. fpatents measure innovation in the subclass that is not from U.S. inventors. We observe the same subclasses at each point in time from 1875 to 1939. Thus, we have a panel dataset.

To fit the model, we first xtset our data at the subclass level, classid:

```
. use https://www.stata-press.com/data/r18/patents
(Excerpt from Moser and Voena (2012))
. xtset classid
Panel variable: classid (balanced)
```

Below, we fit a DID model for the number of patents granted to U.S. inventors in a subclass, controlling for the number of non-TWEA patents granted to foreign inventors. The treatment indicator gotpatent is 1 if the subclass received a TWEA patent after 1918 and is 0 otherwise. The model includes year fixed effects and subclass fixed effects.
. xtdidregress (uspatents fpatents) (gotpatent), group(classid) time(year)

Treatment and time information

Time variable: year
Control: gotpatent = 0
Treatment: gotpatent = 1

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>classid</td>
<td>6912</td>
<td>336</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1875</td>
<td>1919</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Number of obs = 471,120
Data type: Longitudinal
(Std. err. adjusted for 7,248 clusters in classid)

| uspatents | Coefficient | std. err. | t     | P>|t|    | [95% conf. interval] |
|-----------|-------------|-----------|-------|--------|----------------------|
| ATET gotpatent (Patent vs None) | .150516    | .0356081  | 4.23  | 0.000  | .0807137     .2203183 |

Note: ATET estimate adjusted for covariates, panel effects, and time effects.

The ATET is 0.15, which means that in subclasses that were awarded one or more patents, domestic inventors produced an average of 0.15 additional patents after the TWEA compared with the scenario in which no patents are awarded for those subclasses.

Graphical diagnostics and tests

Example 4: DID diagnostic graphs and tests

As illustrated in example 1, when conducting a DID study, it is common to complement the regression analysis with graphical diagnostics and tests that provide evidence of whether an estimated effect can be given a causal interpretation. As discussed in [CAUSAL] DID intro, we would like to observe that the treated and control groups had mean outcomes that evolved similarly to each other over time prior to the treatment. This is usually referred to as a parallel-trends or common-trends assumption. We would also like to ascertain that neither the control nor the treatment group changed their behavior in anticipation of the treatment. This is assessed using a Granger-type test.

Below, we use simulated data to illustrate the diagnostics and tests available after didregress and xtdidregress. Simulated data helps us know exactly what we should expect and how to interpret it.

Suppose we have a panel dataset with 10 time points, denoted by t1, where a treatment has taken place between t1 = 5 and t1 = 6. We have a set of covariates, x1 and x2, and an outcome, y1. The outcome could be something like patient satisfaction, as in example 1, or the number of patents filed by U.S. inventors, as in example 3.
We fit the model:

```
. use https://www.stata-press.com/data/r18/parallelt
(Simulated data to test parallel-trends assumption)
. xtset id1
Panel variable: id1 (unbalanced)
. xtdidregress (y1 c.x1##c.x2) (treated1), group(id1) time(t1)
Treatment and time information
Time variable: t1
Control: treated1 = 0
Treatment: treated1 = 1
```

<table>
<thead>
<tr>
<th></th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>id1 102</td>
<td>98</td>
</tr>
<tr>
<td>Time</td>
<td>Minimum 1</td>
<td>6</td>
</tr>
<tr>
<td></td>
<td>Maximum 1</td>
<td>6</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Data type: Longitudinal

(Std. err. adjusted for 200 clusters in id1)

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>y1</td>
<td>Coefficient</td>
<td>std. err.</td>
<td>t</td>
<td>P&gt;</td>
<td>t</td>
</tr>
<tr>
<td>ATET</td>
<td>treated1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(Treated vs</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Untreated)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>.5069426</td>
<td>.0220218</td>
<td>23.02</td>
<td>0.000</td>
<td>.4635166</td>
</tr>
</tbody>
</table>

Note: ATET estimate adjusted for covariates, panel effects, and time effects.

Is this result valid? We can first explore the assumption of parallel trends graphically, comparing the trajectories of the outcome variable for the control and treatment groups prior to the date of treatment. We can check this assumption by plotting the means of the outcome over time for both groups or by visualizing the results of the linear-trends model. We can perform both of these diagnostic checks by using `estat trendplots`. 
Looking at the plotted observed means (left side of figure 2), the outcome trajectories in the control and treatment groups prior to the treatment are somewhat different. While we can observe declining trends in both groups, it looks as though the decline is progressing more rapidly in the control group, especially between time points three and five. We can get a clearer picture of this by looking at the results of the linear-trends model on the right side of figure 2. The group-level trajectories are shown with respect to a common reference point, \( t_1 = 1 \), which makes it easy to discern whether they are parallel. In this case, we can see that they are not. The differences between the treatment and control groups are growing larger over time up to \( t_1 = 5 \) (posttreatment time periods are not relevant when assessing the parallel-trends assumption). Judging by figure 2 alone, we should be concerned about whether the parallel-trends assumption holds for our effect estimate.

A more formal way to assess whether the pretreatment trajectories are parallel is to perform a test on the linear-trends model coefficient that captures the differences in the trends between treated and controls. If the pretreatment trends are actually linear in both groups, then this coefficient will be 0 because there are no differences in the slopes between the two groups. Thus, by testing this coefficient against 0, we have a test of the null hypothesis that the pretreatment period trajectories are parallel. We can perform this test by using `estat ptrends`:

```
. estat ptrends
Parallel-trends test (pretreatment time period)
H0: Linear trends are parallel
F(1, 199) = 39.97
Prob > F = 0.0000
```

We reject the null hypothesis of the linear trends being parallel.

We can also think of nonparallel as an indication of an anticipatory treatment effect. We saw that the trends were not parallel before the treatment took place, which could indicate a treatment effect even before the treatment is implemented. Thus, another way to state our parallel-trends assumption is that there should be no treatment effect in anticipation of the treatment. To test this assumption, we could fit a Granger-type causality model where we augment our model with dummies that indicate future treatment status for each time period prior to the treatment. A joint test of the coefficients on these dummies against 0 can be used as a test of the null hypothesis that no anticipatory effects have taken place. We can perform this test by using `estat granger`:
The result suggests that we reject the null hypothesis of no anticipatory effects prior to treatment. Thus, based on the results of both *estat ptrends* and *estat granger*, we conclude that we should be concerned about identification of $ATET$.

Notice that the parallel-trends $F$ test consumes only 1 numerator degree of freedom, while the Granger causality $F$ test consumes 4. That is because we have five pretreatment time periods and thus four coefficients to test. In this example, both tests would be appropriate, but the parallel-trends test has higher statistical power. However, as we will see below, the less-powered Granger test is more flexible and can be used in situations where the differences between treatment and control groups are nonlinear in pretreatment periods.

### Example 5: DID diagnostics and tests with nonlinear mean outcome differences

We now look at a different outcome and policy using the same simulated dataset as in example 4. We fit the same model but with the new variables:

```stata
. xtset id2
Panel variable: id2 (balanced)
. xtdidregress (y2 c.z1##c.z2) (treated2), group(id2) time(t2)
```

<table>
<thead>
<tr>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>id2</td>
<td>480</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Data type: Longitudinal

(Std. err. adjusted for 1,000 clusters in id2)

| y2 | Robust Coefficient std. err. | t | P>|t| | [95% conf. interval] |
|----|--------------------------------|---|-------|-----------------------|
| ATET | treated2 (Treated vs Untreated) | .2636651 | .0097188 | 27.13 | 0.000 | .2445936 | .2827367 |

Note: $ATET$ estimate adjusted for covariates, panel effects, and time effects.
Here is the diagnostic plot produced by `estat trendplots`:

![Graphical diagnostics for parallel trends](image)

Figure 3.

Inspecting the observed means over the pretreatment time periods, we can see that no considerable change has occurred in the outcome of the treatment group over time. For the control group, on the other hand, we observe a somewhat U-shaped trajectory in advance of the treatment. However, if we look at the plot from the linear-trends model, both trajectories appear somewhat U-shaped, and the trends appear to be almost parallel. Indeed, if we use `estat ptrends`, we fail to reject the null hypothesis of parallel trends:

```
. estat ptrends
Parallel-trends test (pretreatment time period)
H0: Linear trends are parallel
F(1, 999) = 2.13
Prob > F = 0.1446
```

The Granger causality test can handle cases in which the trajectories are nonlinear and for which `estat ptrends` will fail to reject the null hypothesis when it should:

```
. estat granger
Granger causality test
H0: No effect in anticipation of treatment
F(4, 999) = 9.86
Prob > F = 0.0000
```

We correctly reject the null hypothesis of no effect in anticipation of the treatment.
Example 6: Diagnostics and tests when parallel-trends assumption is satisfied

Finally, we look at a case where the assumption of parallel trends is satisfied. Again, we use the simulated dataset from example 4. We fit the following model:

```stata
. xtset id3
Panel variable: id3 (balanced)
. xtdidregress (y3 c.w1##c.w2) (treated3), group(id3) time(t3)
Treatment and time information
Time variable: t3
Control: treated3 = 0
Treatment: treated3 = 1
```

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>id3</td>
<td>502</td>
<td>498</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>6</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Data type: Longitudinal

(Std. err. adjusted for 1,000 clusters in id3)

| y3     | Coefficient | std. err. | t   | P>|t| | [95% conf. interval] |
|--------|-------------|-----------|-----|-----|--------------------|
| ATET   |             |           |     |     |                    |
| treated3 | .4996049    | .0102458 | 48.76 | .000 | .4794991 -.5197107 |

Note: ATET estimate adjusted for covariates, panel effects, and time effects.

Using estat trendplots, we obtain

Graphical diagnostics for parallel trends

This time, the trajectories of the observed means appear to be parallel before the treatment occurs.
Both follow a declining trend up to the last pretreatment time point. Looking at the results from the linear-trends model, the pretreatment trajectories appear to be the same. Using `estat ptrends` yields the following result:

```
. estat ptrends
Parallel-trends test (pretreatment time period)
H0: Linear trends are parallel
F(1, 999) = 0.00
Prob > F = 0.9688
```

We cannot reject the null hypothesis of parallel linear trends. Likewise, using `estat granger`, we do not reject the null hypothesis of the absence of anticipatory effects:

```
. estat granger
Granger causality test
H0: No effect in anticipation of treatment
F(4, 999) = 0.52
Prob > F = 0.7220
```

Example 7: Time-specific treatment effects

As we have seen in the previous examples, we can use `estat granger` to test for treatment effects occurring prior to the point at which treatment has been administered. Beyond the particular purpose of this test, however, it can be instructive to inspect not only pretreatment effects but also posttreatment effects that are allowed to vary over time. That is, rather than relying on a single treatment-effect estimate that is assumed to be constant over time, it can be instructive to check whether treatment effects change as time unfolds. We can fit a model that includes lags and leads of an indicator for the time period in which the treatment began. The coefficients on the lags can be used to assess whether there is any change in the treatment effect during posttreatment time periods. We can use the postestimation command `estat grangerplot` to fit such a model and plot the resulting coefficients. This model is also sometimes referred to as an event study model in the literature; see Schmidheiny and Siegloch (2019) and Clarke and Tapia-Schythe (2021).
We start by fitting the following DID model:

```
. use https://www.stata-press.com/data/r18/hospdd
(Artificial hospital admission procedure data)
. didregress (satis) (procedure), group(hospital) time(month)
```

### Treatment and time information

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>hospital</td>
<td>28</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

### Difference-in-differences regression

| satis   | Robust Coefficient | std. err. | t     | P>|t|   | [95% conf. interval] |
|---------|--------------------|-----------|-------|-------|----------------------|
| ATET    | procedure (New vs Old) | .8479879 | .0321121 | 26.41 | 0.000 | .7833108 .912665 |

Note: ATET estimate adjusted for group effects and time effects.

We can now simply use `estat grangerplot` to plot pre- and postintervention treatment effects:

```
. estat grangerplot
```

This yields the following graph:

![Graph showing effects over time, leads and lags with 95% CIs](image-url)
By default, `estat grangerplot` uses all available leads and lags, and in this case, we have estimates for two leads and four lags. For identification purposes, we must omit one of the indicator variables. By default, `estat grangerplot` omits the first lead, which sets the coefficient to 0 that corresponds to the time period prior to treatment administration. Thus, the plotted effects are normalized with respect to that time period. If we wanted, we could change the baseline period by using the `baseline()` option. We can see that the coefficients on the leads that correspond to the first two months are both close to 0, which shows that there are no substantial effects in the pretreatment era. The posttreatment effects for months 4 to 7 range between around 0.8 and 0.9. When we account for the uncertainty of these estimates, as indicated by the plotted 95% confidence intervals, it appears as though the treatment effects are rather stable over time in this example.

Rather than just plotting the coefficients, we could be interested in looking at a table with the numeric results. We can obtain the underlying results by specifying the `verbose` option. We are also using the `nodraw` option to see only the numeric results and not draw the graph again:

```
. estat grangerplot, nodraw verbose
```

```
Linear regression, absorbing indicators
Absorbed variable: hospital

Number of obs = 7,368
No. of categories = 46

F(12, 45) = 94.68
Prob > F = 0.0000
R-squared = 0.5334
Adj R-squared = 0.5298
Root MSE = 0.7240
(Std. err. adjusted for 46 clusters in hospital)

|        | Coefficient | std. err. | t     | P>|t|    | [95% conf. interval] |
|--------|-------------|-----------|-------|--------|---------------------|
| satis  |             |           |       |        |                     |
| month  |             |           |       |        |                     |
| February | -.007044 | .0263953 | -0.27 | 0.791  | -.0602068             | .0461188 |
| March   | .0335696   | .0255925 | 1.31  | 0.196  | -.0179764             | .0851156 |
| April   | .0187852   | .0250623 | 0.75  | 0.457  | -.0316927             | .0692632 |
| May     | -.0211152  | .0269569 | -0.78 | 0.438  | -.0754092             | .0331788 |
| June    | .0091208   | .0179016 | 0.51  | 0.613  | -.026935              | .0451766 |
| July    | -.0203444  | .0306266 | -0.66 | 0.510  | -.0820296             | .0413407 |
| _lead3  | .027897    | .035569  | 0.78  | 0.437  | -.0437426             | .0995367 |
| _lead2  | .0217322   | .0380076 | 0.57  | 0.570  | -.054819              | .0982833 |
| _lag0   | .8228153   | .0494933 | 16.62 | 0.000  | .7231307              | .9224999 |
| _lag1   | .9040498   | .0469682 | 19.25 | 0.000  | .8094511              | .9986486 |
| _lag2   | .844724    | .0608006 | 13.89 | 0.000  | .7222654              | .9671826 |
| _lag3   | .8978885   | .0511588 | 17.55 | 0.000  | .7948494              | 1.000928 |
| _cons   | 3.433074   | .0198449 | 173.00| 0.000  | 3.393104              | 3.473044 |
```

The coefficients on the variables that begin with `_lead` and `_lag` are the ones that are plotted in our graph.

As an interesting side note, notice that our original DID model is a special case of the model that we fit with `estat grangerplot`. Specifically, it is the model that includes only a single lead and no lags. To reproduce our original DID result, we can use the `nleads()` and `nlags()` options:
As we can see, the results match the ones we obtained earlier with *didregress*.

Notice also that the model used by *estat granger* is a special case as well; it is the model with all available leads and no lags. To re-create the test result from *estat granger*, we use the *post* option to store the results:

```
. estat grangerplot, nodraw verbose post nlags(0)
### Snippet above suppressed for brevity...
```

| satis     | Coefficient | std. err. | t     | P>|t|  | [95% conf. interval] |
|-----------|-------------|-----------|-------|------|----------------------|
| month     |             |           |       |      |                      |
| February  | -.007044    | .0263899  | -0.27 | 0.791| -.0601959            |
| March     | .0353696    | .0255873  | 1.31  | 0.196| -.0179658            |
| April     | .0002573    | .0245347  | 0.01  | 0.992| -.0487955            |
| May       | -.0058616   | .0249982  | -0.23 | 0.816| -.0562105            |
| June      | -.0002964   | .0199414  | -0.01 | 0.988| -.0404605            |
| July      | -.007653    | .0251828  | -0.30 | 0.763| -.0583738            |
| _lag0     | .027897     | .0355617  | 0.78  | 0.437| -.043728             |
| _lead2    | .0217322    | .0379998  | 0.57  | 0.570| -.0548033            |
| _lag0     | .8673694    | .0424929  | 20.41 | 0.000| .7817844             |
| _cons     | 3.433074    | .0198408  | 173.03| 0.000| 3.393113             |
We can now test whether the coefficients on the leads are jointly 0:

```
. test _lead3 _lead2
( 1) _lead3 = 0
( 2) _lead2 = 0
    F(  2,  45) =  0.33
    Prob > F =  0.7239
```

The test result indicates that we cannot reject the null hypothesis that the coefficients are jointly 0. We fit our original model again to compare the results with the ones from `estat granger`:

```
. quietly didregress (satis) (procedure), group(hospital) time(month)
. estat granger
```

Granger causality test

H0: No effect in anticipation of treatment

```
F(2, 45) =  0.33
Prob > F =  0.7239
```

As we can see, the results are the same. Using `estat grangerplot` followed by `test` allows us to perform additional tests of lags and leads that are not available through `estat granger`.

### Specifying a 2-by-2 DID

#### Example 8: Specifying a 2-by-2 DID

`didregress` and `xtdidregress` by default fit generalized DID models, also known as two-way fixed-effects models. Yet DID models sometimes are viewed from a two-period and two-group perspective, a 2-by-2 DID. You can also fit a 2-by-2 DID using `didregress` and `xtdidregress`. In fact, you will get equivalent results using a 2-by-2 DID or a generalized DID. The generalized DID is the default because it allows for a wider range of specifications that would not be feasible within the 2-by-2 framework, such as cases when the intervention occurs at different points in time.

In example 1, we got the ATET for hospitals that instituted a new admissions procedure. We typed the following:

```
. use https://www.stata-press.com/data/r18/hospdd
. didregress (satis) (procedure), group(hospital) time(month)
```

This implies that we are regressing `satis` on `procedure` and indicators for hospitals and for months. The indicators are created and added as regressors to our model by default. To fit a 2-by-2 model, we need to omit the hospital and month indicators and instead add an indicator for the period after treatment and an indicator for groups that are treated.

This is what we do below. We create the treatment-group indicator based on the fact that the variable `procedure` is 1 for individuals that experienced the new procedure and 0 otherwise; this variable identifies treated hospitals. We create the posttreatment indicator based on the fact that the new procedures are established after March, which has a value of 3 in the data.

```
. bysort hospital: egen treated = max(procedure)
. generate post = month>3
```
We now fit

```
.didregress (satis i.treated i.post) (procedure), nogteffects
> group(hospital) time(month)
```

Treatment and time information

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>hospital</td>
<td>28</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Data type: Repeated cross-sectional

(Number of obs = 7,368

(Std. err. adjusted for 46 clusters in hospital)

| satis      | Robust Coefficient | std. err. | t      | P>|t|   | [95% conf. interval] |
|------------|---------------------|-----------|--------|-------|----------------------|
| ATET       |                     |           |        |       |                      |
| procedure  | (New vs Old)        |           |        |       |                      |
|            | .8479879            | .0320051  | 26.50  | 0.000 | .7835263 .9124494    |

Note: ATET estimate adjusted for covariates.

We add the new indicators to our list of covariates and use the `nogteffects` option to exclude the group and time indicators that were included by default.

The point estimates are identical. What changes are the standard errors. They change because we have a different number of regressors.

---

**Standard error considerations**

**Example 9: Standard errors and data aggregation**

In example 3, we had 336 treated subclasses and 7,248 subclasses in total. However, many studies face the challenge of having very few elements per group. In fact, sometimes the data analyzed consist of only two groups—the treatment group in which treatment is administered to members of the group and the controls group in which no treatment is administered. Think, for example, of an analysis at the state level where one state is treated and one state is controlled. As discussed in [CAUSAL DID intro], these scenarios with few elements pose a challenge for inference. For a good discussion on these issues, see MacKinnon (2019).

`didregress` and `xtdidregress` provide alternatives in such cases. One alternative is to compute standard errors by using the wild cluster–bootstrap. Another alternative is to use bias-corrected clustered standard errors with the degrees-of-freedom adjustment proposed by Bell and McCaffrey (2002). A final alternative is to aggregate your data and then compute effects and standard errors such as proposed by Donald and Lang (2007) or use bias-corrected standard errors.
Below, we explore these options using simulated data. We created a dataset with 2,000 individuals and five time periods. The treatment occurs at the county level and there are six counties. Two of the counties, county 1 and county 2, receive the treatment and the remaining four counties do not. The true value of the ATET is $-1.0$. As we see below, the groups are unbalanced.

```
. use https://www.stata-press.com/data/r18/smallg
(Simulated data with a small number of groups)
. tab county

<table>
<thead>
<tr>
<th>County</th>
<th>Freq.</th>
<th>Percent</th>
<th>Cum.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>715</td>
<td>7.15</td>
<td>7.15</td>
</tr>
<tr>
<td>2</td>
<td>2,570</td>
<td>25.70</td>
<td>32.85</td>
</tr>
<tr>
<td>3</td>
<td>3,410</td>
<td>34.10</td>
<td>66.95</td>
</tr>
<tr>
<td>4</td>
<td>2,285</td>
<td>22.85</td>
<td>89.80</td>
</tr>
<tr>
<td>5</td>
<td>920</td>
<td>9.20</td>
<td>99.00</td>
</tr>
<tr>
<td>6</td>
<td>100</td>
<td>1.00</td>
<td>100.00</td>
</tr>
<tr>
<td>Total</td>
<td>10,000</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>
```

First, we fit the model using the default standard errors, which perform well with many balanced groups but not necessarily well with data like these. We compare the other results with these results.

```
. didregress (outcome x i.b) (treated), group(county) time(year)
Treatment and time information
Time variable: year
Control: treated = 0
Treatment: treated = 1

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>county</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>Time</td>
<td>Minimum</td>
<td>Maximum</td>
</tr>
<tr>
<td></td>
<td>2011</td>
<td>2011</td>
</tr>
<tr>
<td></td>
<td>2013</td>
<td>2013</td>
</tr>
</tbody>
</table>

Difference-in-differences regression
Data type: Repeated cross-sectional
(Std. err. adjusted for 6 clusters in county)

| outcome | Robust Coefficient | std. err. | t     | P>|t| | [95% conf. interval] |
|---------|--------------------|-----------|-------|-----|---------------------|
| ATET    | treated (Treated vs Untreated) | -.9394987 | .0884134 | -10.63 | 0.000 | [-1.166773, -.7122247] |

Note: ATET estimate adjusted for covariates, group effects, and time effects.

For this draw, the point estimates are reasonably close to the true value of $-1.0$, and the true values are contained inside the confidence interval.

Next, we use the wild cluster-bootstrap. For a good introduction to the methodology, see Cameron, Gelbach, and Miller (2008), MacKinnon and Webb (2018), and Roodman et al. (2019). The wild cluster-bootstrap works well in scenarios like the one above, where there are few groups.
We use the wild cluster bootstrap to construct \( p \)-values and confidence intervals, imposing the null hypothesis that the ATET is 0. We describe both computations in *Methods and formulas*. Here are the results:

```
. didregress (outcome x i.b) (treated), group(county) time(year)
> wildbootstrap(rseed(123) errorweight(webb))
Performing 1,000 replications for p-value for constraint
   treated = 0 ...
Computing confidence interval for treated
   Lower bound: ........10.........20...... done (26)
note: lower-bound CI achieved \( F(-1.25) = 0.0240 \), but target is \( F(x) = .025 \).
   note: the sorted bootstrap t statistics have at least two tied values
      adjacent to the t statistic under the null; this prevents the CI bound
      from converging to the target.
   Upper bound: ...... done (6)
```

Now we look at two data aggregation methods. In both cases, data aggregation occurs in two steps. In the first step, we regress the outcome on the set of covariates that vary at the individual, group, and time levels. We then estimate the group–time fixed effects from this procedure to use in a second stage as the new dependent variable. In the second stage, we aggregate the remaining covariates and the new dependent variable at the group and time levels and run a regression. This aggregation strategy is described in more detail in [Bertrand, Duflo, and Mullainathan (2004)](https://dx.doi.org/10.1086/375252), [Donald](https://dx.doi.org/10.1086/519025).
and Lang (2007), and Cameron and Miller (2015). We also describe the aggregation methods in *Methods and formulas*.

Below, we aggregate the data as described above and estimate bias-corrected standard errors by using the degrees-of-freedom adjustment suggested by Bell and McCaffrey (2002). This method of getting standard errors is computationally intensive, so it is well suited for cases where the dimensionality of the problem is reduced via aggregation.

```bash
.didregress (outcome x i.b) (treated), group(county) time(year)
> aggregate(standard) vce(hc2)
Computing degrees of freedom ... 
Treatment and time information
Time variable: year
Control: treated = 0
Treatment: treated = 1

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>county</td>
<td>4</td>
<td>2</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>year</td>
<td>2011</td>
<td>2013</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Data type: Repeated cross-sectional
Aggregation: Standard

| outcome | Robust HC2 | Coefficient | std. err. | t | P>|t| | [95% conf. interval] |
|---------|------------|-------------|-----------|---|-----|----------------------|
| ATET    | treated (Treated vs Untreated) | -.9958521 | .1373277 | -7.25 | 0.017 | -1.566242 - .4254624 |

Note: ATET estimate adjusted for covariates, group effects, and time effects.

We see that the confidence intervals are again wider than with the default standard errors. It is also worth noticing that we went from 10,000 observations to 30. This is because we aggregated at the county and year levels. We have six counties and five years.
We could also aggregate our data and compute the $t$ statistics using the degrees of freedom proposed by Donald and Lang (2007). This gives us

```
. didregress (outcome x i.b) (treated), group(county) time(year)
> aggregate(dlang)
```

Treatment and time information

<table>
<thead>
<tr>
<th></th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Group</td>
<td>county</td>
<td>4</td>
</tr>
<tr>
<td>Time</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Minimum</td>
<td></td>
<td>2011</td>
</tr>
<tr>
<td>Maximum</td>
<td></td>
<td>2011</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Data type: Repeated cross-sectional
Aggregation: Donald-Lang

| outcome | Coefficient | Std. err. | $t$   | $P>|t|$ | [95% conf. interval] |
|---------|-------------|-----------|-------|---------|----------------------|
| ATET    | treated     | -.9958521 | .1224496 | -8.13  | 0.000               |
|         | (Treated vs Untreated) | | | | -1.248576 - .7431287 |}

Note: ATET estimate adjusted for covariates, group effects, and time effects.

Again, we see wider confidence intervals.

Whenever you have few elements for each group and the groups are unbalanced, as in this example, you should be careful not to base your conclusions solely on the default cluster–robust standard errors. As was mentioned in [CAUSAL] DID intro, you should validate your conclusions by using one of the standard error computations suggested above.
Default cluster–robust standard errors

Example 10: Default cluster–robust standard errors with didregress and xtdidregress

We have stated that didregress was designed to handle repeated cross-sections and xtdidregress was designed to handle panel datasets. However, you can use xtdidregress when you have repeated cross-sections. For instance, for the hospital dataset in example 1, you may have typed

```
. use https://www.stata-press.com/data/r18/hospdd
(Artificial hospital admission procedure data)
. xtset hospital
Panel variable: hospital (unbalanced)
. xtdidregress (satis) (procedure), group(hospital) time(month)
Treatment and time information
Time variable: month
Control: procedure = 0
Treatment: procedure = 1
```

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>hospital</td>
<td>28</td>
<td>18</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minimum</td>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>Maximum</td>
<td>1</td>
<td>4</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Number of obs = 7,368
Data type: Longitudinal
(Std. err. adjusted for 46 clusters in hospital)

| satis    | Robust Coefficient | std. err. | t      | P>|t|  | [95% conf. interval] |
|----------|--------------------|-----------|--------|------|---------------------|
| ATET     | procedure          | (.8479879)| .0320138| 26.49 | 0.000               | .7835088   | .9124669 |

Note: ATET estimate adjusted for panel effects and time effects.

You will get the same point estimate as you would get with didregress but different standard errors. The reason is that xtdidregress with cluster–robust standard errors does not count the group variables as regressors in the degrees-of-freedom correction used to compute standard errors. It relies on the asymptotic theory of fixed-effects regression, where the number of group effects are expected to grow with the sample size. Put differently, xtdidregress is using xtreg, fe to compute the default cluster–robust standard errors. didregress uses areg, which assumes the number of groups is fixed and counts them in the degrees-of-freedom computation.

As a practical matter, these standard errors are going to be close to each other when the number of observations per cluster is large, as is the case for this example. As the number of observations per cluster becomes smaller, however, the standard errors will differ more, with standard errors of didregress tending to be larger.

Which estimator to use depends on the assumptions you would like to make about your data. If you think your data should be treated as panel data, with the number of groups growing with the sample size, then even if you have repeated cross-sections, you should use xtdidregress.
More in-depth discussions about cluster–robust standard errors can be found in Cameron and Miller (2015) and Wooldridge (2010).

**Stored results**

didregress stores the following in e():

Scalars

- `e(N)` number of observations
- `e(N_clust)` number of clusters
- `e(tmin)` minimum of first observed treatment time across groups
- `e(tmax)` maximum of first observed treatment time across groups
- `e(N_reps)` number of bootstrap replications
- `e(df_r)` residual degrees of freedom
- `e(blocksize)` block size used in wild bootstrap computations

Macros

- `e(cmd)` didregress
- `e(cmdline)` command as typed
- `e(depvar)` name of outcome variable
- `e(treatment)` indicator for treated observations
- `e(treatment_type)` binary or continuous
- `e(wtype)` weight type
- `e(wdep)` weight expression
- `e(wb_weight)` wild bootstrap error weight
- `e(datatype)` data type
- `e(groupvars)` group variables
- `e(clustvar)` name of cluster variable
- `e(timevar)` time variable
- `e(vcero)` vcero specified in vce()
- `e(vcetype)` title used to label Std. err.
- `e(properties)` b V
- `e(estat_cmd)` program used to implement estat
- `e(predict)` program used to implement predict
- `e(marginsnotok)` predictions disallowed by margins
- `e(_contrast_not_ok)` prediction disallowed by contrast

Matrices

- `e(b)` coefficient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(group_count)` matrix with number of groups and treatment time
- `e(fwboot)` wild bootstrap estimates
- `e(vboot)` wild bootstrap statistics and constraint
- `e(aggmethod)` aggregation method

Functions

- `e(sample)` marks estimation sample

In addition to the above, the following is stored in r():

Matrices

- `r(table)` matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.
**xtdidregress** stores the following in **\( e() \):**

**Scalars**
- \( e(N) \): number of observations
- \( e(N_{clust}) \): number of clusters
- \( e(t_{min}) \): minimum of first observed treatment time across groups
- \( e(t_{max}) \): maximum of first observed treatment time across groups
- \( e(N_{reps}) \): number of bootstrap replications
- \( e(df_r) \): residual degrees of freedom
- \( e(blocksize) \): block size used in wild bootstrap computations

**Macros**
- \( e(cmd) \): \( xtdidregress \)
- \( e(cmdline) \): command as typed
- \( e(depvar) \): name of outcome variable
- \( e(treatment) \): indicator for treated observations
- \( e(treatment\_type) \): binary or continuous
- \( e(wg) \): weight type
- \( e(wexp) \): weight expression
- \( e(wboots) \): wild bootstrap error weight
- \( e(datatype) \): data type
- \( e(groupvars) \): group variables
- \( e(clustvar) \): name of cluster variable
- \( e(panelvar) \): panel variable
- \( e(timevar) \): time variable
- \( e(vce) \): \( vcetype \) specified in \( vce() \)
- \( e(vcetype) \): title used to label Std. err.
- \( e(properties) \): \( b \ V \)
- \( e(estat\_cmd) \): program used to implement \( estat \)
- \( e(predict) \): program used to implement \( predict \)
- \( e(margins\_notok) \): predictions disallowed by \( margins \)
- \( e(_contrast\_not\_ok) \): prediction disallowed by \( contrast \)

**Matrices**
- \( e(b) \): coefficient vector
- \( e(V) \): variance–covariance matrix of the estimators
- \( e(group\_count) \): matrix with number of groups and treatment time
- \( e(fwboot) \): wild bootstrap estimates
- \( e(wboots) \): wild bootstrap statistics and constraint
- \( e(aggmethod) \): aggregation method

**Functions**
- \( e(sample) \): marks estimation sample

In addition to the above, the following is stored in **\( r() \):**

**Matrices**
- \( r(table) \): matrix containing the coefficients with their standard errors, test statistics, \( p \)-values, and confidence intervals

Note that results stored in **\( r() \)** are updated when the command is replayed and will be replaced when any \( r \)-class command is run after the estimation command.

## Methods and formulas

Methods and formulas are presented under the following headings:

- **DID for repeated cross-sectional data**
- **DDD model**
- **DID and DDD models with longitudinal data**
- **Aggregation estimators**
- **Wild bootstrap confidence intervals and \( p \)-values**
- **Bias-corrected clustered standard error**
DID for repeated cross-sectional data

The DID model for repeated cross-sectional data fit by \texttt{didregress} is given by

\[ y_{ist} = \gamma_s + \gamma_t + z_{ist}\beta + D_{st}\delta + \varepsilon_{ist} \]  

(1)

where \( i \) is the observation-level index, \( s \) is a group-level index, and \( t \) is a time-level index. For example, we might have yearly repeated cross-sectional data for individuals living in different states; in this case, \( i \) denotes the individual, \( s \) the state, and \( t \) the year. In (1) above, \( \gamma_s \) are group fixed effects and \( \gamma_t \) are time fixed effects. \( z_{ist} \) are the covariates, and \( \varepsilon_{ist} \) is the error term. \( D_{st} \) is the treatment that varies at the group and time levels. \( D_{st} \) could be binary or continuous.

If \( y_{ist} \) is \( y \) in the data, then \( z_{ist} \) are \( z1 \) and \( z2 \), \( D_{st} \) is \( d \), the group is \( \text{state} \), and time is \( \text{year} \). To fit the model, you would type

\begin{verbatim}
didregress (y z1 z2) (d), group(state) time(year)
\end{verbatim}

which is equivalent to typing

\begin{verbatim}
areg y z1 z2 i.year d, absorb(state) vce(cluster state)
\end{verbatim}

The methods and formulas for this model are those of a linear regression and can be found in \textit{Methods and formulas} in [R] \texttt{areg}.

DDD model

The DDD model is given by

\[ y_{isgt} = \gamma_s + \gamma_g + \gamma_t + \gamma_g\gamma_s + \gamma_g\gamma_t + \gamma_s\gamma_g + z_{isgt}\beta + D_{sigt}\delta + \varepsilon_{isgt} \]  

(2)

where \( i \) is the observation-level index, \( s \) and \( g \) are group-level indices, and \( t \) is a time-level index. For example, we might have yearly repeated cross-sectional data for older and younger individuals living in different states. In this case, \( i \) denotes the individual, \( s \) the state, \( g \) age group, and \( t \) the year. In (2) above, \( \gamma_s \) are group \( s \) fixed effects, \( \gamma_g \) are group \( g \) fixed effects, and \( \gamma_t \) are time fixed effects. \( z_{ist} \) are the covariates, and \( \varepsilon_{isgt} \) is the error term. \( D_{sigt} \) denotes the treatment that varies at the group \( s \), group \( g \), and time levels. \( D_{sigt} \) could be binary or continuous.

Say you now want to fit a DDD, where the treatment occurs for \text{group2} in some of the states, \text{state}. You would type

\begin{verbatim}
didregress (y z1 z2) (d), group(state group2) time(year)
\end{verbatim}

\texttt{didregress} would construct the group and time interactions in (2) and fit a linear regression. The methods and formulas for the specification are the same as those in \textit{Methods and formulas} in [R] \texttt{areg}.

DID and DDD models with longitudinal data

The DID model for longitudinal data fit by \texttt{xtdidregress} is given by

\[ y_{ist} = \alpha_i + \gamma_t + z_{ist}\beta + D_{st}\delta + \varepsilon_{ist} \]  

(3)

where \( i \) is the observation-level index, \( s \) is a group-level index, and \( t \) is a time-level index. For example, we might have a longitudinal dataset of individuals over time living in a given state; in this case, \( i \) denotes the individual, \( s \) the state, and \( t \) the year. In (3) above, \( \alpha_i \) are individual fixed effects and \( \gamma_t \) are time fixed effects. \( z_{ist} \) are the covariates, and \( \varepsilon_{ist} \) is the error term. \( D_{st} \) denotes the treatment that varies at the group and time levels. \( D_{st} \) could be binary or continuous. Individuals, \( i \), are assumed to be nested within the group; thus, group effects are subsumed by the individual effects.
If $y_{ist}$ is $y$ in the data, then $z_{ist}$ are $z1$ and $z2$. $D_{st}$ is $d$, the group is state, time is year, and individuals are denoted by $id$. To fit the model, you would type

\[
\text{xtset id year} \\
\text{xtdidregress} \ (y \ z1 \ z2) \ (d), \ \text{group(state)} \ \text{time(year)}
\]

which is equivalent to typing

\[
\text{xtreg} \ y \ z1 \ z2 \ i.\text{year} \ d, \ \text{fe vce(cluster state)}
\]

The methods and formulas for this model are those of a within estimator computed by \textit{xtreg}, \textit{fe} and can be found in Methods and formulas in \textit{XT} \textit{xtreg}.

The \textit{DDD} model is given by

\[
y_{isgt} = \alpha_i + \gamma_t + \gamma_s \gamma_g + z_{ist} \beta + D_{st} \delta + \varepsilon_{isgt}
\]

**Aggregation estimators**

To discuss the aggregation estimators, it is instructive to rewrite the covariate vector $z_{ist}$ as consisting of group and time invariant components $z_{1ist}$ and time-, group-, and individual-varying components $z_{2ist}$. The DID model can now be expressed as

\[
y_{ist} = \gamma_s + \gamma_t + z_{1ist} \beta_1 + z_{2ist} \beta_2 + D_{st} \delta + \varepsilon_{ist}
\]  

\[
y_{ist} = z_{2ist} \beta_2 + C_{st} + \epsilon_{ist}
\]  

\[
C_{st} = \gamma_s + \gamma_t + z_{1ist} \beta_1 + D_{st} \delta + \nu_{st}
\]  

The standard and \textit{dlang}, constant aggregation methods regress $y_{ist}$ on $z_{2ist}$ in (4) and then obtain an estimate of $C_{st}$, $\hat{C}_{st}$. After we regress $y_{ist}$ on $z_{2ist}$, we get an estimate of the group–time effects $\hat{C}_{st}$ from $y_{ist} - z_{2ist} \hat{\beta}_2 - \hat{\epsilon}_{ist}$. These effect estimates come from using \textit{predict} with option \textit{d} after \textit{areg}. We then aggregate the data at the $s$ and $t$ levels. With the aggregated data, we run a regression of $\hat{C}_{st}$ on $z_{1ist}$ and on $D_{st}$ using group and time fixed effects in (5). The \textit{dlang} method computes the ordinary least-squares standard errors from this second stage. The standard method uses cluster–robust standard errors clustered at the group level by default.

The \textit{dlang}, varying method runs a regression for each group defined by $s$ and $t$ and obtains an estimate of $C_{st}$ as the set of constants for each regression. Whereas in the first case $\beta_2$ was the same vector across the sample, in this case, we will get a different estimate of the slope coefficient for each group. The second step is the same as for \textit{dlang}, constant.

**Wild bootstrap confidence intervals and p-values**

The wild cluster–bootstrap is a bit different from the conventional bootstrap in that it keeps the covariates fixed and constructs the new dependent variables using the residuals from the original regression. The wild bootstrap procedure proceeds as follows:

1. Fit a restricted model, where the ATET is constrained to be 0. In other words, imposing the null hypothesis that the ATET is 0. Obtain the predicted values, $\hat{y}_r$, and residuals, $\hat{\epsilon}_r$, from the restricted model, where the subscript $r$ refers to the bootstrap replicates. Also fit the unrestricted model and compute the $t$ statistic, $t$, testing the null hypothesis ATET = 0.
2. At each of the subsequent $B - 1$ bootstrap steps, compute $y^* = \hat{y}_r + \hat{u}_r \odot (Kw)$, where $w$ is a column vector of length $S$, the number of clusters, containing the wild bootstrap weights, and $K$ is an $N \times S$ matrix with elements $k_{i,j} = 1$ if observation $i, i = 1, \ldots, N$, is in group $j$ and 0 otherwise. The operator $\odot$ is the Hadamard product that performs elementwise multiplication. By default, $w_j = 1$ with probability 0.5 and $w_j = -1$ with probability 0.5, that is, the rademacher error weights.

Alternatively, use one of the following error weights $w$:

- **mammen**, which is $1 - \phi$ with probability $\phi/\sqrt{5}$ and $\phi$ otherwise, where $\phi = (1 + \sqrt{5})/2$.
- **webb**, which takes the values $-\sqrt{3}/2, -\sqrt{2}/2, -\sqrt{1}/2, \sqrt{1}/2, \sqrt{2}/2$, and $\sqrt{3}/2$, each with probability 1/6.
- **normal**, which is a normal distribution with the first four moments given by 0, 1, 0, and 3.
- **gamma**, which is a gamma distribution with shape parameter 4 and scale parameter $1/2$.

3. For each bootstrap sample, compute the unrestricted model and the null hypothesis $t$ statistic, $t^*$. Include the observed $t$ statistic, $t$ in this set.

4. Given the $B$ bootstrapped $t$ statistics $t^*_b, b = 1, \ldots, B$, compute the wild bootstrap lower-tail $p$-value

$$P_l(t) = \frac{1}{B} \sum_{b=1}^{B} I(|t| < |t^*_b|)$$

and the upper-tail $p$-value

$$P_u(t) = \frac{1}{B} \sum_{b=1}^{B} I(|t| > |t^*_b|)$$

The reported equal-tail $p$-value is $P_{et}(t) = 2 \min\{P_l(t), P_u(t)\}$.

5. Compute the wild bootstrap confidence interval by searching for the lower limit and upper limit separately using the bisection root-finding algorithm. In doing so, find the quantities $\text{ATET} = c_l$ and $\text{ATET} = c_u$, $c_l < c_u$, such that $(100 - L)/2\%$ of the bootstrapped distribution of the $t$ statistics testing $\text{ATET} = c$ is contained in either the lower or the upper tail. By default, $L = 95$. The search algorithm does not always generate an $L\%$ confidence interval. One reason for this situation is if $B(100 - L)/200$ is not an integer, then there is not a slot in the sorted vector $t^*$ that corresponds to the upper- or lower-tail area $(100 - L)/200$. See Roodman et al. (2019, 8) for a good discussion.

Also, although a low-probability event, if at least one of the computed $t^*$ is (numerically) equal to the $t$ testing $\text{ATET} = c$, or if there are ties between other computed values in $t^*$ adjacent to the $t$ testing $\text{ATET} = c$, then the algorithm will not achieve the optimal coverage. In this case, we choose the confidence interval that is more conservative than requested. The achieved confidence level is reported in the $\text{ATET}$ coefficient table.

The chance of ties in $t^*$ increases if the number of clusters, $S$, is small and the rademacher error weights are used, because there are $2^S$ possible combinations of the two values 1 and $-1$ in the weight vector $w$. For example, if $S = 10$, then there are 1,024 possible combinations.
Bias-corrected clustered standard error

Let there be $S$ clusters, $s = 1, \ldots, S$, each with $N_s$ observations, and let $N$ be the number of observations in the data. Let $X_s$ be the covariate matrix for cluster $s$, and let $X$ be the covariate matrix for all observations with dimension $N \times L$. Also, define $I_{N_s}$ to be an identity matrix of dimension $N_s$, and define $P_{ss} = X_s (X'X)^{-1} X_s'$ as a projection matrix for cluster $s$. Finally, let $\hat{\varepsilon}_s$ be the residuals corresponding to cluster $s$. The bias-corrected cluster variance–covariance matrix is given by the following:

$$V = \left( \sum_{s=1}^{S} X_s'X_s \right)^{-1} \sum_{s=1}^{S} X_s' \left( I_{N_s} - P_{ss} \right)^{-1/2} \hat{\varepsilon}_s \hat{\varepsilon}_s' \left( I_{N_s} - P_{ss} \right)^{-1/2} X_s \left( \sum_{s=1}^{S} X_s'X_s \right)^{-1} \tag{6}$$

Let $P$ be the projection matrix for all the data, and let $(I_N - P)_s$ be an $N_s \times N$ matrix of rows of the $N \times N$ matrix $(I_N - P)$. Finally, let $e_{L,k}$ be an $L$ vector with $k$th component equal to 1 and equal to 0 elsewhere. We define the matrix $G$ as the matrix with $s$th column given by

$$G_s = (I_N - P)_s^t (I_{N_s} - P_{ss})^{-1/2} X_s (X'X)^{-1} e_{L,k}$$

We compute the degrees-of-freedom adjustment proposed by Bell and McCaffrey (2002), $K_{BM}$, as follows,

$$K_{BM} = \frac{\text{tr}(G'G)^2}{\text{tr}\left\{ (G'G)^2 \right\}} = \frac{\left( \sum_{i=1}^{S} \lambda_i \right)^2}{\sum_{i=1}^{S} \lambda_i^2}$$

where $\text{tr}(\cdot)$ is the trace function and $\lambda_i$ is the $i$th eigenvalue of the matrix $G'G$.

When $S$ is large, computing the eigenvalues can be time consuming. We define

$$a_s = (I_{N_s} - P_{ss})^{-1/2} X_s (X'X)^{-1} e_{L,k}$$

$$b_s = P_{ss} a_s$$

$$A = (a_1, a_1, \ldots, a_S a_S)^t = (A_1, A_2, \ldots, A_S)^t$$

$$B = (b_1, \ldots, b_S) = \begin{pmatrix} B_{1,1} & B_{1,2} & \cdots & B_{1,S} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N,1} & B_{N,2} & \cdots & B_{N,S} \end{pmatrix}$$

where $P_s = X(X'X)^{-1}X_s'$ and $B_{i,s}, i = 1, \ldots, N$, are the elements of the vectors $b_s, s = 1, \ldots, S$. Then $G'G = \text{diag}(A) - B'B$. We now express the adjusted degrees of freedom (Kolesár 2021) as

$$K_{BM} = \frac{\left( \sum_{s=1}^{S} A_s - \sum_{i=1}^{N} \sum_{s=1}^{S} B_{i,s}^2 \right)^2}{\sum_{s=1}^{S} A_s^2 - 2 \sum_{s=1}^{S} A_s \sum_{i=1}^{N} B_{i,s}^2 + \sum_{s=1}^{S} \sum_{s=2}^{S} (b_{s1} b_{s2})^2}$$
which can be computed efficiently in Mata and using QR decomposition. For example, by decomposing $X = QR$, where $Q$ is $n \times k$ and orthonormal and $R$ is $k \times k$ and upper triangular. The matrix $B$ can be rewritten so that it has dimension $k \times S$ instead of $N \times S$. We can do even better if the column rank of $X$ is $r \leq k$ by taking advantage of QR column pivoting.

When weights are specified, we use the weighted-covariate matrix $\tilde{X} = \text{diag}(w)^{\frac{1}{2}} X$ and its corresponding projection matrix $\tilde{P}$, as well as the cluster covariance matrices $\tilde{X}_s$, their projection matrices $\tilde{P}_{ss}$, and weighted residuals $\tilde{\varepsilon}_s = \text{diag}(w_s)^{\frac{1}{2}} \hat{\varepsilon}_s$.

For the ATET coefficient, we use the standard errors from (6) and the degrees of freedom in $K_{BM}$ to construct confidence intervals.

Acknowledgment

The algorithm used to compute the wild bootstrap was made faster using techniques described in Roodman et al. (2019). We thank David Roodman for bringing this to our attention.

References


**Also see**

[CAUSAL] didregress postestimation — Postestimation tools for didregress and xtdidregress

[CAUSAL] DID intro — Introduction to difference-in-differences estimation

[CAUSAL] hdidregress — Heterogeneous difference in differences

[CAUSAL] tfeffects intro — Introduction to treatment effects for observational data

[CAUSAL] tfeffects intro advanced — Advanced introduction to treatment effects for observational data

[CAUSAL] xthdidregress — Heterogeneous difference in differences for panel data

[U] 20 Estimation and postestimation commands
**Postestimation commands**

The following postestimation commands are of special interest after `didregress` and `xtdidregress`:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>estat trendplots</code></td>
<td>graphical diagnostics for parallel trends</td>
</tr>
<tr>
<td><code>estat ptrends</code></td>
<td>parallel-trends test</td>
</tr>
<tr>
<td><code>estat granger</code></td>
<td>Granger causality test</td>
</tr>
<tr>
<td><code>estat grangerplot</code></td>
<td>time-specific treatment effects</td>
</tr>
<tr>
<td><code>estat bdecomp</code></td>
<td>treatment-effect decomposition</td>
</tr>
</tbody>
</table>

The following standard postestimation commands are also available:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>estat summarize</code></td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td><code>estat vce</code></td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td><code>estimates</code></td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td><code>etable</code></td>
<td>table of estimation results</td>
</tr>
<tr>
<td><code>forecast</code></td>
<td>dynamic forecasts and simulations</td>
</tr>
<tr>
<td><code>lincom</code></td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td><code>nlcom</code></td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td><code>predict</code></td>
<td>linear predictions and residuals</td>
</tr>
<tr>
<td><code>predictnl</code></td>
<td>point estimates, standard errors, testing, and inference for generalized predictions</td>
</tr>
<tr>
<td><code>test</code></td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td><code>testnl</code></td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>
predict

Description for predict

predict creates a new variable containing predictions such as linear predictors and residuals.

Menu for predict

Statistics > Postestimation

Syntax for predict

predict [type] newvar [if] [in] [ , statistic ]

statistic | Description
---|---
Main | xb
| linear predictor; the default
residuals | residuals

Options for predict

xb, the default, calculates the linear predictor. It excludes the effect of the first group or of the panel identifier. All other effects, including the time fixed effects, are included in the linear predictor.

residuals calculates the overall residuals. It is the difference of the outcome and the linear predictor, including all group, panel, and time effects. In other words, it is not just the difference of the outcome and the linear predictor.
estat

Description for estat

estat trendplots produces two diagnostic plots for assessing the parallel-trends assumption that is required for consistent estimation of the ATET using didregress or xtdidregress. The first plot consists of two lines showing the mean of the outcome over time for the treatment and the control groups. The second plot augments the DID model to include interactions of time with an indicator of treatment and plots the predicted values of this augmented model for the treatment and control groups. Both plots include a vertical line one period before treatment.

estat ptrends performs a test of whether the linear trends in the outcome variable are parallel between control and treatment groups during the pretreatment period.

estat granger performs a test of whether treatment effects can be observed in anticipation of the treatment.

estat grangerplot produces a graph of time-specific treatment effects by plotting coefficients from leads and lags of the treatment indicator variable.

estat bdecomp performs a treatment-effect (Bacon) decomposition into all 2-by-2 treatment-effect components when there are multiple cohorts, each with a different treatment timing. Optionally, the components can be plotted against their weights. estat bdecomp requires that the data be strongly balanced. That is, units defined by the group variable must have the same number of observations, and observation times must be the same across all units.

Menu for estat

Statistics > Postestimation

Syntax for estat

Graphical diagnostics for parallel trends

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Parallel-trends test

estat ptrends

Granger causality test

estat granger

Time-specific treatment effects

estat grangerplot [, grangerplot_options]

Treatment-effect decomposition

estat bdecomp [, bdecomp_options]
### trend_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>omeans</code></td>
</tr>
<tr>
<td><code>omeans(plot_options)</code></td>
</tr>
<tr>
<td><code>ltrends</code></td>
</tr>
<tr>
<td><code>ltrends(plot_options)</code></td>
</tr>
<tr>
<td><code>notitle</code></td>
</tr>
<tr>
<td><code>noxline</code></td>
</tr>
<tr>
<td><code>nocommonlegend</code></td>
</tr>
<tr>
<td><code>legendfrom(#)</code></td>
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</table>

### plot_options

<table>
<thead>
<tr>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td>Plot</td>
</tr>
<tr>
<td><code>cline_options</code></td>
</tr>
<tr>
<td><code>line1opts(cline_options)</code></td>
</tr>
<tr>
<td><code>line2opts(cline_options)</code></td>
</tr>
<tr>
<td><code>twoway_options</code></td>
</tr>
</tbody>
</table>

### grangerplot_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>nleads(#)</code></td>
</tr>
<tr>
<td><code>nlags(#)</code></td>
</tr>
<tr>
<td><code>baseline(#)</code></td>
</tr>
<tr>
<td><code>lagview</code></td>
</tr>
<tr>
<td><code>verbose</code></td>
</tr>
<tr>
<td><code>post</code></td>
</tr>
<tr>
<td><code>noci</code></td>
</tr>
<tr>
<td><code>noyline</code></td>
</tr>
<tr>
<td>Cl plot</td>
</tr>
<tr>
<td><code>recastci(plottype)</code></td>
</tr>
<tr>
<td><code>ciopts(rcap_options)</code></td>
</tr>
</tbody>
</table>

**Add plots**

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>addplot(plot)</code></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>twoway_options</code></td>
</tr>
</tbody>
</table>
### bdecomp_options

<table>
<thead>
<tr>
<th>Options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>summaryonly</td>
<td>show decomposition summary only</td>
</tr>
<tr>
<td>fulldecomonly</td>
<td>show full decomposition only</td>
</tr>
<tr>
<td>noheader</td>
<td>suppress the table header in the output</td>
</tr>
<tr>
<td>notable</td>
<td>suppress the table in the output</td>
</tr>
<tr>
<td>graph</td>
<td>plot individual components against their weights</td>
</tr>
</tbody>
</table>

#### Plot

<table>
<thead>
<tr>
<th>Options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>copts(marker_options)</td>
<td>change look of markers for cohorts</td>
</tr>
<tr>
<td>tnopts(marker_options)</td>
<td>change look of markers for treated versus never treated</td>
</tr>
<tr>
<td>laopts(marker_options)</td>
<td>change look of markers for later versus always treated</td>
</tr>
<tr>
<td>leopts(marker_options)</td>
<td>change look of markers for later versus earlier treated</td>
</tr>
<tr>
<td>elopts(marker_options)</td>
<td>change look of markers for earlier versus later treated</td>
</tr>
<tr>
<td>atetlineopts(cline_options)</td>
<td>change look of the ATET reference line</td>
</tr>
<tr>
<td>noatetline</td>
<td>suppress ATET reference line</td>
</tr>
</tbody>
</table>

**Y axis, X axis, Titles, Legend, Overall**

<table>
<thead>
<tr>
<th>Options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>twoway_options</td>
<td>any options other than by() documented in [G-3] twoway_options</td>
</tr>
</tbody>
</table>

### Options for estat trendplots

omeans, omeans(plot_options), 1trends, and 1trends(plot_options) specify which graphs are to be included and how they should be individually rendered. The default is omeans 1trends, meaning that both graphs are included without any modifications.

omeans specifies that the observed-means graph be included. Specifying omeans suppresses the linear-trends model graph unless 1trends or 1trends(plot_options) is also specified.

omeans(plot_options) specifies that the observed-means graph be included and affects its rendition. Specifying omeans(plot_options) suppresses the linear-trends model graph unless 1trends or 1trends(plot_options) is also specified.

1trends specifies that the linear-trends model graph be included. Specifying 1trends suppresses the observed-means graph unless omeans or omeans(plot_options) is also specified.

1trends(plot_options) specifies that the linear-trends model graph be included and affects its rendition. Specifying 1trends(plot_options) suppresses the observed-means graph unless omeans or omeans(plot_options) is also specified.

notitle suppresses the overall title of the rendered graph.

noxline suppresses rendering of the treatment-time reference line.

nocommonlegend suppresses the display of one common legend and renders two individual legends.

legendfrom(#) specifies which legend to use; the default is legendfrom(1), which refers to the legend of the first plot (observed means). legendfrom(#) is not allowed with the nocommonlegend option.

Plot

cline_options affect the rendition of the plotted trend lines, including their style, size, and color; see [G-3] cline_options.
line1opts(cline_options) affect the rendition of the plotted trend lines for the group of controls, including their style, size, and color; see [G-3] cline_options.

line2opts(cline_options) affect the rendition of the plotted trend lines for the group of treated, including their style, size, and color; see [G-3] cline_options.

twoway_options are any of the options documented in [G-3] twoway_options, excluding by(). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

Options for estat grangerplot

nleads(#) specifies the number of leads to be included in the model and plotted. By default, all available leads are included. The number of leads must be greater than 0.

nlags(#) specifies the number of lags to be included in the model and plotted. By default, all available lags are included. The number of lags must be greater than or equal to 0.

baseline(#) specifies the baseline period for which the corresponding lead or lag is omitted. By default, the first lead is omitted, which corresponds to the time period prior to intervention.

lagview specifies to show the values of the x axis in terms of lags. If this option is not specified, time values are shown.

verbose specifies to display the output of the underlying regression model.

post posts the results of the underlying regression as estimation results in e().

noci removes the pointwise confidence intervals. The default is to plot the confidence intervals.

noyline suppresses rendering of the reference line.

recastci(plottype) specifies that confidence intervals be plotted using plottype. plottype may be rarea, rbar, rspike, rcap, rcapsym, rline, rconnected, or rscatter; see [G-2] graph twoway. When recastci() is specified, the plot-rendition options appropriate to the specified plottype may be used in lieu of rcap_options. For details on those options, follow the appropriate link from [G-2] graph twoway.

ciopts(rcap_options) affects the rendition of confidence intervals; see [G-3] rcap_options.

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

addplot(plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

twoway_options are any of the options documented in [G-3] twoway_options, excluding by(). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).
Options for estat bdecomp

`summaryonly` displays the decomposition summary only.
`fulldecomponly` displays the results of the full decomposition only.
`noheader` suppresses the table header in the output.
`notable` suppresses the results table in the output.
`graph` specifies to plot the treatment-effect components against their weights.

`copts(marker_options)` affect the rendition of the markers for the plotted cohort coefficients, including their style, size, and color; see [G-3] `marker_options`.
`tnopts(marker_options)` affect the rendition of the markers for the plotted coefficients for treated versus never treated, including their style, size, and color; see [G-3] `marker_options`.
`laopts(marker_options)` affect the rendition of the markers for the plotted coefficients for later versus always treated, including their style, size, and color; see [G-3] `marker_options`.
`lopts(marker_options)` affect the rendition of the markers for the plotted coefficients for later versus earlier treated, including their style, size, and color; see [G-3] `marker_options`.
`elopts(marker_options)` affect the rendition of the markers for the plotted coefficients for earlier versus later treated, including their style, size, and color; see [G-3] `marker_options`.
`atetlineopts(cline_options)` affect the rendition of the plotted ATET reference line, including its style, size, and color; see [G-3] `cline_options`.
`noatetline` suppresses rendering of the ATET reference line.

`ttitle_options` are any of the options documented in [G-3] `twoway_options`, excluding `by()`. These include options for titling the graph (see [G-3] `title_options`) and for saving the graph to disk (see [G-3] `saving_option`).
Remarks and examples

In the following, we discuss the treatment-effect decomposition approach implemented in `estat bdecomp`. To see examples of `estat trendplots`, `estat ptrends`, and `estat granger`, see examples 1, 4, 5, and 6 in `[CAUSAL] didregress`. An example of `estat grangerplot` is shown in example 7 in `[CAUSAL] didregress`.

A simple 2-by-2 DID design is characterized by two time periods (a period before and a period after a treatment or intervention occurs) and two groups (a treatment and a control group). Under this design, all treated units receive the treatment at the same time. A staggered DID design, on the other hand, involves multiple time points at which the treatment is administered. Goodman-Bacon (2021) has shown that, in this case, the ATET is a weighted average of all possible 2-by-2 DID estimators that compare treatment cohorts with each other. Decomposing an overall ATET into these 2-by-2 components provides a diagnostic tool to assess the validity of the two-way fixed-effects DID estimator implemented in `didregress` and `xtdidregress` when treatment timing varies.

Consider the case of a DID design with two treatment times so that the data consist of three groups: an earlier-treated group, a later-treated group, and a never-treated group. Under this scenario, there would be four 2-by-2 comparisons. Two of these come from comparing the group that was treated first with the never-treated group and comparing the group that was treated second with the never-treated group. The third compares the earlier-treated group with the later-treated group (before the later-treated group was treated). The fourth compares the later-treated group with the earlier-treated group (beginning at the point where the earlier-treated group received treatment). This fourth comparison is of particular concern if the 2-by-2 DID estimate substantially differs from the overall ATET estimate and if the corresponding weight is considerably large. In that case, this 2-by-2 estimate induces bias in the overall ATET estimate. The Bacon decomposition implemented in `estat bdecomp` offers insight into which 2-by-2 comparisons are driving the overall ATET estimate. With this decomposition, we can identify sources of variation that potentially bias the overall effect estimate. In cases of suspected bias, users may consider using `hdidregress` or `xthdidregress` to account for treatment-effect heterogeneity.

The decomposition requires that the data be strongly balanced with respect to the observed times per unit. Specifically, units defined by the group variable that is specified in the `group()` option of `didregress` or `xtdidregress` must have an equal number of observations, and observation times given by the variable specified in the `time()` option of `didregress` or `xtdidregress` must be the same across all units.

Example 1: DID treatment-effect decomposition for designs with variation in treatment timing

We use a (fictional) dataset where the units are dog breeds for which registration numbers at the American Kennel Club are observed over the course of 10 years. We wish to find out whether the registration numbers change in response to a breed being featured in a widely shown movie. Here is an excerpt of the data:

```
. use https://www.stata-press.com/data/r18/akc
   (Fictional dog breed and AKC registration data)
. list in 981/1000, sepby(breed) abbreviate(10) noobs
```

<table>
<thead>
<tr>
<th>year</th>
<th>breed</th>
<th>movie</th>
<th>best</th>
<th>registered</th>
</tr>
</thead>
<tbody>
<tr>
<td>2031</td>
<td>Old English Sheepdog</td>
<td>0</td>
<td>0</td>
<td>1324</td>
</tr>
<tr>
<td>2032</td>
<td>Old English Sheepdog</td>
<td>0</td>
<td>0</td>
<td>878</td>
</tr>
<tr>
<td>2033</td>
<td>Old English Sheepdog</td>
<td>0</td>
<td>0</td>
<td>699</td>
</tr>
<tr>
<td>2034</td>
<td>Old English Sheepdog</td>
<td>0</td>
<td>0</td>
<td>645</td>
</tr>
<tr>
<td>2035</td>
<td>Old English Sheepdog</td>
<td>0</td>
<td>0</td>
<td>1762</td>
</tr>
<tr>
<td>2036</td>
<td>Old English Sheepdog</td>
<td>0</td>
<td>0</td>
<td>1182</td>
</tr>
<tr>
<td>2037</td>
<td>Old English Sheepdog</td>
<td>1</td>
<td>0</td>
<td>1816</td>
</tr>
<tr>
<td>2038</td>
<td>Old English Sheepdog</td>
<td>1</td>
<td>0</td>
<td>4171</td>
</tr>
<tr>
<td>2039</td>
<td>Old English Sheepdog</td>
<td>1</td>
<td>0</td>
<td>3755</td>
</tr>
<tr>
<td>2040</td>
<td>Old English Sheepdog</td>
<td>1</td>
<td>0</td>
<td>4054</td>
</tr>
<tr>
<td>2031</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>692</td>
</tr>
<tr>
<td>2032</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>598</td>
</tr>
<tr>
<td>2033</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>670</td>
</tr>
<tr>
<td>2034</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>1642</td>
</tr>
<tr>
<td>2035</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>1311</td>
</tr>
<tr>
<td>2036</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>572</td>
</tr>
<tr>
<td>2037</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>1167</td>
</tr>
<tr>
<td>2038</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>620</td>
</tr>
<tr>
<td>2039</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>511</td>
</tr>
<tr>
<td>2040</td>
<td>Otterhound</td>
<td>0</td>
<td>0</td>
<td>1065</td>
</tr>
</tbody>
</table>
We list the data for two breeds, the Old English Sheepdog and the Otterhound. We can see that the Old English Sheepdog is featured in a movie that is released in the year 2037 and, thus, is a treated unit. The Otterhound breed has no such luck and belongs to the group that is never treated. Our outcome variable registered measures the number of registrations at the American Kennel Club. We use xtdidregress to fit a two-way fixed-effects DID model:

```
. xtset breed
Panel variable: breed (balanced)
. xtdidregress (registered) (movie), group(breed) time(year)
```

Treatment and time information

<table>
<thead>
<tr>
<th>Group</th>
<th>Control</th>
<th>Treatment</th>
</tr>
</thead>
<tbody>
<tr>
<td>breed</td>
<td></td>
<td>119</td>
</tr>
<tr>
<td></td>
<td></td>
<td>22</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Time</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>movie</td>
<td>2031</td>
<td>2034</td>
</tr>
<tr>
<td></td>
<td>2031</td>
<td>2037</td>
</tr>
</tbody>
</table>

Difference-in-differences regression

Data type: Longitudinal

(Number of obs = 1,410)

(Std. err. adjusted for 141 clusters in breed)

| registered | Coefficient | std. err. | t    | P>|t| | [95% conf. interval] |
|------------|-------------|-----------|------|-----|-------------------|
| ATET       |             |           |      |     |                   |
| movie      |             |           |      |     |                   |
| (1 vs 0)   | 2129.655    | 78.16241  | 27.25| 0.000| 1975.124          |
|            |             |           |      |     | 2284.186          |

Note: ATET estimate adjusted for panel effects and time effects.
Note: Treatment occurs at different times.
We can see that our overall ATET estimate is around 2,130, which is to say that we expect the number of dog registrations to increase by 2,130 if a member of their breed is featured as a movie protagonist. We can now use *estat bdecomp* to decompose this number into its constituent 2-by-2 parts:

```
. estat bdecomp
DID treatment-effect decomposition
ATET = 2129.655

Number of obs = 1,410
Number of groups = 141
Number of cohorts = 4

ATET decomposition summary

<table>
<thead>
<tr>
<th>Component</th>
<th>ATET</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treated vs never treated</td>
<td>2166.021</td>
<td>0.970810</td>
</tr>
<tr>
<td>Treated earlier vs later</td>
<td>936.70729</td>
<td>0.013157</td>
</tr>
<tr>
<td>Treated later vs earlier</td>
<td>906.60588</td>
<td>0.016033</td>
</tr>
</tbody>
</table>

Full ATET decomposition

<table>
<thead>
<tr>
<th>Component</th>
<th>2x2 coefficient</th>
<th>Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>Treated vs never treated</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2034 vs never treated</td>
<td>1673.5688</td>
<td>0.157125</td>
</tr>
<tr>
<td>2036 vs never treated</td>
<td>1999.2515</td>
<td>0.140290</td>
</tr>
<tr>
<td>2037 vs never treated</td>
<td>2315.6702</td>
<td>0.673394</td>
</tr>
<tr>
<td>Treated earlier vs later</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2034 vs 2036</td>
<td>580.59722</td>
<td>0.001132</td>
</tr>
<tr>
<td>2034 vs 2037</td>
<td>836.17778</td>
<td>0.008488</td>
</tr>
<tr>
<td>2036 vs 2037</td>
<td>1291.9333</td>
<td>0.003537</td>
</tr>
<tr>
<td>Treated later vs earlier</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2036 vs 2034</td>
<td>542.825</td>
<td>0.001886</td>
</tr>
<tr>
<td>2037 vs 2034</td>
<td>831.70833</td>
<td>0.011318</td>
</tr>
<tr>
<td>2037 vs 2036</td>
<td>1448.7167</td>
<td>0.002829</td>
</tr>
</tbody>
</table>
```

Note: Number of cohorts includes never treated.

Note: The ATET reported by *xtdidregress* is a weighted average of the ATET components. If any component is substantially different from the ATET reported by *xtdidregress* and the weight is large, consider accounting for treatment-effect heterogeneity by using *xthdidregress*.

In the table header, we see that we have 1,410 observations from 141 groups (breeds) and that we have four cohorts. The four cohorts consist of groups of breeds that are treated at three different times (2034, 2036, and 2037) and a group that is never treated. The first table shows a decomposition summary where the estimated ATET is decomposed into components due to comparisons between treated and never treated, between earlier treated and later treated, and between later treated and earlier treated. The results in the second table decompose the summary results further into the individual 2-by-2 comparisons.

From the summary results, we see that the component due to comparisons between treated and never treated is close to the overall effect estimate and that its weight is large. Specifically, the weight is around 0.97, which indicates that 97% of the overall ATET estimate is due to comparisons between treated and never treated. The other two components are not close to the overall effect. This would be a potential concern, especially for the later- versus earlier-treated comparisons, if the corresponding weight was considerably large. However, the weights are small here. The later- versus earlier-treated and earlier- versus later-treated components together account for only 3% of the overall ATET.

In the full decomposition results, we see the three components from the summary broken down further. For example, if we look at the treated versus never-treated component, we see that around two thirds of that component is driven by the group that received treatment last. We also see that the 2-by-2 coefficients are increasing, which provides some indication that the treatment effect may be changing over time.
In this example, the number of treatment times is relatively small. However, if that number is large, it can be instructive to plot the 2-by-2 coefficients against their weights. We do this by using the `graph` option of `estat bdecomp`. Because we do not need to see the numerical results again, we also specify the `notable` and `noheader` options:

```
    . estat bdecomp, graph notable noheader
```

The blue circles are the 2-by-2 coefficients from the treated versus never-treated component; the red diamonds are the 2-by-2 coefficients from the earlier- versus later-treated component; and the green triangles are the 2-by-2 coefficients from the later- versus earlier-treated component. The triangles and diamonds, while being substantially different from the overall ATET (indicated by the dashed reference line), are all very close to zero weight. This indicates that the contribution of these coefficients to the overall ATET is negligible.
Stored results

`estat ptrends` stores the following results for the test of linear trends in \( r() \):

Scalars
- \( r(N) \)  number of observations
- \( r(F) \)  test statistic
- \( r(df_r) \)  number of degrees of freedom of the residuals for the \( F \) distribution under \( H_0 \)
- \( r(p) \)  \( p \)-value
- \( r(df_m) \)  number of degrees of freedom of the test for the \( F \) distribution under \( H_0 \)

`estat granger` stores the following results for the test of treatment anticipation in \( r() \):

Scalars
- \( r(N) \)  number of observations
- \( r(F) \)  test statistic
- \( r(df_r) \)  number of degrees of freedom of the residuals for the \( F \) distribution under \( H_0 \)
- \( r(p) \)  \( p \)-value
- \( r(df_m) \)  number of degrees of freedom of the test for the \( F \) distribution under \( H_0 \)

`estat grangerplot`, when used with option `post`, stores results from the underlying regression model in \( e() \) and \( r() \).

`estat bdecomp` stores the following results in \( r() \):

Scalars
- \( r(N\_groups) \)  number of panels
- \( r(N\_obs) \)  number of observations
- \( r(N\_cohorts) \)  number of cohorts
- \( r(atet) \)  overall ATET
- \( r(atet\_between\_b) \)  between component
- \( r(atet\_within\_b) \)  within component
- \( r(atet\_between\_w) \)  weight of between component
- \( r(atet\_within\_w) \)  weight of within component

Macros
- \( r(ttimes) \)  times of treatment

Matrices
- \( r(atet\_comp\_b) \)  ATET components
- \( r(atet\_comp\_w) \)  weights of ATET components
- \( r(coefs) \)  individual 2-by-2 coefficients
- \( r(wgts) \)  weights of individual 2-by-2 coefficients

Methods and formulas

The tests performed with `estat ptrends` and `estat granger` are based on augmented difference-in-differences (DID) models. With `estat ptrends`, we augment the DID model with terms that capture the differences in slopes between treated and controls. With `estat granger`, we augment the model by interacting the dummy variable that marks treated observations with dummy variables for time periods prior to the treatment to capture any potential anticipatory treatment effects.

Let’s consider the case of panel data for individuals over time, in which individuals belong to a group \( s \). Groups could be states, occupational categories, districts, etc. Let \( y_{its} \) be the outcome of individual \( i \), who belongs to group \( s \), at time \( t \), where \( i = 1, \ldots, N, t = 1, \ldots, T, \) and \( s = 1, \ldots, S \).

We can write the DID model for such setups as follows:

\[
y_{ist} = \gamma_i + \gamma_t + x_{ist}\beta + D_{st}\delta + \epsilon_{ist}
\]
Here $\gamma_i$ are individual fixed effects, $\gamma_t$ are time fixed effects, $x_{ist}$ are covariates, $D_{st}$ is a variable that is 1 if an individual belongs to a group $s$ that is treated at time $t$ and is 0 otherwise, and $\epsilon_{ist}$ is an error term. The coefficient $\delta$ represents the average treatment effect on the treated (ATET).

To simplify the exposition below, we rewrite the model as follows:

$$y_{ist} = \gamma_i + \gamma_t + x_{ist}\beta + D_{st}\delta + \epsilon_{ist}$$

$$y_{ist} = \text{DID}_{ist} + \epsilon_{ist} \quad \text{(1)}$$

The linear-trends model that is used for the parallel-trends test with `estat ptrends` augments the above model with two more terms. Let $d_{t,0} = 1(d_t = 0)$ be a variable indicating pretreatment time periods, and let $d_{t,1} = 1(d_t = 1)$ be a variable indicating posttreatment time periods. Also, let $w_i$ be a variable that is 1 if the individual belongs to a treated group and is 0 otherwise. The augmentation terms then consist of two 3-way interactions between $d_{t,0}$, $w_i$, and $t$, and $d_{t,1}$, $w_i$, and $t$:

$$y_{ist} = \text{DID}_{ist} + w_i d_{t,0} t \zeta_1 + w_i d_{t,1} t \zeta_2 + \epsilon_{ist} \quad \text{(2)}$$

Under this specification, the coefficient $\zeta_1$ captures the differences in slopes between treatment group and control group in pretreatment periods, while $\zeta_2$ captures the differences in slopes in posttreatment periods. The same is true for $\zeta_2$ with respect to the posttreatment period; however, posttreatment differences in trends are not relevant for assessing the parallel-trends assumption. `estat ptrends` uses a Wald test of $\zeta_1$ against 0 to assess whether the linear trends are parallel prior to treatment. Thus, the null hypothesis of this test is that the linear trends are parallel.

`estat granger` performs a Granger-type causality test to assess whether treatment effects are observed prior to the treatment. To illustrate this, suppose the treatment took place at time $t = j$. We could express $D_{st}$ as $D_{st} = 1(t \geq j)w_i$. The Granger-type test augments the model with counterfactual treatment-time indicators. For example, if the treatment occurred at time $j - 1$, then we could construct a new treatment as $1(t_{it} \geq j - 1)w_i$, and if we have sufficient time points, we could construct another counterfactual treatment as $1(t_{it} \geq j - 2)w_i$, and so on. These terms are referred to as leads in the DID literature. The model used by `estat granger` uses the model in (1) and augments it with all leads leaving out one for identification purposes. Let $J$ index the time at which the treatment occurs.

$$y_{ist} = \text{DID}_{ist} + \sum_{j=2}^{J-1} 1(t_{it} \geq j)w_i \lambda_j + \nu_{ist} \quad \text{(3)}$$

The test result is then obtained by performing a joint Wald test on the coefficients $\lambda_j$. Thus, the null hypothesis for this test is that the coefficients in $\lambda_j$ are jointly 0, which is to say there are no anticipatory effects.

`estat grangerplot` fits a generalization of the DID model in (1) and plots the estimated coefficients from this model, including their 95% confidence intervals. The model is similar to (3), but this model parameterizes the leads differently and includes lags in addition to leads. Let $I_s$ be the time of treatment, $m < 0$ be the number of time periods prior to $I_s$, $q \geq 0$ be the number of periods after $I_s$, and $b$ be the baseline period. The model is

$$y_{ist} = \gamma_i + \gamma_t + x_{ist}\beta + \sum_{k=m,k \neq b}^{q} B_{st}^k w_i \lambda_k + \epsilon_{ist}$$
where

\[ B^k_{st} = \begin{cases} 1(t_{it} \leq I_s + k), & \text{if } k = m \\ 1(t_{it} = I_s + k), & \text{if } m < k < q \\ 1(t_{it} \geq I_s + k), & \text{if } k = q \end{cases} \]

This yields a model with \(|m|\) leads and \(q\) lags. By default, \texttt{estat grangerplot} uses all available leads and lags. If, without loss of generality, we set the base to the period prior to treatment, \(b = -1\) (the default), then with \(t = 1, \ldots, T\) and \(I_s = J\), a maximum of \(n_{\text{leads}} = J - 2\) leads and \(n_{\text{lags}} = T - J + 1\) lags is available. Notice that, if all available leads and lags are used, \(B^k_{st}\) reduces to \(B^k_{st} = 1(t_{it} = I_s + k)\) because there are no periods before or after \(I_s + k\). With fewer than available leads and lags, that is, \(|m| < n_{\text{leads}}\) or \(q < n_{\text{lags}}\), notice that the corresponding indicator variables capture the periods beyond the endpoints that correspond to \(m\) and \(q\).

At a minimum, the model has to include a single lead. In that case, we have that \(m = -1\) and \(q = 0\). After omitting the base, we have that \(k = 0\) and \(B^k_{st}\) reduces to \(B^k_{st} = 1(t_{it} \geq I_s)\). Notice that \(B^k_{si} w_i\) now yields our original treatment indicator \(D_{st}\). In other words, the model with a single lead and no lags yields the special case of the DID model in (1). Notice also that the model in (3) is a special case, too. It is equivalent to the model fit by \texttt{estat grangerplot} with all available leads and no lags. However, (3) uses a different parameterization because the indicator variables are constructed differently. \texttt{estat grangerplot} plots the coefficients \(\lambda_k\) against the corresponding time periods.

The \texttt{estat trendplots} command produces two plots. The first plot is simply plotting the observed means for each treatment group at each point in time. The second plot is based on the model in (2), which is the model used for the parallel-trends test, but this model centers the continuous time variable around its minimum value:

\[ y_{ist} = \text{DID}_{ist} + w_i d_{t,0}(t - \min(t))\zeta_1 + w_i d_{t,1}(t - \min(t))\zeta_2 + \mu_{ist} \]

Centering around the minimum time value provides a common reference point at the first observed time point such that deviations from parallelism are easily detectable. The graph then shows the predicted values from this model, evaluated at all observed time points for each of the treatment groups and at the means of the covariates.

While the formulas above are shown for the case of panel data, these methods work the same way for data that consist of repeated cross-sections.

The \texttt{estat bdecomp} command may be used in cases where units get treatment at different times. The command performs a decomposition of the estimated DID treatment effect into components that come from all possible 2-by-2 cohort comparisons. Cohorts are defined by the times of treatment administration and also include a group of never treated and potentially a group of always treated. Cohorts are also referred to as timing groups. \texttt{estat bdecomp} results are based on the derivation in Goodman-Bacon (2021).

For a DID model without covariates, we have the two-way fixed-effects model

\[ y_{it} = \gamma_i + \gamma_t + D_{it}\delta^{DD} + \epsilon_{it} \]

where \(\delta^{DD}\) is the overall treatment effect from the DID model. Let \(t_i\) denote the time when unit \(i\) receives treatment. If unit \(i\) was never treated, we set \(t_i = U > T\) and say that the unit belongs to cohort \(U\) (where \(T\) is the latest time period for which we have data). If unit \(i\) was treated in period \(k \in \{1, 2, \ldots T\}\), that is, \(t_i = k\), we say that the unit belongs to cohort \(k\). Let \(K\) denote the set of all cohorts (including \(U\)).
Goodman-Bacon (2021) show that we can decompose the estimated $\hat{\delta}^{DD}$ into the weighted sum of the DID coefficients that comes from all possible 2-by-2 cohort pairs,

$$\hat{\delta}^{DD} = \sum_{k \neq U} w_{kU} \hat{\delta}_{kU}^{2 \times 2} + \sum_{k \neq U} \sum_{l > k} \left[ w_{kl} \hat{\delta}_{kl}^{2 \times 2,k} + w_{kl} \hat{\delta}_{kl}^{2 \times 2,l} \right]$$

where $\hat{\delta}_{kU}^{2 \times 2}$ is the coefficient from the 2-by-2 DID model that uses cohorts $k$ and $U$ as treatment and control, respectively, and $\hat{\delta}_{kl}^{2 \times 2,k}$ is the coefficient from the model that uses cohorts $k$ and $l$ as treatment and control, respectively. The weights $w_t$ are given by

$$w_{kU} = \frac{(s_k + s_U)^2 s_{kU} (1 - s_{kU}) D_k (1 - \bar{D}_k)}{V^D}$$
$$w_{kl} = \frac{[(s_k + s_l)(1 - \bar{D}_l)]^2 s_{kl} (1 - s_{kl}) D_k \bar{D}_l}{V^D}$$

where $s_j = \sum_i 1(t_i = j)/N$ is the share of cohort $j \in K$, $s_{ab} = s_a/(s_a + s_b)$ is the relative size of cohort $a$ with respect to cohort $b$, $\bar{D}_j = \sum_t 1(t \geq j)/T$ is the proportion of time spent in treatment by cohort $j \in K$, and $V^D$ is the sample variance of the treatment indicator demeaned with respect to both group and time fixed effects. The weights $w_t$ sum to unity: $\sum_{k \neq U} w_{kU} + \sum_{k \neq U} \sum_{l > k} [w_{kl}^k + w_{kl}^l] = 1$.

For a model that includes time-varying covariates, $x_{it}$, a similar decomposition result holds. Consider the model

$$y_{it} = \gamma_i + \gamma_t + x_{it} \beta + D_{it} \delta^{DD}x + \epsilon_{it}$$

where $\delta^{DD|x}$ denotes the covariate-adjusted ATET. In this case, we can decompose $\hat{\delta}^{DD|x}$ into a within and a between component,

$$\hat{\delta}^{DD|x} = \Omega \hat{\delta}^p_w + (1 - \Omega) \hat{\delta}^d_b$$

To estimate the within and between components, we first fit a linear model that partials the covariates out of $D_{it}$; that is,

$$\tilde{D}_{it} = \tilde{x}_{it} \tilde{\Gamma} + \tilde{d}_{it} = \tilde{p}_{it} + \tilde{d}_{it}$$

where $\tilde{D}_{it}$ and $\tilde{x}_{it}$ are respectively the treatment indicator and the covariates demeaned with respect to both group and time fixed effects. The resulting residuals can then be separated into a within component, $d_{i(k)t}$, and a between component, $d_{kt}$, so that $d_{it} = d_{i(k)t} + d_{kt}$.

The within component of $\tilde{d}_{it}$ is defined as $\tilde{d}_{i(k)t} = (d_{it} - \bar{d}_i) - (d_{kt} - \bar{d}_k)$, where $d_{it} = D_{it} - \tilde{\Gamma} x_{it}$ is the treatment status adjusted by covariates only; and $\tilde{d}_i$, $\bar{d}_i$, $\tilde{d}_k$, and $\bar{d}_k$ are respectively the averages of $d_{it}$ by individual, cohort, and cohort by year. This component captures deviations of $d_{it}$ from cohort averages and cohort-by-year averages.

The between component of $\tilde{d}_{it}$ is defined as $\tilde{d}_{kt} = (d_{kt} - \bar{d}_k)(\bar{d}_t - \bar{d})$, where $\bar{d}_t$ and $\bar{d}d$ are the averages of $d_{it}$ by year and in the whole sample, respectively.

The within and between components of the DID treatment effect, $\hat{\delta}_w^p$ and $\hat{\delta}_b^d$, can be obtained by separately regressing the outcome variable $y_{it}$ on $\tilde{d}_{i(k)t}$ and $\tilde{d}_{kt}$, respectively. Thus, $y_{it} = a_w + \tilde{d}_{i(k)t} \hat{\delta}_w^p + \nu_{it}$ and $y_{it} = a_b + \tilde{d}_{kt} \hat{\delta}_b^d + \eta_{it}$. The weight $\Omega$ is given by the ratio of the variances $\Omega = \tilde{V}^d_w / \tilde{V}^d$, where $\tilde{V}^d_w$ and $\tilde{V}^d$ are the sample variances of $\tilde{d}_{i(k)t}$ and $\tilde{d}_{it}$, respectively.
The between component $\hat{\delta}^d_b$ can be decomposed further into 2-by-2 components

$$\hat{\delta}^d_b = \sum_k \sum_{l>k} w^{b|x}_{kl} \hat{\delta}^{2 \times 2|d}_{kl}$$

where

$$\hat{\delta}^{2 \times 2|d}_{kl} = \frac{\hat{V}^D_{kl} \hat{\delta}^{2 \times 2}_{kl} - \hat{V}^p_{b,kl} \hat{\delta}^p_{b,kl}}{\hat{V}^d_{b,kl}}$$

and

$$w^{b|x}_{kl} = (s_k + s_l)^2 \frac{\hat{V}^d_{b,kl}}{\hat{V}^d_{b,kl}}$$

In the formulas above, $\hat{V}^d_b$ and $\hat{V}^d_{b,kl}$ are the sample variances of $\tilde{D}_{kt}$ in the whole sample and in the subsample consisting of cohorts $k$ and $l$, respectively. $\hat{V}^D_{kl}$ and $\hat{V}^p_{b,kl}$ are the sample variances of $\tilde{D}_{it}$ and $\tilde{p}_{it}$, respectively, in the subsample consisting of cohorts $k$ and $l$. Finally, $\hat{\delta}^{2 \times 2}_{kl}$ and $\hat{\delta}^p_{b,kl}$ are the coefficients from a regression of $y_{it}$ on $\tilde{D}_{it}$ and from a regression of $y_{it}$ on $\tilde{p}_{it}$, respectively, in the subsample consisting of cohorts $k$ and $l$.

Thus, the full decomposition for a DID specification with controls is

$$\hat{\delta}^{DD|x} = \Omega \hat{\delta}^p_w + (1 - \Omega) \sum_k \sum_{l>k} w^{b|x}_{kl} \hat{\delta}^{2 \times 2|d}_{kl}$$

References


Also see

[CAUSAL] didregress — Difference-in-differences estimation

[CAUSAL] DID intro — Introduction to difference-in-differences estimation

[U] 20 Estimation and postestimation commands
eteffects estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational data when treatment assignment is correlated with the potential outcomes. It allows for continuous, binary, count, fractional, and nonnegative outcomes and requires a binary treatment. To control for the endogeneity of the treatment assignment, the estimator includes residuals from the treatment model in the models for the potential outcomes, known as a control-function approach.

**Quick start**

ATE of binary treatment `treat` using a linear model for outcome `y1` on `x` and the residuals from a probit model for `treat` on `x` and `z`

```
eteffects (y1 x) (treat x z)
```

Same as above, but estimate ATET

```
eteffects (y1 x) (treat x z), atet
```

Same as above, but estimate POMs

```
eteffects (y1 x) (treat x z), pomeans
```

Same as above, and show parameters from auxiliary equations

```
eteffects (y1 x) (treat x z), pomeans aequations
```

ATE of `treat` using an exponential-mean model for `y1`

```
eteffects (y1 x, exponential) (treat x z)
```

Same as above, but for count outcome `y2`

```
eteffects (y2 x, exponential) (treat x z)
```

Same as above, but use a probit model for binary outcome `y3`

```
eteffects (y3 x, probit) (treat x z)
```

Same as above, but use a fractional probit model for `y4` ranging from 0 to 1

```
eteffects (y4 x, fractional) (treat x z)
```

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Menu

Statistics > Causal inference/treatment effects > Endogenous treatment > Control function estimator > Continuous outcomes
Statistics > Causal inference/treatment effects > Endogenous treatment > Control function estimator > Binary outcomes
Statistics > Causal inference/treatment effects > Endogenous treatment > Control function estimator > Count outcomes
Statistics > Causal inference/treatment effects > Endogenous treatment > Control function estimator > Fractional outcomes
Statistics > Causal inference/treatment effects > Endogenous treatment > Control function estimator > Nonnegative outcomes

Syntax

```
eteffects (ovar omvarlist [ , omodel noconstant ]) 
  (tvar tmvarlist [ , noconstant ]) [ if ] [ in ] [ weight ] [ , stat options ]
```

`ovar` is the `depvar` of the outcome model.

`omvarlist` is the list of exogenous `indepvars` in the outcome model.

`tvar` is the binary treatment variable.

`tmvarlist` is the list of covariates that predict treatment assignment.

<table>
<thead>
<tr>
<th><code>omodel</code></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>linear outcome model; the default</td>
</tr>
<tr>
<td>fractional</td>
<td>fractional probit outcome model</td>
</tr>
<tr>
<td>probit</td>
<td>probit outcome model</td>
</tr>
<tr>
<td>exponential</td>
<td>exponential-mean outcome model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><code>stat</code></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
</tr>
<tr>
<td>pomeans</td>
<td>estimate potential-outcome means</td>
</tr>
</tbody>
</table>
Options

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
<td></td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant term</td>
</tr>
<tr>
<td><strong>SE/Robust</strong></td>
<td></td>
</tr>
<tr>
<td>vce(vcetype)</td>
<td>vcetype may be robust, cluster clustvar, bootstrap, or jackknife</td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
<td></td>
</tr>
<tr>
<td>level(#)</td>
<td>set confidence level; default is level(95)</td>
</tr>
<tr>
<td>aequations</td>
<td>display auxiliary-equation results</td>
</tr>
<tr>
<td>display_options</td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td><strong>Maximization</strong></td>
<td></td>
</tr>
<tr>
<td>maximize_options</td>
<td>control the maximization process; seldom used</td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td></td>
</tr>
<tr>
<td>pstolerance(#)</td>
<td>set tolerance for overlap assumption</td>
</tr>
<tr>
<td>osample(newvar)</td>
<td>generate newvar to mark observations that violate the overlap assumption</td>
</tr>
<tr>
<td>coeflegend</td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>

*omvarlist* and *tmvarlist* may contain factor variables; see [U] 11.4.3 Factor variables.

bootstrap, by, collect, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

**Options**

- **Model**
  - noconstant; see [R] Estimation options.

  *stat* is one of three statistics: *ate*, *atet*, or *pomeans*. *ate* is the default.

  - *ate* specifies that the average treatment effect be estimated.
  - *atet* specifies that the average treatment effect on the treated be estimated.
  - *pomeans* specifies that the potential-outcome means for each treatment level be estimated.

- **SE/Robust**
  - vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

- **Reporting**
  - level(#); see [R] Estimation options.

  aequations specifies that the results for the outcome-model or the treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.
display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofylabel, fvwrap(#), fvwrapon(style), cformat(%,fmt), pformat(%,fmt), sformat(%,fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize_options: iterate(#), [no]log, and from(init_specs); see [R] Maximize. These options are seldom used.

init_specs is one of

matname [, skip copy ]
# [, # ...], copy

Advanced

pстolerance(#) specifies the tolerance used to check the overlap assumption. The default value is pстolerance(1e-5). eteffects will exit with an error if an observation has an estimated propensity score smaller than that specified by pстolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that violate the overlap assumption.

The following option is available with eteffects but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

Remarks and examples

If you are unfamiliar with treatment-effects estimators for observational data or the teffects commands, we recommend that you look at [CAUSAL] teffects intro. For the intuition behind some of the concepts discussed below, we recommend that you read Defining treatment effects in [CAUSAL] teffects intro advanced.

The estimators implemented in eteffects extend the regression adjustment (RA) estimators implemented in teffects ra to allow for endogenous treatments, that is, when treatment assignment is not independent of outcomes. This endogeneity is a violation of the conditional mean independence assumption used by teffects ra, as discussed in The potential-outcome model in [CAUSAL] teffects intro advanced.

eteffects estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs). It uses a linear, a probit, a fractional probit, or an exponential-mean model for the potential outcomes and a probit model for treatment assignment. After conditioning on the observable covariates, eteffects allows some remaining unobservable components to affect both treatment assignment and the potential outcomes. The treatment assignment process is endogenous because these unobservable components affect both treatment assignment and the potential outcomes.

To control for the endogeneity of the treatment assignment, eteffects uses a control-function approach. This method controls for endogeneity by including the residuals from the treatment-assignment model as a regressor in the models for the potential outcome. The implementation in eteffects follows Wooldridge (2010), who provides an excellent discussion of the control-function approach that addresses endogeneity problems in a treatment-effects context.

The control-function approach estimates the parameters of the conditional means of the potential outcomes. Sample averages of the conditional means are used to estimate the unconditional ATE, ATET, or POMs. This method is known as RA.
Taken collectively, the estimators implemented in \texttt{eteffects} are control-function RA estimators. See \textit{Methods and formulas} below for details about the estimation procedure.

\textbf{Example 1: Linear outcome estimates for ATE}

Suppose we want to know the effect of a mother smoking while pregnant on the birthweight of her infant. We use an extract from Cattaneo (2010) in which \texttt{bweight} records the baby’s birthweight and \texttt{mbsmoke} is the variable (0 or 1) indicating whether a mother smoked while pregnant.

We may believe that birthweight (the potential outcome) is influenced by whether the mother had a prenatal exam in the first trimester, whether the mother is married, the mother’s age, whether this is the first birth, and the education level of the father. We may also believe that the smoking decision (the treatment) is influenced by the mother’s marital status, the education level of the mother, her age, whether she had a prenatal exam in the first trimester, and whether this baby is her first baby.

Thus we condition on different sets of covariates in the models for treatment assignment and the potential outcomes. In the probit model for smoking status (\texttt{mbsmoke}), we condition on marital status (\texttt{mmarried}), age (\texttt{mage}), mother’s education level (\texttt{medu}), father’s education level (\texttt{fedu}), and whether it was the mother’s first baby (\texttt{fbaby}). We model birthweight (\texttt{bweight}) as a linear function of whether the mother had a first-trimester prenatal exam (\texttt{prenatal1}), \texttt{mmarried}, \texttt{mage}, and \texttt{fbaby}. We can estimate the ATE of smoking status using one of the \texttt{teffects} estimators if we believe that there are no unobservable components that affect both the decision to smoke while pregnant and the potential birthweights.

If we believe there is some unobservable factor that affects both assignment to treatment and the potential outcome, we must select another estimator. For example, we do not observe a mother’s health consciousness, which affects both the smoking decision and each potential birthweight through other behaviors such as intake of prenatal vitamins. Under these assumptions, the estimators in \texttt{teffects} consistently estimate the ATE, but the estimators in [CAUSAL] \texttt{teffects} yield inconsistent estimates.

```
. use https://www.stata-press.com/data/r18/cattaneo2
. eteffects (bweight i.prenatal1 i.mmarried mage i fbaby)
   > (mbsmoke i.mmarried mage i fbaby medu fedu)
Iteration 0: EE criterion = 4.704e-24
Iteration 1: EE criterion = 1.223e-25
Endogenous treatment-effects estimation Number of obs = 4,642
Outcome model: linear
Treatment model: probit

| bweight | Robust Coefficient | std. err. | z   | P>|z|   | [95% conf. interval]   |
|---------|--------------------|-----------|-----|-------|------------------------|
| ATE     |                    |           |     |       |                        |
| mbsmoke |                   |           |     |       |                        |
| (Smoker vs Nonsmoker) |   | -455.9119 | 212.4393 | -2.15 | 0.032  | -872.2853  | -39.53852 |
| PMean   |                   |           |     |       |                        |
| mbsmoke |                   |           |     |       |                        |
| Nonsmoker |                 | 3437.964  | 31.21145 | 110.15 | 0.000  | 3376.791   | 3499.138 |
```

When no mother smokes, the average birthweight is 3,438 grams. The average birthweight is 456 grams less when all mothers smoke than when no mother smokes.
We can compare these results with those obtained if we ignore the endogeneity of the smoking decision. Below we estimate the ATE using the inverse-probability-weighted regression-adjustment estimator in \texttt{[CAUSAL] teffects ipwra}.

\begin{verbatim}
. teffects ipwra (bweight i.prenatal1 i.mmarried mage i.fbaby)
> (mbsmoke i.mmarried mage i.fbaby medu fedu)
Iteration 0: EE criterion = 3.036e-22
Iteration 1: EE criterion = 3.755e-26
Treatment-effects estimation Number of obs = 4,642
Estimator : IPW regression adjustment
Outcome model : linear
Treatment model: logit

| bweight     | Robust Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|-------------|--------------------|-----------|------|-------|---------------------|
| ATE mbsmoke (Smoker vs Nonsmoker) | -233.6835 | 25.07695 | -9.32 | 0.000 | -282.8335 -184.5336 |
| POmean mbsmoke Nonsmoker | 3403.191 | 9.529709 | 357.11 | 0.000 | 3384.513 3421.869 |
\end{verbatim}

In magnitude, the estimated ATE is more than half the estimate that allows for endogenous treatment assignment. If there is endogeneity, disregarding it underestimates the effect of smoking on birthweight. We show how to test for endogeneity in example 1 of \texttt{[CAUSAL] eteffects postestimation}.

\section*{Example 2: Estimating the ATET}

Continuing example 1, we can use the \texttt{atet} option to estimate the ATET.

\begin{verbatim}
. eteffects (bweight i.prenatal1 i.mmarried mage i.fbaby)
> (mbsmoke i.mmarried mage i.fbaby medu fedu), atet
Iteration 0: EE criterion = 4.688e-24
Iteration 1: EE criterion = 8.479e-26
Endogenous treatment-effects estimation Number of obs = 4,642
Outcome model: linear
Treatment model: probit

| bweight     | Robust Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|-------------|--------------------|-----------|------|-------|---------------------|
| ATET mbsmoke (Smoker vs Nonsmoker) | -409.8527 | 161.4816 | -2.54 | 0.011 | -726.3507 -93.35466 |
| POmean mbsmoke Nonsmoker | 3547.512 | 160.0595 | 22.16 | 0.000 | 3233.801 3861.223 |
\end{verbatim}
In the population of mothers who smoke, the average infant birthweight would be 3,548 grams if none of these mothers smoked. For the mothers who smoke, the average infant birthweight is 410 grams less than if none of these mothers smoked.

Example 3: Exponential-mean outcomes

We estimate the ATE of living in an urban area on monthly earnings (wage), using a subset of the National Longitudinal Survey in 1980 found in Wooldridge (2010). We assume that once we condition on work experience (exper), whether education level attained is college or higher (college), and IQ (iq), individual wages follow an exponential mean. The variables used to predict residence in an urban area (urban) are college and whether the respondent’s father attained a bachelor’s degree or higher (fcollege).

```
use https://www.stata-press.com/data/r18/nlsy80
. eteffects (wage exper iq i.college, exponential nocons) (urban i.college fcollege)
Iteration 0: EE criterion = 2.903e-25
Iteration 1: EE criterion = 2.903e-25 (backed up)
Endogenous treatment-effects estimation
Number of obs = 935
Outcome model: exponential
Treatment model: probit

                      ATE            POmean
                   urban (1 vs 0) urban 0
wage

          Coefficient std. err.     z     P>|z|     [95% conf. interval]
--- ------------------ -------- ------- --------- -------- ------------------
ATE
urban
   (1 vs 0)  481.0465  31.74882  15.15  0.000  418.82    543.2731
POmean
urban
   0      233.8083  13.51028  17.31  0.000  207.3286    260.288
```

When everyone lives outside urban areas, wages are $234 a month on average. Wages are $481 a month greater, on average, when everyone lives in urban areas.

Stored results

eteffects stores the following in e():

Scalars
- e(N): number of observations
- e(nj): number of observations for treatment level j
- e(k_eq): number of equations in e(b)
- e(k_levels): number of levels in treatment variable
- e(rank): rank of e(V)
- e(converged): 1 if converged, 0 otherwise

Macros
- e(cmd): eteffects
- e(cmdline): command as typed
- e(depvar): name of outcome variable
- e(tvar): name of treatment variable
eteffects — Endogenous treatment-effects estimation

The treatment-effects models considered in eteffects are given by

\[ y_{i0} = E(y_{i0} | x_i) + \epsilon_{i0} \]  \hspace{1cm} (1)
\[ y_{i1} = E(y_{i1} | x_i) + \epsilon_{i1} \]  \hspace{1cm} (2)
\[ t_i = E(t_i | z_i) + \nu_i \]  \hspace{1cm} (3)
\[ y_i = t_iy_{i1} + (1 - t_i)y_{i0} \]  \hspace{1cm} (4)
\[ E(\epsilon_{ij} | x_i, z_i) = E(\epsilon_{ij} | z_i) = E(\epsilon_{ij} | x_i) = 0 \text{ for } j \in \{0, 1\} \]  \hspace{1cm} (5)
\[ E(\epsilon_{ij} | t) \neq 0 \text{ for } j \in \{0, 1\} \]  \hspace{1cm} (6)

where the subscript \( i \) denotes individual level observations, \( y_{i1} \) is the potential outcome of receiving the treatment, \( y_{i0} \) is the potential outcome when the treatment is not received, \( t_i \) is the observed binary treatment, and \( y_i \) is the observed outcome. Each one of the potential outcomes is determined by its expected value conditional on a set of regressors \( x_i \) and an unobserved random component \( \epsilon_{ij} \), for \( j \in \{0, 1\} \). Similarly, the treatment is given by its expectation conditional on a set of regressors \( z_i \), which does not need to differ from \( x_i \), and an unobserved component \( \nu_i \).

Equations (1)–(5) describe the parametric treatment-effects models in [CAUSAL] teffects. Equation (6) adds endogeneity to the framework. It states that the unobservables in the potential-outcome equations are correlated to treatment status. For our birthweight example, this would happen if mothers who do not smoke are more health conscious than those who smoke and if we do not observe health awareness in our data. If we do not observe health awareness, the decision to smoke or not to smoke is not independent of the infant’s birthweight.
Equations (3), (5), and (6) are the basis of the control-function estimator implemented by eteffects. Equation (5) states that the unobserved components in the potential outcome are independent of $z_i$. Therefore, the correlation between $t_i$ and the unobserved components must be equivalent to the correlation between $\epsilon_{ij}$ and $\nu_i$. Another way of stating this is

\[
E(\epsilon_{ij}|t_i) = E(\epsilon_{ij}|E(t|z_i) + \nu_i) = E(\epsilon_{ij}|\nu_i) = \nu_i\beta_{2j}
\]

from (3)

\[
E(y_{ij}|x_i, \nu_i, t_i = j) = x_i'\beta_{1j} + \nu_i\beta_{2j} \quad \text{for } j \in \{0, 1\}
\]

(7)

For the probit and exponential-mean cases, respectively, we have the following:

\[
E(y_{ij}|x_i, \nu_i, t_i = j) = \Phi(x_i'\beta_{1j} + \nu_i\beta_{2j})
\]

(8)

\[
E(y_{ij}|x_i, \nu_i, t_i = j) = \exp(x_i'\beta_{1j} + \nu_i\beta_{2j})
\]

(9)

The parameters of (3) and (7)–(9), and the ATE, ATET, and POMs are estimated using the generalized method of moments (GMM). The moment equations used in GMM are the sample analogs of $E \{w_i'\epsilon_i(\theta)\} = 0$, where $w_i$ are the instruments, $\epsilon_i(\theta)$ are residuals, and $\theta$ are the parameters of the model (see [R] gmm). The moment conditions in the GMM estimation for the linear model are given by

\[
\frac{1}{n} \sum_{i=1}^{n} x_i'(y_i - x_i'\hat{\beta}_{1j} + \hat{\nu}_i\beta_{2j})t_i = 0
\]

(10)

\[
\frac{1}{n} \sum_{i=1}^{n} x_i'(y_i - x_i'\hat{\beta}_{1j} + \hat{\nu}_i\beta_{2j})(1 - t_i) = 0
\]

(11)

\[
\frac{1}{n} \sum_{i=1}^{n} x_i' \left\{ t_i \frac{\phi(z_i'\hat{\pi})}{\Phi(z_i'\hat{\pi})} - (1 - t_i) \frac{\phi(z_i'\hat{\pi})}{1 - \Phi(z_i'\hat{\pi})} \right\} = 0
\]

(12)

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ (x_i'\hat{\beta}_{10} + \hat{\nu}_i\hat{\beta}_{20}) - \text{POM0} \right\} = 0
\]

(13)

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ (x_i'\hat{\beta}_{11} + \hat{\nu}_i\hat{\beta}_{21}) - \text{POM0} - \text{ATE} \right\} = 0
\]

(14)

where $\hat{\nu}_i = t_i - \Phi(z_i'\hat{\pi})$, $n$ is the number of observations, and $\hat{\beta}_{11}, \hat{\beta}_{10}, \hat{\beta}_{21}, \hat{\beta}_{20}, \hat{\pi}, \hat{\text{ATE}},$ and $\hat{\text{POM0}}$ are the parameters. If we want to estimate the ATET, we replace (14) with
\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ \left( x_i' \hat{\beta}_{11} + \hat{\nu}_i \hat{\beta}_{21} \right) - \frac{n}{n_t} \hat{\text{POM}0} - \hat{\text{ATET}} \right\} = 0
\] (15)

and if we want to estimate the potential-outcome means, we replace (14) with

\[
\frac{1}{n} \sum_{i=1}^{n} \left\{ \left( x_i' \hat{\beta}_{11} + \hat{\nu}_i \hat{\beta}_{21} \right) - \hat{\text{POM}1} \right\} = 0
\] (16)

where \( \hat{\text{ATET}} \) and \( \hat{\text{POM}1} \) are the parameters of the model, and \( n_t \) is the number of treated units.

For the exponential-mean outcome model, we replace \( x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j} \) with \( \exp(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j}) \) to obtain the residual equations in (10)–(16). For the probit outcome model, we replace (10) and (11) with the following:

\[
\frac{1}{n} \sum_{i=1}^{n} t_i x_i' \left\{ \frac{\phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})}{\Phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})} - (1 - y_i) \frac{\phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})}{1 - \Phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})} \right\} = 0
\]

\[
\frac{1}{n} \sum_{i=1}^{n} (1 - t_i) x_i' \left\{ \frac{\phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})}{\Phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})} - (1 - y_i) \frac{\phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})}{1 - \Phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j})} \right\} = 0
\]

For the remaining equations, \( x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j} \) is replaced with \( \Phi(x_i' \hat{\beta}_{1j} + \hat{\nu}_i \hat{\beta}_{2j}) \). The fractional probit model uses the same moment conditions as the probit model.

**Acknowledgment**

We thank Jeffrey Wooldridge of Michigan State University for helpful conversations and advice about the literature and methods in this area.

**References**


Also see

[CAUSAL] eteffects postestimation — Postestimation tools for eteffects
[CAUSAL] etregress — Linear regression with endogenous treatment effects
[CAUSAL] teffects — Treatment-effects estimation for observational data
[R] gmm — Generalized method of moments estimation
[R] probit — Probit regression
[R] regress — Linear regression
[U] 20 Estimation and postestimation commands
Postestimation commands

The following postestimation command is of special interest after `eteffects`:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>estat endogenous</code></td>
<td>perform tests of endogeneity</td>
</tr>
</tbody>
</table>

The following standard postestimation commands are available after `eteffects`:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>estat summarize</code></td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td><code>estat vce</code></td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td><code>estimates</code></td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td><code>etable</code></td>
<td>table of estimation results</td>
</tr>
<tr>
<td><code>hausman</code></td>
<td>Hausman’s specification test</td>
</tr>
<tr>
<td><code>lincom</code></td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td><code>nlcom</code></td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td><code>predict</code></td>
<td>treatment effects, conditional means at treatment, propensity scores, etc.</td>
</tr>
<tr>
<td><code>predictnl</code></td>
<td>point estimates, standard errors, testing, and inference for generalized predictions</td>
</tr>
<tr>
<td><code>test</code></td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td><code>testnl</code></td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>
predict

Description for predict

predict creates a new variable containing predictions such as treatment effects, conditional means, propensity scores, and linear predictions.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [ if ] [ in ] [, statistic tlevel ]
```

```
predict [type] stub* [ if ] [ in ], scores
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main</td>
<td></td>
</tr>
<tr>
<td>te</td>
<td>treatment effect; the default</td>
</tr>
<tr>
<td>cmean</td>
<td>conditional mean at treatment level</td>
</tr>
<tr>
<td>ps</td>
<td>propensity score</td>
</tr>
<tr>
<td>xb</td>
<td>linear prediction</td>
</tr>
<tr>
<td>pxbxb</td>
<td>linear prediction for propensity score</td>
</tr>
<tr>
<td>xbtotal</td>
<td>linear prediction, using residuals from treatment model</td>
</tr>
</tbody>
</table>

Specify one new variable with te; specify one or two new variables with cmean, ps, and xb.

Options for predict

```
te, the default, calculates the treatment effect.
cmean calculates the conditional mean for the control group. To also obtain the conditional mean for the treatment group, specify two variables. If you want the conditional mean for only the treatment group, specify the tlevel option.
pbs calculates the probability of being in the control group. To also obtain the probability of being in the treatment group, specify two variables. If you want the probability of being in the treatment group only, specify the tlevel option.
xb calculates the linear prediction for the control group. To also obtain the linear prediction for the treatment group, specify two variables. If you want the linear prediction for only the treatment group, specify the tlevel option.
pxbxb calculates the linear prediction for the propensity score.
xbtotal calculates the linear prediction for the control group, including the residuals from the treatment model as regressors. To also obtain the linear prediction for the treatment group, specify two variables. If you want the linear prediction, including the residuals from the treatment model as regressors, only for the treatment group, specify the tlevel option.
tlevel specifies that the statistic be calculated for the treatment group; the default is to calculate the statistic for the control group.

scores calculates the score variables. For eteffects, this is the same as the residuals in the moment conditions used by the generalized method of moments (see \texttt{[R] gmm}). For the average treatment effect, the average treatment effect on the treated, and the potential-outcome means, parameter-level scores are computed. For the auxiliary equations, equation-level scores are computed.

\textbf{estat}

\textbf{Description for estat}

\texttt{estat endogenous} performs a Wald test to determine whether the estimated correlations between the treatment-assignment and potential-outcome models are different from zero. The null hypothesis is that the correlations are jointly zero. Rejection of the null hypothesis suggests endogeneity.

\textbf{Menu for estat}

Statistics $>$ Postestimation

\textbf{Syntax for estat}

\begin{verbatim}
estat endogenous
\end{verbatim}

\texttt{collect} is allowed with \texttt{estat endogenous}; see \texttt{[U] 11.1.10 Prefix commands}.

\textbf{Remarks and examples}

\textbf{Example 1: Testing for endogeneity}

In \texttt{example 3 of [CAUSAL] eteffects}, endogeneity could arise if unobservable factors that determine wages are correlated with the decision to live in an urban area. If there is no endogeneity, we would prefer to use one of the \texttt{teffects} estimators because they will give us the more efficient standard errors. The control-function approach used by \texttt{eteffects} allows us to test for endogeneity.

The control-function approach estimates the correlation between the unobservables of the treatment-assignment and potential-outcome models. If there is no correlation between the unobservables, then there is no endogeneity. We test for correlation, and thus for endogeneity, by typing

\begin{verbatim}
.use https://www.stata-press.com/data/r18/nlsy80
. eteffects (wage exper iq i.college, exponential nocons)
> (urban i.college fcollege)
(output omitted)
. estat endogenous
Test of endogeneity
H0: Treatment and outcome unobservables are uncorrelated
   chi2(  2) =   275.36
     Prob > chi2 =   0.0000
\end{verbatim}
We reject the null hypothesis of no endogeneity. This suggests that unobservable factors that determine wages mediate the decision to live in an urban area.

---

### Technical note

The estimated correlations between the unobservables of the treatment-assignment and potential-outcome models are auxiliary parameters. They appear under the headings $TEOM0$ and $TEOM1$, which refer to treatment residuals (TE) for outcome model 0 (OM0) and outcome model 1 (OM1), when the option `aequations` is specified.

For the model in example 3 of [CAUSAL] _eteffects_ with the `aequations` option, the results are the following:

```
. eteffects (wage exper iq i.college, exponential nocons)
> (urban i.college fcollege), aequations
Iteration 0: EE criterion = 2.90e-25
Iteration 1: EE criterion = 2.90e-25 (backed up)
Endogenous treatment-effects estimation
Number of obs = 935
Outcome model: exponential
Treatment model: probit

|               | Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|---------------|-------------|-----------|------|------|---------------------|
| **ATE** urban (1 vs 0) | 481.0465    | 31.74882  | 15.15| 0.000| 418.82 543.2731 |
| **POmean** urban 0 | 233.8083    | 13.51028  | 17.31| 0.000| 207.3286 260.288 |
| **TME1** college 1 | .195811     | .1012119  | 1.93 | 0.053| -.0025607 .3941827 |
|               | .1069748    | .0992075  | 1.08 | 0.281| -.0874683 .3014179 |
|               | .498012     | .056408   | 8.83 | 0.000| .3874543 .6085698 |
| **OME0** exper 1 | .0193244    | .0085633  | 2.26 | 0.024| .0025405 .0361082 |
|               | .0099473    | .00386949 | 2.69 | 0.007| .0027053 .0171892 |
| college 1 | -.3718598   | .2678636  | -1.39| 0.165| -.8968629 .1531433 |
| **OME1** exper 1 | .0238566    | .017597   | 1.36 | 0.175| -.0106329 .058346 |
|               | .0148581    | .0113311  | 1.31 | 0.190| -.0073505 .0370667 |
| college 1 | 1.236947     | .6401383  | 1.93 | 0.053| -.0177013 2.491595 |
| **TEOM0** _cons | -7.771932   | .6406251  | -12.13| 0.000| -.9.027534 .651633 |
| **TEOM1** _cons | 16.7739     | 4.777519  | 3.51 | 0.000| 7.410131 26.13766 |
```
Among other things, we can use these correlations to test the joint significance of the coefficients on the residuals from the treatment-assignment models. This is equivalent to the endogeneity test in example 1. We type

```
. test [TEOM0]_cons [TEOM1]_cons
   ( 1) [TEOM0]_cons = 0
   ( 2) [TEOM1]_cons = 0
    chi2(  2) = 275.36
    Prob > chi2 = 0.0000
```

Also see

[CAUSAL] eteffects — Endogenous treatment-effects estimation

[U] 20 Estimation and postestimation commands
etpoisson estimates the parameters of a Poisson regression model in which one of the regressors is an endogenous binary treatment. Both the average treatment effect and the average treatment effect on the treated can be estimated with etpoisson.

Quick start

Poisson model of \( y \) on \( x \) and endogenous binary treatment \( \text{treat} \) modeled by \( x \) and \( w \)

\[
etpoisson \ y \ x, \ treat(treat = x \ w)
\]

With robust standard errors

\[
etpoisson \ y \ x, \ treat(treat = x \ w) \ vce(robust)
\]

Average treatment effect after etpoisson with the required vce(robust) option

\[
margins \ r.treat, \ vce(unconditional)
\]

Same as above, but calculate average treatment effect on the treated

\[
margins, \ vce(unconditional) \ predict(cte) \ subpop(if \ treat==1)
\]
etpoisson — Poisson regression with endogenous treatment effects

Syntax

```
etpoisson depvar [ indepvars ] [ if ] [ in ] [ weight ],
    treat(depvar_t = indepvars_t [, noconstant offset(varname_o)]) [ options ]
```

```
options
```

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>treat()</strong></td>
<td>equation for treatment effects</td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant term</td>
</tr>
<tr>
<td>exposure(varname_o)</td>
<td>include ln(varname_o) in model with coefficient constrained to 1</td>
</tr>
<tr>
<td>offset(varname_o)</td>
<td>include varname_o in model with coefficient constrained to 1</td>
</tr>
<tr>
<td>constraints(constraints)</td>
<td>apply specified linear constraints</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>SE/Robust</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>vce(vcetype)</td>
<td>vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackknife</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Reporting</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>level(#)</td>
<td>set confidence level; default is level(95)</td>
</tr>
<tr>
<td>irr</td>
<td>report incidence-rate ratios</td>
</tr>
<tr>
<td>noconsreport</td>
<td>do not display constraints</td>
</tr>
<tr>
<td>display_options</td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Integration</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>intpoints(#)</td>
<td>use # Gauss–Hermite quadrature points; default is intpoints(24)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Maximization</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>maximize_options</td>
<td>control the maximization process; seldom used</td>
</tr>
<tr>
<td>collinear</td>
<td>keep collinear variables</td>
</tr>
<tr>
<td>coeflegend</td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>

*`treat()` is required.

The full specification is `treat(depvar_t = indepvars_t [, noconstant offset(varname_o)])`.

`indepvars` and `indepvars_t` may contain factor variables; see [U] 11.4.3 Factor variables.
`depvar`, `depvar_t`, `indepvars`, and `indepvars_t` may contain time-series operators; see [U] 11.4.4 Time-series varlists.
`bootstrap`, `by`, `collect`, `jackknife`, `rolling`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the `bootstrap` prefix; see [R] bootstrap.
`aweights` are not allowed with the `jackknife` prefix; see [R] jackknife.
`vce()` and weights are not allowed with the `svy` prefix; see [SVY] svy.
`fweights`, `aweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 weight.
collinear and coeflegend do not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
treat(\textit{depvar}_t = \textit{indepvars}_t [, \textit{noconstant offset(varname}_o)] ) specifies the variables and options for the treatment equation. It is an integral part of specifying a treatment-effects model and is required.

The indicator of treatment, \textit{depvar}_t, should be coded as 0 or 1.

noconstant, exposure(\textit{varname}_e), offset(\textit{varname}_o), constraints(\textit{constraints}); see [R] Estimation options.

\textit{vce(vcetype)} specifies the type of standard error reported, which includes types that are derived from asymptotic theory (\textit{oim, opg}), that are robust to some kinds of misspecification (\textit{robust}), that allow for intragroup correlation (\textit{cluster clustvar}), and that use bootstrap or jackknife methods (\textit{bootstrap, jackknife}); see [R] \textit{vce} option.

level(#); see [R] Estimation options.

\textit{irr} reports estimated coefficients transformed to incidence-rate ratios, that is, \(e^{\beta_i}\) rather than \(\beta_i\). Standard errors and confidence intervals are similarly transformed. This option affects how results are displayed, not how they are estimated or stored. \textit{irr} may be specified at estimation or when replaying previously estimated results.

\textit{nocsnsreport}; see [R] Estimation options.

\textit{display_options}: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(\%fmt), pformat(\%fmt), sformat(\%fmt), and nolstretch; see [R] Estimation options.

\textit{intpoints(#)} specifies the number of integration points to use for integration by quadrature. The default is \textit{intpoints(24)}; the maximum is \textit{intpoints(128)}. Increasing this value improves the accuracy but also increases computation time. Computation time is roughly proportional to its value.

\textit{maximize_options}: difficult, technique(algorithm_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init_specs); see [R] Maximize. These options are seldom used.

Setting the optimization type to technique(bhhh) resets the default \textit{vcetype} to \textit{vce(opg)}.

The following options are available with \textit{etpoisson} but are not shown in the dialog box:

\textit{collinear}, coeflegend; see [R] Estimation options.
etpoisson — Poisson regression with endogenous treatment effects

Remarks and examples

Remarks are presented under the following headings:

Overview
Basic example
Average treatment effect (ATE)
Average treatment effect on the treated (ATET)

Overview

etpoisson estimates the parameters of a Poisson regression model that includes an endogenous binary-treatment variable. The dependent variable must be a Poisson distributed count. The parameters estimated by etpoisson can be used to estimate the average treatment effect (ATE) and average treatment effect on the treated (ATET).

We call the model fit by etpoisson an endogenous treatment-regression model, although it is also known as an endogenous binary-variable model or as an endogenous dummy-variable model. The endogenous treatment-regression model fit by etpoisson is a specific endogenous treatment-effects model; it uses a nonlinear model for the outcome and a constrained normal distribution to model the deviation from the conditional independence assumption imposed by the estimators implemented by teffects; see [CAUSAL] teffects intro. In treatment-effects jargon, the endogenous binary-variable model fit by etpoisson is a nonlinear potential-outcome model that allows for a specific correlation structure between the unobservables that affect the treatment and the unobservables that affect the potential outcomes. See [CAUSAL] etregress for an estimator that allows for a linear-outcome model and a similar model for the endogeneity of the treatment.

More formally, we have an equation for outcome $y_j$ and an equation for treatment $t_j$:

$$
E(y_j | x_j, t_j, \epsilon_j) = \exp(x_j \beta + \delta t_j + \epsilon_j)
$$

$$
t_j = \begin{cases} 
1, & w_j \gamma + u_j > 0 \\
0, & \text{otherwise}
\end{cases}
$$

The $x_j$ are the covariates used to model the outcome, $w_j$ are the covariates used to model treatment assignment, and error terms $\epsilon_j$ and $u_j$ are bivariate normal with mean 0 and covariance matrix

$$
\begin{bmatrix}
\sigma^2 & \sigma \rho \\
\sigma \rho & 1
\end{bmatrix}
$$

The covariates $x_j$ and $w_j$ are unrelated to the error terms; in other words, they are exogenous. Note that $y_j$ may be a count or continuous and nonnegative in this specification.

Terza (1998) describes the maximum likelihood estimator used in etpoisson. Terza (1998) categorized the model fit by etpoisson as an endogenous-switching model. These models involve a binary switch that is endogenous for the outcome. Calculation of the maximum likelihood estimate involves numeric approximation of integrals via Gauss–Hermite quadrature. This is computationally intensive, but the computational costs are reasonable on modern computers.
Basic example

Example 1

In this example, we observe a simulated random sample of 5,000 households. The outcome of interest is the number of trips taken by members of the household in the 24-hour period immediately prior to the interview time.

We have fictional household level data on the following variables: number of trips taken in the past 24 hours (trips), distance to the central business district from the household (cbd), distance from the household to a public transit node (ptn), an indicator of whether there is a full-time worker in the household (worker), an indicator of whether the examined period is on a weekend (weekend), the ratio of the household income to the median income of the census tract (realinc), and an indicator of car ownership (owncar). We suspect that unobservables that affect the number of trips also affect the household’s propensity to own a car.

We use \texttt{etpoisson} to estimate the parameters of a Poisson regression model for the number of trips with car ownership as an endogenous treatment. In subsequent examples, we will use \texttt{margins} (see \texttt{[R] margins}) to estimate the ATE and the ATET of car ownership on the number of trips taken by the household. In the \texttt{etpoisson} command below, we specify the \texttt{vce(robust)} option because we need to specify \texttt{vce(unconditional)} when we use \texttt{margins} later.
. use https://www.stata-press.com/data/r18/trip1
(Household trips, car ownership)
. etpoisson trips cbd ptn worker weekend,
> treat(owncar = cbd ptn worker realinc) vce(robust)
Iteration 0: Log pseudolikelihood = -14845.147 (not concave)
Iteration 1: Log pseudolikelihood = -14562.997 (not concave)
Iteration 2: Log pseudolikelihood = -13655.592 (not concave)
Iteration 3: Log pseudolikelihood = -12847.219 (not concave)
Iteration 4: Log pseudolikelihood = -12566.037
Iteration 5: Log pseudolikelihood = -12440.974
Iteration 6: Log pseudolikelihood = -12413.485
Iteration 7: Log pseudolikelihood = -12412.699
Iteration 8: Log pseudolikelihood = -12412.696
Iteration 9: Log pseudolikelihood = -12412.696
Poisson regression with endogenous treatment
Number of obs = 5,000
(24 quadrature points) Wald chi2(5) = 397.94
Log pseudolikelihood = -12412.696 Prob > chi2 = 0.0000

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
<th></th>
<th></th>
<th></th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>std. err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>trips</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cbd</td>
<td>-.0100919</td>
<td>.0020071</td>
<td>-5.03</td>
<td>0.000</td>
<td>-.0140258</td>
</tr>
<tr>
<td>ptn</td>
<td>-.0204038</td>
<td>.0020289</td>
<td>-10.06</td>
<td>0.000</td>
<td>-.0243805</td>
</tr>
<tr>
<td>worker</td>
<td>.692301</td>
<td>.0548559</td>
<td>12.62</td>
<td>0.000</td>
<td>.5847854</td>
</tr>
<tr>
<td>weekend</td>
<td>.0930517</td>
<td>.034538</td>
<td>2.69</td>
<td>0.007</td>
<td>.0525385</td>
</tr>
<tr>
<td>1.owncar</td>
<td>.5264713</td>
<td>.1124157</td>
<td>4.68</td>
<td>0.000</td>
<td>.3061406</td>
</tr>
<tr>
<td>_cons</td>
<td>-.2340772</td>
<td>.0810812</td>
<td>-2.89</td>
<td>0.004</td>
<td>-.3929934</td>
</tr>
<tr>
<td>owncar</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>cbd</td>
<td>.007218</td>
<td>.00239</td>
<td>3.02</td>
<td>0.003</td>
<td>.0025337</td>
</tr>
<tr>
<td>ptn</td>
<td>.0084769</td>
<td>.0024518</td>
<td>3.46</td>
<td>0.001</td>
<td>.0036714</td>
</tr>
<tr>
<td>worker</td>
<td>.543643</td>
<td>.0504267</td>
<td>10.78</td>
<td>0.000</td>
<td>.4448085</td>
</tr>
<tr>
<td>realinc</td>
<td>.176479</td>
<td>.0108746</td>
<td>16.23</td>
<td>0.000</td>
<td>.1551652</td>
</tr>
<tr>
<td>_cons</td>
<td>-.4611246</td>
<td>.0592161</td>
<td>-7.79</td>
<td>0.000</td>
<td>-.5771859</td>
</tr>
<tr>
<td>/athrho</td>
<td>.5741169</td>
<td>.0957832</td>
<td>5.99</td>
<td>0.000</td>
<td>.3863852</td>
</tr>
<tr>
<td>/lnsigma</td>
<td>-.2182037</td>
<td>.0256281</td>
<td>-8.51</td>
<td>0.000</td>
<td>-.2684338</td>
</tr>
<tr>
<td>rho</td>
<td>.5183763</td>
<td>.0700449</td>
<td>7.32</td>
<td>0.000</td>
<td>.3682398</td>
</tr>
<tr>
<td>sigma</td>
<td>.8093617</td>
<td>.020604</td>
<td>39.63</td>
<td>0.000</td>
<td>.764576</td>
</tr>
</tbody>
</table>

Wald test of indep. eqns. (rho = 0): chi2(1) =   35.93  Prob > chi2 = 0.0000

The Wald test in the header is highly significant, indicating a good model fit. All the covariates are statistically significant, and the Wald test in the footer indicates that we can reject the null hypothesis of no correlation between the treatment errors and the outcome errors.

We can interpret the coefficient on 1.owncar as the logarithm of the ratio of the treatment potential-outcome mean to the control potential-outcome mean. The treatment variable did not interact with any of the outcome covariates, so the effect of each regressor is the same in the two regimes and will cancel from the ratio of potential-outcome means. This means the ratio is equivalent to the exponentiated coefficient on 1.owncar. After discussing the other parameters, we will use lincom to obtain this ratio. See [R] lincom for more information.

The estimated correlation between the treatment-assignment errors and the outcome errors is 0.518, indicating that unobservables that increase the number of trips tend to occur with unobservables that increase the chance of car ownership.
The results for the two ancillary parameters require explanation. `etpoisson` estimates the inverse hyperbolic tangent of $\rho$,

\[
\text{atanh } \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)
\]

and $\ln \sigma$ rather than $\rho$ and $\sigma$. For numerical stability during optimization, `etpoisson` does not directly estimate $\rho$ and $\sigma$.

Now we use `lincom` and the `eform` option to estimate the exponentiated coefficient for `1.owncar`. This corresponds to the ratio of the treatment regime potential-outcome mean to the control regime potential-outcome mean.

```
. lincom [trips]_b[1.owncar], eform
   ( 1) [trips]1.owncar = 0

| exp(b) | Std. err. | z     | P>|z| | [95% conf. interval] |
|-------|-----------|-------|------|----------------------|
| (1)   | 1.692948  | .1903139 | 4.68 | 0.000 | 1.358173 2.110241 |
```

The potential-outcome mean for the treatment regime is 1.69 times the potential-outcome mean for the control regime. So the average number of trips in the treatment regime is over one and a half times the average number of trips in the control regime.

By interacting the treatment, `owncar`, with the other regressors, we could estimate different coefficients for the regressors in the treatment and control regimes. In the current model, there are no treatment interactions, so the coefficients are the same in each regime.

### Average treatment effect (ATE)

The parameter estimates from `etpoisson` can be used by `margins` to estimate the ATE, the average difference of the treatment and control potential outcomes.

#### Example 2

Continuing with example 1, we use `margins` to estimate the ATE of car ownership on the number of trips taken in a 24-hour period.

We can estimate the ATE of car ownership by using the potential-outcome means obtained through the `predict, pomean` command and the `margins` command; see Methods and formulas below and [CAUSAL] `etpoisson postestimation` for more details about the use of `predict` after `etpoisson`.

The `r.` notation indicates that the potential-outcome means for treatment and control will be contrasted. We specify the `contrast(nowald)` option to suppress the Wald tests that `margins` displays by default for contrasts.

```
. margins r.owncar, vce(unconditional) contrast(nowald)
Contrasts of predictive margins
Expression: Potential-outcome mean, predict()
Number of obs = 5,000

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
<th></th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Contrast</td>
<td>std. err.</td>
<td></td>
</tr>
<tr>
<td>owncar</td>
<td>(1 vs 0)</td>
<td>1.058914</td>
<td>.1922909 .6820309 1.435797</td>
</tr>
</tbody>
</table>
```
The estimated ATE of car ownership on the number of trips taken is 1.06. The average household will take 1.06 more trips when it owns a car.

### Average treatment effect on the treated (ATET)

The parameter estimates from `etpoisson` can be used by `margins` to estimate the ATET, the average difference of the treatment and control potential outcomes in the treated population.

#### Example 3

Continuing with the previous example, we use `margins` to estimate the ATET of car ownership on the number of trips taken in a 24-hour period.

We can estimate the ATET of car ownership by using the conditional treatment effect (conditional on exogenous covariates and treatment level) obtained through the `predict, cte` command and the `margins` command; see Methods and formulas below and [CAUSAL] `etpoisson postestimation` for more details about the use of `predict` after `etpoisson`.

We estimate the ATET with `margins`. We specify `cte` in the `predict()` option. Estimation is restricted to the treated subpopulation by specifying `owncar` in the `subpop()` option.

```
. margins, predict(cte) vce(unconditional) subpop(owncar)
```

```
Predictive margins
Expression: Conditional treatment effect, predict(cte)

Number of obs = 5,000
Subpop. no. obs = 3,504

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Margin std. err.</td>
</tr>
<tr>
<td>_cons</td>
<td>1.251971</td>
</tr>
</tbody>
</table>
```

The estimated ATET of car ownership on the number of trips taken is 1.25. Thus the average household in the treated population will take 1.25 more trips than it would if it did not own a car. This number is higher than the ATE. In this model, the ATE and ATET will only coincide when there is no correlation between the treatment errors and outcome errors and the exogenous covariates $x$ have the same distribution in the general population and treated subpopulation. See Methods and formulas for more details.
etpoisson — Poisson regression with endogenous treatment effects

Stored results

etpoisson stores the following in e():

Scalars

- `e(N)` number of observations
- `e(k)` number of parameters
- `e(k_eq)` number of equations in `e(b)`
- `e(k_eq_model)` number of equations in overall model test
- `e(k_aux)` number of auxiliary parameters
- `e(k_dv)` number of dependent variables
- `e(df_m)` model degrees of freedom
- `e(ll)` log likelihood
- `e(N_clust)` number of clusters
- `e(chi2)` \( \chi^2 \)
- `e(chi2_c)` \( \chi^2 \) for comparison, \( \rho=0 \) test
- `e(n_quad)` number of quadrature points
- `e(p)` \( p \)-value for model test
- `e(p_c)` \( p \)-value for comparison test
- `e(rank)` rank of `e(V)`
- `e(ic)` number of iterations
- `e(rc)` return code
- `e(converged)` 1 if converged, 0 otherwise

Macros

- `e(cmd)` `etpoisson`
- `e(cmdline)` command as typed
- `e(depvar)` name of dependent variable
- `e(wtype)` weight type
- `e(wexp)` weight expression
- `e(title)` title in estimation output
- `e(title2)` secondary title in estimation output
- `e(clustvar)` name of cluster variable
- `e(offset1)` offset for regression equation
- `e(offset2)` offset for treatment equation
- `e(chi2type)` Wald; type of model \( \chi^2 \) test
- `e(chi2_ct)` Wald; type of comparison \( \chi^2 \) test
- `e(vcetype)` vcetype specified in vce()
- `e(vctype)` title used to label Std. err.
- `e(opt)` type of optimization
- `e(which)` max or min; whether optimizer is to perform maximization or minimization
- `e(ml_method)` type of ml method
- `e(user)` name of likelihood-evaluator program
- `e(technique)` maximization technique
- `e(properties)` b V
- `e(predict)` program used to implement predict
- `e(marginsok)` predictions allowed by margins
- `e(asbalanced)` factor variables fvset as asbalanced
- `e(asobserved)` factor variables fvset as asobserved

Matrices

- `e(b)` coefficient vector
- `e(Cns)` constraints matrix
- `e(ilog)` iteration log (up to 20 iterations)
- `e(gradient)` gradient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(V_modelbased)` model-based variance

Functions

- `e(sample)` marks estimation sample
In addition to the above, the following is stored in \( r() \):

- \( \mathbf{r(table)} \): matrix containing the coefficients with their standard errors, test statistics, \( p \)-values, and confidence intervals

Note that results stored in \( r() \) are updated when the command is replayed and will be replaced when any \( r \)-class command is run after the estimation command.

### Methods and formulas

Terza (1998) derives the maximum likelihood estimator implemented here. We provide some details of the derivation and then explain how the model is nested in a more general potential-outcomes model. Then the ATE and ATET are derived.

Let \( \mathbf{x}_j \) be the covariates used to model the outcome, and let \( \mathbf{w}_j \) be the covariates used to model treatment assignment. Define \( \mathbf{z}_j = (\mathbf{w}_j, \mathbf{x}_j) \). The vector \( \mathbf{z}_j \) contains all the exogenous covariates in the model. When offsets \( o_j^\beta \) are used in the outcome variable equation, the following formulas apply with \( \mathbf{x}_j \beta \) changed to \( \mathbf{x}_j \beta + o_j^\beta \). Similarly, when offsets \( o_j^\gamma \) are used in the endogenous treatment equation, the following formulas apply with \( \mathbf{w}_j \gamma \) changed to \( \mathbf{w}_j \gamma + o_j^\gamma \). If offsets are used in either equation, they are included in the vector of exogenous covariates \( \mathbf{z}_j \).

For treatment \( t_j \), \( \mathbf{z}_j \), and \( \epsilon_j \), outcome \( y_j \) of this model has conditional mean

\[
E(y_j|\mathbf{x}_j, t_j, \epsilon_j) = \exp(\mathbf{x}_j \beta + \delta t_j + \epsilon_j)
\]  

(1)

The probability density function of \( y_j \) for this model, conditioned on treatment \( t_j \), \( \mathbf{z}_j \), and \( \epsilon_j \), is given by

\[
f(y_j|\mathbf{z}_j, t_j, \epsilon_j) = \frac{\exp\{-\exp(\mathbf{x}_j \beta + \delta t_j + \epsilon_j)\}\{\exp(\mathbf{x}_j \beta + \delta t_j + \epsilon_j)\}^{y_j}}{y_j!}
\]

The treatment \( t_j \) is determined by

\[
t_j = \begin{cases} 
1, & \text{if } \mathbf{w}_j \gamma + u_j > 0 \\
0, & \text{otherwise}
\end{cases}
\]

The error terms \( \epsilon_j \) and \( u_j \) are bivariate normal with mean zero and covariance matrix

\[
\begin{bmatrix}
\sigma^2 & \sigma \rho \\
\sigma \rho & 1
\end{bmatrix}
\]

Conditional on \( \epsilon_j \), \( u_j \) is normal with mean \( \epsilon_j \rho / \sigma \) and variance \((1 - \rho^2)\); thus we obtain the following conditional probability density for \( t_j \):

\[
\Pr(t_j|\mathbf{z}_j, \epsilon_j) = t_j \Phi \left\{ \frac{\mathbf{w}_j \gamma + (\rho/\sigma) \epsilon_j}{\sqrt{1 - \rho^2}} \right\} + (1 - t_j) \left[ 1 - \Phi \left\{ \frac{\mathbf{w}_j \gamma + (\rho/\sigma) \epsilon_j}{\sqrt{1 - \rho^2}} \right\} \right]
\]

\( \Phi \) denotes the standard normal cumulative distribution function. This leads to the following joint density of \( y_j \), \( t_j \), and \( \epsilon_j \):

\[
f(y_j, t_j, \epsilon_j|\mathbf{z}_j) = f(y_j|\mathbf{z}_j, t_j, \epsilon_j)P(t_j|\mathbf{z}_j, \epsilon_j)f(\epsilon_j)
\]

The density of \( y_j \) and \( t_j \), conditioned on \( \mathbf{z}_j \), is obtained by integrating the above with respect to \( \epsilon_j \). Recall that \( \epsilon_j \) is normal with mean 0 and variance \( \sigma^2 \).

\[
f(y_j, t_j|\mathbf{z}_j) = \int_{-\infty}^{\infty} f(y_j|\mathbf{z}_j, t_j, \epsilon_j)P(t_j|\mathbf{z}_j, \epsilon_j)\frac{1}{\sigma \sqrt{2\pi}} \exp \left\{ - \frac{\epsilon_j^2}{2} \right\} d\epsilon_j
\]
\( f(y_j, t_j | z_j) \) cannot be evaluated in a closed form. We change the variable of integration from \( \epsilon_j \) to \( \eta_j = \epsilon_j / (\sigma \sqrt{2}) \), which yields

\[
f(y_j, t_j | z_j) = \frac{1}{\sqrt{\pi}} \int_{-\infty}^{\infty} f(y_j | z_j, t_j, \sqrt{2} \sigma \eta_j) P(t_j | z_j, \sqrt{2} \sigma \eta_j) \exp(-\eta_j^2) \, d\eta_j
\]

We approximate this integral by Gauss–Hermite quadrature. Observing a sample of \( t_j, y_j, \) and \( z_j \), we calculate the log likelihood as the following:

\[
\ln L = \sum_{j=1}^{n} w_j \ln \{ f(y_j, t_j | z_j) \}
\]

The \( w_j \) terms denote optional weights.

In the maximum likelihood estimation, \( \sigma \) and \( \rho \) are not directly estimated. Directly estimated are \( \ln \sigma \) and \( \text{atanh} \rho \):

\[
\text{atanh} \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)
\]

Now we present formulas for the ATE and ATET. First, we nest the endogenous-treatment Poisson regression model in a potential-outcome model. A potential-outcome model specifies what each individual would obtain in each treatment level.

A potential-outcome model that nests the endogenous-treatment Poisson regression fit by \texttt{etpoisson} is

\[
E(y_{0j} | x_j, \epsilon_j) = \exp(x_j \beta_0 + \epsilon_{0j})
\]

\[
E(y_{1j} | x_j, \epsilon_j) = \exp(x_j \beta_1 + \epsilon_{1j})
\]

\[
t_j = \begin{cases} 
1, & \text{if } w_j \gamma + u_j > 0 \\
0, & \text{otherwise}
\end{cases}
\]

where \( y_{0j} \) is the outcome that person \( j \) obtains if person \( j \) selects treatment 0, and \( y_{1j} \) is the outcome that person \( j \) obtains if person \( j \) selects treatment 1. This formulation allows differing coefficients for the control (\( \beta_0 \)) and treatment (\( \beta_1 \)) regimes. The constant intercept for the control group is \( \beta_{00} \). The constant intercept for the treatment group is \( \beta_{11} = \beta_{00} + \delta \), where \( \delta \) is the coefficient for treatment \( t_j \) in the outcome (1). The remaining notation was defined above.

We may allow other coefficients to differ across regimes in the outcome (1) by adding interactions between the treatment \( t_j \) and covariates \( x_j \) to the model. To be concise, we use two coefficient vectors \( \beta_0 \) and \( \beta_1 \) here rather than a single coefficient vector with interactions between the treatment \( t_j \) and covariates \( x_j \). The two formulations are equivalent.

We never observe both \( y_{0j} \) and \( y_{1j} \), only one or the other. We observe

\[
y_j = t_j y_{1j} + (1 - t_j) y_{0j}
\]

The vector of error terms \( (\epsilon_{0j}, \epsilon_{1j}, u_j)' \) comes from a mean zero trivariate normal distribution with covariance matrix

\[
\begin{bmatrix}
\sigma^2 & \theta & \sigma \rho \\
\theta & \sigma^2 & \sigma \rho \\
\sigma \rho & \sigma \rho & 1
\end{bmatrix}
\]
The parameters $\sigma$ and $\rho$ were discussed earlier. The parameter $\theta$ is the covariance between the two potential outcomes. We cannot estimate $\theta$ because we have no observations in which an individual is observed in both potential outcomes. Fortunately, $\theta$ is not required for the calculations that we present.

The ATE is the difference in means of the potential outcomes. The mean of each potential outcome accounts for each individual’s contribution, regardless of whether that individual selects that treatment level.

The conditional means of the potential outcomes $y_{tj}$, $t \in (0, 1)$ for exogenous covariates $z_j$ are

$$E(y_{tj}|z_j) = \exp \left( x_j \beta_t + \frac{\sigma^2}{2} \right)$$

We can see that when the coefficients are the same across the regimes, the ratio of potential-outcome means will be equal to $\exp(\delta)$; this is true of the conditional and marginal potential-outcome means.

The difference in potential-outcome means or treatment effect at exogenous covariates $z_j$ is

$$E(y_{1j} - y_{0j}|z_j) = \{ \exp(x_j \beta_1) - \exp(x_j \beta_0) \} \exp \left( \frac{\sigma^2}{2} \right)$$

By the law of iterated expectations, the ATE is

$$E(y_{1j} - y_{0j}) = E\{E(y_{1j} - y_{0j}|z_j)\}$$

$$= E \left[ \{ \exp(x_j \beta_1) - \exp(x_j \beta_0) \} \exp \left( \frac{\sigma^2}{2} \right) \right]$$

This expectation can be estimated as a predictive margin.

Now we will derive an expression for the ATET.

The conditional means of the potential outcomes $y_{tj}$, $t \in (0, 1)$ for exogenous covariates $z_j$ and treatment $t_j$ are

$$E(y_{tj}|z_j, t_j) = \exp \left( x_j \beta_t + \frac{\sigma^2}{2} \right) \left\{ \frac{\Phi(\rho \sigma + w_j \gamma)}{\Phi(w_j \gamma)} \right\}^{t_j} \left\{ \frac{1 - \Phi(\rho \sigma + w_j \gamma)}{1 - \Phi(w_j \gamma)} \right\}^{1-t_j}$$

Rather than the conditional potential-outcome means, the conditional mean of the observed outcome may be of interest. The conditional mean of the observed outcome $y_j$ for endogenous treatment indicator $t_j$ and exogenous covariates $z_j$ is given by

$$E(y_j|z_j, t_j) = t_j \exp \left( x_j \beta_1 + \frac{\sigma^2}{2} \right) \frac{\Phi(\rho \sigma + w_j \gamma)}{\Phi(w_j \gamma)}$$

$$+ (1 - t_j) \exp \left( x_j \beta_0 + \frac{\sigma^2}{2} \right) \frac{1 - \Phi(\rho \sigma + w_j \gamma)}{1 - \Phi(w_j \gamma)}$$
The treatment effect at exogenous covariates $z_j$ and treatment $t_j$ is

$$E(y_{1j} - y_{0j}|z_j, t_j) =$$

$$\{ \exp(x_j\beta_1) - \exp(x_j\beta_0) \} \exp \left( \frac{\sigma^2}{2} \right) \left\{ \frac{\Phi(\rho\sigma + w_j\gamma)}{\Phi(w_j\gamma)} \right\}^{t_j} \left\{ \frac{1 - \Phi(\rho\sigma + w_j\gamma)}{1 - \Phi(w_j\gamma)} \right\}^{1-t_j}$$

By the law of iterated expectations, the ATET is

$$E(y_{1j} - y_{0j}|t_j = 1) = E\{E(y_{1j} - y_{0j}|z_j, t_j = 1)|t_j = 1\}$$

$$= E \left[ \{ \exp(x_j\beta_1) - \exp(x_j\beta_0) \} \exp \left( \frac{\sigma^2}{2} \right) \frac{\Phi(\rho\sigma + w_j\gamma)}{\Phi(w_j\gamma)} \bigg| t_j = 1 \right]$$

This can be estimated as a predictive margin on the treated subpopulation.

We note that when $\rho = 0$, the correction factor involving $\Phi$ will disappear from the ATET. Then the ATE and ATET will be equivalent if the distribution of $x_j$ under the treated population is identical to the distribution over the entire population.

The probability of $y_j$ conditional on $t_j$ and $z_j$ is

$$Pr(y_j = n|z_j, t_j) = \frac{f(y_j = n, t_j|z_j)}{\Phi(w_j\gamma)^{t_j} \Phi(-w_j\gamma)^{1-t_j}}$$

As discussed earlier, we approximate $f(y_j, t_j|z_j)$ using Gauss–Hermite quadrature.

References


Also see

[C] *etpoisson postestimation* — Postestimation tools for etpoisson

[C] *etregress* — Linear regression with endogenous treatment effects

[R] *heckpoisson* — Poisson regression with sample selection

[R] *ivpoisson* — Poisson model with continuous endogenous covariates

[R] *ivprobit* — Probit model with continuous endogenous covariates

[R] *ivregress* — Single-equation instrumental-variables regression

[R] *ivtobit* — Tobit model with continuous endogenous covariates

[R] *poisson* — Poisson regression

[SVY] *svy estimation* — Estimation commands for survey data

[U] *20 Estimation and postestimation commands*
etpoisson postestimation — Postestimation tools for etpoisson

Postestimation commands

The following standard postestimation commands are available after `etpoisson`:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>contrast</td>
<td>contrasts and ANOVA-style joint tests of estimates</td>
</tr>
<tr>
<td>estat ic</td>
<td>Akaike’s, consistent Akaike’s, corrected Akaike’s, and Schwarz’s Bayesian information criteria (AIC, CAIC, AICC, and BIC)</td>
</tr>
<tr>
<td>estat summarize</td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td>estat vce</td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td>estat (svy)</td>
<td>postestimation statistics for survey data</td>
</tr>
<tr>
<td>estimates</td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td>etable</td>
<td>table of estimation results</td>
</tr>
<tr>
<td>*hausman</td>
<td>Hausman’s specification test</td>
</tr>
<tr>
<td>lincom</td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td>*lrtest</td>
<td>likelihood-ratio test</td>
</tr>
<tr>
<td>margins</td>
<td>marginal means, predictive margins, marginal effects, and average marginal effects</td>
</tr>
<tr>
<td>marginsplot</td>
<td>graph the results from margins (profile plots, interaction plots, etc.)</td>
</tr>
<tr>
<td>nlcom</td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td>predict</td>
<td>potential-outcome means, observed-outcome means, conditional treatment effects, etc.</td>
</tr>
<tr>
<td>predictnl</td>
<td>point estimates, standard errors, testing, and inference for generalized predictions</td>
</tr>
<tr>
<td>pwcompare</td>
<td>pairwise comparisons of estimates</td>
</tr>
<tr>
<td>suest</td>
<td>seemingly unrelated estimation</td>
</tr>
<tr>
<td>test</td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td>testnl</td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>

*`hausman` and `lrtest` are not appropriate with `svy` estimation results.
predict

Description for predict

predict creates a new variable containing predictions such as counts, conditional treatment effects, probabilities, and linear predictions.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] newvar [if] [in] [, statistic nooffset]

predict [type] stub* [if] [in], scores
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pomean</td>
<td>potential-outcome mean (the predicted count); the default</td>
</tr>
<tr>
<td>omean</td>
<td>observed-outcome mean (the predicted count)</td>
</tr>
<tr>
<td>cte</td>
<td>conditional treatment effect at treatment level</td>
</tr>
<tr>
<td>pr(n)</td>
<td>probability ( \Pr(y_j = n) )</td>
</tr>
<tr>
<td>pr(a,b)</td>
<td>probability ( \Pr(a \leq y_j \leq b) )</td>
</tr>
<tr>
<td>xb</td>
<td>linear prediction</td>
</tr>
<tr>
<td>xbtreat</td>
<td>linear prediction for treatment equation</td>
</tr>
</tbody>
</table>

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

```
Main
```

- `pomean`, the default, calculates the potential-outcome mean.
- `omean` calculates the observed-outcome mean.
- `cte` calculates the treatment effect, the difference of potential-outcome means, conditioned on treatment level.
\( \text{pr}(n) \) calculates the probability \( \Pr(y_j = n) \), where \( n \) is a nonnegative integer that may be specified as a number or a variable.

\( \text{pr}(a,b) \) calculates the probability \( \Pr(a \leq y_j \leq b) \), where \( a \) and \( b \) are nonnegative integers that may be specified as numbers or variables;

- \( b \) missing (\( b \geq . \)) means \( +\infty \);
- \( \text{pr}(20,.) \) calculates \( \Pr(y_j \geq 20) \);
- \( \text{pr}(20,b) \) calculates \( \Pr(y_j \geq 20) \) in observations for which \( b \geq . \) and calculates \( \Pr(20 \leq y_j \leq b) \) elsewhere.

\( \text{pr}(.,b) \) produces a syntax error. A missing value in an observation of the variable \( a \) causes a missing value in that observation for \( \text{pr}(a,b) \).

\( \text{xb} \) calculates the linear prediction for the dependent count variable, which is \( x_j \beta \) if neither \( \text{offset}() \) nor \( \text{exposure}() \) was specified; \( x_j \beta + \text{offset}_j \) if \( \text{offset}() \) was specified; or \( x_j \beta + \ln(\text{exposure}_j) \) if \( \text{exposure}() \) was specified.

\( \text{xbtreat} \) calculates the linear prediction for the endogenous treatment equation, which is \( w_j \gamma \) if \( \text{offset}() \) was not specified in \( \text{treat}() \) and \( w_j \gamma + \text{offset}_j \) if \( \text{offset}() \) was specified in \( \text{treat}() \).

\( \text{nooffset} \) is relevant only if you specified \( \text{offset}() \) or \( \text{exposure}() \) when you fit the model. It modifies the calculations made by \( \text{predict} \) so that they ignore the offset or exposure variable. \( \text{nooffset} \) removes the offset from calculations involving both the \( \text{treat}() \) equation and the dependent count variable.

\( \text{scores} \) calculates equation-level score variables.

- The first new variable will contain \( \partial \ln L / \partial (x_j \beta) \).
- The second new variable will contain \( \partial \ln L / \partial (w_j \gamma) \).
- The third new variable will contain \( \partial \ln L / \partial \text{atanh } \rho \).
- The fourth new variable will contain \( \partial \ln L / \partial \ln \sigma \).
margins

Description for margins

margins estimates margins of response for counts, conditional treatment effects, probabilities, and linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

\[ \text{margins} \left[ \text{marginlist} \right] \left[ , \text{options} \right] \]
\[ \text{margins} \left[ \text{marginlist} \right] , \text{predict} ( \text{statistic} \ldots) \left[ \text{predict} ( \text{statistic} \ldots) \ldots \right] \left[ \text{options} \right] \]

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>pomean</td>
<td>potential-outcome mean (the predicted count); the default</td>
</tr>
<tr>
<td>omean</td>
<td>observed-outcome mean (the predicted count)</td>
</tr>
<tr>
<td>cte</td>
<td>conditional treatment effect at treatment level</td>
</tr>
<tr>
<td>pr(n)</td>
<td>probability ( \Pr(y_j = n) )</td>
</tr>
<tr>
<td>pr(a,b)</td>
<td>probability ( \Pr(a \leq y_j \leq b) )</td>
</tr>
<tr>
<td>xb</td>
<td>linear prediction</td>
</tr>
<tr>
<td>xbtreat</td>
<td>linear prediction for treatment equation</td>
</tr>
</tbody>
</table>

Statistics not allowed with margins are functions of stochastic quantities other than \( e(b) \).

For the full syntax, see \([R]\) margins.

Remarks and examples

The average treatment effect (ATE) and the average treatment effect on the treated (ATET) are the parameters most frequently estimated by postestimation techniques after etpoisson.

You can use the margins command (see \([R]\) margins) after etpoisson to estimate the ATE or ATET. See example 2 of \([CAUSAL]\) etpoisson for an example of ATE estimation. See example 3 of \([CAUSAL]\) etpoisson for an example of ATET estimation.

See example 1 of \([CAUSAL]\) etpoisson for an example using lincom after etpoisson.

Methods and formulas

See Methods and formulas of \([CAUSAL]\) etpoisson for details.

Also see

\([CAUSAL]\) etpoisson — Poisson regression with endogenous treatment effects
\([U]\) 20 Estimation and postestimation commands
etregress estimates an average treatment effect (ATE) and the other parameters of a linear regression model augmented with an endogenous binary-treatment variable. Estimation is by full maximum likelihood, a two-step consistent estimator, or a control-function estimator.

In addition to the ATE, etregress can be used to estimate the average treatment effect on the treated (ATET) when the outcome may not be conditionally independent of the treatment.

**Quick start**

ATE and ATET from a linear regression model of $y$ on $x$ and endogenous binary treatment $treat$ modeled by $x$ and $w$

$$\text{etregress } y \ x, \ treat(treat = x \ w)$$

Same as above, but use a control-function estimator

$$\text{etregress } y \ x, \ treat(treat = x \ w) \ cfunction$$

With robust standard errors

$$\text{etregress } y \ x, \ treat(treat = x \ w) \ vce(\text{robust})$$

Add the interaction between $treat$ and continuous covariate $x$ using factor variables

$$\text{etregress } y \ x \ i.treat#c.x, \ treat(treat = x \ w) \ vce(\text{robust})$$

ATE after etregress with the required vce(robust) option and endogenous treatment interaction terms

$$\text{margins r.treat, vce(conditional)}$$

Same as above, but calculate ATET

$$\text{margins, vce(conditional) predict(cte) subpop(if treat==1)}$$

**Menu**

Statistics > Causal inference/treatment effects > Endogenous treatment > Maximum likelihood estimator > Continuous outcomes
Syntax

Basic syntax

```
etregress depvar [ indepvars ], treat(depvar_t = indepvars_t) [ twostep | cfunction]
```

Full syntax for maximum likelihood estimates only

```
etregress depvar [ indepvars ] [ if ] [ in ] [ weight ],
  treat(depvar_t = indepvars_t [, noconstant]) [ etregress_ml_options ]
```

Full syntax for two-step consistent estimates only

```
etregress depvar [ indepvars ] [ if ] [ in ],
  treat(depvar_t = indepvars_t [, noconstant]) twostep [ etregress_ts_options ]
```

Full syntax for control-function estimates only

```
etregress depvar [ indepvars ] [ if ] [ in ],
  treat(depvar_t = indepvars_t [, noconstant]) cfunction [ etregress_cf_options ]
```
### etregress_ml_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td><em><code>treat()</code></em></td>
</tr>
<tr>
<td><code>noconstant</code></td>
</tr>
<tr>
<td><code>poutcomes</code></td>
</tr>
<tr>
<td><code>constraints(constraints)</code></td>
</tr>
<tr>
<td><strong>SE/Robust</strong></td>
</tr>
<tr>
<td><code>vce(vcetype)</code></td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
</tr>
<tr>
<td><code>level(#)</code></td>
</tr>
<tr>
<td><code>first</code></td>
</tr>
<tr>
<td><code>hazard(newvar)</code></td>
</tr>
<tr>
<td><code>lrmodel</code></td>
</tr>
<tr>
<td><code>nocnsreport</code></td>
</tr>
<tr>
<td><code>display_options</code></td>
</tr>
<tr>
<td><strong>Maximization</strong></td>
</tr>
<tr>
<td><code>maximize_options</code></td>
</tr>
<tr>
<td><code>collinear</code></td>
</tr>
<tr>
<td><code>coeflegend</code></td>
</tr>
</tbody>
</table>

*`treat(depvar_t = indepvars_t [, noconstant])` is required.*

### etregress_ts_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td><em><code>treat()</code></em></td>
</tr>
<tr>
<td><em><code>twostep</code></em></td>
</tr>
<tr>
<td><code>noconstant</code></td>
</tr>
<tr>
<td><strong>SE</strong></td>
</tr>
<tr>
<td><code>vce(vcetype)</code></td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
</tr>
<tr>
<td><code>level(#)</code></td>
</tr>
<tr>
<td><code>first</code></td>
</tr>
<tr>
<td><code>hazard(newvar)</code></td>
</tr>
<tr>
<td><code>display_options</code></td>
</tr>
<tr>
<td><code>coeflegend</code></td>
</tr>
</tbody>
</table>

*`treat(depvar_t = indepvars_t [, noconstant])` and `twostep` are required.*
**etregress — Linear regression with endogenous treatment effects**

<table>
<thead>
<tr>
<th>etregress_cf_options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
<td></td>
</tr>
<tr>
<td>*treat()</td>
<td>equation for treatment effects</td>
</tr>
<tr>
<td>*cfunction</td>
<td>produce control-function estimate</td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant term</td>
</tr>
<tr>
<td>poutcomes</td>
<td>use potential-outcome model with separate treatment and control group variance and correlation parameters</td>
</tr>
<tr>
<td><strong>SE</strong></td>
<td></td>
</tr>
<tr>
<td>vce(vcetype)</td>
<td>vcetype may be robust, bootstrap, or jackknife</td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
<td></td>
</tr>
<tr>
<td>level(#)</td>
<td>set confidence level; default is level(95)</td>
</tr>
<tr>
<td>first</td>
<td>report first-step probit estimates</td>
</tr>
<tr>
<td>hazard(newvar)</td>
<td>create newvar containing hazard from treatment equation</td>
</tr>
<tr>
<td>display_options</td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td><strong>Maximization</strong></td>
<td></td>
</tr>
<tr>
<td>maximize_options</td>
<td>control the maximization process; seldom used</td>
</tr>
<tr>
<td>coeflegend</td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>

*treat(depvar = indepvars, noconstant) and cfunction are required.

**indepvars and indepvars_t may contain factor variables; see [U] 11.4.3 Factor variables.**

**depvar, indepvars, depvar_t, and indepvars_t may contain time-series operators; see [U] 11.4.4 Time-series varlists.**

**bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands.**

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

aweights are not allowed with the jackknife prefix; see [R] jackknife.

twostep, cfunction, vce(), first, hazard(), lrmodel, and weights are not allowed with the svy prefix; see [SVY] svy.

**pweights, aweights, fweights, and iweights are allowed with both maximum likelihood and control-function estimation; see [U] 11.1.6 weight. No weights are allowed if twostep is specified.**

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
Model

\( \text{treat} (\text{depvar}_t = \text{indepvars}_t[, \text{noconstant}]) \) specifies the variables and options for the treatment equation. It is an integral part of specifying a treatment-effects model and is required. 

\text{noconstant}; \text{see [R] Estimation options.}

\text{poutcomes} specifies that a potential-outcome model with separate variance and correlation parameters for each of the treatment and control groups be used.

\text{constraints} (\text{constraints}); \text{see [R] Estimation options.}

SE/Robust

\text{vce} (\text{vcetype}) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (\text{oim}, \text{opg}), that are robust to some kinds of misspecification (\text{robust}), that allow for intragroup correlation (\text{cluster clustvar}), and that use bootstrap or jackknife methods (\text{bootstrap}, \text{jackknife}); \text{see [R] vce_option.}

Reporting

\text{level} (#); \text{see [R] Estimation options.}

\text{first} specifies that the first-step probit estimates of the treatment equation be displayed before estimation.

\text{hazard} (\text{newvar}) will create a new variable containing the hazard from the treatment equation. The hazard is computed from the estimated parameters of the treatment equation.

\text{lrmodel}, \text{nocnsreport}; \text{see [R] Estimation options.}

display_options: \text{noci}, \text{nopvalues}, \text{noomitted}, \text{vsquish}, \text{noemptycells}, \text{baselevels}, \text{allbaselevels}, \text{nofvlabel}, \text{fvwrap(#)}, \text{fvwrapon(style)}, \text{cformat(\text{\%fmt})}, \text{pformat(\text{\%fmt})}, \text{sformat(\text{\%fmt})}, \text{and nostretch}; \text{see [R] Estimation options.}

Maximization

\text{maximize_options}: \text{difficult}, \text{technique(algorithm_spec)}, \text{iterate(#)}, [\text{no}] \text{log}, \text{trace}, \text{gradient}, \text{showstep}, \text{hessian}, \text{showtolerance}, \text{tolerance(#)}, \text{ltolerance(#)}, \text{nrtolerance(#)}, \text{nonrtolerance}, \text{and from(init_specs)}; \text{see [R] Maximize. These options are seldom used.}

Setting the optimization type to \text{technique(bhhh)} resets the default \text{vcetype} to \text{vce(opg)}.

The following options are available with \text{etregress} but are not shown in the dialog box: 
\text{collinear}, \text{coeflegend}; \text{see [R] Estimation options.}
Options for two-step consistent estimates

\[ \text{treat}(\text{depvar}_t = \text{indepvars}_t[, \text{noconstant}]) \] specifies the variables and options for the treatment equation. It is an integral part of specifying a treatment-effects model and is required.

twostep specifies that two-step consistent estimates of the parameters, standard errors, and covariance matrix be produced, instead of the default maximum likelihood estimates.

noconstant; see \text{[R] Estimation options}.

\[ \text{vce(vcetype)} \] specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see \text{[R] vce_option}.

\text{vce(conventional)}, the default, uses the conventionally derived variance estimator for the two-step estimator of the treatment-effects model.

\[ \text{level(#)}; \text{see \text{[R] Estimation options}.} \]

first specifies that the first-step probit estimates of the treatment equation be displayed before estimation.

hazard(newvar) will create a new variable containing the hazard from the treatment equation. The hazard is computed from the estimated parameters of the treatment equation.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nolabels, fvwrap(#), fvwrapon(style), cformat(\%/fmt), pformat(\%/fmt), sformat(\%/fmt), and nolstretch; see \text{[R] Estimation options}. The following option is available with \text{etregress} but is not shown in the dialog box:

\text{coeflegend; see \text{[R] Estimation options}.}

Options for control-function estimates

\[ \text{treat}(\text{depvar}_t = \text{indepvars}_t[, \text{noconstant}]) \] specifies the variables and options for the treatment equation. It is an integral part of specifying a treatment-effects model and is required.

cfunction specifies that control-function estimates of the parameters, standard errors, and covariance matrix be produced instead of the default maximum likelihood estimates. cfunction is required.

noconstant; see \text{[R] Estimation options}.

poutcomes specifies that a potential-outcome model with separate variance and correlation parameters for each of the treatment and control groups be used.

\[ \text{vce(vcetype)} \] specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust) and that use bootstrap or jackknife methods (bootstrap, jackknife); see \text{[R] vce_option}.
level(#) \text{; see [R] Estimation options.}

\textit{first} specifies that the first-step probit estimates of the treatment equation be displayed before estimation.

\texttt{hazard(newvar)} will create a new variable containing the hazard from the treatment equation. The hazard is computed from the estimated parameters of the treatment equation.

display_options: \texttt{noci}, \texttt{nopvalues}, \texttt{noomitted}, \texttt{vsquish}, \texttt{noemptycells}, \texttt{baselevels}, \texttt{allbaselevels}, \texttt{nofvlabel}, \texttt{fvwrap(#)}, \texttt{fvwrapon(style)}, \texttt{cformat(\%fmt)}, \texttt{pformat(\%fmt)}, \texttt{sformat(\%fmt)}, and \texttt{nolstretch}; see [R] Estimation options.

maximize_options: \texttt{iterate(#)}, \texttt{[no]log}, and \texttt{from(init_specs)}; see [R] Maximize. These options are seldom used.

\texttt{init_specs} is one of

\begin{itemize}
  \item \texttt{matname [, skip copy]}
  \item # [\# \ldots] copy
\end{itemize}

The following option is available with \texttt{etregress} but is not shown in the dialog box:

\texttt{coeflegend}; see [R] Estimation options.

\section*{Remarks and examples}

Remarks are presented under the following headings:

\begin{itemize}
  \item Overview
  \item Basic examples
  \item Average treatment effect (ATE)
  \item Average treatment effect on the treated (ATET)
\end{itemize}

\subsection*{Overview}

\texttt{etregress} estimates an ATE and the other parameters of a linear regression model that also includes an endogenous binary-treatment variable. In addition to the ATE, the parameters estimated by \texttt{etregress} can be used to estimate the ATET when the outcome is not conditionally independent of the treatment.

We call the model fit by \texttt{etregress} an endogenous treatment-regression model, although it is also known as an endogenous binary-variable model or as an endogenous dummy-variable model. The endogenous treatment-regression model is a specific endogenous treatment-effects model; it uses a linear model for the outcome and a normal distribution to model the deviation from the conditional independence assumption imposed by the estimators implemented in \texttt{teffects}; see [CAUSAL] \texttt{teffects intro}. In treatment-effects jargon, the endogenous binary-variable model is a linear potential-outcome model that allows for a specific correlation structure between the unobservables that affect the treatment and the unobservables that affect the potential outcomes. See [CAUSAL] \texttt{etpoisson} for an estimator that allows for a nonlinear outcome model and a similar model for the endogeneity of the treatment.
Heckman (1976, 1978) brought this model into the modern literature. Maddala (1983) derives the maximum likelihood and the control-function (CF) estimators of the model. Maddala (1983) also reviews some empirical applications and describes it as an endogenous-switching model. Barnow, Cain, and Goldberger (1981) provide another useful derivation of this model. They concentrate on deriving the conditions for which the self-selection bias of the simple OLS estimator of the treatment effect, $\delta$, is nonzero and of a specific sign. Cameron and Trivedi (2005, sec. 16.7 and 25.3.4) and Wooldridge (2010, sec. 21.4.1) discuss the endogenous binary-variable model as an endogenous treatment-effects model and link it to recent work.

`etregress` performs CF estimation in one step by using the generalized method of moments (GMM) with stacked moments. See Newey (1984) and Wooldridge (2010, sec. 14.2) for a description of this technique. Many econometric and statistical models can be expressed as conditions on the population moments. The parameter estimates produced by GMM estimators make the sample-moment conditions as true as possible given the data. See [R] `gmm` for further information on GMM estimation and how Stata performs it. Two-step CF estimation is also supported by `etregress`.

Formally, the endogenous treatment-regression model is composed of an equation for the outcome $y_j$ and an equation for the endogenous treatment $t_j$. The variables $x_j$ are used to model the outcome. When there are no interactions between $t_j$ and $x_j$, we have

$$y_j = x_j \beta + \delta t_j + \epsilon_j$$

$$t_j = \begin{cases} 
1, & \text{if } w_j \gamma + u_j > 0 \\
0 , & \text{otherwise}
\end{cases}$$

where $w_j$ are the covariates used to model treatment assignment, and the error terms $\epsilon_j$ and $u_j$ are bivariate normal with mean zero and covariance matrix

$$\begin{bmatrix}
\sigma^2 & \rho \sigma \\
\rho \sigma & 1
\end{bmatrix}$$

The covariates $x_j$ and $w_j$ are unrelated to the error terms; in other words, they are exogenous. We call this the constrained model because the variance and correlation parameters are identical across the treatment and control groups.

This model can be generalized to a potential-outcome model with separate variance and correlation parameters for the treatment and control groups. The generalized model is

$$y_{0j} = x_j \beta_0 + \epsilon_{0j}$$

$$y_{1j} = x_j \beta_1 + \epsilon_{1j}$$

$$t_j = \begin{cases} 
1, & \text{if } w_j \gamma + u_j > 0 \\
0 , & \text{otherwise}
\end{cases}$$

where $y_{0j}$ is the outcome that person $j$ obtains if person $j$ selects treatment 0, and $y_{1j}$ is the outcome that person $j$ obtains if person $j$ selects treatment 1. We never observe both $y_{0j}$ and $y_{1j}$, only one or the other. We observe

$$y_j = t_j y_{1j} + (1 - t_j) y_{0j}$$
In this unconstrained model, the vector of error terms \((\epsilon_{0j}, \epsilon_{1j}, u_j)’\) comes from a mean zero trivariate normal distribution with covariance matrix

\[
\begin{bmatrix}
\sigma_0^2 & \sigma_{01} & \sigma_{0}\rho_0 \\
\sigma_{01} & \sigma_1^2 & \sigma_{1}\rho_1 \\
\sigma_{0}\rho_0 & \sigma_{1}\rho_1 & 1
\end{bmatrix}
\]

The covariance \(\sigma_{01}\) cannot be identified because we never observe both \(y_{1j}\) and \(y_{0j}\). However, identification of \(\sigma_{01}\) is not necessary to estimate the other parameters because all covariates and the outcome are observed in observations from each group. We normalize the treatment error variance to be 1 because we observe only whether an outcome occurs under treatment. More details are found in *Methods and formulas*.

Rather than showing two separate regression equations, *etregress* reports one outcome equation with interaction terms between the treatment and outcome covariates. *etregress* can fit the constrained and generalized potential-outcome models using either the default maximum likelihood estimator or the one-step CF estimator obtained with option cfunction. The two-step CF estimator provides consistent estimates for the constrained model.

### Basic examples

When there are no interactions between the treatment variable and the outcome covariates in the constrained model, *etregress* directly estimates the ATE and the ATET.

#### Example 1: Basic example

We estimate the ATE of being a union member on wages of women in 1972 from a nonrepresentative extract of the National Longitudinal Survey on young women who were ages 14–26 in 1968. We will use the variables *wage* (wage), *grade* (years of schooling completed), *smsa* (an indicator for living in an SMSA—standard metropolitan statistical area), *black* (an indicator for being African-American), *tenure* (tenure at current job), and *south* (an indicator for living in the South).
We use `etregress` to estimate the parameters of the endogenous treatment-regression model.

```
. use https://www.stata-press.com/data/r18/union3
(NLS Women 14-24 in 1968)
. etregress wage age grade smsa black tenure, treat(union = south black tenure)
```

| Coefficient | Std. err. | z   | P>|z| | [95% conf. interval] |
|-------------|-----------|-----|-----|----------------------|
| wage        |           |     |     |                      |
| age         | .1487409  | .0193291 | 7.70 | 0.000 |
| grade       | .4205658  | .0293577 | 14.33 | 0.000 |
| smsa        | .9117044  | .1249041 | 7.30  | 0.000 |
| black       | -.7882471 | .0740521 | -10.76 | 0.000 |
| tenure      | .1524015  | .0369596 | 4.12  | 0.000 |
| 1.union     | 2.945815  | .2749621 | 10.76 | 0.000 |
| _cons       | -4.351572 | .5283952 | -8.24 | 0.000 |
| union       |           |     |     |                      |
| south       | -.5807419 | .0851111 | -6.82 | 0.000 |
| black       | .4557499  | .0958042 | 4.76  | 0.000 |
| tenure      | .0871536  | .0232483 | 3.75  | 0.000 |
| _cons       | -.8855758 | .0724506 | -12.22 | 0.000 |

```
/athrho     -.6544347  .0910314  -7.19  0.000  -.832853  -.4760164
/lnsigma    .7026769  .0293372  23.95  0.000   .645177   .7601767
```

\[
\text{LR test of indep. eqns. (rho = 0): chi2(1) = 19.84 Prob > chi2 = 0.0000}
\]

The likelihood-ratio test in the footer indicates that we can reject the null hypothesis of no correlation between the treatment-assignment errors and the outcome errors. The estimated correlation between the treatment-assignment errors and the outcome errors, \( \rho \), is \(-0.575\). The negative relationship indicates that unobservables that raise observed wages tend to occur with unobservables that lower union membership. We discuss some details about this parameter in the technical note below.

The estimated ATE of being a union member is 2.95. The ATET is the same as the ATE in this case because the treatment indicator variable has not been interacted with any of the outcome covariates, and the correlation and variance parameters are identical across the control and treatment groups.
Technical note

The results for the ancillary parameters $\rho$ and $\sigma$ require explanation. For numerical stability during optimization, `etregress` does not directly estimate $\rho$ or $\sigma$. Instead, `etregress` estimates the inverse hyperbolic tangent of $\rho$,

$$\text{atanh}\,\rho = \frac{1}{2}\ln\left(\frac{1 + \rho}{1 - \rho}\right)$$

and $\ln\sigma$. Also `etregress` reports $\lambda = \rho\sigma$, along with an estimate of the standard error of the estimate and the confidence interval.

In contrast to the constrained model, `etregress` directly estimates the ATE only when there are no interactions between the treatment variable and the outcome covariates in the unconstrained model.

Example 2: Allowing group-specific variance and correlation

We estimate the ATE of having health insurance on the natural logarithm of total out-of-pocket prescription drug expenditures from a simulated random sample of individuals between the ages of 25 and 64. We will use the variables `lndrug` (natural logarithm of spending on prescription drugs), `age` (age of the individual), `chron` (whether the individual has a chronic condition), `lninc` (natural logarithm of income), `married` (marriage status), and `work` (employment status). Our treatment is whether the person has health insurance, `ins`. We allow the outcome error variance and correlation parameters to vary between the treated (insured) and control (uninsured) groups in this example, rather than constraining them to be equal as in example 1.

We use `etregress` to estimate the parameters of the endogenous treatment-effects model. To estimate separate variance and correlation parameters for each of the control and treatment groups, we specify the `poutcomes` option. We specify the `cfunction` option to use the CF estimator.
. use https://www.stata-press.com/data/r18/drugexp
(Prescription drug expenditures)
. etregress lndrug chron age lninc, treat(ins=age married lninc work) poutcomes
> cfunction
Iteration 0: GMM criterion Q(b) = 2.279e-15
Iteration 1: GMM criterion Q(b) = 6.358e-30
Linear regression with endogenous treatment
Number of obs = 6,000
Estimator: Control function

| Coefficient | std. err. | z   | P>|z| | [95% conf. interval] |
|-------------|-----------|-----|-----|----------------------|
| lndrug      | .4671725  | .0319731 | 14.61 | 0.000 | .4045064 .5298387 |
| chron       | .1021359  | .00292 | 34.98 | 0.000 | .0964128 .1078589 |
| age         | .0550672  | .0225036 | 2.45 | 0.014 | .0109609 .0991735 |
| lninc       | -.08598836 | .3483648 | -2.47 | 0.014 | -1.542666 -.1771011 |
| ins         | 1.665539  | .2527527 | 6.59 | 0.000 | 1.170153 2.160925 |
| age         | .021142   | .0022961 | 9.21 | 0.000 | .0166416 .0256424 |
| married     | .084631   | .0359713 | 2.35 | 0.019 | .0414286 .1551334 |
| lninc       | .1023032  | .0225009 | 4.55 | 0.000 | .0582022 .1464041 |
| work        | .288418   | .0372281 | 7.75 | 0.000 | .2154522 .3613837 |
| _cons       | -0.622993 | .087956 | -5.73 | 0.000 | -.8362273 -.4097587 |
| /athrho0    | .4035094  | .1724539 | 2.34 | 0.019 | .0655059 .7415129 |
| /lnsigma0   | .3159269  | .0500476 | 6.31 | 0.000 | .2178353 .4140184 |
| /athrho1    | .7929459  | .2986601 | 2.66 | 0.008 | .2075829 .1378309 |
| /lnsigma1   | .1865347  | .0613124 | 3.04 | 0.002 | .0663646 .3067048 |
| rho0        | .3829477  | .1471637 | 2.64 | 0.009 | .0654124 .630583 |
| sigma0      | 1.37153   | .0686418 | 1.243382 | 1.512885 |
| lambda0     | .5252243  | .226367  | 0.0815532 | 0.9688954 |
| rho1        | .6600746  | .1685343 | 0.2046518 | 0.880572 |
| sigma1      | 1.205066  | .0738855 | 1.068616 | 1.35894 |
| lambda1     | .7954338  | .2513036 | 3.028878 | 1.28798 |

Wald test of indep. (rho0 = rho1 = 0): chi2(2) = 8.88 Prob > chi2 = 0.0118

The Wald test reported in the footer indicates that we can reject the null hypothesis of no correlation between the treatment-assignment errors and the outcome errors for the control and treatment groups. The estimate of the correlation of the treatment-assignment errors for the control group (ρ₀) is positive, indicating that unobservables that increase spending on prescription drugs tend to occur with unobservables that increase health insurance coverage. Because ρ₁ is also positive, we make the same interpretation for individuals with insurance. The estimate ρ₁ is larger than the estimate ρ₀, indicating a stronger relationship between the unobservables and treatment outcomes in the treated group.

The estimated ATE of having health insurance is −0.86. Note that while the ATE and ATET were the same in example 1, that is not the case here. We show how to calculate the ATET for a potential-outcome model in example 6.

The estimate of the outcome error standard-deviation parameter for the control group (σ₀) is slightly larger than that of the treatment-group parameter (σ₁), indicating a greater variability in the unobservables among the untreated group.
Average treatment effect (ATE)

When there is a treatment variable and outcome covariate interaction, the parameter estimates from **etregress** can be used by **margins** to estimate the ATE, the average difference of the treatment potential outcomes and the control potential outcomes.

- **Example 3: Allowing interactions between treatment and outcome covariates, ATE**

In **example 1**, the coefficients on the outcome covariates do not vary by treatment level. The differences in wages between union members and nonmembers are modeled as a level shift captured by the coefficient on the indicator for union membership. In this example, we use factor-variable notation to allow some of the coefficients to vary over treatment level and then use **margins** (see [R] **margins**) to estimate the ATE. (See [U] 11.4.3 Factor variables for an introduction to factor-variable notation.)

We begin by estimating the parameters of the model in which the coefficients on **black** and **tenure** differ for union members and nonmembers. We specify the **vce(robust)** option because we need to specify **vce(unconditional)** when we use **margins** below.
. use https://www.stata-press.com/data/r18/union3
  (NLS Women 14-24 in 1968)
. etregress wage age grade smsa i.union#c.(black tenure),
> treat(union = south black tenure) vce(robust)

Iteration 0:  Log pseudolikelihood =  -3614.6714
Iteration 1:  Log pseudolikelihood =  -3218.8152
Iteration 2:  Log pseudolikelihood =  -3057.0115
Iteration 3:  Log pseudolikelihood =  -3049.3081
Iteration 4:  Log pseudolikelihood =  -3049.2838
Iteration 5:  Log pseudolikelihood =  -3049.2838

Linear regression with endogenous treatment
Number of obs =  1,210
Estimator: Maximum likelihood
Wald chi2(8)  =    493.40
Log pseudolikelihood =  -3049.2838  Prob > chi2 =    0.0000

|                  | Robust Coefficient | std. err. | z     | P>|z|     | [95% conf. interval] |
|------------------|--------------------|-----------|-------|---------|----------------------|
| wage             |                    |           |       |         |                      |
| age              | .1489075           | .0207283  | 7.18  | 0.000   | .1082809 .1895342    |
| grade            | .4200493           | .0377621  | 11.12 | 0.000   | .3460371 .4940616    |
| smsa             | .9232615           | .1201486  | 7.68  | 0.000   | .6877746 1.158748    |
| union#           |                    |           |       |         |                      |
| c.black          |                    |           |       |         |                      |
| 0                | -.6685582          | .1444213  | -4.63 | 0.000   | -.9516187 -.3854977  |
| 1                | -1.1831            | .2574817  | -4.59 | 0.000   | -1.687755 -.6784455  |
| union#           |                    |           |       |         |                      |
| c.tenure         |                    |           |       |         |                      |
| 0                | .168746            | .0503107  | 3.35  | 0.001   | .0701388 .2673532    |
| 1                | .0836367           | .0903669  | 0.93  | 0.355   | -.0934792 .2607526   |
| 1.union          | 3.342859           | .5586863  | 5.98  | 0.000   | 2.247854 4.437864    |
| _cons            | -4.42566           | .6493003  | -6.82 | 0.000   | -5.698265 -3.153055  |
| union            |                    |           |       |         |                      |
| south            | -.5844678          | .0833069  | -7.02 | 0.000   | -.7477464 -.4211893  |
| black            | .4740688           | .093241   | 5.08  | 0.000   | .2913197 .658178     |
| tenure           | .0874297           | .0258392  | 3.44  | 0.001   | .0376678 .1371916    |
| _cons            | -.8910484          | .0746329  | -11.94| 0.000   | -1.037326 -.7447706  |
| /athrho          | -.6733149          | .2215328  | -3.04 | 0.002   | -1.107511 -.2391185  |
| /lnsigma         | .7055907           | .0749711  | 9.41  | 0.000   | .55865 .8525313      |
| rho              | -.5871562          | .1451589  | -4.07 | 0.000   | -.8031809 -.234663   |
| sigma            | 2.025042           | .1518197  | 1.34  | 0.090   | 1.748311 2.345577    |
| lambda           | -1.189016          | .3631079  | -1.90 | 0.056   | -.900695 -.4773378   |

Wald test of indep. eqns. (rho = 0): chi2(1)  =   9.24  Prob > chi2 = 0.0024

The results indicate that the coefficients on black differ by union membership and that the coefficient on tenure for nonmembers is positive, while the coefficient on tenure for members is 0. The model fits well overall, so we proceed with interpretation. Because we interacted the treatment variable with two of the covariates, the estimated coefficient on the treatment level is not an estimate of the ATE. Below we use margins to estimate the ATE from these results. We specify the vce(unconditional) option to obtain the standard errors for the population ATE instead of the sample ATE. We specify the contrast(nowald) option to suppress the Wald tests, which margins displays by default for contrasts.
. margins r.union, vce(unconditional) contrast(nowald)

Contrasts of predictive margins

Expression: Linear prediction, predict()

<table>
<thead>
<tr>
<th></th>
<th>Unconditional</th>
<th>std. err.</th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>union</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1 vs 0)</td>
<td>3.042688</td>
<td>.5305151</td>
<td>2.002898 4.082478</td>
</tr>
</tbody>
</table>

The ATE estimate is essentially the same as the one produced by the constrained model in example 1.

We can use the same methods above to obtain the ATE in an unconstrained model.

Example 4: Treatment interactions and group-specific variance and correlation, ATE

In example 2, the coefficients on the outcome covariates do not vary by treatment level. Suppose we believe that the effect of having a chronic condition on out-of-pocket spending differs between the insured and uninsured. Again, we use an interaction term. Because we are using a CF estimator, the variance–covariance of the estimator (VCE) is already robust so we do not specify vce(robust).
. use https://www.stata-press.com/data/r18/drugexp
(Prescription drug expenditures)
. etregress lndrug i.ins#i.chron age lninc, treat(ins=age married lninc work) > poutcomes cfunction
Iteration 0:  GMM criterion Q(b) =  2.279e-15
Iteration 1:  GMM criterion Q(b) =  1.561e-28
Linear regression with endogenous treatment Number of obs = 6,000
Estimator: Control function

| Coefficient std. err. | z     | P>|z|   | [95% conf. interval] |
|-----------------------|-------|------|----------------------|
| lndrug                |       |      |                      |
| ins#chron             |       |      |                      |
| 0 1                   | .3798705 .0720713  5.27  0.000  .2386132 .5211277 |
| 1 1                   | .4957773 .0352571 14.06  0.000  .4266746 .5648801 |
| age                   | .1022045 .0029228 34.97  0.000  .0964758 .1079331 |
| lninc                 | .0548917 .0225219  2.44  0.015  .0107497 .0990337 |
| 1.ins                 | -.89703 .3493058 -2.57  0.010 -1.581657 -.2124031 |
| _cons                 | 1.691336 .2531222  6.68  0.000  1.195225 2.187446 |
| ins                   |       |      |                      |
| age                   | .021142 .0022961  9.21  0.000  .0166416 .0256424 |
| married               | .084631 .0369713  2.35  0.019  .0414286 .1511334 |
| lninc                 | .1023032 .0225009  4.55  0.000  .0582208 .1464041 |
| work                  | .288418 .0342281  7.75  0.000  .2154522 .3613837 |
| _cons                 | -.622993 .10795  -5.73  0.000  -.8362273 -.4097587 |
| /athrho0              | .4046007 .1725597  2.34  0.019  .0663899 .7428115 |
| /lnsigma0             | .3157598 .0205195  6.29  0.000  .0912561 .6402634 |
| /athrho1              | .7950592 .2992825  2.66  0.008  .2084763 1.381642 |
| /lnsigma1             | .1868903 .0614281  3.04  0.002  .0664934 .3072871 |
| rho0                  | .3838786 .1471308  2.64  0.009  .0662925 .6038408 |
| sigma0                | 1.371296 .0688329  2.00  0.044  1.012481 1.730112 |
| lambda0               | .5264111 .2264197  2.34  0.019  .0826366 .9701856 |
| rho1                  | .6612655 .1684146  3.93  0.000  .3305076 .9919731 |
| sigma1                | 1.205495 .0740512  1.66  0.097  1.068754 1.342237 |
| lambda1               | .7971523 .2514293  3.15  0.002  .3043599 1.289945 |

Wald test of indep. (rho0 = rho1 = 0): chi2(2) = 8.90  Prob > chi2 = 0.0117

The results indicate that the coefficient on chron differs by whether an individual has insurance. The model fits well overall, so we proceed with interpretation.

Because we interacted the treatment variable with one of the covariates, the estimated coefficient on the treatment level is not an estimate of the ATE. Below we use margins to estimate the ATE from these results. We specify the vce(unconditional) option to obtain the standard errors for the population ATE instead of the sample ATE. We specify the contrast(nowald) option to suppress the Wald tests.
Contrasts of predictive margins

<table>
<thead>
<tr>
<th>Contrast</th>
<th>std. err.</th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ins (1 vs 0)</td>
<td>-0.8632045</td>
<td>.3484924</td>
</tr>
</tbody>
</table>

The ATE estimate is similar to the one produced by the constrained model in example 2.

**Average treatment effect on the treated (ATET)**

When there is a treatment variable and outcome covariate interaction, the parameter estimates from `etregress` can be used by `margins` to estimate the ATET, the average difference of the treatment potential outcomes and the control potential outcomes on the treated population.

**Example 5: Allowing interactions between treatment and outcome covariates, ATET**

The ATET may differ from the ATE in example 3 because the interaction between the treatment variable and some outcome covariates makes the ATE and the ATET vary over outcome covariate values. Below we use `margins` to estimate the ATET by specifying the `subpop(union)` option, which restricts the sample used by `margins` to union members.

```stata
.margins r.union, vce(unconditional) contrast(nowald) subpop(union)
```

<table>
<thead>
<tr>
<th>Union</th>
<th>std. err.</th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1 vs 0)</td>
<td>2.968977</td>
<td>.5358457</td>
</tr>
</tbody>
</table>

The estimated ATET and ATE are close, indicating that the average predicted outcome for the treatment group is similar to the average predicted outcome for the whole population.
Example 6: Treatment interactions and group-specific variance and correlation, ATET

The ATET may differ from the ATE in example 4 because the interaction between the treatment variable and some outcome covariates makes the ATE and the ATET vary over values of the covariate in the outcome equation. Even if there is no interaction between treatment assignment and a covariate in the outcome equation, the estimated ATE and ATET will differ if the variances of the outcome errors and their correlations with the treatment-assignment errors differ across the control and treatment groups.

We can estimate the ATET of having health insurance by using the conditional treatment effect (conditional on exogenous covariates and treatment level) obtained using the predict, cte and the margins commands; see Methods and formulas below and [CAUSAL] etregress postestimation for more details about the use of predict after etregress.

We restrict estimation to the treated subpopulation by specifying the subpop(ins) option with margins.

```
. use https://www.stata-press.com/data/r18/drugexp
   (Prescription drug expenditures)
. etregress lndrug i.ins#i.chron age lninc, treat(ins = age married lninc work) poutcomes cfunction
   (output omitted)
. margins, predict(cte) subpop(ins) vce(unconditional)
Predictive margins
Expression: Conditional treatment effect, predict(cte)

| Unconditional Margin | std. err. | z    | P>|z|  | 95% conf. interval |
|----------------------|-----------|------|------|------------------|
| _cons                | -.7558373 | .3827579 | -1.97 | 0.048          | -1.506029 - .0056457 |
```

In absolute value, the treatment effect on the treated of −0.76 is smaller than the population average effect of −0.86 that we found in example 4.
**Stored results**

`etregress` (maximum likelihood) stores the following in `e()`:

**Scalars**
- `e(N)` number of observations
- `e(k)` number of parameters
- `e(k_eq)` number of equations in `e(b)`
- `e(k_eq_model)` number of equations in overall model test
- `e(k_aux)` number of auxiliary parameters
- `e(k_dv)` number of dependent variables
- `e(df_m)` model degrees of freedom
- `e(ll)` log likelihood
- `e(ll_0)` log likelihood, constant-only model (lrmodel only)
- `e(N_clust)` number of clusters
- `e(lambda)` estimate of $\lambda$ in constrained model
- `e(selambda)` standard error of $\lambda$ in constrained model
- `e(sigma)` estimate of $\sigma$ in constrained model
- `e(sigma0)` estimate of $\sigma_0$ in potential-outcome model
- `e(lambda0)` estimate of $\lambda_0$ in potential-outcome model
- `e(selambda0)` standard error of $\lambda_0$ in potential-outcome model
- `e(sigma0)` estimate of $\sigma_0$ in potential-outcome model
- `e(lambda1)` estimate of $\lambda_1$ in potential-outcome model
- `e(selambda1)` standard error of $\lambda_1$ in potential-outcome model
- `e(sigma1)` estimate of $\sigma_1$ in potential-outcome model
- `e(chi2)` $\chi^2$
- `e(chi2_c)` $\chi^2$ for comparison test
- `e(p)` $p$-value for model test
- `e(p_c)` $p$-value for comparison test
- `e(rho)` estimate of $\rho$ in constrained model
- `e(rho0)` estimate of $\rho_0$ in potential-outcome model
- `e(rho1)` estimate of $\rho_1$ in potential-outcome model
- `e(rank)` rank of `e(V)`
- `e(rank0)` rank of `e(V)` for constant-only model
- `e(ic)` number of iterations
- `e(rc)` return code
- `e(converged)` 1 if converged, 0 otherwise

**Macros**
- `e(cmd)` `etregress`
- `e(cmdline)` command as typed
- `e(depvar)` name of dependent variable
- `e(hazard)` variable containing hazard
- `e(wtype)` weight type
- `e(wexp)` weight expression
- `e(title)` title in estimation output
- `e(title2)` secondary title in estimation output
- `e(clustvar)` name of cluster variable
- `e(chi2type)` Wald or LR: type of model $\chi^2$ test
- `e(chi2_ct)` Wald or LR: type of model $\chi^2$ test corresponding to `e(chi2_c)`
- `e(vce)` `vctype` specified in `vce()`
- `e(vcetype)` title used to label Std. err.
- `e(opt)` type of optimization
- `e(which)` `max` or `min`: whether optimizer is to perform maximization or minimization
- `e(method)` `ml`
- `e(ml_method)` type of `ml` method
- `e(user)` name of likelihood-evaluator program
- `e(technique)` maximization technique
- `e(properties)` `b V`
- `e(predict)` program used to implement `predict`
- `e(footnote)` program used to implement the footnote display
- `e(marginsok)` predictions allowed by `margins`
- `e(asbalanced)` factor variables `fvset` as `asbalanced`
- `e(asobserved)` factor variables `fvset` as `asobserved`
etregress — Linear regression with endogenous treatment effects

Matrices
- `e(b)` coefficient vector
- `e(Cns)` constraints matrix
- `e(ilog)` iteration log (up to 20 iterations)
- `e(gradient)` gradient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(V_modelbased)` model-based variance

Functions
- `e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices
- `r(table)` matrix containing the coefficients with their standard errors, test statistics, \( p \)-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.

etregress (two-step) stores the following in `e()`:

Scalars
- `e(N)` number of observations
- `e(df_m)` model degrees of freedom
- `e(lambda)` \( \lambda \)
- `e(selambda)` standard error of \( \lambda \)
- `e(sigma)` estimate of sigma
- `e(chi2)` \( \chi^2 \)
- `e(p)` \( p \)-value for model test
- `e(rho)` \( \rho \)
- `e(rank)` rank of `e(V)`

Macros
- `e(cmd)` etregress
- `e(cmdline)` command as typed
- `e(depvar)` name of dependent variable
- `e(hazard)` variable containing hazard
- `e(title)` title in estimation output
- `e(title2)` secondary title in estimation output
- `e(chi2type)` Wald or LR; type of model \( \chi^2 \) test
- `e(vce)` `vce` type specified in `vce()`
- `e(method)` twostep
- `e(properties)` `b V`
- `e(predict)` program used to implement `predict`
- `e(footnote)` program used to implement the footnote display
- `e(marginsok)` predictions allowed by `margins`
- `e(marginsnotok)` predictions disallowed by `margins`
- `e(asbalanced)` factor variables `fvset` as `asbalanced`
- `e(asobserved)` factor variables `fvset` as `asobserved`

Matrices
- `e(b)` coefficient vector
- `e(V)` variance–covariance matrix of the estimators

Functions
- `e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices
- `r(table)` matrix containing the coefficients with their standard errors, test statistics, \( p \)-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r`-class command is run after the estimation command.
etregress (control-function) stores the following in e():

Scalars

- `e(N)`  
  number of observations
- `e(k)`  
  number of parameters
- `e(k_eq)`  
  number of equations in `e(b)`
- `e(k_aux)`  
  number of auxiliary parameters
- `e(k_dv)`  
  number of dependent variables
- `e(lambda)`  
  estimate of $\lambda$ in constrained model
- `e(lambda0)`  
  estimate of $\lambda_0$ in potential-outcome model
- `e(lambda1)`  
  estimate of $\lambda_1$ in potential-outcome model
- `e(sigma)`  
  estimate of $\sigma$ in constrained model
- `e(sigma0)`  
  estimate of $\sigma_0$ in potential-outcome model
- `e(sigma1)`  
  estimate of $\sigma_1$ in potential-outcome model
- `e(chi2_c)`  
  $\chi^2$ for comparison test
- `e(p_c)`  
  $p$-value for comparison test
- `e(rho)`  
  estimate of $\rho$ in constrained model
- `e(rho0)`  
  estimate of $\rho_0$ in potential-outcome model
- `e(rho1)`  
  estimate of $\rho_1$ in potential-outcome model
- `e(rank)`  
  rank of `e(V)`
- `e(converged)`  
  1 if converged, 0 otherwise

Macros

- `e(cmd)`  
  `etregress`
- `e(cmdline)`  
  command as typed
- `e(depvar)`  
  name of dependent variable
- `e(hazard)`  
  variable containing hazard
- `e(wtype)`  
  weight type
- `e(wexp)`  
  weight expression
- `e(title)`  
  title in estimation output
- `e(title2)`  
  secondary title in estimation output
- `e(chi2_ct)`  
  Wald; type of model $\chi^2$ test corresponding to `e(chi2_c)`
- `e(vcetype)`  
  vcetype specified in `vce()`
- `e(vcetitle)`  
  title used to label Std. err.
- `e(method)`  
  cfunction
- `e(properties)`  
  `b V`
- `e(predict)`  
  program used to implement `predict`
- `e(footnote)`  
  program used to implement the footnote display
- `e(marginsok)`  
  predictions allowed by `margins`
- `e(asbalanced)`  
  factor variables `fvset` as `asbalanced`
- `e(asobserved)`  
  factor variables `fvset` as `asobserved`

Matrices

- `e(b)`  
  coefficient vector
- `e(V)`  
  variance–covariance matrix of the estimators

Functions

- `e(sample)`  
  marks estimation sample

In addition to the above, the following is stored in r():

Matrices

- `r(table)`  
  matrix containing the coefficients with their standard errors, test statistics, $p$-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.
Methods and formulas

Maddala (1983, 117–122 and 223–228) derives both the maximum likelihood and the CF estimators implemented here. Greene (2012, 890–894) also provides an introduction to the treatment-effects model. Cameron and Trivedi (2005, sections 16.7 and 25.3.4) and Wooldridge (2010, section 21.4.1) discuss the endogenous binary-variable model as an endogenous treatment-effects model and link it to recent work.

Methods and formulas are presented under the following headings:

Constrained model
General potential-outcome model
Average treatment effect
Average treatment effect on the treated

Constrained model

The primary regression equation of interest is

\[ y_j = x_j \beta + \delta t_j + \epsilon_j \]  

(1)

where \( t_j \) is a binary-treatment variable that is assumed to stem from an unobservable latent variable:

\[ t_j^* = w_j \gamma + u_j \]

The decision to obtain the treatment is made according to the rule

\[ t_j = \begin{cases} 1, & \text{if } t_j^* > 0 \\ 0, & \text{otherwise} \end{cases} \]

where \( \epsilon \) and \( u \) are bivariate normal with mean zero and covariance matrix

\[
\begin{bmatrix}
\sigma^2 & \rho \sigma \\
\rho \sigma & 1
\end{bmatrix}
\]

Interactions between \( x_j \) and the treatment \( t_j \) are also allowed in (1). The likelihood function for this model is given in Maddala (1983, 122). Greene (2000, 180) discusses the standard method of reducing a bivariate normal to a function of a univariate normal and the correlation \( \rho \). The following is the log likelihood for observation \( j \),

\[
\ln L_j = \begin{cases} 
\ln \Phi \left( \frac{w_j \gamma + (y_j - x_j \beta - \delta) \rho / \sigma}{\sqrt{1 - \rho^2}} \right) - \frac{1}{2} \left( \frac{y_j - x_j \beta - \delta}{\sigma} \right)^2 - \ln(\sqrt{2\pi}\sigma) & t_j = 1 \\
\ln \Phi \left( -\frac{w_j \gamma - (y_j - x_j \beta) \rho / \sigma}{\sqrt{1 - \rho^2}} \right) - \frac{1}{2} \left( \frac{y_j - x_j \beta}{\sigma} \right)^2 - \ln(\sqrt{2\pi}\sigma) & t_j = 0
\end{cases}
\]

where \( \Phi(\cdot) \) is the cumulative distribution function of the standard normal distribution.

In the maximum likelihood estimation, \( \sigma \) and \( \rho \) are not directly estimated. Rather \( \ln \sigma \) and \( \text{atanh } \rho \) are directly estimated, where

\[
\text{atanh } \rho = \frac{1}{2} \ln \left( \frac{1 + \rho}{1 - \rho} \right)
\]
The standard error of $\lambda = \rho \sigma$ is approximated through the delta method, which is given by

$$\text{Var}(\lambda) \approx D \text{Var}\{(\text{atanh} \; \rho \; \ln \sigma)\} \; D'$$

where $D$ is the Jacobian of $\lambda$ with respect to atanh $\rho$ and ln $\sigma$.

Maddala (1983, 120–122) also derives the CF estimator as a two-step estimator. This estimator is implemented here. We will discuss it and then discuss the one-step CF estimator that is also implemented.

For the two-step estimator, probit estimates of the treatment equation

$$\Pr(t_j = 1 \mid w_j) = \Phi(w_j \gamma)$$

are obtained in the first stage. From these estimates, the hazard, $h_j$, for each observation $j$ is computed as

$$h_j = \begin{cases} \phi(w_j \hat{\gamma})/\Phi(w_j \hat{\gamma}) & t_j = 1 \\ -\phi(w_j \hat{\gamma})/(1 - \Phi(w_j \hat{\gamma})) & t_j = 0 \end{cases}$$

where $\phi$ is the standard normal density function. If

$$d_j = h_j (h_j + w_j \hat{\gamma})$$

then

$$E(y_j \mid t_j, x_j, w_j) = x_j \beta + \delta t_j + \rho \sigma h_j$$

$$\text{Var}(y_j \mid t_j, x_j, w_j) = \sigma^2 (1 - \rho^2 d_j)$$

The two-step parameter estimates of $\beta$ and $\delta$ are obtained by augmenting the regression equation with the hazard $h$. Thus the regressors become $[x \; t \; h]$, and the additional parameter estimate $\beta_h$ is obtained on the variable containing the hazard. A consistent estimate of the regression disturbance variance is obtained using the residuals from the augmented regression and the parameter estimate on the hazard

$$\hat{\sigma}^2 = e' e + \beta_h^2 \sum_{j=1}^N d_j$$

The two-step estimate of $\rho$ is then

$$\hat{\rho} = \frac{\beta_h}{\hat{\sigma}}$$

To understand how the consistent estimates of the coefficient covariance matrix based on the augmented regression are derived, let $A = [x \; t \; h]$ and $D$ be a square diagonal matrix of size $N$ with $(1 - \rho^2 d_j)$ on the diagonal elements. The conventional VCE is

$$V_{\text{twostep}} = \hat{\sigma}^2 (A'A)^{-1}(A'DA + Q)(A'A)^{-1}$$

where

$$Q = \hat{\rho}^2 (A'DA)V_p(A'DA)$$

and $V_p$ is the variance–covariance estimate from the probit estimation of the treatment equation.

The one-step CF estimator is a GMM estimator with stacked moments. See Newey (1984) and Wooldridge (2010, sec. 14.2) for a description of this technique. Many econometric and statistical models can be expressed as conditions on the population moments. The parameter estimates produced by GMM estimators make the sample-moment conditions as true as possible given the data.
Under CF estimation, as in maximum likelihood estimation, we directly estimate \( \text{atanh} \rho \) and \( \ln \sigma \) rather than \( \rho \) and \( \sigma \), so the parameter vector is

\[
\theta = (\beta', \delta, \gamma', \text{atanh} \rho, \ln \sigma)' 
\]

In this case, we have separate error functions for the treatment assignment

\[
u_t(t_j, w_j, \theta) = \begin{cases} 
\phi(w_j \gamma) / \Phi(w_j \gamma) & t_j = 1 \\
-\phi(w_j \gamma) / \{1 - \Phi(w_j \gamma)\} & t_j = 0
\end{cases}
\]

for the outcome mean

\[
u_m(y_j, t_j, x_j, w_j, \theta) = y_i - x_j \beta - \delta t_j - \rho \sigma u_{t,j}
\]

and for the outcome variance

\[
u_v(y_j, t_j, x_j, w_j, \theta) = u_{m,j}^2 - \sigma^2 \left[ 1 - \rho^2 \{ u_{t,j} (u_{t,j} + w_j \gamma) \} \right]
\]

We calculate the hazard, \( h_j \), prior to estimation from a probit regression of the treatment \( t_j \) on the treatment covariates \( w_j \). Let \( \tilde{z}_j = (x_j, t_j, h_j) \). Now we define

\[
Z_j = \begin{bmatrix} \tilde{z}_j & 0 & 0 \\
0 & w_j & 0 \\
0 & 0 & 1 \end{bmatrix}
\]

and

\[
s_j(y_j, t_j, x_j, w_j, \theta) = Z_j' \begin{bmatrix} u_{m,j} \\
u_{t,j} \\
u_{v,j} \end{bmatrix}
\]

The CF estimator \( \hat{\theta} \) is the value of \( \theta \) that satisfies the sample-moment conditions

\[
0 = \frac{1}{N} \sum_i s_j(y_j, t_j, x_j, w_j, \theta)
\]

The Huber/White/robust sandwich estimator is consistent for the VCE. See Wooldridge (2010, chap. 14), Cameron and Trivedi (2005, chap. 6), and Newey and McFadden (1994).

The formula is

\[
\hat{V} = (1/N) \bar{G} \bar{S} \bar{G}'
\]

where

\[
\bar{G} = \left\{ (1/N) \sum_i \frac{\partial s_j(y_j, t_j, x_j, w_j, \theta)}{\partial \theta} \right\}^{-1}
\]

and

\[
\bar{S} = (1/N) \sum_i s_j(y_j, t_j, x_j, w_j, \theta) s_j(y_j, t_j, x_j, w_j, \theta)'
\]

The matrix \( \bar{G} \) is not symmetric because our estimator comes from stacking the moment conditions instead of optimizing one objective function. The implication is that the robust formula should always be used because, even under correct specification, the nonsymmetric \( \bar{G} \) and the symmetric \( \bar{S} \) converge to different matrices.
General potential-outcome model

Equation (1) can be generalized to a potential-outcome model with separate variance and correlation parameters for the control and treatment groups.

The generalized model is

\[
\begin{align*}
y_{0j} &= \mathbf{x}_j \beta_0 + \epsilon_{0j} \\
y_{1j} &= \mathbf{x}_j \beta_1 + \epsilon_{1j} \\
t_j &= \begin{cases} 
1, & \text{if } w_j \gamma + u_j > 0 \\
0, & \text{otherwise}
\end{cases}
\end{align*}
\]

where \(y_{0j}\) is the outcome that person \(j\) obtains if person \(j\) selects treatment 0, and \(y_{1j}\) is the outcome that person \(j\) obtains if person \(j\) selects treatment 1. We never observe both \(y_{0j}\) and \(y_{1j}\), only one or the other. We observe

\[y_j = t_j y_{1j} + (1 - t_j) y_{0j}\]

In this unconstrained model, the vector of error terms \((\epsilon_{0j}, \epsilon_{1j}, u_j)'\) comes from a mean zero trivariate normal distribution with covariance matrix

\[
\begin{bmatrix}
\sigma_0^2 & \sigma_{01} & \sigma_{0}\rho_0 \\
\sigma_{01} & \sigma_1^2 & \sigma_1\rho_1 \\
\sigma_{0}\rho_0 & \sigma_1\rho_1 & 1
\end{bmatrix}
\]

The likelihood function for this model is given in Maddala (1983, 224).

\[
\ln f_j = \left\{ \begin{array}{ll}
\ln \Phi \left( \frac{w_j \gamma + (y_j - \mathbf{x}_j \beta_1) \rho_1 / \sigma_1}{\sqrt{1 - \rho_1^2}} \right) - \frac{1}{2} \left( \frac{y_j - \mathbf{x}_j \beta_1}{\sigma_1} \right)^2 - \ln(\sqrt{2\pi} \sigma_1), & t_j = 1 \\
\ln \Phi \left( \frac{-w_j \gamma - (y_j - \mathbf{x}_j \beta_0) \rho_0 / \sigma_0}{\sqrt{1 - \rho_0^2}} \right) - \frac{1}{2} \left( \frac{y_j - \mathbf{x}_j \beta_0}{\sigma_0} \right)^2 - \ln(\sqrt{2\pi} \sigma_0), & t_j = 0
\end{array} \right.
\]

\[
\ln L = \sum_{j=1}^{n} w_j \ln f_j
\]

where \(\Phi(\cdot)\) is the cumulative distribution function of the standard normal distribution, and \(w_j\) is an optional weight. The covariance between \(\epsilon_{0j}\) and \(\epsilon_{1j}\), \(\sigma_{01}\), cannot be estimated because the potential outcomes \(y_{0j}\) and \(y_{1j}\) are never observed simultaneously.

As in the constrained model, \(\sigma_0\) and \(\sigma_1\) are not directly estimated in the maximum likelihood estimation; rather, \(\ln \sigma_0\) and \(\ln \sigma_1\) are estimated.

The parameters \(\rho_0\) and \(\rho_1\) are also not directly estimated; rather, \(\text{atanh} \rho_0\) and \(\text{atanh} \rho_1\) are directly estimated.

The new parameter vector is

\[
\theta = (\beta_0', \beta_1', \gamma', \text{atanh} \rho_0, \ln \sigma_0, \text{atanh} \rho_1, \ln \sigma_1)'
\]

The CF estimator for this potential-outcome model uses new error functions for the outcome mean

\[
u_m(y_j, t_j, \mathbf{x}_j, w_j, \theta) = y_i - t_j (x_j \beta_1 + \rho_1 \sigma_1 u_i, j) - (1 - t_j) (x_j \beta_0 + \rho_0 \sigma_0 u_i, j)
\]
and for the outcome variances

\[ u_{v,0}(y_j, t_j, x_j, w_j, \theta) = (1 - t_j) \left( u_{m,j}^2 - \sigma_0^2 \left[ 1 - \rho_0^2 \{ u_{t,j}(u_{t,j} + w_j \gamma) \} \right] \right) \]

\[ u_{v,1}(y_j, t_j, x_j, w_j, \theta) = t_j \left( u_{m,j}^2 - \sigma_1^2 \left[ 1 - \rho_1^2 \{ u_{t,j}(u_{t,j} + w_j \gamma) \} \right] \right) \]

These error functions are derived based on the identities

\[ E(y_j | t_j, x_j, w_j) = t_j (x_j \beta_1 + \rho_1 \sigma_1 u_{t,j}) + (1 - t_j)(x_j \beta_0 + \rho_0 \sigma_0 u_{t,j}) \]

\[ \text{Var}(y_j | t_j = 0, x_j, w_j) = \sigma_0^2 \left[ 1 - \rho_0^2 \{ u_{t,j}(u_{t,j} + w_j \gamma) \} \right] \]

\[ \text{Var}(y_j | t_j = 1, x_j, w_j) = \sigma_1^2 \left[ 1 - \rho_1^2 \{ u_{t,j}(u_{t,j} + w_j \gamma) \} \right] \]

We calculate the hazard, \( h_j \), prior to estimation from a probit regression of the treatment, \( t_j \), on the treatment covariates, \( w_j \). Let \( \tilde{z}_j = \{ x_j, t_j h_j, (1 - t_j)h_j \} \). Now we define

\[
Z_j = \begin{bmatrix}
\tilde{z}_j & 0 & 0 & 0 \\
0 & w_j & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

and

\[ s_j(y_j, t_j, x_j, w_j, \theta) = Z_j' \begin{bmatrix}
u_{m,j} \\
u_{t,j} \\
u_{v,0,j} \\
u_{v,1,j}
\end{bmatrix} \]

The CF estimator \( \hat{\theta} \) is the value of \( \theta \) that satisfies the sample-moment conditions

\[ 0 = \frac{1}{N} \sum_i s_j(y_j, t_j, x_j, w_j, \theta) \]

The Huber/White/robust sandwich estimator is consistent for the VCE. See Wooldridge (2010, chap. 14), Cameron and Trivedi (2005, chap. 6), and Newey and McFadden (1994).

The formula is

\[ \hat{V} = (1/N) \overline{G} \overline{S} \overline{G}' \]

where

\[ \overline{G} = \left\{ (1/N) \sum_i \frac{\partial s_j(y_j, t_j, x_j, w_j, \theta)}{\partial \theta} \right\}^{-1} \]

and

\[ \overline{S} = (1/N) \sum_i s_j(y_j, t_j, x_j, w_j, \theta)s_j(y_j, t_j, x_j, w_j, \theta)' \]

The matrix \( \overline{G} \) is not symmetric because our estimator comes from stacking the moment conditions instead of optimizing one objective function. The implication is that the robust formula should always be used because, even under correct specification, the nonsymmetric \( \overline{G} \) and the symmetric \( \overline{S} \) converge to different matrices.
Average treatment effect

The ATE is the average difference of the treated potential outcomes and the control potential outcomes.

By the law of iterated expectations, the ATE is

\[ E(y_{1j} - y_{0j}) = E\{E(y_{1j} - y_{0j}|x_j, \epsilon_{0j}, \epsilon_{1j})\} \]
\[ = E(x_j \beta_1 + \epsilon_1 - x_j \beta_0 - \epsilon_0) \]
\[ = E\{x_j(\beta_1 - \beta_0)\} \]

This expectation can be estimated as a predictive margin when \(x_j(\beta_1 - \beta_0)\) varies in \(x_j\). Otherwise, the ATE is estimated as the coefficient of \(t_j\) in the model.

Average treatment effect on the treated

The ATE is the average difference of the treated potential outcomes and the control potential outcomes on the treated population.

The conditional means of the potential outcomes \(y_{tj}, t \in (0, 1)\) for exogenous covariates \(x_j\) and treatment covariates \(w_j\) at treatment \(t_j = 1\) are

\[ E(y_{tj}|x_j, w_j, t_j = 1) = x_j \beta_t + \rho_t \sigma_t \phi(w_j \gamma)/\Phi(w_j \gamma) \]

By the law of iterated expectations, the ATET is

\[ E(y_{1j} - y_{0j}|t_j = 1) = E\{E(y_{1j} - y_{0j}|x_j, w_j, t_j = 1)\} \]
\[ = E\{x_j(\beta_1 - \beta_0) + (\rho_1 \sigma_1 - \rho_0 \sigma_0) \phi(w_j \gamma)/\Phi(w_j \gamma)|t_j = 1\} \]

This expectation can be estimated as a predictive margin on the treated population when \(x_j(\beta_1 - \beta_0)\) varies in \(x_j\) or when the variance and correlation parameters differ by treatment group. Otherwise, the ATET is estimated as the coefficient of \(t_j\) in the model.

References


Heckman, J. J. 1976. The common structure of statistical models of truncation, sample selection and limited dependent variables and a simple estimator for such models. *Annals of Economic and Social Measurement* 5: 475–492.


---

**Also see**

[Causal] `etregress postestimation` — Postestimation tools for etregress

[Causal] `etpoisson` — Poisson regression with endogenous treatment effects

[ERM] `eregress` — Extended linear regression

[R] `heckman` — Heckman selection model

[R] `probit` — Probit regression

[R] `regress` — Linear regression

[SVY] `svy estimation` — Estimation commands for survey data

[U] 20 Estimation and postestimation commands
etregress postestimation — Postestimation tools for etregress

Postestimation commands

The following standard postestimation commands are available after etregress:

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<tr>
<td>estimates</td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td>etable</td>
<td>table of estimation results</td>
</tr>
<tr>
<td>*hausman</td>
<td>Hausman’s specification test</td>
</tr>
<tr>
<td>lincom</td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td>*lrtest</td>
<td>likelihood-ratio test</td>
</tr>
<tr>
<td>margins</td>
<td>marginal means, predictive margins, marginal effects, and average marginal effects</td>
</tr>
<tr>
<td>marginsplot</td>
<td>graph the results from margins (profile plots, interaction plots, etc.)</td>
</tr>
<tr>
<td>nlcom</td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td>predict</td>
<td>conditional treatment effects, linear predictions and their SEs, etc.</td>
</tr>
<tr>
<td>predictnl</td>
<td>point estimates, standard errors, testing, and inference for generalized predictions</td>
</tr>
<tr>
<td>pwcompare</td>
<td>pairwise comparisons of estimates</td>
</tr>
<tr>
<td>*suest</td>
<td>seemingly unrelated estimation</td>
</tr>
<tr>
<td>test</td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td>testnl</td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>

*estat ic, lrtest, and suest are not appropriate after etregress, twostep or etregress, cfunction.
hausman and lrtest are not appropriate with svy estimation results.
predict

Description for predict

predict creates a new variable containing predictions such as linear predictions, conditional treatment effects, standard errors, expected values, and probabilities.

Menu for predict

Statistics > Postestimation

Syntax for predict

After ML, twostep, or cfunction

    predict [type] newvar [if] [in] [ , statistic]

After ML or cfunction

    predict [type] stub* [if] [in], scores

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>xb</td>
<td>linear prediction; the default</td>
</tr>
<tr>
<td>cte</td>
<td>conditional treatment effect at treatment level</td>
</tr>
<tr>
<td>stdp</td>
<td>standard error of the prediction</td>
</tr>
<tr>
<td>stdf</td>
<td>standard error of the forecast</td>
</tr>
<tr>
<td>yctrt</td>
<td>$E(y_j</td>
</tr>
<tr>
<td>ycntrt</td>
<td>$E(y_j</td>
</tr>
<tr>
<td>ptrt</td>
<td>$\Pr(\text{treatment} = 1)$</td>
</tr>
<tr>
<td>xbttrt</td>
<td>linear prediction for treatment equation</td>
</tr>
<tr>
<td>stdptrt</td>
<td>standard error of the linear prediction for treatment equation</td>
</tr>
</tbody>
</table>

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

stdf is not allowed with svy estimation results.

Options for predict

xb, the default, calculates the linear prediction, $x_j \hat{b}$.

cte calculates the treatment effect, the difference of potential-outcome means, conditioned on treatment level.

stdp calculates the standard error of the prediction, which can be thought of as the standard error of the predicted expected value or mean for the observation’s covariate pattern. The standard error of the prediction is also referred to as the standard error of the fitted value.
etregress postestimation — Postestimation tools for etregress

stdf calculates the standard error of the forecast, which is the standard error of the point prediction for one observation. It is commonly referred to as the standard error of the future or forecast value. By construction, the standard errors produced by stdf are always larger than those produced by stdp; see Methods and formulas in [R] regress postestimation.

yctrt calculates the expected value of the dependent variable conditional on the presence of the treatment: \( E(y_j \mid \text{treatment} = 1) \).

ycntrt calculates the expected value of the dependent variable conditional on the absence of the treatment: \( E(y_j \mid \text{treatment} = 0) \).

ptrt calculates the probability of the presence of the treatment:

\[
Pr(\text{treatment} = 1) = Pr(w_j \gamma + u_j > 0).
\]

xbtrt calculates the linear prediction for the treatment equation.

stdptrt calculates the standard error of the linear prediction for the treatment equation.

scores, not available with twostep, calculates equation-level score variables.

- The first new variable will contain \( \partial \ln L / \partial (x_j \beta) \).
- The second new variable will contain \( \partial \ln L / \partial (w_j \gamma) \).
- Under the constrained model, the third new variable will contain \( \partial \ln L / \partial \tanh \rho \).
- Under the constrained model, the fourth new variable will contain \( \partial \ln L / \partial \ln \sigma \).
- Under the general potential-outcome model, the third new variable will contain \( \partial \ln L / \partial \tanh \rho_0 \).
- Under the general potential-outcome model, the fourth new variable will contain \( \partial \ln L / \partial \ln \sigma_0 \).
- Under the general potential-outcome model, the fifth new variable will contain \( \partial \ln L / \partial \tanh \rho_1 \).
- Under the general potential-outcome model, the sixth new variable will contain \( \partial \ln L / \partial \ln \sigma_1 \).
margins

Description for margins

margins estimates margins of response for linear predictions, conditional treatment effects, expected values, and probabilities.

Menu for margins

Statistics > Postestimation

Syntax for margins

margins [marginlist] [ , options ]
margins [marginlist] , predict(statistic ...) [ predict(statistic ...) ... ] [ options ]

Maximum likelihood and control-function estimation results

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>xb</td>
<td>linear prediction; the default</td>
</tr>
<tr>
<td>cte</td>
<td>conditional treatment effect at treatment level</td>
</tr>
<tr>
<td>yactrt</td>
<td>$E(y_j</td>
</tr>
<tr>
<td>ycntrt</td>
<td>$E(y_j</td>
</tr>
<tr>
<td>ptrt</td>
<td>$Pr(\text{treatment} = 1)$</td>
</tr>
<tr>
<td>xbrtrt</td>
<td>linear prediction for treatment equation</td>
</tr>
<tr>
<td>stdp</td>
<td>not allowed with margins</td>
</tr>
<tr>
<td>stdf</td>
<td>not allowed with margins</td>
</tr>
<tr>
<td>stdptrt</td>
<td>not allowed with margins</td>
</tr>
</tbody>
</table>

Two-step estimation results

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>xb</td>
<td>linear prediction; the default</td>
</tr>
<tr>
<td>ptrt</td>
<td>$Pr(\text{treatment} = 1)$</td>
</tr>
<tr>
<td>xbrtrt</td>
<td>linear prediction for treatment equation</td>
</tr>
<tr>
<td>cte</td>
<td>not allowed with margins</td>
</tr>
<tr>
<td>yactrt</td>
<td>not allowed with margins</td>
</tr>
<tr>
<td>ycntrt</td>
<td>not allowed with margins</td>
</tr>
<tr>
<td>stdp</td>
<td>not allowed with margins</td>
</tr>
<tr>
<td>stdf</td>
<td>not allowed with margins</td>
</tr>
<tr>
<td>stdptrt</td>
<td>not allowed with margins</td>
</tr>
</tbody>
</table>

Statistics not allowed with margins are functions of stochastic quantities other than $e(b)$.

For the full syntax, see [R] margins.
Remarks and examples

The average treatment effect (ATE) and the average treatment effect on the treated (ATET) are the parameters most frequently estimated by postestimation techniques after `etregress`.

When there are no interactions between the treatment variable and the outcome covariates in the constrained model, `etregress` directly estimates the ATE and the ATET; see example 1 of [CAUSAL] `etregress`.

When there are no interactions between the treatment variable and the outcome covariates in the general potential-outcome model, `etregress` directly estimates the ATE; see example 2 of [CAUSAL] `etregress`.

When there are interactions between the treatment variable and the outcome covariates, you can use `margins` after `etregress` to estimate the ATE. See example 3 and example 4 of [CAUSAL] `etregress` for examples of ATE estimation.

When there are interactions between the treatment variable and the outcome covariates in the constrained model, you can use `margins` after `etregress` to estimate the ATET. See example 5 of [CAUSAL] `etregress` for an example of ATET estimation in the constrained model.

In the general potential-outcome model, you can use `margins` after `etregress` to estimate the ATET. See example 6 of [CAUSAL] `etregress` for an example of ATET estimation in the general potential-outcome model.

Also see

[CAUSAL] `etregress` — Linear regression with endogenous treatment effects

[U] 20 Estimation and postestimation commands
hdidregress estimates average treatment effects on the treated (ATETs) that may vary over time and over treatment cohorts. Treatment cohorts are groups subject to treatment at different points in time. hdidregress provides four estimators: extended two-way fixed effects (TWFE), regression adjustment (RA), inverse-probability weighting (IPW), and augmented inverse-probability weighting (AIPW). See [CAUSAL] teffects intro for a discussion of RA, IPW, and AIPW estimators.

hdidregress is for repeated cross-sectional data. For panel data, see [CAUSAL] xthdidregress.

Quick start

Estimate ATETs of treatment treat on outcome y with group grpvar and time tvar; use the RA estimator and model y using covariate x

```
hdidregress ra (y x) (treat), group(grpvar) time(tvar)
```

Same as above, but use the TWFE estimator

```
hdidregress twfe (y x) (treat), group(grpvar) time(tvar)
```

Use the IPW estimator and model treat using covariate z

```
hdidregress ipw (y) (treat z), group(grpvar) time(tvar)
```

Use the AIPW estimator, model y using covariate x, and model treat using covariate z

```
hdidregress aipw (y x) (treat z), group(grpvar) time(tvar)
```

Same as above, but use the not-yet-treated group as the control group

```
hdidregress aipw (y x) (treat z), group(grpvar) time(tvar) ///
controlgroup(notyet)
```

Same as above, but cluster at the county level

```
hdidregress aipw (y x) (treat z), group(grpvar) time(tvar) ///
controlgroup(notyet) vce(cluster county)
```
Syntax

Two-way fixed effects

\texttt{hdidregress twfe (ovar [omvarlist]) (tvar) [if] [in] [weight],}
\texttt{group(groupvar) time(timevar) [options]}

Regression adjustment

\texttt{hdidregress ra (ovar [omvarlist]) (tvar) [if] [in] [weight],}
\texttt{group(groupvar) time(timevar) [options]}

Inverse-probability weighting

\texttt{hdidregress ipw (ovar) (tvar [tmvarlist]) [if] [in] [weight],}
\texttt{group(groupvar) time(timevar) [options]}

Augmented inverse-probability weighting

\texttt{hdidregress aipw (ovar [omvarlist]) (tvar [tmvarlist]) [if] [in] [weight],}
\texttt{group(groupvar) time(timevar) [options]}

\texttt{ovar} is a continuous outcome of interest.

\texttt{omvarlist} specifies the covariates in the outcome model and may contain factor variables; see \texttt{\textnormal{[U] 11.4.3 Factor variables.}}

\texttt{tvar} must be a binary variable indicating observations subject to treatment.

\texttt{tmvarlist} specifies the covariates in the treatment model and may contain factor variables; see \texttt{\textnormal{[U] 11.4.3 Factor variables.}}

\texttt{groupvar} is a categorical variable that indicates the group level at which the treatment occurs.

\texttt{timevar} is a time variable.
### options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Model</strong></td>
</tr>
<tr>
<td><em>group</em>(<em>groupvar</em>)</td>
</tr>
<tr>
<td><em>time</em>(<em>timevar</em>)</td>
</tr>
<tr>
<td><strong>controlgroup</strong>(<em>cgtype</em>)</td>
</tr>
<tr>
<td><strong>cohortvar</strong>(<em>cvar</em>, <em>replace</em>)</td>
</tr>
<tr>
<td>†hettype(<em>hetspec</em>)</td>
</tr>
<tr>
<td><strong>SE/Robust</strong></td>
</tr>
<tr>
<td>vce(<em>vcetype</em>)</td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
</tr>
<tr>
<td><em>level</em>(#)</td>
</tr>
<tr>
<td>[no]log</td>
</tr>
<tr>
<td>nodots</td>
</tr>
<tr>
<td><strong>display_options</strong></td>
</tr>
</tbody>
</table>

| coeflegend | display legend instead of statistics |

*^group*(*groupvar*) and *time*(*timevar*) are required.

†This option may be specified only when **twfe** is specified.

### cgtype

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>never</td>
</tr>
<tr>
<td>notyet</td>
</tr>
</tbody>
</table>

### hetspec

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>timecohort</td>
</tr>
<tr>
<td>time</td>
</tr>
<tr>
<td>cohort</td>
</tr>
</tbody>
</table>

by, collect, and statsby are allowed; see [U] 11.1.10 Prefix commands.

fweights, aweights, and pweights are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
Options

**Model**

- **group(groupvar)** specifies a group variable that indicates the group level at which the treatment occurs. `groupvar` may be, for example, states, counties, or hospitals. `group()` also defines the level clustering for the default cluster–robust standard errors. `group()` is required. You may specify only one group variable.

- **time(timevar)** specifies the time variable used to define treatment-time cohorts. `time()` is required.

- **controlgroup(cgtype)** specifies the type of control group. A control group can be either a never-treated group or a not-yet-treated group. A never-treated group refers to the units that are untreated from the first to the last period. A not-yet-treated group refers to the units that are untreated up to a specific period. `cgtype` can be one of `never`, referring to the never-treated group, or `notyet`, referring to the not-yet-treated group. By default, `cgtype` is `never`.

- **cohortvar(cvar [, replace])** specifies the variable name `cvar` for the generated cohort variable. The cohort variable is a categorical variable indicating the period when the unit is first treated. By default, `_did_coherence` is used as the name of the cohort variable. If `_did_coherence` already exists in the dataset, it is replaced if option `cohortvar()` is not specified.

  If suboption `replace` is specified, `cvar` is replaced.

- **hettype(hetspec)** specifies time or cohort heterogeneity for the `twfe` estimator. By default, treatment is interacted with time and cohort. You may choose to keep one of time or cohort interactions using `hetspec`.

  `hetspec` may be one of `timecohort` for heterogeneous treatment effects over both time and cohort, `time` for heterogeneous treatment effects over time only, or `cohort` for heterogeneous treatment effects over cohort only. By default, `hetspec` is `timecohort`.

**SE/Robust**

- **vce(vcetype)** specifies the type of standard error reported, which includes types that allow for intragroup correlation (`cluster clustvar`), that are robust to intragroup correlation among group variable (`robust`), and that use bootstrap or jackknife sampling done at the individual level (`bootstrap`, `jackknife`); see [R] `vce_option`.

  `vce(cluster clustvar)`, the default, uses the variable specified in `group(groupvar)`.

**Reporting**

- **level(#)**; see [R] *Estimation options*.

  `log` and `nolog` specify whether to display the iteration log. The iteration log is displayed by default unless you used `set iterlog off` to suppress it; see `set iterlog` in [R] `set iter`.

  `nodots` suppresses display of the replication dots.

  `display_options`: `noci`, `nopvalues`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] *Estimation options*.

The following option is available with `hdidregress` but is not shown in the dialog box:

- **coeflegend**; see [R] *Estimation options*.
Remarks and examples

It is common to study the effects of a treatment, for example, a policy or intervention, on a group. `hdidregress` is for data where the treated groups are subject to the treatment at different points in time and they remain exposed to the treatment. For example, a health policy such as an increase in the age to purchase cigarettes is implemented in a given region, and over time, other regions decide to imitate the initiative. Another example is change in work policies across industries. Perhaps airlines implement a minimum number of hours between shifts for safety reasons. The policy is subsequently adopted by other similar industries. Some similar industries may never adopt the policy, remaining untreated, or it might be that all similar industries eventually adopt the policy.

`hdidregress` estimates ATET parameters that change over time and treatment cohorts (groups treated at different points in time). Each one of these ATETs has the same interpretation that the parameters of a two-time two-group difference-in-differences (DID) parameter would have. Because there are multiple DID parameters, we refer to them as heterogeneous treatment effects or as heterogeneous DID. This is in contrast to estimating only one ATET, which assumes there is no variation across time or cohort. If you assume no variation across time or cohort, you may use `didregress`; see [CAUSAL] `didregress`.

`hdidregress` provides four estimators: TWFE, outlined in Wooldridge (2021); RA, IPW, and AIPW, outlined in Callaway and Sant’Anna (2021). Each one of these estimators fits a model for the outcome of interest, a model for the treatment, or a model for both. For example, RA and TWFE model the outcome; IPW models the treatment; and AIPW models both. If the model for the outcome is correctly specified, RA and TWFE are best, with TWFE being more efficient. If the treatment model is correctly specified, IPW should be best. AIPW models both treatment and outcome. If at least one of the models is correctly specified, it provides consistent estimates. Thus, it allows us to misspecify one of the models and still get consistent estimates, a property called double robustness. See [CAUSAL] `teffects intro` for a discussion of RA, IPW, and AIPW estimators.

`hdidregress` is for repeated cross-sectional data. For panel data, see [CAUSAL] `xthdidregress`. Below, we illustrate how to use `hdidregress`. For a general overview of DID and more information about the methods used below, see [CAUSAL] `DID intro`. For general discussions about the methods, see Roth et al. (2022) and de Chaisemartin and D’Haultfœuille (forthcoming) and the references therein.

### Example 1: Heterogeneous DID for repeated cross-sections

We are interested in knowing if a school-district-level program, Healthy Habits, reduces the body mass index (BMI) for students in the school district. We have fictional data on the Healthy Habits program. This program incorporates more exercise time and augments the intakes of fruits and vegetables. Our data are at the school-district level and include information on whether a school participates in the program, `hhabit`, and the BMI of students in the district, `bmi`. We have repeated samples of students ages 11 to 14 from 40 school districts from the year 2032 to the year 2040.

```stata
use https://www.stata-press.com/data/r18/hhabits
(Fictional children BMI and school district data)
```

We are going to use the `aipw` estimator, which allows us to model the outcome and the treatment. If we had selected another estimator and specified the outcome incorrectly, the treatment effects would be inconsistent; see [CAUSAL] `teffects aipw`. With the `aipw` estimator, as long as one of the treatment or outcome model is correctly specified, we will get a consistent estimate of the ATET—a property called double robustness.

We model `hhabit` using the number of parks in the district, `parksd`. We conjecture that school districts with more parks consider exercise spaces more important in their urban planning than those with fewer parks. These districts are therefore more amenable to the Healthy Habits program.
For the outcome variable, we believe that mother’s education, medu, is a good predictor of the health habits of children. We also believe that participation in sports, sports, affects bmi. Finally, we control for whether the student is a girl to account for behavioral differences and differences in body types of boys and girls at this age.

In the first set of parentheses, we define the outcome, bmi, and any covariates that affect the outcome directly. In the second set of parentheses, we define the observation-level treatment variable, hhabit, and the covariates that affect it. After the comma, we must define the group variable in group(); this is a required option. The group variable defines at which level the treatment occurs and also identifies the clustering variable, which in this case is schools. We also need to specify a time variable in time(). We fit the following model:

\[
.\text{hdidregress aipw (bmi medu i.girl i.sports) (hhabit parksd),}
>\text{ group(schools) time(year)}
\]

\[
\text{note: variable _did_cohort, containing cohort indicators formed by treatment variable hhabit and group variable schools, was added to the dataset.}
\]

Computing ATET for each cohort and time:

<table>
<thead>
<tr>
<th>Cohort 2034 (8):</th>
<th>done</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cohort 2036 (8):</td>
<td>done</td>
</tr>
<tr>
<td>Cohort 2038 (8):</td>
<td>done</td>
</tr>
</tbody>
</table>

Treatment and time information

<table>
<thead>
<tr>
<th>Treatment and time information</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time variable: year</td>
</tr>
<tr>
<td>Time interval: 2032 to 2040</td>
</tr>
<tr>
<td>Control: _did_cohort = 0</td>
</tr>
<tr>
<td>Treatment: _did_cohort &gt; 0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>_did_cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cohorts</td>
</tr>
<tr>
<td>Number of obs</td>
</tr>
<tr>
<td>Never treated</td>
</tr>
<tr>
<td>2034</td>
</tr>
<tr>
<td>2036</td>
</tr>
<tr>
<td>2038</td>
</tr>
</tbody>
</table>
### Heterogeneous treatment-effects regression

**Estimator:** Augmented IPW  
**Treatment level:** schools  
**Control group:** Never treated

(Std. err. adjusted for 40 clusters in schools)

| Cohort | ATET  | Robust std. err. | z     | P>|z|   | [95% conf. interval] |
|--------|-------|------------------|-------|-------|---------------------|
|        |       |                   |       |       |                     |
| 2034   |       |                   |       |       |                     |
| 2033   | .6544681 | .5946048         | 1.10  | 0.271 | -.5109359          | 1.819872    |
| 2034   | -1.226451 | .379168          | -3.23 | 0.001 | -1.969607          | -.4832957   |
| 2035   | -2.491842 | .4169657         | -5.98 | 0.000 | -3.30908           | -1.674605   |
| 2036   | -2.72486 | .2363878         | -11.53| 0.000 | -3.188171          | -2.261548   |
| 2037   | -2.786634 | .667267          | -4.18 | 0.000 | -4.094492          | -1.478776   |
| 2038   | -3.880456 | .2993279         | -13.30| 0.000 | -4.567123          | -3.393794   |
| 2039   | -.604415 | .5929199         | -1.02 | 0.308 | -1.766517          | .5576866    |
| 2040   | -.652272 | .3640416         | -1.79 | 0.073 | -1.365736          | .0612812    |
| 2036   |       |                   |       |       |                     |
| 2033   | .6635794 | .3089663         | 2.15  | 0.032 | .0580167           | 1.269142    |
| 2034   | -1.3933  | .3871204         | -3.60 | 0.000 | -2.152042          | -.6345582   |
| 2035   | .5947865 | .4065947         | 1.46  | 0.144 | -.2021245          | 1.391697    |
| 2036   | -1.71427 | .4565384         | -3.75 | 0.000 | -2.609069          | -.8194714   |
| 2037   | -3.170542 | .5221368        | -6.07 | 0.000 | -4.193912          | -2.147173   |
| 2038   | -2.967701 | .4247053        | -6.99 | 0.000 | -3.800108          | -2.135294   |
| 2039   | .0360098 | .6868764         | 0.05  | 0.958 | -1.310243          | 1.382263    |
| 2040   | -.957117 | .3510986         | -2.73 | 0.006 | -1.645258          | -.2689763   |
| 2038   |       |                   |       |       |                     |
| 2033   | -1.434451 | .5163232        | -2.78 | 0.005 | -2.446426          | -.422476    |
| 2034   | 1.010288 | .4808165        | 2.10  | 0.036 | .067905            | 1.952671    |
| 2035   | -.3809733 | .4336764      | -0.88 | 0.380 | -1.230963          | .4690169    |
| 2036   | .5199519 | .4849723        | 1.07  | 0.284 | -.4305763          | 1.47048     |
| 2037   | -.0315794 | .5863875      | -0.05 | 0.957 | -1.180878          | 1.117719    |
| 2038   | -3.602114 | .3498692      | -10.30| 0.000 | -4.287845          | -2.916383   |
| 2039   | -1.388906 | .6765493      | -2.05 | 0.040 | -2.714919          | -.0628943   |
| 2040   | -.6222491 | .5510466      | -1.13 | 0.259 | -1.70228          | .4577824     |

Note: ATET computed using covariates.

Notice the note below the command. A variable with the name _did_cohort has been generated. Using the group variable and the observation-level treatment, hdidregress generated treatment-time cohorts. The new variable creates treatment groups based on the time when a group was first treated. For instance, if two schools adopt the Healthy Habits program in 2034, they are grouped in the 2034 cohort. The variable also contains a category for a control group. In this case, the control group is formed by the schools that never participate in the program. Cohorts are an important input for estimation and for postestimation commands. You do not need to adhere to the default name, _did_cohort, and may provide your own name using the cohortvar() option.
Next appears a table that gives you a sense of the treatment groups and time. You see the time variable, `year`, and its range, 2032 to 2040. Then we see what defines a treated or a control group. The table after provides group-level information about the cohort-time groups. The first row tells you the number of cohorts. Following the number of cohorts is a tabulation showing how many observations are in each cohort. For instance, 11,355 observations are never treated in our data. The table gives you a sense of the amount of information available in each cohort and might hint at the variability of cohort-level estimates.

The next table presents the ATET estimates. The first panel shows the ATETs for the 2034 cohort. We first have the 2033 ATET of 0.65, and the confidence interval includes 0. This is as expected; before treatment, the effect should be 0. We should interpret the ATET to mean that among the school districts that adopted the Healthy Habits program in 2034, the expected `bmi` is 0.65 higher than if the districts had never participated in the program. At treatment onset, in 2034, we observe a treatment effect is a decrease of the `bmi` of 1.23. In the last two periods, the effect of the treatment has diminished for the 2034 cohort; the confidence intervals for the effects in 2039 and 2040 again include 0. We interpret the results for the other cohorts similarly.

### Example 2: Visualizing estimation results

In the example above, we had four cohorts and nine time periods. There is a lot of information to process, and it can get even more daunting if we had more cohorts and time periods. To better visualize the results, we can use `estat atetplot`:

```
. estat atetplot
```

![Figure 1. ATETs by cohort over time](image)

The graph shows the pretreatment and the posttreatment ATETs for each cohort and their pointwise confidence intervals. For the 2034 cohort, we see that the program reduces `bmi` by approximately 2 to 4 points but this tendency seems to start reverting in 2038. A similar pattern emerges over the other two cohorts.
Example 3: Less heterogeneity; aggregating and summarizing treatment effects

So far, we have allowed treatment effects to change over cohort and over time. But we might want to obtain only one treatment effect for each cohort, abstracting away from time variation within cohorts. You would get this using the postestimation command `estat aggregation`.

```
  . estat aggregation, cohort
  ATET over cohort
  (Std. err. adjusted for 40 clusters in schools)
```

| Cohort | ATET | std. err. | z   | P>|z| | [95% conf. interval] |
|--------|------|-----------|-----|-----|----------------------|
| 2034   | -2.065755 | .1999412 | -10.33 | 0.000 | -2.457633 | -1.673877 |
| 2036   | -1.7781 | .4013978 | -4.43 | 0.000 | -2.564825 | -.9913744 |
| 2038   | -1.869405 | .4650349 | -4.02 | 0.000 | -2.780857 | -.9579538 |

Note that aggregation occurs only for the posttreatment periods and not for the pretreatment periods. The 2034 estimate is a weighted average of all the treatment-effect estimates after 2034 for the 2034 cohort; see [CAUSAL] `hdidregress postestimation` for more details.

Aggregated estimates are easier to digest; now we have 3 treatment effects to analyze instead of 24. For the 2034 cohort, we have a treatment effect of $-2.1$. For the 2036 cohort, the effect is $-1.8$, and for the 2038 cohort, it is $-1.9$. We cannot see how the treatment evolves over time for each cohort, but we have a sense of the average effect over time for each of them.

We could instead want to see the treatment effect at each point in time, abstracting from cohort-level variation.

```
  . estat aggregation, time
  ATET over time
  (Std. err. adjusted for 40 clusters in schools)
```

| Time | ATET | std. err. | z   | P>|z| | [95% conf. interval] |
|------|------|-----------|-----|-----|----------------------|
| 2034 | -1.226451 | .379168 | -3.23 | 0.001 | -1.969607 | -.4832957 |
| 2035 | -2.491842 | .4169657 | -5.98 | 0.000 | -3.30908 | -1.674605 |
| 2036 | -2.111619 | .3654785 | -5.78 | 0.000 | -2.827943 | -.1395294 |
| 2037 | -3.028686 | .4278557 | -7.08 | 0.000 | -3.867268 | -2.190104 |
| 2038 | -3.449829 | .2670184 | -12.92 | 0.000 | -3.973176 | -2.926483 |
| 2039 | -0.6624494 | .44865 | -1.48 | 0.140 | -1.541787 | .2168884 |
| 2040 | -0.7575068 | .2816374 | -2.69 | 0.007 | -1.309506 | -.2055078 |
We see the treatment effects for each one of the posttreatment periods. As before, we have the option to look at the effects graphically. We just need to use the `graph` option.

```
. estat aggregation, time graph
```

![figure](image.png)

**Figure 2. ATETs over time**

**Example 4: Dynamic treatment effects**

We could also ask what the evolution of the treatment effect is after treatment. For instance, we might want to know what happens one period after the onset of treatment, two periods after treatment, and so forth. It might be the case that treatment effects vanish over time or even change patterns. We might also want to see whether, before treatment, we observe a treatment effect or a pattern that might suggest that there is anticipation of treatment. `estat aggregation` allows us to answer these questions by using the `dynamic` option.

```
. estat aggregation, dynamic graph
```

```
Duration of exposure ATET Number of obs = 16,725
(Std. err. adjusted for 40 clusters in schools)

| Exposure | ATET | Robust std. err. | z     | P>|z|  | [95% conf. interval] |
|----------|------|------------------|-------|------|-------------------|
| -5       | -1.434451 | .5163232 | -2.78 | 0.005 | -2.446426 to -.422476 |
| -4       | 1.010288  | .4808165  | 2.10  | 0.036 | .067905 to 1.952671 |
| -3       | .1338267  | .3091619  | 0.43  | 0.665 | -.4721195 to .739773 |
| -2       | -.4256324 | .4292553  | -0.99 | 0.321 | -1.266957 to .4156925 |
| -1       | .3727141  | .3197563  | 1.17  | 0.244 | -.2539967 to .999425 |
| 0        | -2.285098 | .3827362  | -5.97 | 0.000 | -3.035248 to -1.534949 |
| 1        | -2.344265 | .3829047  | -6.12 | 0.000 | -3.094744 to -1.593785 |
| 2        | -2.045521 | .3911543  | -5.23 | 0.000 | -2.81217 to -1.278873 |
| 3        | -1.045601 | .6840119  | -1.53 | 0.126 | -2.38624 to .2950372 |
| 4        | -2.145004 | .5952526  | -3.60 | 0.000 | -3.311678 to -.978331 |
| 5        | -.604415  | .5929199  | -1.02 | 0.308 | -1.766517 to .5576866 |
| 6        | -.6522272 | .3640416  | -1.79 | 0.073 | -1.365736 to .0612812 |
```

Note: Exposure is the number of periods since the first treatment time.
In the three periods prior to treatment, there is no effect. This suggests no anticipation to treatment. At the onset, the program reduces \( \text{bmi} \), but the effect decreases for school districts that remain for more than four years in the program.

Example 5: TWFE estimation

The literature on heterogeneous DID started by pointing out the problems that arise when one assumes erroneously that the treatment effects are homogeneous. It suggested that TWFE estimation was inadequate. Wooldridge (2021) suggests that fixed-effects estimation can be used if we extend it to include interactions between treatment-time cohorts and time.

Another important insight of Wooldridge (2021) is that you can use pooled ordinary least squares and add panel-level averages of covariates and obtain the same point estimates as one would get with fixed-effects estimation in the context of DID estimation. This is an extension of the intuition by Mundlak (1978). \texttt{xthdidregress} and \texttt{hdidregress} fit pooled ordinary least-squares models using these ideas. Below, we present the results we obtain using the \texttt{twfe} estimator.
. *hdidregress twfe (bmi medu i.girl i.sports) (hhabit), group(schools) time(year)*
note: variable _did_cohort, containing cohort indicators formed by treatment
variable hhabit and group variable schools, was added to the dataset.
Computing ATETs using *margins* ...

Treatment and time information
Time variable: year
Time interval: 2032 to 2040
Control: _did_cohort = 0
Treatment: _did_cohort > 0

<table>
<thead>
<tr>
<th>_did_cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cohorts</td>
</tr>
<tr>
<td>Number of obs</td>
</tr>
<tr>
<td>Never treated</td>
</tr>
<tr>
<td>2034</td>
</tr>
<tr>
<td>2036</td>
</tr>
<tr>
<td>2038</td>
</tr>
</tbody>
</table>

Heterogeneous treatment-effects regression

Data type: Repeated cross-sectional
Estimator: Two-way fixed effects
Treatment level: schools
Control group: Never treated
Heterogeneity: Cohort and time

(Std. err. adjusted for 40 clusters in schools)

| Cohort | Robust ATET | std. err. | t | P>|t| | [95% conf. interval] |
|--------|-------------|-----------|---|-----|---------------------|
| 2034   |             |           |   |     |                     |
| year   |             |           |   |     |                     |
| 2034   | -.8057824   | .2723491  | -2.96 | 0.005 | -1.35666 - .2549045 |
| 2035   | -1.951481   | .2098279  | -9.30 | 0.000 | -2.375898 -1.527064 |
| 2036   | -2.091438   | .2081903  | -10.05 | 0.000 | -2.512542 -1.670333 |
| 2037   | -2.329408   | .4674253  | -4.98 | 0.000 | -3.274865 -1.383952 |
| 2038   | -3.623645   | .4658056  | -7.78 | 0.000 | -4.565826 -2.681464 |
| 2039   | -1.729334   | .7543583  | -0.23 | 0.820 | -1.698767  1.3529 |
| 2040   | -.2267266   | .3344035  | -0.68 | 0.502 | -.9031216  .4496684 |

| 2036   |             |           |   |     |                     |
| year   |             |           |   |     |                     |
| 2036   | -1.671963   | .3424563  | -4.88 | 0.000 | -2.364646 -.9792798 |
| 2037   | -3.27542    | .3496365  | -9.37 | 0.000 | -3.982627 -2.568213 |
| 2038   | -2.995124   | .2853544  | -10.50 | 0.000 | -3.572308 -2.41794 |
| 2039   | -.0792949   | .5152787  | -0.15 | 0.878 | -1.121544 .9629547 |
| 2040   | -.9852905   | .1856743  | -5.31 | 0.000 | -1.360852 -.6097289 |

| 2038   |             |           |   |     |                     |
| year   |             |           |   |     |                     |
| 2038   | -3.389082   | .154181   | -21.98 | 0.000 | -3.700942 -3.077221 |
| 2039   | -.7309226   | .5173441  | -1.41 | 0.166 | -1.77735 - .3155046 |
| 2040   | -.6942153   | .3558486  | -1.95 | 0.058 | -1.413987 .0255683 |

Note: ATET computed using covariates.
The output is almost the same as the one for the ra estimator in example 1. There are a couple of noteworthy differences. First, the estimator fits an extended TWFE regression and then uses margins to compute the treatment effects. The command tells us Computing ATETs using margins when it begins this more computationally intensive calculation.

Second, the ATET parameters are shown for each cohort only at the time of treatment exposure and for the periods thereafter but not for the pretreatment periods. As discussed in Wooldridge (2021), these are the parameters identified using the parallel-trends assumption he derives.

As we did before, we could use estat aggregation to explore different ways of looking at our treatment effects and estat atetplot to visualize the estimated ATETs.

Example 6: Reducing model complexity

When we fit the aipw model, we had to estimate ATET parameters for each cohort over time. The complexity of the model grows with the number of cohorts and the number of time periods. As is described in Methods and formulas, the aipw estimator uses a different subset of the data to obtain each parameter. To get a reliable estimator of each parameter, you need sufficient data for each subsample. Sometimes, there are few observations for a given cohort in a given set of time periods.

We can ameliorate this problem by reducing the amount of heterogeneity we assume. For the twfe estimator, the complexity of the model comes from the interactions between the observation-level treatment with cohort and time and the interactions between the observation-level treatment, cohort, time, and covariates. This allows us to decide which interactions to include in our model. We could, for instance, allow for heterogeneity at the cohort level instead of at the cohort and time level. We use the hettype() option with the argument cohort() to do this:
. hdidregress twfe (bmi medu i.girl i.sports) (hhabit), group(schools) > time(year) hettype(cohort)

note: variable _did_cohort, containing cohort indicators formed by treatment variable hhabit and group variable schools, was added to the dataset.

Computing ATETs using margins ...

Treatment and time information

Time variable: year
Time interval: 2032 to 2040
Control: _did_cohort = 0
Treatment: _did_cohort > 0

<table>
<thead>
<tr>
<th>_did_cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cohorts</td>
</tr>
<tr>
<td>Number of obs</td>
</tr>
<tr>
<td>Never treated</td>
</tr>
<tr>
<td>2034</td>
</tr>
<tr>
<td>2036</td>
</tr>
<tr>
<td>2038</td>
</tr>
</tbody>
</table>

Heterogeneous treatment-effects regression

Data type: Repeated cross-sectional
Estimator: Two-way fixed effects
Treatment level: schools
Control group: Never treated
Heterogeneity: Cohort

(Std. err. adjusted for 40 clusters in schools)

| Cohort | ATET | Robust std. err. | t | P>|t| | [95% conf. interval] |
|--------|------|------------------|---|------|---------------------|
| 2034   | -1.619553 | .2223114 | -7.29 | 0.000 | -2.069221 | -1.169886 |
| 2036   | -1.832602 | .1954433 | -9.38 | 0.000 | -2.227924 | -1.437281 |
| 2038   | -1.739144 | .2152765 | -8.08 | 0.000 | -2.174582 | -1.303706 |

Note: ATET computed using covariates.

You fit a regression model with fewer terms and obtain treatment effects only at the cohort level. You could also have the treatment effect change over time but not over cohort by typing cohort(time).

For the estimators proposed by Callaway and Sant’Anna (2021), heterogeneity is built in, so we need to estimate all the ATET parameters.
Stored results

`hdidregress` stores the following in `e()`:

Scalars
- `e(N)` number of observations
- `e(N_clust)` number of clusters
- `e(tmin)` first time period
- `e(tmax)` last time period
- `e(rank)` rank of `e(V)`

Macros
- `e(cmd)` `hdidregress`
- `e(cmdline)` command as typed
- `e(clustvar)` name of cluster variable
- `e(control_group)` control group
- `e(het_type)` heterogeneity type for `twfe` estimator
- `e(cohortvar)` name of cohort variable
- `e(ovar)` name of outcome variable
- `e(weight)` weight type
- `e(wexp)` weight expression
- `e(marginsnotok)` predictions disallowed by `margins`
- `e(timevar)` time variable
- `e(treatname)` name of treatment variable
- `e(estat_cmd)` program used to implement `estat`
- `e(vce)` `vcetype` specified in `vce()`
- `e(vctype)` title used to label Std. err.
- `e(method)` estimator method
- `e(properties) b V`

Matrices
- `e(b)` coefficient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(cohort_count)` matrix with cohort count information

Functions
- `e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices
- `r(table)` matrix containing the coefficients with their standard errors, test statistics, `p`-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any `r-class` command is run after the estimation command.

Methods and formulas

Methods and formulas are presented under the following headings:

- Introduction
- The RA, IPW, and AIPW estimators
- The TWFE estimator

Introduction

`hdidregress` for repeated cross-sectional data implements the RA, IPW, and AIPW estimators, outlined in Callaway and Sant’Anna (2021), and the TWFE estimator, outlined in Wooldridge (2021).
To reveal how the heterogeneous treatment effects evolve across cohorts and time, we are interested in estimating the ATET for each combination of cohort and time. Cohorts are defined by the time a group is treated, where time is denoted by \( t \), where \( t = 1, \ldots, T \). We denote a cohort by \( g \) and the individuals in our sample by \( i \), where \( i = 1, \ldots, N \). Let \( G_{ig} \) be an indicator that equals one if unit \( i \) is first treated at time \( g \). Then the units in cohort \( g \) can be denoted by \( G_{ig} = 1 \). When a unit \( i \) is never treated, we denote \( G_{i0} = 1 \). Thus, cohort 0 indicates all the units that are never treated. We assume that once a unit is treated, it will remain treated.

Let \( \theta(g, t) \) be the ATET for cohort \( g \) at time \( t \), which is defined as

\[
\theta(g, t) = E\{y_t(g) - y_t(0)|G_g = 1\}
\]

where \( y_t(g) \) is the potential outcome at time \( t \) for those first treated at time \( g \), \( y_t(0) \) is the potential outcome for those that are never treated, and \( G_g \) equals 1 if a unit belongs to cohort \( g \). All the four estimators provided in \texttt{hdidregress} estimate \( \theta(g, t) \) in equation \text{(ATET)}. We cannot directly estimate \( \theta(g, t) \) using equation \text{(ATET)} because the potential outcomes \( y_t(g) \) and \( y_t(0) \) are not observable.

Next, we will describe the RA, IPW, and AIPW estimators.

### The RA, IPW, and AIPW estimators

To define treatment effect, we need a control group. There are two ways to define the control group. One way is to use the units that are never treated as the control group. Let \( C_{ig} \) be an indicator that equals one if a unit belongs to the never-treated group. In particular, \( C_{ig} = 1 \). When a unit \( i \) is never treated, we denote \( G_{i0} = 1 \). Thus, cohort 0 indicates all the units that are never treated. We assume that once a unit is treated, it will remain treated.

Let \( \theta(g, t) \) be the ATET for cohort \( g \) at time \( t \), which is defined as

\[
\theta(g, t) = E\{y_t(g) - y_t(0)|G_g = 1\}
\]

where \( y_t(g) \) is the potential outcome at time \( t \) for those first treated at time \( g \), \( y_t(0) \) is the potential outcome for those that are never treated, and \( G_g \) equals 1 if a unit belongs to cohort \( g \). All the four estimators provided in \texttt{hdidregress} estimate \( \theta(g, t) \) in equation \text{(ATET)}. We cannot directly estimate \( \theta(g, t) \) using equation \text{(ATET)} because the potential outcomes \( y_t(g) \) and \( y_t(0) \) are not observable.

The definitions of the RA, IPW, and AIPW estimators depend on the definition of \( C_{g,t} \), which can either be \( \text{C}^\text{NEV} \) or \( \text{C}^\text{NY} \). However, regardless of the control group’s choice, the estimators’ definitions can always be written using the general notation \( C_{g,t} \).

For each unit \( i \) in the pooled sample, we observe \( \{\tau_i, y_{i,\tau_i}, x_{i,\tau_i}, d_{i,\tau_i}, z_{i,\tau_i}\} \), where \( y_i \) is the outcome, \( x_i \) are pretreatment covariates for the outcome model, \( d_i \) is a treatment indicator, \( z_i \) are covariates for the treatment assignment model, and \( \tau_i \in \{1, \ldots, T\} \) is a categorical variable indicating the time when unit \( i \) is observed. Let \( T_i \) equal one if the unit is observed at time \( t \) and zero otherwise.

The estimands also require the following notation,

\[
m_{g,s}(x) = E(y|x, G_g = 1, \tau = s) \\
m_{g,s,t}(x) = E(y|x, C_{g,t} = 1, \tau = s) \\
\varphi_{g,s}^{\text{treat}} = \frac{T_s G_g}{E(T_s G_g)} \\
\varphi_{g,s}^{\text{comp}}(z) = \frac{T_s p_{g,t}(z) C_{g,t}^*}{1 - p_{g,t}(z)} \\
\]

where \( p_{g,t}(z) \) is defined by

\[
p_{g,t}(z) = Pr(G_g = 1|z, G_g + C_{g,t}^* = 1) \quad \text{(Pz)}
\]

and the superscript refers to the group we are conditioning on, either the treated group (treat) or the control or comparison group (comp).
The RA estimand is
\[
\theta_{RA}(g,t) = E\left( \frac{G_g}{E(G_g)} \left[ \{m_{g,t}^{\text{treat}}(x) - m_{g,g-1}^{\text{treat}}(x)\} - \{m_{g,g-1,t}^{\text{comp}}(x) - m_{g,g-1}(x)\} \right] \right) \quad (RA)
\]

The IPW estimand is
\[
\theta_{IPW}(g,t) = E \left\{ \left( w_{g,t}^{\text{treat}} - w_{g,g-1}^{\text{treat}} \right) y \right\} - E \left\{ \left( w_{g,t,t}^{\text{comp}}(z) - w_{g,g-1,t}^{\text{comp}}(z) \right) y \right\} \quad (IPW)
\]

The AIPW estimand is
\[
\theta_{AIPW}(g,t) = E \left( \frac{G_g}{E(G_g)} \left[ \{m_{g,t}^{\text{treat}}(x) - m_{g,g-1}^{\text{treat}}(x)\} - \{m_{g,t,t}^{\text{comp}}(x) - m_{g,g-1,t}^{\text{comp}}(x)\} \right] \right)
+ E \left[ w_{g,t}^{\text{treat}} \left\{ y - m_{g,t}^{\text{treat}}(x) \right\} - w_{g,g-1}^{\text{treat}} \left\{ y - m_{g,g-1}^{\text{treat}}(x) \right\} \right]
- E \left[ w_{g,t,t}^{\text{comp}}(z) \left\{ y - m_{g,t,t}^{\text{comp}}(x) \right\} - w_{g,g-1,t}^{\text{comp}}(z) \left\{ y - m_{g,g-1,t}^{\text{comp}}(x) \right\} \right] \quad (AIPW)
\]

Under some regularity conditions, Callaway and Sant’Anna (2021) showed that the estimand for RA, IPW, and AIPW is the same as \( \theta(g,t) \) in equation (ATE). In other words,
\[
\theta(g,t) = \theta_{RA}(g,t) = \theta_{IPW}(g,t) = \theta_{AIPW}(g,t)
\]

Furthermore, the estimands in equations (RA), (IPW), and (AIPW) are estimable because they are all based on observed variables. The identification of the estimators sheds light on how to estimate \( \theta(g,t) \). The estimator can be generally divided into three steps:

1. Restrict the sample to time \( t \) and \( t_0 \), and keep only the units in cohort \( g \) or in control group \( \mathcal{C}_{g,t}^* \), where \( t_0 = g - 1 \) if \( t \geq g \) or \( t_0 = t - 1 \) if \( t < g \).

2. Use a parametric model to estimate the nuisance functions.
   a. For outcomes: linear regression to estimate \( m_{g,t}^{\text{treat}}(x) \), \( m_{g,t_0}^{\text{treat}}(x) \), \( m_{g,t}^{\text{comp}}(x) \), and \( m_{g,t_0}^{\text{comp}}(x) \).
   b. For propensity: logit regression to estimate \( p_{g,t}(z) \).
   c. For probability weights: \( w_{g,t}^{\text{treat}}(x) \), \( w_{g,t_0}^{\text{treat}}(x) \), \( w_{g,t}^{\text{comp}}(x) \), and \( w_{g,t_0}^{\text{comp}}(x) \) to estimate using propensity scores \( T_t \) and \( G_g \).

3. Plug in the nuisance function estimates into the estimating equation in equations (RA), (IPW), or (AIPW). Notice that the expectation operator \( E(\cdot) \) is replaced by the sample average.

The variance–covariance matrix for the estimates is computed using the influence-function approach proposed in Callaway and Sant’Anna (2021). The influence function approach is numerically equivalent to the generalized method of moments approach. However, it is much faster because it avoids computing the covariance matrix for the parameters in the nuisance functions. For more discussions on influence functions, see Hampel et al. (1986), Newey and McFadden (1994), and Jann (2020).
The TWFE estimator

A TWFE estimator for repeated cross-sections fits

\[ y_i = \alpha_h + \gamma_t + x_i \beta + d_i \tau + \varepsilon_i \]

Above, \( h \) denotes the group level at which treatment occurs. Wooldridge (2021) extends this model to incorporate interactions between the observation-level treatment, \( d_i \), cohort, \( G_{ig} \), posttreatment periods, and covariates. We define indicators for the posttreatment period as \( f_s \) with \( s \) going from \( q \) to \( T \), where \( q \) is the first time period we observe treatment. For instance, \( f_q \) equals 1 if we are in time-period \( q \) and 0 otherwise. To simplify this notation, we show the model without covariates. The extended fixed-effects model is given by

\[ y_i = \eta + \sum_{g=q}^{T} G_{ig} \theta_g + \sum_{s=2}^{T} f_s \gamma_s + \sum_{g=q}^{T} \sum_{s=g}^{T} d_i G_{ig} f_s \tau gs + \varepsilon_i \]  

(TWFE)

We can fit equation (TWFE) using pooled ordinary least squares or a within estimator. We are going to use the estimator proposed by Mundlak (1978). This gives the same point estimates as using the within estimator with \( h \) as the panel level for the parameters in equation (TWFE) but has different degrees of freedom because of the additional terms added by the Mundlak approach. Unlike within estimation, the Mundlak approach works for both repeated cross-sectional data as well as for panel data. Also, it has good properties to obtain partial effect under various data-generating processes, as pointed out in Wooldridge (2019).

Above, the \( \tau gs \) are the cohort-time treatment effects. When we have covariates, we interact them with all the relevant variables in the model. To get the treatment effects in this case, we need to control for the variation in the covariates. We can obtain both effects using margins.

In particular, we are going to type

\[ . \text{margins, dydx(d) at(year=q ... year=T) over(cohort) vce(unconditional)} \]

where \( d \) is the treatment indicator, \( year \) indicates treatment times at which treatment will be evaluated using at(), and cohort is the treatment-time cohorts. We use vce(unconditional) to account for the variation in the covariates.

With the hettype() option, we reduce the complexity of (TWFE). In particular, if we ask for hettype(time), we have

\[ y_i = \eta + \sum_{g=q}^{T} G_{ig} \theta_g + \sum_{s=2}^{T} f_s \gamma_s + \sum_{s=q}^{T} d_i f_s \tau_s + \varepsilon_i \]

Now treatment varies over time but not over cohort, that is, \( \tau_s \). If we use the hettype(cohort) option, we have

\[ y_i = \eta + \sum_{g=q}^{T} G_{ig} \theta_g + \sum_{s=2}^{T} f_s \gamma_s + \sum_{g=q}^{T} d_i G_{ig} \tau_g + \varepsilon_{it} \]

Now treatment varies over cohort but not over time, that is, \( \tau_g \).
When the controlgroup(notyet) option is specified, the $G_{ig}$ indicator excludes the last treated cohort. As discussed in Wooldridge (2021), when every group is eventually treated, we cannot identify the treatment effect for this cohort. It is therefore sensible to use the last treated cohort as a control group. When some of the units in our sample are never treated, we can always identify all cohorts, and the twfe estimator will always revert to using controlgroup(never).

Acknowledgments

We thank Fernando Rios-Avila of the Levy Economics Institute of Bard College for helpful conversations and advice and for his contributions in developing heterogeneous treatment-effects software and accompanying material. His community-contributed commands csdid and drdid implement some of the methods in this entry and a few others we have yet to include.

We thank Pedro H. C. Sant’Anna of Vanderbilt University for helpful conversations and advice and for his continuing contributions to the heterogeneous treatment-effects literature.

We thank Jeff Wooldridge of Michigan State University for helpful conversations about extended two-way fixed-effects estimation.

We also thank Austin Nichols of Amazon for his advice, for his insights, and for organizing the WHYDC causal inference conference.

Stata has an active research community adding features to the area of heterogeneous DID. We would like to acknowledge their previous and ongoing contributions to this area: csdid by Fernando Rios-Avila; did2s by Kyle Butts; did_multiplegt by Clement de Chaisemartin and Xavier D’Haultfœuille; did_imputation by Kirill Borusyak, Xavier Jaravel, and Jann Spiess; eventstudyinteract by Liyang Sun and Sarah Abraham; jwdid by Fernando Rios-Avila; and many more. Type search heterogeneous difference in differences to see Stata’s official and community-contributed features for heterogeneous DID.

References


Also see

[Causal] **hdidregress postestimation** — Postestimation tools for hdidregress and xthdidregress

[Causal] **xthdidregress** — Heterogeneous difference in differences for panel data

[Causal] **DID intro** — Introduction to difference-in-differences estimation

[Causal] **didregress** — Difference-in-differences estimation

[Causal] **teffects intro** — Introduction to treatment effects for observational data

[Causal] **teffects intro advanced** — Advanced introduction to treatment effects for observational data

[U] 20 Estimation and postestimation commands
**hdidregress postestimation** — Postestimation tools for hdidregress and xthdidregress

**Postestimation commands**

The following postestimation commands are of special interest after `hdidregress` and `xthdidregress`:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>estat ptrends</code></td>
<td>parallel-trends test</td>
</tr>
<tr>
<td><code>estat atetplot</code></td>
<td>plot the coefficients of ATET for each cohort</td>
</tr>
<tr>
<td>‡<code>estat aggregation</code></td>
<td>aggregate the ATETs to characterize the heterogeneity of treatment effects</td>
</tr>
<tr>
<td>∗<code>estat sci</code></td>
<td>multiplier bootstrap for simultaneous confidence intervals</td>
</tr>
</tbody>
</table>

‡`estat aggregation` is not allowed after estimation with bootstrap or jackknife standard errors.

∗`estat sci` may not be used after estimation using TWFE.

The following postestimation commands are also available:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>estat summarize</code></td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td><code>estat vce</code></td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td><code>estimates</code></td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td><code>etable</code></td>
<td>table of estimation results</td>
</tr>
<tr>
<td><code>lincom</code></td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td><code>nlcom</code></td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td><code>test</code></td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td><code>testnl</code></td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>
estat

Description for estat

estat ptrends tests that all pretreatment periods are equal to zero.
estat atetplot plots the coefficients of ATET for each cohort across different periods.
estat aggregation aggregates the cohort-period ATETs to characterize the heterogeneity of treatment effects. Aggregation may be within cohorts, time periods, time exposed to treatment, or within cohort and time periods. You may display the output of estat aggregation simultaneously as a table and a graph. The default is the tabular output.
estat sci provides the simultaneous confidence intervals for ATETs using the multiplier bootstrap method proposed in Callaway and Sant’Anna (2021). It may not be used after estimation using the TWFE estimator.

Menu for estat

Statistics > Postestimation

Syntax for estat

Tests that all pretreatment periods are zero
estat ptrends

Plot coefficients for ATETs
estat atetplot [cohort_list] [, atetplot_options]

Aggregate ATETs
estat aggregation [, aggregation_options]

Simultaneous confidence intervals
estat sci [, level(#) sci_options]

cohort_list is a subset of all the cohorts when estimating the ATETs. By default, the cohort_list contains all the cohorts. cohort_list is not allowed when the TWFE estimator is combined with option hettype(time) or hettype(cohort).
## atetplot_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>level(#)</strong></td>
</tr>
<tr>
<td><strong>sci</strong>(sci_options)</td>
</tr>
</tbody>
</table>

## Graph options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main</strong></td>
</tr>
<tr>
<td><strong>noci</strong></td>
</tr>
<tr>
<td><strong>preteo</strong>(scatter_opts)</td>
</tr>
<tr>
<td><strong>postteo</strong>(scatter_opts)</td>
</tr>
<tr>
<td>[no]zeroline</td>
</tr>
<tr>
<td><strong>zerolineo</strong>(refline_opts)</td>
</tr>
<tr>
<td>‡[no]cohortline</td>
</tr>
<tr>
<td>‡cohortlineo**(refline_opts)</td>
</tr>
</tbody>
</table>

## CI plot

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ci</strong>(area_opts)</td>
</tr>
</tbody>
</table>

## Y axis, X axis, Titles, Legend, Overall

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>‡by**(byo**gs)</td>
</tr>
<tr>
<td><strong>twoway_options</strong></td>
</tr>
</tbody>
</table>

*These options are not allowed for the TWFE estimators.

‡These options are not allowed when the TWFE estimator is combined with option hettype(time) or hettype(cohort).

## aggregation_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>overall</strong></td>
</tr>
<tr>
<td><strong>dynamic</strong>(event_list)</td>
</tr>
<tr>
<td><strong>time</strong>(time_list)</td>
</tr>
<tr>
<td><strong>cohort</strong>(cohort_list)</td>
</tr>
<tr>
<td>[no]graph</td>
</tr>
<tr>
<td><strong>graph</strong>(graph_opts)</td>
</tr>
<tr>
<td><strong>level(#)</strong></td>
</tr>
<tr>
<td><strong>sci</strong>(sci_options)</td>
</tr>
</tbody>
</table>

Only one of overall, dynamic(), cohort(), or time() is allowed.

*This option is not allowed after the TWFE estimator.

## sci_options

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>rseed(#)</strong></td>
</tr>
<tr>
<td><strong>reps(#)</strong></td>
</tr>
</tbody>
</table>
### scatter_opts

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>connect_options</td>
<td>change the look of lines or connecting method</td>
</tr>
<tr>
<td>marker_options</td>
<td>change the look of markers (color, size, etc.)</td>
</tr>
</tbody>
</table>

### refine_opts

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>style(addedlinestyle)</td>
<td>overall style of added line</td>
</tr>
<tr>
<td>no</td>
<td>extend line through plot region’s margins</td>
</tr>
<tr>
<td>lstyle(linestyle)</td>
<td>overall style of line</td>
</tr>
<tr>
<td>lpattern(linepatternstyle)</td>
<td>line pattern (solid, dashed, etc.)</td>
</tr>
<tr>
<td>lwidth(linewidthstyle)</td>
<td>thickness of line</td>
</tr>
<tr>
<td>lcolor(colorstyle)</td>
<td>color and opacity of line</td>
</tr>
</tbody>
</table>

### graph_opts

#### Options for estat

Options for **estat** are presented under the following headings:

- Options for **estat atetplot**
- Options for **estat aggregation**
- Options for **estat sci**
Options for estat atetplot

**level(##)** specifies the confidence level, as a percentage, for CIs. The default is `level(95)` or as set by `set level`; see [U] 20.8 Specifying the width of confidence intervals.

sci or `sci(sci_options)` plots the simultaneous confidence intervals (SCIs) using the multiplier bootstrap method proposed in Callaway and Sant’Anna (2021). SCIs simultaneously cover the true values of all the ATETs with a predefined probability level. By default, specifying `sci` implies using 999 bootstrap replications to construct the SCIs.

`sci(sci_options)` specifies the number of replications and the seed for the multiplier bootstrap when computing SCIs. `sci_options` may be `rseed(##)` or `reps(##)`. For the definition of these options, see Options for estat sci.

Option `sci` or `sci()` is not allowed after the TWFE estimator in `hdidregress` and `xthdidregress`. In addition, it is not allowed after estimation with bootstrap or jackknife standard errors for RA, IPW, and AIPW estimators.

By default, `estat atetplot` plots the pointwise CIs.

---

**noci** removes plots of the CIs. The default is to plot the CIs.

**preteopts(scatter_opts)** affects the rendition of the scatterplot for pretreatment periods. This option is not allowed after the TWFE estimator in `hdidregress` and `xthdidregress`. `scatter_opts` may be the following:

*connect_options* specify how points on a graph are to be connected; [G-3] `connect_options`.

*marker_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] `marker_options`.

**postteopts(scatter_opts)** affects the rendition of the scatterplot for posttreatment periods. `scatter_opts` may be the following:

*connect_options* specify how points on a graph are to be connected; [G-3] `connect_options`.

*marker_options* affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] `marker_options`.

**nozeroline** suppresses the y-axis reference line passing through zero. After estimation with `hdidregress` and the RA, IPW, or AIPW estimator, the default is to plot this reference line. After estimation with the TWFE estimator, the default is not to plot this reference line.

**zerolineopts(refline_opts)** affects the rendition of the reference line passing through zero. `refline_opts` may be the following:

*style(addedlinestyle)* specifies the overall style of the added line, which includes `noextend` and `lstyle(linestyle)` documented below. See [G-4] `addedlinestyle`. The `noextend` and `lstyle()` options allow you to change the added line’s attributes individually, but `style()` is the starting point.

You need not specify `style()` just because there is something that you want to change, and in fact, most people seldom specify the `style()` option. You specify `style()` when another style exists that is exactly what you desire or when another style would allow you to specify fewer changes to obtain what you want.

*extend* and *noextend* specify whether the line should extend through the plot region’s margin and touch the axis; see [G-3] `region_options`. Usually, *noextend* is the default, and *extend*
is the option, but that is determined by the overall \texttt{style()} and, of course, the scheme; see \cite{G-4} Schemes intro.

\texttt{Istyle(linestyle), lpattern(linepatternstyle), lwidth(linewidthstyle), lalign(linealignmentstyle), and lcolor(colorstyle)} specify the look of the line; see \cite{G-2} graph twoway line.

\texttt{nocohortline} suppresses the $x$-axis reference line passing through the time when the treatment began for each cohort. The default is to plot this reference line. This option is not allowed after the TWFE estimator.

\texttt{cohortlineopts(refline_opts)} affects the rendition of the reference line passing through the time when the treatment began for each cohort. This option is not allowed after the TWFE estimator. \texttt{refline_opts} may be the following:

\texttt{style(addedlinestyle)} specifies the overall style of the added line, which includes \texttt{[no]extend} and \texttt{lstyle(linestyle)} documented below. See \cite{G-4} \texttt{addedlinestyle}. The \texttt{[no]extend} and \texttt{lstyle()} options allow you to change the added line’s attributes individually, but \texttt{style()} is the starting point.

You need not specify \texttt{style()} just because there is something that you want to change, and in fact, most people seldom specify the \texttt{style()} option. You specify \texttt{style()} when another style exists that is exactly what you desire or when another style would allow you to specify fewer changes to obtain what you want.

\texttt{extend} and \texttt{noextend} specify whether the line should extend through the plot region’s margin and touch the axis; see \cite{G-3} region options. Usually, \texttt{noextend} is the default, and \texttt{extend} is the option, but that is determined by the overall \texttt{style()} and, of course, the scheme; see \cite{G-4} Schemes intro.

\texttt{lstyle(linestyle), lpattern(linepatternstyle), lwidth(linewidthstyle), lalign(linealignmentstyle), and lcolor(colorstyle)} specify the look of the line; see \cite{G-2} graph twoway line.

\texttt{CI plot}

\texttt{ciplot(areaoptions)} affects the rendition of the CIs; see \cite{G-3} area options.

\texttt{Y axis, X axis, Titles, Legend, Overall}

\texttt{byplot(byopts)} affects the rendition of the graph combined by cohorts. For \texttt{byplot}, see \cite{G-3} \texttt{by_option}. This option is not allowed after the TWFE estimator.

\texttt{twoway_options} are any of the options documented in \cite{G-3} twoway options, excluding \texttt{by().} These include options for titling the graph (see \cite{G-3} title options) and for saving the graph to disk (see \cite{G-3} saving option).

**Options for estat aggregation**

\texttt{overall} aggregates ATETs within all the cohorts and time periods; it is the default.

\texttt{dynamic} or \texttt{dynamic(event_list)} aggregates ATETs within exposure to the treatment. For example, two periods of exposure to the treatment means two periods after the treatment started. Specifying \texttt{dynamic} implies aggregating ATETs within all the estimable exposures to the treatment.

\texttt{dynamic(event_list)} aggregates ATETs within the exposure to the treatment specified by \texttt{event_list}. \texttt{event_list} is a \texttt{numlist} specifying length of exposures to the treatment.
time or time(time_list) aggregates ATETs within time periods. Specifying time implies aggregating ATETs within all the estimable time periods.

time(time_list) aggregates ATETs within the time specified by time_list. time_list is a numlist specifying time periods.

cohort or cohort(cohort_list) aggregates ATETs within cohort. Specifying cohort implies aggregating ATETs within all the estimable cohorts.

cohort(cohort_list) aggregates ATETs within the cohorts specified by cohort_list. cohort_list is a numlist specifying cohorts.

nograph and graph specifies whether to suppress or display the plot of aggregation of ATETs. nograph is the default.

graph(graph_opts) affects the rendition of the aggregation plot. graph_opts may be the following:

  noci removes plots of the CIs. The default is to plot the CIs.
  connect_options specify how points on a graph are to be connected; [G-3] connect_options.
  marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] marker_options.
  ciopts(area_options) affects the rendition of the CIs; see [G-3] area_options.
  twoway_options are any of the options documented in [G-3] twoway_options, excluding by(). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

level(#) specifies the confidence level, as a percentage, for CIs. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

sci or sci(sci_options) plots the simultaneous confidence intervals (SCIs) using the multiplier bootstrap method proposed in Callaway and Sant’Anna (2021). SCIs simultaneously cover the true values of aggregations of ATETs with a predefined probability level. By default, specifying sci implies using 999 bootstrap replications to construct the SCIs.

sci(sci_options) specifies the number of replications and the seed for the multiplier bootstrap when computing SCIs. sci_options may be rseed(#) or reps(#). For the definition of these options, see Options for estat sci.

Option sci or sci() is not allowed after the TWFE estimator in hdidregress and xthdidregress. By default, estat aggregation plots the pointwise CIs if option graph() is specified.

Options for estat sci

level(#) specifies the confidence level, as a percentage, for CIs. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

rseed(#) sets the random-number seed. Specifying this option makes the results reproducible because the critical values are drawn from a bootstrap sample.

reps(#) specifies the number of bootstrap replications to get the critical values of the test. The default is reps(999).
Remarks and examples

For examples of the `estat` commands above, see [CAUSAL] `hdidregress` and [CAUSAL] `xthdidregress`. Both entries have examples that illustrate how the estimation and postestimation commands work together.

Stored results

`estat ptrends` stores the following in `r()`:

Scalars
- `r(F)` \( F \) statistic
- `r(chi2)` \( \chi^2 \)
- `r(df)` test constraints degrees of freedom
- `r(p)` two-sided \( p \)-value
- `r(df_r)` residual degrees of freedom
- `r(drop)` 1 if constraints were dropped, 0 otherwise

`estat aggregation` stores the following in `r()`:

Scalars
- `r(reps)` number of replications

Macros
- `r(agg_type)` aggregation type

Matrices
- `r(b)` coefficient vector
- `r(V)` variance–covariance matrix of the estimators
- `r(table)` matrix containing test statistics and critical values

`estat atetplot` stores the following in `r()`:

Macros
- `r(table)` matrix containing test statistics and critical values

`estat sci` stores the following in `r()`:

Scalars
- `r(reps)` number of replications

Matrices
- `r(table)` matrix containing coefficients, bootstrap standard errors, and SCIs

Methods and formulas

Methods and formulas are presented under the following headings:

- Test for all pretreatment period ATETs being zero
- Aggregations for the RA, IPW, and AIPW estimators
- Aggregations for the TWFE estimator
- SCIs

Test for all pretreatment period ATETs being zero

`estat ptrends` tests that all pretreatment period ATETs are zero. This should be satisfied if both parallel trends and no anticipation of treatment hold for the pretreatment period.
For the RA, IPW, and AIPW estimators, `estat ptrends` is equivalent to a Wald test of all the pretreatment ATET estimates equaling zero. For methods and formulas on the Wald test, see Methods and formulas in [R] `test`.

Below, we will use the notation from Methods and formulas in [CAUSAL] `xthdidregress`. For the TWFE estimator, we fit the augmented model:

\[
y_{it} = \eta + \sum_{g=q}^{T} G_{ig} \alpha_{g} + \sum_{s=q}^{T} f_{s} \gamma_{s} + \sum_{g=2}^{q-1} \sum_{s=g}^{q-1} d_{it} G_{ig} f_{s} \omega_{gs} + \sum_{g=q}^{T} \sum_{s=g}^{T} d_{it} G_{ig} f_{s} \delta_{gs} + \varepsilon_{it}
\]

We then jointly test if the \( \omega_{gs} \) terms are zero by using `test`.

### Aggregations for the RA, IPW, and AIPW estimators

Denote \( \theta(g,t) \) as ATET\((g,t)\). These are the parameters computed during estimation. Instead of looking at all of these parameters, we can aggregate them to explore heterogeneity in different dimensions. We denote \( \theta \) as aggregations of ATETs. Regardless of whether we use cohort, time, or dynamic aggregation, we can always write \( \theta \) as a weighted sum of \( \theta(g,t) \) as follows

\[
\theta = \sum_{g \in G} \sum_{t=2}^{T} w(g,t) \theta(g,t)
\]

where \( G \) is the set of all the possible cohort values and \( w(g,t) \) is the cohort-time weights. The type of questions of interest determines the definitions of \( w(g,t) \).

One popular question in DID with multiple time periods set up is to study the dynamics of treatment effects: how do the average treatment effects vary with the length of exposure to the treatment? In literature, it is also known as the event study. Let \( e = t - g \) be the length of exposure to the treatment. We can summarize ATETs as

\[
\theta_d(e) = \sum_{g \in G} \mathbb{I}\{g + e \leq T\} P\{G = g|G + e \leq T\} \theta(g, g + e)
\]

where \( \mathbb{I}(\cdot) \) is an indicator function and \( G \) is a random categorical variable for a cohort. \( \theta_d(e) \) is computed when the dynamic option is specified.

To account for the heterogeneous treatment effects across cohorts, we consider the following aggregation:

\[
\theta_c(g) = \sum_{t=g}^{T} \theta(g,t) P(G = g|G = g, t \geq g)
\]

\( \theta_c(g) \) is computed when the cohort option is specified.

Time effects characterize treatment-effects heterogeneity across time. The average effect of participating in the treatment in a period \( t \) (among cohorts that are treated by time \( t \)) is

\[
\theta_t(t) = \sum_{g \in G} \mathbb{I}(t \geq g) P(G = g|G \leq t) \theta(g,t)
\]

\( \theta_t(t) \) is computed when the time option is specified.
The overall aggregation is the average of all the identified posttreatment ATETs. It is defined as

\[ \theta_o = \frac{1}{\kappa} \sum_{g \in G} \sum_{t=2}^{T} I(t \geq g) P(G = g|G \leq T) \theta(g, t) \]

where \( \kappa = \sum_{g \in G} \sum_{t=2}^{T} I(t \geq g) P(G = g|G \leq T) \). \( \theta_o \) is computed when the overall option is specified.

The variance–covariance matrix for the estimates of \( \theta \) is computed using the influence function approach outlined in section 4.2 in Callaway and Sant’Anna (2021).

When the sci option is specified, the SCIs are computed using the multiplier bootstrap proposed in section 4.2 in Callaway and Sant’Anna (2021).

**Aggregations for the TWFE estimator**

Aggregation after TWFE uses margins after the Mundlak estimation of the model. Let treat denote the observation-level treatment, cohort denote the variable that contains treat-time cohorts, and exposure denote a variable that indicates the time exposed to treatment.

For estat aggregation, overall:

\[ . \text{ margins r.treat, subpop(if treat==1) vce(unconditional)} \]

For estat aggregation, cohort:

\[ . \text{ margins, subpop(if treat==1) dydx(treat) over(cohort) vce(unconditional)} \]

For estat aggregation, time:

\[ . \text{ margins, subpop(if treat==1) dydx(treat) over(time) vce(unconditional)} \]

For estat aggregation, dynamic:

\[ . \text{ margins, subpop(if treat==1) dydx(treat) over(exposure) vce(unconditional)} \]

**SCIs**

After the RA, IPW, and AIPW estimators, estat sci can provide the SCIs that are guaranteed to cover all the ATETs with a specified probability. estat sci computes the SCIs using the multiplier bootstrap approach outlined in section 4.1 in Callaway and Sant’Anna (2021).

Unlike the traditional bootstrap, the multiplier bootstrap resamples the influence functions (which are already computed in the estimation step). Thus, the multiplier bootstrap is much faster than the traditional bootstrap because there is no need to recompute the estimators.

The influence function is a linear representation of the estimator. Let \( \hat{\theta}(g, t) \) be the RA, IPW, and AIPW estimators, and denote \( \theta(g, t) \) the true ATET for cohort \( g \) at time \( t \). Then the linear representation of these estimators can be written as

\[ \hat{\theta}(g, t) - \theta(g, t) = \frac{1}{n} \sum_{i=1}^{n} \psi_{g,t}(w_i) + o_p(1) \]

where \( \psi_{g,t}(\cdot) \) is the influence function, \( n \) is the sample size of the estimation sample for \( \hat{\theta}(g, t) \), \( w_i \) are the data, and \( o_p(1) \) is a term that vanishes to zero in probability as \( n \) grows. For a more detailed discussion on influence functions, see section 4.1 in Callaway and Sant’Anna (2021).
Denote \( \hat{\theta} \) as estimates of all the ATETs, and let \( \hat{\Psi} \) be estimates of the influence functions for \( \hat{\theta} \). Let \( \hat{\theta}^b \) be the \( b \)th bootstrap draw, which is defined as

\[
\hat{\theta}^b = \hat{\theta} + \frac{1}{n} \sum_{i=1}^{n} V_i \cdot \hat{\Psi}_i
\]

where \( \{V_i\} \) is a Bernoulli draw with \( P(V = 1 - \eta) = \eta/\sqrt{5}, \ P(V = \eta) = 1 - \eta/\sqrt{5} \) and \( \eta = (\sqrt{5} + 1)/2 \). Then the SCIs can be computed in the following steps:

1. Draw \( B \) samples of \( \{V_i\}_{i=1,...,n} \), and compute \( \hat{\theta}^b \) using each sample.
2. Compute the bootstrap diagonal of \( \Sigma^{1/2} \) as

\[
\hat{\Sigma}_{g,t}^{1/2} = \frac{q_{0.75}(g,t) - q_{0.25}(g,t)}{z_{0.75} - z_{0.25}}
\]

where \( q_p(g,t) \) is the \( p \)th sample quantile of \( \hat{R}_{g,t}^b = \sqrt{n} \left\{ \hat{\theta}^b(g,t) - \hat{\theta}(g,t) \right\} \) in \( B \) draws and \( z_p \) is the \( p \)th sample quantile of standard normal distribution.
3. For each bootstrap draw, compute the \( t \) test\(^b \) as

\[
test^b = \max_{(g,t)} |\hat{R}_{g,t}^b| \hat{\Sigma}_{g,t}^{-1/2}
\]
4. Compute the critical values \( \hat{c}_{1 - \alpha/2} \) as the \( 1 - \alpha/2 \) quantile of the \( B \) draws of \( t \) test\(^b \).
5. Construct the simultaneous bootstrap confidence intervals for \( \hat{\theta}(g,t) \) as

\[
\hat{C}(g,t) = \left\{ \hat{\theta}(g,t) - \hat{c}_{1 - \alpha/2} \hat{\Sigma}_{g,t}^{1/2} / \sqrt{n}, \ \hat{\theta}(g,t) + \hat{c}_{1 - \alpha/2} \hat{\Sigma}_{g,t}^{1/2} / \sqrt{n} \right\}
\]

Reference


Also see

[CAUSAL] **hdidregress** — Heterogeneous difference in differences
[CAUSAL] **xthdidregress** — Heterogeneous difference in differences for panel data
[CAUSAL] **DID intro** — Introduction to difference-in-differences estimation
[U] **20 Estimation and postestimation commands**
**mediate — Causal mediation analysis**

**Description**

*mediate* fits causal mediation models and estimates effects of a treatment on an outcome. The treatment effect can occur both directly and indirectly through another variable, a mediator. The outcome and mediator variables may be continuous, binary, or count. The treatment may be binary, multivalued, or continuous. The estimated direct, indirect, and total effects have a causal interpretation provided that assumptions pertaining to causal mediation models are met.

**Quick start**

Fit the mediation model with continuous outcome $y_1$, continuous mediator $m_1$, and categorical treatment $t_1$, and estimate the total effect, natural direct effect, and natural indirect effect

```
mediate (y1) (m1) (t1)
```

Same as above, but with covariates in both the outcome and the mediator equations

```
mediate (y1 x1 x2) (m1 x1 x3) (t1)
```

Same as above, but with probit model for binary outcome $y_2$ and Poisson model for count mediator $m_2$

```
mediate (y2 x1 x2, probit) (m2 x1 x3, poisson) (t1)
```

Same as above, but estimate only the natural indirect effect (NIE)

```
mediate (y2 x1 x2, probit) (m2 x1 x3, poisson) (t1), nie
```

Same as above, but also estimate potential-outcome means

```
mediate (y2 x1 x2, probit) (m2 x1 x3, poisson) (t1), nie pomeans
```

Fit the mediation model with continuous treatment $t_2$, and evaluate at values 0 and 4 of the treatment with 0 as the control

```
mediate (y2 x1 x2, probit) (m2 x1 x3, poisson) (t2, continuous(0 4))
```

**Menu**

- Statistics > Causal inference/treatment effects > Continuous outcomes > Causal mediation
- Statistics > Causal inference/treatment effects > Binary outcomes > Causal mediation
- Statistics > Causal inference/treatment effects > Count outcomes > Causal mediation
- Statistics > Causal inference/treatment effects > Nonnegative outcomes > Causal mediation
# Syntax

```plaintext
mediate (ovar [omvarlist, omodel noconstant])
   (mvar [mmvarlist, mmodel noconstant])
   (tvar [, continuous(numlist)] [if] [in] [weight] [, stat options])
```

*ovar* is a continuous, binary, or count outcome of interest.

*omvarlist* specifies the covariates in the outcome model.

*mvar* is the mediator variable and may be continuous, binary, or count.

*mmvarlist* specifies the covariates in the mediator model.

*tvar* is the treatment variable and may be binary, multivalued, or continuous.

## omodel Description

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>linear model; the default</td>
</tr>
<tr>
<td>expmean</td>
<td>exponential-mean model</td>
</tr>
<tr>
<td>logit</td>
<td>logistic regression model</td>
</tr>
<tr>
<td>probit</td>
<td>probit regression model</td>
</tr>
<tr>
<td>poisson</td>
<td>Poisson model</td>
</tr>
</tbody>
</table>

*omodel* specifies the model for the outcome variable.

## mmodel Description

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>linear model; the default</td>
</tr>
<tr>
<td>expmean</td>
<td>exponential-mean model</td>
</tr>
<tr>
<td>logit</td>
<td>logistic regression model</td>
</tr>
<tr>
<td>probit</td>
<td>probit regression model</td>
</tr>
<tr>
<td>poisson</td>
<td>Poisson model</td>
</tr>
</tbody>
</table>

*mmodel* specifies the model for the mediator variable.

The *logit* outcome model may not be combined with the *linear* or *expmean* mediator model; *probit* rather than *logit* may be used in these cases.
mediate — Causal mediation analysis

<table>
<thead>
<tr>
<th>stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Stat</strong></td>
</tr>
<tr>
<td></td>
<td><strong>Pearl’s labeling of effects</strong></td>
</tr>
<tr>
<td>nie</td>
<td>natural indirect effect</td>
</tr>
<tr>
<td>nde</td>
<td>natural direct effect</td>
</tr>
<tr>
<td>te</td>
<td>total effect</td>
</tr>
<tr>
<td>pnie</td>
<td>pure natural indirect effect</td>
</tr>
<tr>
<td>tnde</td>
<td>total natural direct effect</td>
</tr>
<tr>
<td></td>
<td><strong>ATE labeling of effects</strong></td>
</tr>
<tr>
<td>aite</td>
<td>average indirect treatment effect; synonym for nie</td>
</tr>
<tr>
<td>adte</td>
<td>average direct treatment effect; synonym for nde</td>
</tr>
<tr>
<td>ate</td>
<td>total average treatment effect; synonym for te</td>
</tr>
<tr>
<td>aitec</td>
<td>average indirect treatment effect with respect to controls; synonym for pnie</td>
</tr>
<tr>
<td>adtet</td>
<td>average direct treatment effect with respect to the treated; synonym for tnde</td>
</tr>
<tr>
<td>pomeans</td>
<td>potential-outcome means</td>
</tr>
<tr>
<td>all</td>
<td>all effects and potential-outcome means</td>
</tr>
</tbody>
</table>

Multiple effects may be specified; default is nie nde te.

<table>
<thead>
<tr>
<th>options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><strong>Model</strong></td>
</tr>
<tr>
<td>nointeraction</td>
<td>exclude interaction of mediator and treatment</td>
</tr>
<tr>
<td>control(#</td>
<td>label)</td>
</tr>
<tr>
<td></td>
<td><strong>SE/Robust</strong></td>
</tr>
<tr>
<td>vce(vcetype)</td>
<td>vcetype may be robust, cluster clustvar, bootstrap, or jackknife</td>
</tr>
<tr>
<td>nose</td>
<td>do not estimate standard errors</td>
</tr>
<tr>
<td></td>
<td><strong>Reporting</strong></td>
</tr>
<tr>
<td>level(#)</td>
<td>set confidence level; default is level(95)</td>
</tr>
<tr>
<td>ateterms</td>
<td>use ATE terminology to label effects</td>
</tr>
<tr>
<td>aequations</td>
<td>display auxiliary-equation results</td>
</tr>
<tr>
<td>nolegend</td>
<td>suppress table legend</td>
</tr>
<tr>
<td>display_options</td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td></td>
<td><strong>Optimization</strong></td>
</tr>
<tr>
<td>optimization_options</td>
<td>control the optimization process; seldom used</td>
</tr>
<tr>
<td></td>
<td><strong>Advanced</strong></td>
</tr>
<tr>
<td>force</td>
<td>force estimation when the number of treatment groups exceeds 10</td>
</tr>
<tr>
<td>coeflegend</td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>
omvarelist and mmvarelist may contain factor variables; see [U] 11.4.3 Factor variables.

bootstrap, by, collect, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
pweights, fweights, and iweights are allowed; see [U] 11.1.6 weight.

coloflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

--- Model ---

noconstant; see [R] Estimation options.

continuous(numlist) specifies that the treatment variable is continuous; numlist specifies the values at which the potential-outcome means are to be evaluated, where the first value in the list is taken as the control.

nointeraction excludes the interaction between the treatment and the mediator; by default, the model includes the treatment–mediator interaction.

ccontrol(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with continuous treatments.

--- Stat ---

stat specifies the statistics to be estimated. You may select from among five effects, each of which can be labeled according to terminology used by Pearl and others or by ATE terminology. In addition to effects, you may request that potential-outcome means be reported. The default is nie nde te.

stat may be one or more of the following:

<table>
<thead>
<tr>
<th>stat</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>nie</td>
<td>natural indirect effect</td>
</tr>
<tr>
<td>nde</td>
<td>natural direct effect</td>
</tr>
<tr>
<td>te</td>
<td>total effect</td>
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<td>pnie</td>
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<td>adte</td>
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</tr>
<tr>
<td>ate</td>
<td>average treatment effect; synonym for te</td>
</tr>
<tr>
<td>aitec</td>
<td>average indirect treatment effect with respect to controls; synonym for pnie</td>
</tr>
<tr>
<td>adtet</td>
<td>average direct treatment effect with respect to the treated; synonym for tnde</td>
</tr>
<tr>
<td>pomeans</td>
<td>potential-outcome means</td>
</tr>
</tbody>
</table>

all specifies that all effects and potential-outcome means be estimated; specifying all is equivalent to specifying nie nde te pnie tnde pomeans. When option ateterms is specified, all is equivalent to specifying aite adte ate aitec adtet pomeans.
SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

nose suppresses calculation of the variance–covariance matrix and standard errors.

Reporting

level(#) ; see [R] Estimation options.

ateterms specifies that ATE terminology be used to label effects. ateterms is strictly a labeling option. This option may not be specified on replay.

eaequations specifies that the estimation results for the outcome model and the mediator model be displayed. By default, they are not displayed.

nolegend suppresses the display of the table legend.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Optimization

optimization_options: conv_maxiter(), conv_ptol(), conv_vtol(), tracelevel(), and [no] log. See [M-5] optimize().

conv_maxiter(#) specifies the maximum number of iterations. The default is the number set using set maxiter, which by default is 300.

conv_ptol(#) specifies the convergence criteria for the parameters. The default is conv_ptol(1e-6).

conv_vtol(#) specifies the convergence criteria for the gradient. The default is conv_vtol(1e-7).

tracelevel(tracelevel) allows you to display additional information about the iterative process in the iteration log. tracelevel may be none, value, tolerance, step, params, or gradient. See tracelevel in [M-5] optimize() for details.

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

Advanced

force forces estimation when the number of treatment groups exceeds 10. By default, only 10 groups are allowed for multivalued treatments. Do not use the force option if the treatment is continuous; instead, use the continuous() option.

The following option is available with mediate but is not shown in the dialog box:

coeflegend; see [R] Estimation options.
Remarks and examples

Remarks are presented under the following headings:

- Introduction
- Approaches to mediation analysis
- Workflow for causal mediation
- Forming research questions
- Potential outcomes and effect decompositions
- Evaluating assumptions for causal inference
- Estimation of effects
- Technical overview of causal mediation
- Mediation analysis in the potential-outcomes framework
- Total, direct, and indirect effects
- Comparison of potential outcomes and classical mediation analysis
- Accounting for treatment–mediator interaction
- Assumptions for causal identification
- Examples
  - Example 1: A simple causal mediation model
  - Example 2: Including covariates and relaxing the no-interaction assumption
  - Example 3: Referring to treatment effects using an alternative naming scheme
  - Example 4: Causal mediation model with a binary mediator
  - Example 5: Causal mediation model with a binary outcome
  - Example 6: Causal mediation model with a binary mediator and binary outcome
  - Example 7: Causal mediation model with a count mediator
  - Example 8: Causal mediation model with an exponential-mean outcome
  - Example 9: Causal mediation model with multivalued treatment
  - Example 10: Causal mediation model with continuous treatment
  - Example 11: Estimating controlled direct effects
  - Example 12: Estimating treatment effects on different scales

Introduction

Causal inference is an essential goal in many research areas and aims at identifying and quantifying causal effects. For example, we might wish to find out whether physical exercise leads to an improvement in self-perceived well-being, and if so, to what extent. Causality in this context typically means that there is some cause \( T \) that has an effect on some outcome \( Y \). We could visualize this relation with a simple causal diagram:

\[
\begin{array}{c}
T \\
\rightarrow \\
Y
\end{array}
\]

Figure 1

If \( T \) is a measure of exercise and \( Y \) is well-being, then under certain assumptions, we could use the above causal model to identify the total effect of exercise on well-being (by means of a randomized controlled trial, for instance). However, a question that we cannot answer empirically with our simple causal model is why exercise may increase well-being. Perhaps exercising causes an increase in certain chemicals or hormones in the human body, which in turn affects perceptions of well-being. To assess such intermediary effects, we need to expand our simple causal model by adding variables that lie on the causal pathway between \( T \) and \( Y \):

\[
\begin{array}{c}
T \\
\rightarrow \\
M \\
\rightarrow \\
Y
\end{array}
\]

Figure 2

Suppose that, in our exercise example, the variable \( M \) represents the production of a certain chemical in the human body. With this new model, we now hypothesize that exercising leads to the production of this chemical, which in turn leads to an increase in well-being. However, it might be unrealistic to assume that the effect of exercise on well-being hinges exclusively on the production
of that chemical. Perhaps we would like to allow for the possibility that exercise has an effect on well-being beyond its path through the mediating variable, and so a better model might be

\[ \begin{align*}
T & \rightarrow M \\
M & \rightarrow Y \\
T & \rightarrow Y
\end{align*} \]

Here, we include a direct path from \( T \) to \( Y \) in addition to the indirect path of \( T \) to \( Y \) via \( M \). In other words, we assume that exercise produces a particular chemical that affects well-being, but we also allow for the possibility of a direct effect of exercise on well-being that is not related to the chemical. This is the classical mediation model that decomposes the total effect into a direct and an indirect effect. Causal mediation analysis aims to identify these direct and indirect effects and give them a causal interpretation.

\textbf{Approaches to mediation analysis}

Mediation analysis can be performed in a variety of ways. The classical approach of Baron and Kenny (1986) fits two linear regression models, one for \( M \) and one for \( Y \), and estimates direct, indirect, and total effects as functions of the coefficients. Estimation can be simplified by fitting the models for \( M \) and \( Y \) simultaneously via structural equation modeling as discussed in [SEM] Example 42g. In Stata, you can use \texttt{sem} to fit linear models for the outcome and mediator, and you can then use \texttt{estat teffects} to obtain a decomposition of direct and indirect effects based on the results from \texttt{sem}. Note that this classical approach relies on the specification of a particular model at the outset of the process.

Another approach to mediation analysis is based on the potential-outcomes framework. The potential outcomes are values of the outcome that would be obtained under different conditions, such as when the treatment occurs. Differences in potential outcomes yield direct, indirect, and total effects of interest. This is the approach typically referred to as causal mediation analysis and is the one implemented in \texttt{mediate}.

The causal mediation framework allows much flexibility. In this framework, it is common to allow the mediator and the treatment to interact; thus, we do not assume that the effect of the mediator on the outcome is the same for the treated and untreated groups. The total effect of the treatment on the outcome can be decomposed into direct and indirect effects in two ways, and the researcher can select the decomposition that matches his or her research question. The effects are defined in a model-free manner, so the researcher can select an estimation method that is appropriate for his or her data and then compute estimates of the effects of interest.

In the situation where both the outcome and the mediator are modeled using linear regression and there is no treatment–mediator interaction, the classical approach and causal mediation via the potential-outcomes framework will lead to the same results.

\textbf{Workflow for causal mediation}

The general workflow for researchers performing causal mediation analysis is as follows:

1. Specify your research question.
2. Identify the treatment, mediator, and outcome variables to be analyzed.
3. Determine which effect decomposition can be used to answer your research question.
4. Evaluate whether assumptions for causal interpretation are appropriate.

5. Select a method for estimating the causal effects of interest.

6. Interpret the results.

In our introductory discussion, we provided an example of step 2 by using exercise, chemical production, and well-being as the treatment, mediator, and outcome variables, respectively. Below, we will provide a conceptual introduction to steps 1, 3, 4, and 5. In *Examples*, we will interpret the results in different scenarios.

### Forming research questions

Before performing causal mediation analysis, we must decide which research questions motivated the desire to perform the analysis. Here are some types of research questions that may arise:

1. A scenario in which the primary interest is to determine whether there is an indirect effect and, if so, to quantify it. In our example above, we might assume there will be some direct impact of exercise on well-being, but we also wonder if and to what extent there is an indirect effect, such as exercise increasing production of a chemical which in turn increases well-being. In this case, we would be interested in decomposing the effects according to ATE decomposition 1.

2. A scenario in which the primary interest is to determine whether there is a direct effect and, if so, to quantify it. Continuing with our example, perhaps we expect an indirect effect, but we also wish to determine if there are any other ways in which exercise causes changes in well-being. In this case, we would be interested in decomposing the effects according to ATE decomposition 2.

3. A scenario in which the primary interest is to determine how the total effect can be decomposed into direct and indirect effects, with focus remaining on all effects and not just direct or just indirect effects. In our example, we simply want to explore the breakdown of the total effect of exercise on well-being into all possible direct and indirect effects. In this case, we would likely be interested in looking at both decompositions, ATE decomposition 1 and ATE decomposition 2.

4. A scenario in which we want to determine the effect of the treatment on the outcome when the mediator is set to a specific value. In our example, we might want to know the effect of exercise on well-being for individuals whose level of this particular chemical is 10, which is the mean value in the population. In this case, we would be interested in controlled direct effects.

### Potential outcomes and effect decompositions

Below, we introduce statistics that may be of interest when performing causal mediation analysis. Many of these statistics have a variety of names in the causal mediation literature. See, for instance, Robins and Greenland (1992), VanderWeele (2015), and Pearl and MacKenzie (2018) for some of the various terminology.

1. **Potential-outcome means.** These estimate the population-average value of the outcome that would be expected if everyone was given the treatment (denoted here as $Y[1, M(1)]$) or if everyone was given the control (denoted $Y[0, M(0)]$). In our example, $Y[1, M(1)]$ is the expected average well-being if everyone exercises, and $Y[0, M(0)]$ is the expected average well-being if no one exercises.

In addition, there are two cross-world potential outcomes. These are a bit less intuitive because they correspond to situations that do not exist for any individual in the population. The first is
the expected value of the outcome when everyone is treated but counterfactually experiences
the value of the mediator associated with being untreated (denoted $Y[1, M(0)]$). The second is
the expected value of the outcome when everyone is untreated but counterfactually experiences
the value of the mediator associated with being treated (denoted $Y[0, M(1)]$). In our example,
$Y[1, M(0)]$ is the expected well-being if everyone was treated but experiencing the chemical
level as if untreated. $Y[0, M(1)]$ is the expected value if everyone was untreated but experiencing
the chemical level as if treated.

The `mediate` command reports these potential-outcome means when the `pomeans` option is
specified.

2. **Total effect (TE).** This estimates the average difference in outcomes that we expect when
everyone receives the treatment versus when no one receives the treatment. In our case, it
estimates the improvement in well-being that we would expect if everyone exercises versus if
no one exercises.

   The total effect is also referred to as the average treatment effect (ATE), the total average
treatment effect, or the marginal total effect.

   The `mediate` command reports this statistic when the `te` option or its synonym `ate` is specified.

   The total effect can be decomposed into direct and indirect effects in two ways when we allow
for a treatment–mediator interaction.

**Decomposition 1.** This decomposition separates the direct effect under the untreated mediator
condition from the total indirect effect. Nguyen, Schmid, and Stuart (2021) recommend using
this decomposition when a direct effect is assumed and the researcher is questioning whether a
mediation effect also exists. In our example, we would be interested in this decomposition if we
expect that exercise has a direct effect on well-being but want to determine whether a portion of
the total effect can be attributed to the increase in the chemical (and if so, how much of the total
effect is due to this mediation effect).

3. **Natural direct effect (NDE).** This estimates the average direct effect of the treatment on
the outcome when the mediator is held at its value associated with being untreated. It is the
difference $Y[1, M(0)] - Y[0, M(0)]$.

   This effect is sometimes referred to as the pure natural direct effect or the average direct
treatment effect (ADTE). All remaining effects of the treatment on the outcome are included in
the natural indirect effect.

   The `mediate` command reports this statistic when the `nde` option or its synonym `adte` is specified.

4. **Natural indirect effect (NIE).** This estimates the average indirect effect through a mediator. It
is the difference $Y[1, M(1)] - Y[1, M(0)]$.

   This effect is sometimes referred to as the total natural indirect effect, causal mediation effect,
or average indirect treatment effect (AITE).

   The `mediate` command reports this statistic when the `nie` option or its synonym `aite` is specified.

**Decomposition 2.** This decomposition separates the indirect effect under the untreated condition
from the total direct effect. Nguyen, Schmid, and Stuart (2021) recommend using this decomposition
when an indirect effect is assumed and the researcher is questioning whether a direct effect also
exists. In our example, we would be interested in this decomposition if we believe that exercise
increases production of the chemical which in turn increases well-being but want to determine if
there is also some change in well-being that is not caused by this mediation effect (and if so, how
much of the total effect is not due to the mediation effect).
5. **Pure natural indirect effect (PNIE).** This estimates the average indirect effect of a mediator under the untreated/control condition. It is the difference \( Y[0, M(1)] - Y[0, M(0)] \).

This is sometimes referred to as the average indirect treatment effect with respect to controls (AITEC). All remaining effects of the treatment on the outcome are included in the total natural direct effect.

The `mediate` command reports this statistic when the `pnie` option or its synonym `aitec` is specified.

6. **Total natural direct effect (TNDE).** This estimates the average direct treatment effect when the mediator is held at its value associated with being treated. It is the difference \( Y[1, M(1)] - Y[0, M(1)] \).

This effect is sometimes referred to as the average direct treatment effect with respect to the treated (ADTET).

The `mediate` command reports this statistic when the `tnde` option or its synonym `adtet` is specified.

When no prior assumptions are made about the existence of direct and indirect effects, Nguyen, Schmid, and Stuart (2021) recommend reporting both Decomposition 1 and Decomposition 2.

7. **Controlled direct effects.** These are the direct effects when the mediator is controlled by setting it to a specific value. After fitting your model with `mediate`, you can estimate the average controlled direct effect with the mediator set to your selected value by using `estat cde`; see Example 11: Estimating controlled direct effects. In our well-being example, controlled direct effects provide the direct effect of exercise on well-being when the chemical is assumed to be a specific value.

---

**Evaluating assumptions for causal inference**

Before proceeding to estimation and interpretation of the effects of interest, we need to verify that it is reasonable to give them a causal interpretation in our particular research context.

General assumptions for causal inference are discussed in [CAUSAL] Intro, and more precise definitions in the context of mediation are provided in Assumptions for causal identification below.

To evaluate whether assumptions of causality are met for our mediation model, we must first consider all potential variables, both observed and unobserved, that could affect the relationships among our treatment, mediator, and outcome. If we anticipate that there are confounders (variables that affect both an outcome and a predictor), we must determine whether these confounders will lead to biased results in the estimation of effects from our mediation analysis. In particular, we want to assume that

1. There is no unobserved confounding in the treatment–outcome relationship, and observed confounders are included as covariates in the outcome model.
2. There is no unobserved confounding in the mediator–outcome relationship, and observed confounders are included as covariates in the outcome model.
3. There is no unmeasured confounding in the treatment–mediator relationship, and observed confounders are included as covariates in the mediator model.
4. There are no confounders in the mediator–outcome relationship that are caused by the treatment. No variable exists that affects both the mediator and the outcome and that itself is caused by the treatment.
Estimation of effects

When assumptions are met, the `mediate` command can be used to estimate the causal parameters of interest.

While the effects derived under the potential-outcomes framework required no particular model, we now need to decide how to model our data to obtain estimates.

We first select models for the outcome and the mediator. Outcomes can be continuous, binary, or counts and can be modeled using a linear, exponential-mean, logistic, probit, or Poisson model. Mediators can also be continuous, binary, or counts and can also be modeled using a linear, exponential-mean, logistic, probit, or Poisson model. Covariates can be included in the outcome and mediator models. The treatment may be binary, categorical (multi-valued), or continuous.

As a simple example of the `mediate` command, say that we have a binary outcome $y$, a continuous mediator $m$, and a binary treatment $t$. We can fit a mediation model by typing

```
. mediate (y, probit) (m, linear) (t)
```

The first set of parentheses specifies a model for the outcome. The second set of parentheses specifies the model for the mediator. The third set of parentheses defines the treatment. By default, the TE and its decomposition into NDE and NIE are reported in the output.

If we would instead like to see the second type of decomposition, we can obtain the TE, PNIE, and TNDE by typing

```
. mediate (y, probit) (m, linear) (t), te pnie tnde
```

Many combinations of models and effects can be obtained. See Examples below for additional syntax examples as well as interpretation of the results.

Technical overview of causal mediation

Above, we provided a conceptual introduction to the concepts in causal mediation analysis. Here we more formally define the potential-outcomes framework; explain total, direct, and indirect effects; and introduce the assumptions necessary for causal inference. If you are familiar with the technical aspects of causal mediation and are ready to see `mediate` demonstrated, go directly to Examples.

Mediation analysis in the potential-outcomes framework

The potential-outcomes framework is commonly employed for identifying causal effects. If we go back to the model associated with figure 1 and assume $T$ is a binary treatment, we can identify two sets of potential outcomes, $Y_i(1)$ and $Y_i(0)$. $Y_i(t)$ is the outcome that would be realized if the $i$th individual were exposed to treatment level $t$.

Consider a randomized experiment where the experimental group exercises while the control group spends the same amount of time in a resting state. The outcome is subjective well-being that is measured after exercising/resting. If it were possible to observe an individual in both states at the same time, we would observe one outcome value under treatment, $Y_i(1)$, and one value under the control condition, $Y_i(0)$. Then the treatment effect would be the difference $\tau_i = Y_i(1) - Y_i(0)$. In other words, there is a potential outcome for each treatment level that could be administered. Averaging the difference over all individuals in the sample would yield an estimate of the ATE $\tau = E[Y_i(1) - Y_i(0)] = E[Y_i(1)] - E[Y_i(0)]$. 
However, it is not possible to observe the same individual under both conditions at the same time; we can only observe one of these while the other is missing. If an individual is treated, we observe $Y_i(1)$, and if not, we observe $Y_i(0)$. This has been coined the “fundamental problem of causal inference” (Holland 1986). Much of the treatment effects and causal inference literature deals with the question of how to estimate an ATE in the presence of this problem.

In a simple experiment where treatment is randomly assigned, the potential outcomes are independent of treatment assignment and the missing potential outcomes are missing completely at random. In this situation, we have another potential-outcome means. The treatment effect is identified if $\tau = E[Y_i(1)] - E[Y_i(0)]$ where $E[Y_i(t)] = E(Y_i|T_i = t)$ such that $\tau = E[Y_i(1)] - E[Y_i(0)] = E[Y_i|T_i = 1] - E[Y_i|T_i = 0]$.

With observational rather than experimental data, however, the potential outcomes are not independent of the treatment assignment process, and the causal effect is not identifiable without imposing further assumptions such as conditional independence. Stata’s t/effects suite of commands provides a variety of estimators from this class of treatment-effects estimators.

For further information about identification and estimation in the context of causal models as well as an overview of estimators implemented in Stata, see [CAUSAL] Intro. Here we focus on causal inference and potential outcomes specifically for mediation analysis. In this situation, we have another set of potential outcomes, $M_i(1)$ and $M_i(0)$, because $M$ is also affected by the treatment. That is, we can only observe $M_i(1)$ for the group of individuals who were treated, and we can only observe $M_i(0)$ for the controls. If we let $t$ denote the treatment level with respect to the outcome and let $t'$ be the treatment level with respect to the mediator, then the potential outcomes become $Y_i[t, M_i(t')]$.

Similar to the nonmediation case above, we can define a treatment effect as a difference between potential outcomes. The treatment effect is identified if

$$E[Y_i(t, M_i(t'))] = E_{M_i|T_i=t'}E[Y_i|M_i, T_i = t]$$

where $E_{M_i|T_i=t'}$ is the expectation of the mediator conditional on the treatment taking on the value $t'$ and where $E[Y_i|M_i, T_i = t]$ is the expectation of the outcome conditional on the mediator and treatment taking on the value $t$.

### Total, direct, and indirect effects

In mediation analysis, we are interested not only in the total treatment effect but also in its decomposition into direct effects and indirect effects.

Notice that if $t = t'$ for a given potential outcome, the resulting potential outcome is equivalent to $Y_i(t)$. Assuming again a binary treatment, we have that

$$\tau = E[Y_i(1)] - E[Y_i(0)] = E[Y_i(1, M_i(1))] - E[Y_i(0, M_i(0))]$$

In the context of mediation analysis, this treatment effect is also referred to as the total effect.

The total effect can be decomposed further into direct and indirect effects using contrasts between potential-outcome means. The contrasts yielding direct and indirect effects use potential outcomes for which $t \neq t'$, which means we set the treatment level to $t$ and set the mediator to its potential value under treatment level $t'$.

The natural indirect effect is then defined as

$$\delta(t) \equiv E[Y_i(t, M_i(1))] - E[Y_i(t, M_i(0))], \quad t \in \{0, 1\}$$
Notice that here we “switch” the treatment from on to off in its effect on the mediator but keep the treatment fixed at value $t$ in its effect on the outcome. This natural indirect effect is also sometimes referred to as the causal mediation effect (Imai, Keele, and Tingley 2010).

Likewise, the natural direct effect can be defined as

$$\zeta(t) \equiv E[Y_i(1, M_i(t))] - E[Y_i(0, M_i(t))], \quad t \in \{0, 1\}$$

**Comparison of potential outcomes and classical mediation analysis**

For those familiar with classical mediation analysis for linear models, it may be helpful to see how the calculation of total, direct, and indirect effects in the potential-outcomes framework relates to the classical product-of-coefficients approach.

We first write our mediation model corresponding with figure 3 as

$$Y_i = \beta_0 + \beta_1 M_i + \beta_2 T_i + \epsilon_i$$

$$M_i = \alpha_0 + \alpha_1 T_i + \nu_i$$

where $\epsilon_i$ and $\nu_i$ are uncorrelated error terms with means 0 and variances $\sigma_{\epsilon}^2$ and $\sigma_{\nu}^2$, respectively.

Let’s consider the indirect effect $\delta(1)$. To calculate $\delta(1)$ in the potential-outcomes framework, we need estimates for the potential-outcome means $E[Y_i(1, M_i(1))]$ and $E[Y_i(1, M_i(0))]$. Intuitively, what we want is a world where everyone in the population is exposed to the treatment, that is, $Y_i(1)$, but where we can switch the treatment on and off in regard to the effect of the treatment on the mediator, that is, $M_i(1)$ and $M_i(0)$. The difference when going from the treatment switched on to the treatment switched off will inform us about the effect of the treatment on the outcome that goes through the mediator. First, we write the above model in reduced form:

$$E[Y_i| M_i, T_i] = \beta_0 + \beta_1 (\alpha_0 + \alpha_1 T_i) + \beta_2 T_i$$

This yields the conditional expectation $E[Y_i|M_i, T_i]$ that we can observe from the data.

To obtain the potential-outcome means, we can modify the reduced-form model by replacing $M_i$ with the expectation of $M_i$ that we would observe if $T_i$ had taken on the value $t'$ for every unit in the population. That is,

$$E[Y_i(t, M_i(t'))] = \beta_0 + \beta_1 E[M_i(t')] + \beta_2 t', \quad t \in \{0, 1\}$$

Thus, to compute the potential-outcome mean $E[Y_i(1, M_i(1))]$, we must set the treatment $T_i$ to 1 in both the outcome and the mediator equations. In other words, we fix both $t$ and $t'$ at 1:

$$E[Y_i(1, M_i(1))] = \beta_0 + \beta_1 E[M_i(t')] + \beta_2 t, \quad t = t' = 1$$

$$= \beta_0 + \beta_1 \alpha_0 + \beta_1 \alpha_1 \times 1 + \beta_2 \times 1$$

$$= \beta_0 + \beta_1 \alpha_0 + \beta_1 \alpha_1 + \beta_2$$
However, to compute \( E[Y_i(1, M_i(0))] \), we need to set treatment \( T_i \) to 1 in the outcome equation and need to set it to 0 in the mediator equation. Specifically, we fix \( t' = 0 \) and \( t = 1 \):

\[
E[Y_i(1, M_i(0))] = \beta_0 + \beta_1 E[M_i(t')] + \beta_2 t, \quad t = 1; \quad t' = 0
\]
\[
= \beta_0 + \beta_1 \alpha_0 + \beta_1 \alpha_1 \times 0 + \beta_2 \times 1
\]
\[
= \beta_0 + \beta_1 \alpha_0 + \beta_2
\]

Calculating the difference between these two potential-outcome means yields the indirect treatment effect

\[
\delta(1) = (\beta_0 + \beta_1 \alpha_0 + \beta_1 \alpha_1 + \beta_2) - (\beta_0 + \beta_1 \alpha_0 + \beta_2)
\]
\[
= \beta_0 + \beta_1 \alpha_0 + \beta_1 \alpha_1 + \beta_2 - \beta_0 - \beta_1 \alpha_0 - \beta_2
\]
\[
= \beta_1 \alpha_1
\]

In this case of a linear model, the indirect treatment effect is the product of the treatment coefficient from the mediator equation and the mediator coefficient from the outcome equation. This is congruent with the indirect effect definition in the product-of-coefficients method for mediation as proposed by the classical mediation literature; see Baron and Kenny (1986).

### Accounting for treatment–mediator interaction

Notice that the indirect effect we estimated above would be the same if we had estimated \( \delta(0) \) instead. Thus far, we assumed that the effect of the mediator on the outcome is the same for both treatment groups. Presumably, a more realistic assumption would be to allow the mediator effects to vary by treatment. This can be achieved by including a treatment–mediator interaction term.

When we allow an interaction, \( \delta(0) \neq \delta(1) \). Now we have two indirect effects, one with respect to treatment \([\delta(1)]\) and one with respect to controls \([\delta(0)]\). In the following, we will refer to \( \delta(1) \) as the NIE and to \( \delta(0) \) as the PNIE.

To illustrate computation of the NIE under inclusion of a treatment–mediator interaction, we write a new model

\[
Y_i = \beta_0 + \beta_1 M_i + \beta_2 T_i + \beta_3 M_i T_i + \epsilon_i
\]
\[
M_i = \alpha_0 + \alpha_1 T_i + \nu_i
\]

Here, \( \text{NIE} \equiv E[Y_i(1, M_i(1)) - Y_i(1, M_i(0))] \), whereas \( \text{PNIE} \equiv E[Y_i(0, M_i(1)) - Y_i(0, M_i(0))] \).

As before, to calculate NIE, we need potential-outcome means \( E[Y_i(1, M_i(1))] \) and \( E[Y_i(1, M_i(0))] \). Writing the model in reduced form, we get

\[
E[Y_i|M_i, T_i] = \beta_0 + \beta_2 T_i + \beta_1 (\alpha_0 + \alpha_1 T_i) + \beta_3 T_i (\alpha_0 + \alpha_1 T_i)
\]
\[
= \beta_0 + \beta_2 T_i + (\beta_1 + \beta_3 T_i)(\alpha_0 + \alpha_1 T_i)
\]
Fixing the values for the treatment in both equations accordingly, we have potential-outcome means

\[ E[Y_i(1, M_i(1))] = \beta_0 + \beta_2 \times 1 + (\beta_1 + \beta_3) (\alpha_0 + \alpha_1 \times 1) \]

and

\[ E[Y_i(1, M_i(0))] = \beta_0 + \beta_2 \times 1 + (\beta_1 + \beta_3) (\alpha_0 + \alpha_1 \times 0) \]

Taking the difference yields the NIE

\[ E[Y_i(1, M_i(1))] - E[Y_i(1, M_i(0))] = (\beta_1 + \beta_3) \alpha_1 \]

We could proceed similarly for the other direct and indirect treatment effects. In this case with treatment–mediator interaction, we also have two direct treatment effects. We have NDE \( \equiv E[Y_i(1, M_i(0)) - Y_i(0, M_i(0))] \) and TNDE \( \equiv E[Y_i(1, M_i(1)) - Y_i(0, M_i(1))] \).

Notice that both treatment-effect decompositions—NIE and NDE as well as PNIE and TNDE—sum to the total treatment effect (or, as we will call it, the TE).

\[ \text{TE} \equiv E[Y_i(1, M_i(1)) - Y_i(0, M_i(0))] \]

For further details and discussion on the different direct and indirect effects, as well as a discussion on the differences between causal inference and traditional mediation approaches, see Nguyen, Schmid, and Stuart (2021).

**Assumptions for causal identification**

The above discussion shows that the estimands of interest are the result of contrasts between potential-outcome means, which are conditional expectations of the outcome with respect to counterfactuals in both the outcome and the mediator equations. In other words, once we can estimate \( E[Y_i(t, M_i(t'))|X_i] \), we can estimate all direct and indirect treatment effects of interest.

The general form for causal mediation potential-outcome means, which includes covariates \( X_i \), can be written as the integral of the conditional expectation of the outcome with respect to the conditional distribution of the mediator (see Imai, Keele, and Tingley [2010]):

\[ E[Y_i(t, M_i(t'))|X_i = x] = \int E[Y_i|M_i = m, T_i = t, X_i = x] dF[m|T_i = t', X_i = x] \]

This is a general, nonparametric solution that applies regardless of the underlying outcome and mediator models. Stata’s `mediate` command uses analytical solutions of this integral for a variety of parametric outcome and mediator model combinations. Also, while so far we assumed a binary treatment for simplicity purposes, this approach generalizes straightforwardly to multivalued as well as continuous treatments.

As is the case with nonmediation causal inference, there are assumptions to be met for the estimated effects to be given a causal interpretation. Most notably, a crucial assumption in the nonmediation case is the conditional independence assumption, also known as conditional ignorability assumption, unconfoundedness, or selection on observables. This assumption states that potential outcomes are independent of treatment assignment after conditioning on a set of observed covariates that affect both the outcome and the selection into treatment (see Imbens [2004]). Intuitively, we have a model that resembles an experiment once we account for observable characteristics. More formally, we have that

\[ Y_i(t) \perp T_i|X_i \]
In the mediation case, however, we have an additional selection process because “selection” into the mediator is also typically not based on random assignment. This leads to the following two conditional independence assumptions:

\[
\{Y_i[t,m], M_i(t')\} \perp T_i | X_i = x \\
Y_i[t,m] \perp M_i(t') | T_i = t', X_i = x
\]

The first assumption states that treatment assignment is independent of potential outcomes and potential mediators after conditioning on observed (pretreatment) covariates, or confounders. The second assumption states that potential mediators are independent of the potential outcomes given the observed treatment and observed (pretreatment) covariates. Because these assumptions are being made sequentially, this has also been coined the sequential ignorability assumption (Imai, Keele, and Tingley 2010).

Similarly, there is an additional overlap assumption with causal mediation models. In the nonmediation case, the overlap assumption states that each individual has a positive probability of receiving each treatment:

\[
0 < \Pr(T_i = t | X_i = x), \quad t \in \{0, 1\}
\]

In the mediation case, the same principle applies to the mediator:

\[
0 < p(M_i(t) = m | T_i = t, X_i = x), \quad t \in \{0, 1\}
\]

Finally, as is the case with nonmediation treatment-effects models, causal mediation models rely on the stable unit treatment-value assumption, which states that potential outcomes do not depend on treatments assigned to other individuals. For a detailed overview of effect identification and assumptions for causal mediation analysis, see Nguyen et al. (2022).

Examples

**Example 1: A simple causal mediation model**

Suppose we wish to find out whether exercise affects perceptions of well-being among some population of individuals. To the extent that there is such a causal relationship, we also wish to find out why exercise affects well-being.

We have fictional data from a randomized controlled trial with individuals randomized into two groups—one group performs physical exercise and the other group spends the same amount of time in a resting state. Subjective well-being is measured before and after treatment sessions. In addition, the level of the (fictional) hormone bonotonin is measured. The researchers wish to determine whether exercise leads to an increase in bonotonin levels, which in turn has a positive effect on subjective well-being. Here is an excerpt from our dataset:

```
use https://www.stata-press.com/data/r18/wellbeing
(Fictional well-being data)
list wellbeing bonotonin exercise age gender in 1/5, abbreviate(12)
```

<table>
<thead>
<tr>
<th>wellbeing</th>
<th>bonotonin</th>
<th>exercise</th>
<th>age</th>
<th>gender</th>
</tr>
</thead>
<tbody>
<tr>
<td>71.73816</td>
<td>196.5467</td>
<td>Control</td>
<td>58</td>
<td>Male</td>
</tr>
<tr>
<td>68.66573</td>
<td>195.8572</td>
<td>Exercise</td>
<td>38</td>
<td>Female</td>
</tr>
<tr>
<td>71.05155</td>
<td>228.6035</td>
<td>Exercise</td>
<td>53</td>
<td>Female</td>
</tr>
<tr>
<td>69.44469</td>
<td>206.6651</td>
<td>Exercise</td>
<td>44</td>
<td>Female</td>
</tr>
<tr>
<td>75.62035</td>
<td>261.6855</td>
<td>Exercise</td>
<td>28</td>
<td>Female</td>
</tr>
</tbody>
</table>
To estimate the treatment effects with `mediate`, we specify `wellbeing` as the outcome variable in the first set of parentheses, `bonotonin` as the mediator variable in the second set of parentheses, and `exercise` as the binary treatment variable in the third set of parentheses. Although inclusion of a treatment–mediator interaction is commonly recommended, we specify the `nointeraction` option here to omit the interaction and fit the simplest model possible.

```
. mediate (wellbeing) (bonotonin) (exercise), nointeraction
```

Iteration 0: EE criterion = 1.627e-25
Iteration 1: EE criterion = 3.061e-28

Causal mediation analysis

Outcome model: Linear
Mediator model: Linear
Mediator variable: bonotonin
Treatment type: Binary

| wellbeing | Coefficient | std. err. | z | P>|z|  | [95% conf. interval] |
|-----------|-------------|-----------|---|------|---------------------|
| NIE       |             |           |   |      |                     |
| exercise  |             |           |   |      |                     |
| (Exercise |             |           |   |      |                     |
| vs       |             |           |   |      |                     |
| Control)  |             |           |   |      |                     |
|           | 9.694617    | .377312   | 25.69 | 0.000 | 8.955099 - 10.43413 |
| NDE       |             |           |   |      |                     |
| exercise  |             |           |   |      |                     |
| (Exercise |             |           |   |      |                     |
| vs       |             |           |   |      |                     |
| Control)  |             |           |   |      |                     |
|           | 2.996658    | .2109357  | 14.21 | 0.000 | 2.583231 - 3.410084 |
| TE        |             |           |   |      |                     |
| exercise  |             |           |   |      |                     |
| (Exercise |             |           |   |      |                     |
| vs       |             |           |   |      |                     |
| Control)  |             |           |   |      |                     |
|           | 12.69127    | .4005769  | 31.68 | 0.000 | 11.90616 - 13.47639 |

Note: Outcome equation does not include treatment–mediator interaction.

In the header of the output, we see that `mediate` fit linear models (the default) for both the outcome and the mediator. Three treatment-effect estimates are reported in the table. TE is the total effect of exercise on well-being and is estimated to be 12.7. The interpretation is the same as for the ATE in the nonmediation case: if everyone in the population exercised, their well-being would be, on average, 12.7 points higher than their well-being would be if no one exercised. The decomposition of the TE into direct and indirect effects is of primary interest. The NIE is estimated to be 9.7, whereas the NDE is estimated to be 3.0. These sum to the total effect of 12.7. The indirect effect is much larger than the direct effect, indicating that the effect of exercise on well-being is largely due to exercise affecting bonotonin levels, which in turn affect well-being. The direct effect of 3.0 is the effect of exercise on well-being beyond the effect through bonotonin.

Instead of comparing estimates of the direct and indirect effects, we might ask what proportion of the total effect is due to mediation. We can answer this question by using `estat proportion`.
The indirect effect via bonotonin accounts for 76% of the effect of physical activity on well-being, and the remaining 24% is due to other mechanisms.

**Example 2: Including covariates and relaxing the no-interaction assumption**

The previous example was somewhat unrealistic. For causal inference, we must evaluate the potential of confounding. With causal mediation models, there are three types of confounders we should consider: treatment–outcome confounders, treatment–mediator confounders, and mediator–outcome confounders. A treatment–outcome confounder, for example, is a variable that affects both the selection into treatment and the outcome. If confounders exist and we observe them in our data, we can add them as covariates to the model to prevent biased results.

Above, we noted that the wellbeing data come from a randomized controlled trial. In this case, we do not have to worry about treatment–outcome and treatment–mediator confounders because treatment assignment is random. We do, however, need to consider variables such as age, gender, and hstatus (a person’s health status) that affect both the mediator and the outcome. We include these variables as covariates in the model for well-being. We also make our model a bit more realistic by including baseline well-being in the outcome equation and baseline bonotonin level in the mediator equation. In addition, we omit the nointeraction option to allow the bonotonin coefficients to vary across treatment groups.
. mediate (wellbeing age gender i.hstatus basewell)
> (bonotonin basebono)
> (exercise)
Iteration 0:  EE criterion = 1.664e-25
Iteration 1:  EE criterion = 1.192e-28

Causal mediation analysis
Number of obs = 2,000

Outcome model:  Linear
Mediator model:  Linear
Mediator variable:  bonotonin
Treatment type:  Binary

| wellbeing | Robust Coefficient | std. err. | z   | P>|z| | [95% conf. interval] |
|-----------|--------------------|-----------|-----|------|---------------------|
| NIE       | exercise (Exercise vs Control) | 9.941404 | .2307909 | 43.08 | 0.000 | 9.489062 - 10.39375 |
| NDE       | exercise (Exercise vs Control) | 3.08372 | .1684778 | 18.30 | 0.000 | 2.753509 - 3.41393 |
| TE        | exercise (Exercise vs Control) | 13.02512 | .2356989 | 55.26 | 0.000 | 12.56316 - 13.48709 |

Note: Outcome equation includes treatment-mediated interaction.

The interpretation of the treatment effects is the same as before. The total effect of exercise on well-being is 13.0. Of this effect, 3.1 is attributed to the direct effect, while the remaining 9.9 is due to the indirect path via bonotonin. These results are similar to our simpler model above.
We find that the expected effect of exercise on well-being is 13.0, but what is the expected well-being when everyone exercises? When no one exercises? We can estimate four such potential-outcome means by specifying the `pomeans` option:

```
. mediate (wellbeing age gender i.hstatus basewell)
> (bonotonin basebono)
> (exercise), pomeans
```

Iteration 0:  EE criterion = 1.660e-25
Iteration 1:  EE criterion = 1.473e-28

Causal mediation analysis

Outcome model:  Linear
Mediator model:  Linear
Mediator variable:  bonotonin
Treatment type:  Binary

| wellbeing | Coefficient | std. err. | z     | P>|z|  | [95% conf. interval] |
|-----------|-------------|-----------|-------|-------|---------------------|
| POmeans   |             |           |       |       |                     |
| Y0M0      | 56.94195    | .2300492  | 247.52| 0.000 | 56.49107 57.39284   |
| Y1M0      | 60.02567    | .2571311  | 233.44| 0.000 | 59.52171 60.52964   |
| Y0M1      | 66.78952    | .2642177  | 252.78| 0.000 | 66.27167 67.30738   |
| Y1M1      | 69.96708    | .232508   | 300.92| 0.000 | 69.51137 70.42278   |

Note: Outcome equation includes treatment-mediator interaction.

Y1M1 is an estimate of the potential-outcome mean \( E[Y_i(1, M_i(1))] \). If everyone in the population exercised, we would expect the average of well-being to be around 70. The values labeled Y1M0 and Y0M1 are estimates of the “cross-world” potential-outcome means \( E[Y_i(1, M_i(0))] \) and \( E[Y_i(0, M_i(1))] \). For these, we set different counterfactuals in the outcome and mediator equations. In this case, the Y1M0 estimate tells us the expected average well-being if, for the outcome equation, we assume that everyone in the population exercised, but we assume that no one exercised in regard to the effect of treatment on the mediator. If we compare the Y1M1 and Y1M0 estimates, we imagine a world where everyone received the treatment, except that the treatment is switched on and off in its effect on the mediator. The difference between these is 69.96708 \(-\) 60.02567 = 9.94141, which is our NIE reported above.

By default, the TE, NIE, and NDE are computed, but we can request specific effects. For example, we could estimate only the NIE by typing

```
. mediate (wellbeing age gender i.hstatus basewell)
> (bonotonin basebono)
> (exercise), nie
```
Alternatively, we could estimate all available effects and potential-outcome means at once by specifying the `all` option:

```
> . mediate (wellbeing age gender i.hstatus basewell)
>    (bonotonin basebono)
>    (exercise), all
Iteration 0:  EE criterion = 1.668e-25
Iteration 1:  EE criterion = 1.532e-28
```

Causal mediation analysis

|                  | Coefficient | std. err. | z    | P>|z|    | [95% conf. interval] |
|------------------|-------------|-----------|------|--------|----------------------|
| POmeans          |             |           |      |        |                      |
| Y0M0             | 56.94195    | .2300492  | 247.52| 0.000  | 56.49107             | 57.39284           |
| Y1M0             | 60.02567    | .2571311  | 233.44| 0.000  | 59.52171             | 60.52964           |
| Y0M1             | 66.78952    | .2642177  | 252.78| 0.000  | 66.27167             | 67.30738           |
| Y1M1             | 69.96708    | .232508   | 300.92| 0.000  | 69.51137             | 70.42278           |
| NIE              |             |           |      |        |                      |
| exercise         |             |           |      |        |                      |
| (Exercise vs Control) | 9.941404    | .2307909  | 43.08| 0.000  | 9.489062             | 10.39375           |
| NDE              |             |           |      |        |                      |
| exercise         |             |           |      |        |                      |
| (Exercise vs Control) | 3.08372     | .1684778  | 18.30| 0.000  | 2.753509             | 3.41393            |
| PNIE             |             |           |      |        |                      |
| exercise         |             |           |      |        |                      |
| (Exercise vs Control) | 9.84757     | .2318329  | 42.48| 0.000  | 9.393186             | 10.30195           |
| TNDE             |             |           |      |        |                      |
| exercise         |             |           |      |        |                      |
| (Exercise vs Control) | 3.177554    | .1800896  | 17.64| 0.000  | 2.824585             | 3.530523           |
| TE               |             |           |      |        |                      |
| exercise         |             |           |      |        |                      |
| (Exercise vs Control) | 13.02512    | .2356989  | 55.26| 0.000  | 12.56316             | 13.48709           |

Note: Outcome equation includes treatment–mediator interaction.

Here we obtain estimates for two additional effects, PNIE and TNDE, which provide a different decomposition of the TE into direct and indirect effects. In this case, PNIE and TNDE are similar to NIE and NDE, respectively, because the coefficient on the treatment–mediator interaction term is quite small in the model for well-being. We can see the results for the underlying models, including this small coefficient of 0.002, if we add the `aequations` option to our `mediate` command.
Example 3: Referring to treatment effects using an alternative naming scheme

The effects we have discussed so far are sometimes referred to by different names. The default naming conventions originate in the works of Pearl and others. However, we can instead use terminology more closely tied to ATEs if we specify the ateterms option:

```stata
.mediate (wellbeing age gender i.hstatus basewell)
> (bonotonin basebono)
> (exercise), all ateterms
```

Iteration 0:  EE criterion = 1.668e-25
Iteration 1:  EE criterion = 1.532e-28

Causal mediation analysis

Number of obs = 2,000
Outcome model:  Linear
Mediator model:  Linear
Mediator variable:  bonotonin
Treatment type:  Binary

| wellbeing | Coefficient | std. err. | z | P>|z| | [95% conf. interval] |
|-----------|-------------|-----------|---|------|----------------------|
| Y0M0      | 56.94195    | .2300492  | 247.52 | 0.000 | 56.49107  | 57.39284 |
| Y1M0      | 60.02567    | .2571311  | 233.44 | 0.000 | 59.52171  | 60.52964 |
| Y0M1      | 66.78952    | .2642177  | 252.78 | 0.000 | 66.27167  | 67.30738 |
| Y1M1      | 69.96708    | .232508   | 300.92 | 0.000 | 69.51137  | 70.42278 |

| AITE exercise (Exercise vs Control) | 9.941404 | .2307909 | 43.08 | 0.000 | 9.489062  | 10.39375 |

| ADTE exercise (Exercise vs Control) | 3.08372 | .1684778 | 18.30 | 0.000 | 2.753509  | 3.41393 |

| AITEC exercise (Exercise vs Control) | 9.84757 | .2318329 | 42.48 | 0.000 | 9.393186  | 10.30195 |

| ADTET exercise (Exercise vs Control) | 3.177554 | .1800896 | 17.64 | 0.000 | 2.824585  | 3.530523 |

| ATE exercise (Exercise vs Control) | 13.02512 | .2356989 | 55.26 | 0.000 | 12.56316  | 13.48709 |

Note: Outcome equation includes treatment-mediator interaction.

Using this notation, ATE can be decomposed into AITE and ADTE or into AITEC and ADTET. Notice that the estimates are the same as in the previous example; they now just have different names.
Example 4: Causal mediation model with a binary mediator

In the previous examples, both outcome and mediator variables were continuous. We now look at the case where the mediator variable is binary. To this end, we use the binary variable bbonotonin, an indicator of higher bonotonin levels after exercise, where improvement is defined as an increase of at least 10%. We could use a probit or a logit model for this mediator; we choose a logit model:

```
. mediate (wellbeing age gender i.hstatus basewell)
>    (bbonotonin, logit)
>    (exercise)
```

```
Iteration 0: EE criterion = 8.253e-18
Iteration 1: EE criterion = 8.223e-18 (backed up)
```

```
Causal mediation analysis

Number of obs = 2,000

Outcome model: Linear
Mediator model: Logit
Mediator variable: bbonotonin
Treatment type: Binary
```

| wellbeing | Robust | Coefficient | std. err. | z | P>|z| | [95% conf. interval] |
|-----------|--------|-------------|-----------|---|-----|-----------------------|
| NIE       | exercise (Exercise vs Control) | 4.41435 | .4666635 | 9.46 | 0.000 | 3.499706 | 5.328994 |
| NDE       | exercise (Exercise vs Control) | 8.429238 | .5696256 | 14.80 | 0.000 | 7.312792 | 9.545683 |
| TE        | exercise (Exercise vs Control) | 12.84359 | .3712965 | 34.59 | 0.000 | 12.11586 | 13.57132 |

Note: Outcome equation includes treatment-mediator interaction.

Direct and indirect effect estimates differ from previous results because we used a different bonotonin measure as our mediator variable. However, because we still have the continuous well-being outcome, the interpretation of the effects is the same as before. Here we estimate a total effect of 12.8 with direct and indirect effects of 8.4 and 4.4, respectively. That is, we expect an increase of 12.8 in well-being due to treatment, of which 4.4 is due to an increase in bonotonin levels whereas the remaining 8.4 is due to other mechanisms.
Example 5: Causal mediation model with a binary outcome

Interpretation of effects did not change with a binary mediator, but interpretation does change when we specify a different type of outcome.

To demonstrate, we return to the continuous mediator but use a binary outcome variable. The outcome \texttt{bwellbeing} indicates higher well-being and is defined as an increase in well-being of at least 10% compared with the baseline measurement. Using \texttt{bwellbeing} as the outcome variable and specifying a probit outcome model, we get

```plaintext
 mediation (bwellbeing age gender i.hstatus, probit)  
   (bonotonin basebono, linear)  
   (exercise)
```

Iteration 0:  \( EE \) criterion = 2.177e-25
Iteration 1:  \( EE \) criterion = 9.730e-29

Causal mediation analysis

<table>
<thead>
<tr>
<th>Outcome model:</th>
<th>Probit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mediator model:</td>
<td>Linear</td>
</tr>
<tr>
<td>Mediator variable:</td>
<td>bonotonin</td>
</tr>
<tr>
<td>Treatment type:</td>
<td>Binary</td>
</tr>
</tbody>
</table>

| bwellbeing | Coefficient | std. err. | z  | P>|z| | [95\% conf. interval] |
|------------|-------------|-----------|----|------|---------------------|
| NIE        |             |           |    |      |                     |
| exercise   |             |           |    |      |                     |
| (Exercise vs Control) | .2346259 | .0145763 | 16.10 | 0.000 | .2060568 .263195 |

| NDE        |             |           |    |      |                     |
| exercise   |             |           |    |      |                     |
| (Exercise vs Control) | .033732 | .0237585 | 1.42 | 0.156 | -.0128338 .0802978 |

| TE         |             |           |    |      |                     |
| exercise   |             |           |    |      |                     |
| (Exercise vs Control) | .2683579 | .0200872 | 13.36 | 0.000 | .2289877 .3077281 |

Note: Outcome equation includes treatment-mediator interaction.

We interpret the effects as expected differences measured on the probability scale, sometimes referred to as risk differences. The TE of 0.27 indicates that if everyone in the population exercised, we would expect the probability of increased well-being to be 0.27 higher than the probability of increased well-being if no one exercised. In other words, the chance of experiencing an increase in well-being goes up by 27 percentage points when exposed to the exercise treatment. We can see that about 23 points are due to the indirect path via bonotonin, and about 3 points are due to other mechanisms.
Example 6: Causal mediation model with a binary mediator and binary outcome

We could also have the case where both the outcome and the mediator are binary. Here we use a logit model for both:

```
. mediate (bwellbeing age gender i.hstatus, logit)
> (bbonotonin, logit)
> (exercise)
```

Iteration 0: EE criterion = 4.223e-16
Iteration 1: EE criterion = 2.107e-30

Causal mediation analysis

<table>
<thead>
<tr>
<th>Treatment type</th>
<th>Number of obs = 2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome model</td>
<td>Logit</td>
</tr>
<tr>
<td>Mediator model</td>
<td>Logit</td>
</tr>
<tr>
<td>Mediator variable</td>
<td>bbonotonin</td>
</tr>
<tr>
<td>Treatment type</td>
<td>Binary</td>
</tr>
</tbody>
</table>

| bwellbeing | Robust | z    | P>|z| | [95% conf. interval] |
|------------|--------|------|------|----------------------|
| NIE        |        |      |      |                      |
| exercise   | .0959618 .0288699 3.32 0.001 .0393778 .1525457 |
| (Exercise vs Control) |      |      |      |                      |
| NDE        |        |      |      |                      |
| exercise   | .1676141 .0358902 4.67 0.000 .0972706 .2379577 |
| (Exercise vs Control) |      |      |      |                      |
| TE         |        |      |      |                      |
| exercise   | .2635759 .0212488 12.40 0.000 .221929 .3052228 |
| (Exercise vs Control) |      |      |      |                      |

Note: Outcome equation includes treatment–mediator interaction.

The interpretation is again in terms of differences in probabilities. We observe a TE of around 0.26, which is partially due to the indirect effect via bbonotonin (0.10) and partially due to other mechanisms (0.17).

Example 7: Causal mediation model with a count mediator

We use a fictional dataset on birthweights and demonstrate how to perform causal mediation analysis when using a Poisson model for a count mediator.

We now pretend to have observational data instead of experimental data. The sample includes women who gave birth to a child. We wish to find out whether socioeconomic status and education of the mother affects the child’s health. The outcome variable is the birthweight of the baby (bweight), and the treatment variable is whether or not the mother has a college degree (college). The mediator variable is the number of cigarettes smoked per day during pregnancy (ncigs). The hypothesis is that women with a higher educational degree are likely to smoke fewer cigarettes and that smoking during pregnancy has negative effects on birthweight. Here is an excerpt from the dataset:
We fit a linear model for the outcome `bweight` and a Poisson model for the mediator `ncigs`, and we specify `college` as the binary treatment variable. Because we have fully observational data, where selection into treatment is no longer completely random, we have to be concerned about all confounder types as mentioned in `Example 2: Including covariates and relaxing the no-interaction assumption`. We specify several potential confounders as covariates in both equations.

We do not assume that the adverse effects of smoking are different between women with a college degree and women without a college degree. Therefore, we use the `nointeraction` option.

As before, the type of model we use for the mediator does not affect the interpretation of the estimated treatment effects. Effects are expected differences on the scale of the outcome variable. The TE indicates that if all women had a college degree, the average birthweight of newborn babies would be almost 515 grams higher than the average birthweight if no woman had a college degree. Of this weight increase, around 167 grams are due to women with higher educational degrees smoking less, while 347 grams are due to other mechanisms.
Example 8: Causal mediation model with an exponential-mean outcome

Here we use an exponential-mean model for the outcome \( bweight \).

```
. mediate (bweight sespar c.age##c.age, expmean)
> (ncigs sespar c.age##c.age, poisson)
> (college), nointeract
```

Iteration 0:  EE criterion = 3.250e-13
Iteration 1:  EE criterion = 1.159e-17

Causal mediation analysis  Number of obs = 2,000
Outcome model:  Exponential mean
Mediator model:  Poisson
Mediator variable:  ncigs
Treatment type:  Binary

|                  | Coefficient | std. err. | z     | P>|z|  | [95% conf. interval] |
|------------------|-------------|-----------|-------|------|----------------------|
|                  |             | Robust    |       |      |                      |
| NIE college      |             |           |       |      |                      |
| (Yes vs No)      | 198.978     | 23.53279  | 8.46  | 0.000| 152.8546  245.1014   |
| NDE college      |             |           |       |      |                      |
| (Yes vs No)      | 320.3318    | 34.47792  | 9.29  | 0.000| 252.7563  387.9072   |
| TE college       |             |           |       |      |                      |
| (Yes vs No)      | 519.3098    | 28.70435  | 18.09 | 0.000| 463.0503  575.5693   |

Note: Outcome equation does not include treatment–mediator interaction.

Because we are still modeling a continuous outcome, the interpretation does not change. The TE is about 519 grams, of which 199 grams are due to women with a college degree smoking less.

Example 9: Causal mediation model with multivalued treatment

So far we have only dealt with treatments that are binary. However, experiments often have more than two treatment arms, or an observational treatment could consist of multiple categories. Then we would refer to the treatment as multivalued.

To demonstrate, we return to our well-being data and use treatment variable `mexercise`, which captures three treatment groups: a control group, a group where individuals exercised for 45 minutes, and a group where individuals exercised for 90 minutes. Such a design would allow the researcher to find out whether and how the duration of exercise affects bonotin levels and thereby well-being. Here we use a linear model for both the outcome and the mediator, and we include the multivalued treatment `mexercise` as our treatment variable:
. use https://www.stata-press.com/data/r18/wellbeing, clear
(Fictional well-being data)
. mediate (wellbeing age gender i.hstatus basewell)
> (bonotonin basebono)
> (mexercise)
Iteration 0: EE criterion = 1.697e-25
Iteration 1: EE criterion = 2.577e-26
Causal mediation analysis Number of obs = 2,000
Outcome model: Linear
Mediator model: Linear
Mediator variable: bonotonin
Treatment type: Multivalued

<table>
<thead>
<tr>
<th>wellbeing</th>
<th>Robust</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>std. err.</td>
<td>z</td>
<td>P&gt;</td>
</tr>
<tr>
<td>NIE mexercise</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(45 minutes</td>
<td>5.128899</td>
<td>.3505171</td>
<td>14.63</td>
<td>0.000</td>
</tr>
<tr>
<td>vs Control)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(90 minutes</td>
<td>9.780537</td>
<td>.2880877</td>
<td>33.95</td>
<td>0.000</td>
</tr>
<tr>
<td>vs Control)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>NDE mexercise</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(45 minutes</td>
<td>1.197498</td>
<td>.1750038</td>
<td>6.84</td>
<td>0.000</td>
</tr>
<tr>
<td>vs Control)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(90 minutes</td>
<td>3.051084</td>
<td>.2071236</td>
<td>14.73</td>
<td>0.000</td>
</tr>
<tr>
<td>vs Control)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>TE mexercise</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(45 minutes</td>
<td>6.326396</td>
<td>.3894269</td>
<td>16.25</td>
<td>0.000</td>
</tr>
<tr>
<td>vs Control)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(90 minutes</td>
<td>12.83162</td>
<td>.2967962</td>
<td>43.23</td>
<td>0.000</td>
</tr>
<tr>
<td>vs Control)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Outcome equation includes treatment-mediator interaction.

We now have two effects per estimand because we compare the two treated groups to the control group. Starting with the TE, we expect nearly a 13-point increase in well-being if everyone in the population exercised for 90 minutes. Of these 13 points, around 10 points are due to the increase in bonotonin levels and 3 points are due to other mechanisms. The results for the 45-minute treatment arm, though expectedly smaller in magnitude, are interpreted similarly.

**Example 10: Causal mediation model with continuous treatment**

Instead of a binary or multivalued treatment, we could have a continuous treatment variable. With continuous treatments, we have to specify at least two values, one to be the treatment and another to be the control. We return to our birthweight data and use socioeconomic status (ses) as our continuous treatment variable. Here are some summary statistics for ses:
We can see that `ses` ranges from around 1 to 16 and has a mean of about 8. These values, however, do not tell us much because the variable is measured on an arbitrary scale. Therefore, we standardize it so that the resulting variable has a mean of 0 and a standard deviation of 1:

```
. generate std_ses = (ses-r(mean))/r(sd)
```

We will use the new variable, `std_ses`, as our treatment variable. We include the `continuous()` option within the third set of parentheses where we define the treatment. This option tells `mediate` to treat the variable as continuous and to use the values specified within the option as the control and treatment points. The first value is the control, and the remaining values are treatments that are compared with the control. Here we will specify one standard deviation below the mean as our control value and one standard deviation above the mean as our treatment value:

```
. mediate (bweight sespar c.age##c.age, expmean)
> (ncigs sespar c.age##c.age, poisson)
> (std_ses, continuous(-1 1)), nointeract
```

```
Coefficient std. err. z P>|z| [95% conf. interval]
NIE std_ses (1 vs 0) 171.3015 14.68778 11.66 0.000 142.514 200.089
NDE std_ses (1 vs 0) 170.0598 32.14841 5.29 0.000 107.05 233.0695
TE std_ses (1 vs 0) 341.3613 31.73741 10.76 0.000 279.1571 403.5655
```

Even though we used a continuous treatment variable, we interpret the results as before: if everyone in the population had a socioeconomic status one standard deviation above the mean, the birthweight of newborn children would be about 341 grams higher than the birthweight if everyone’s status value is one standard deviation below the mean. Of these 341 grams, roughly half is due to women with a higher status smoking less, and the other half is due to other mechanisms.
We could also evaluate the treatment effects at more than two values. Here we use the mean (0) of the standardized variable as the base, and we evaluate the treatment effects at −2, −1, 1, and 2:

```
. mediate (bweight sespar c.age##c.age, expmean)
> (ncigs sespar c.age##c.age, poisson)
> (std_ses, continuous(0 -2 -1 1 2)), nointeract
Iteration 0: EE criterion = 1.470e-12
Iteration 1: EE criterion = 2.773e-17
Causal mediation analysis

Outcome model: Exponential mean
Mediator model: Poisson
Mediator variable: ncigs
Treatment type: Continuous
Continuous treatment levels:
  0: std_ses = 0 (control)
  1: std_ses = -2
  2: std_ses = -1
  3: std_ses = 1
  4: std_ses = 2

| bweight | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|---------|-------------|-----------|------|------|----------------------|
| NIE     |             |           |      |      |                      |
| std_ses |             |           |      |      |                      |
| (1 vs 0)| -276.2757   | 27.69004  | -9.98| 0.000| -330.5471 -222.0042  |
| (2 vs 0)| -100.1155   | 9.170566  | -10.92| 0.000| -118.0894 -82.14148  |
| (3 vs 0)| 65.84585    | 5.423096  | 12.14| 0.000| 55.21678 76.47493   |
| (4 vs 0)| 110.1346    | 8.724232  | 12.62| 0.000| 93.03538 127.2337   |
| NDE     |             |           |      |      |                      |
| std_ses |             |           |      |      |                      |
| (1 vs 0)| -170.9012   | 31.33649  | -5.45| 0.000| -232.3196 -109.4828  |
| (2 vs 0)| -86.56069   | 16.08129  | -5.38| 0.000| -118.0794 -55.04193  |
| (3 vs 0)| 88.83929    | 16.94031  | 5.24 | 0.000| 55.6369 122.0417    |
| (4 vs 0)| 180.0172    | 34.77372  | 5.18 | 0.000| 111.8619 248.1724   |
| TE      |             |           |      |      |                      |
| std_ses |             |           |      |      |                      |
| (1 vs 0)| -447.1769   | 35.41401  | -12.63| 0.000| -516.5871 -377.7667  |
| (2 vs 0)| -186.6761   | 15.73291  | -11.87| 0.000| -217.5121 -155.8402  |
| (3 vs 0)| 154.6851    | 16.31969  | 9.48 | 0.000| 122.6991 186.6712   |
| (4 vs 0)| 290.1517    | 33.85571  | 8.57 | 0.000| 223.7958 356.5077   |

Note: Outcome equation does not include treatment-mediator interaction.
We now get four effect estimates for each treatment effect, which capture the expected differences in the outcome with respect to the control point. With multiple effect estimates, it can be convenient to plot the results. We use the postestimation command `estat effectsplot` to do so:

```
. estat effectsplot
```

For more information about `estat effectsplot`, see `[CAUSAL] mediate postestimation`.

**Example 11: Estimating controlled direct effects**

Controlled direct effects (CDEs) are different from the other estimands we have dealt with so far. Here, rather than having potential outcomes of the form \( Y_i(t, M_i(t')) \), we have potential outcomes of the form \( Y_i(t|M_i = m) \). That is, we have potential outcomes for each treatment level that are evaluated at set values of the mediator. Thus, CDEs only use the results of the outcome equation. Assuming a binary treatment, the CDE for value \( m \) of the mediator is \( \text{CDE}(m) = Y_i(1|M_i = m) - Y_i(0|M_i = m) \). CDEs can be estimated using the postestimation command `estat cde`. 
To demonstrate, we begin by fitting a mediation model using the well-being data:

```stata
. use https://www.stata-press.com/data/r18/wellbeing, clear
   (Fictional well-being data)
. mediate (bwellbeing age gender i.hstatus, probit)
   > (bbonotonin, probit)
   > (exercise)
Iteration 0: EE criterion = 4.326e-19
Iteration 1: EE criterion = 5.424e-32
Causal mediation analysis
   Number of obs = 2,000
Outcome model:  Probit
Mediator model:  Probit
Mediator variable:  bbonotonin
Treatment type:  Binary

|              | Robust Coefficient | std. err. | z     | P>|z|  | [95% conf. interval] |
|--------------|-------------------|-----------|-------|------|---------------------|
| NIE          | exercise          | .0962832  | .0288905 | 3.33 | 0.001  | .0396588 .1529076  |
|              | (Exercise vs     |           |        |      |         |                     |
|              | Control)          |           |        |      |         |                     |
| NDE          | exercise          | .1672304  | .0358936 | 4.66 | 0.000  | .0968803 .2375805  |
|              | (Exercise vs     |           |        |      |         |                     |
|              | Control)          |           |        |      |         |                     |
| TE           | exercise          | .2635137  | .0212346 | 12.41| 0.000  | .2218946 .3051327  |
|              | (Exercise vs     |           |        |      |         |                     |
|              | Control)          |           |        |      |         |                     |

Note: Outcome equation includes treatment–mediator interaction.
```

We fit probit models for the outcome and the mediator, but the type of model is not important here; we can use `estat cde` after any model fit with `mediate`. 

What is the TE if everyone in the population has a mediator value of 0 (no improvement in bonotonin levels)? To find out, we estimate $CDE(0)$ by specifying the `mvalue(0)` option with `estat cde`:

```stata
. estat cde, mvalue(0)
Controlled direct effect
   Number of obs = 2,000
Mediator variable:  bbonotonin
Mediator value = 0

|              | Delta-method CDE | std. err. | z     | P>|z|  | [95% conf. interval] |
|--------------|-----------------|-----------|-------|------|---------------------|
| exercise     | (Exercise vs    | .1605355  | .039731 | 4.04 | 0.000  | .0826641 .2384068  |
|              | Control)        |           |        |      |         |                     |
```
This CDE is around 0.16. The probability of increased well-being when everyone exercises is 0.16 higher than the probability of increased well-being when no one exercises, provided that no one in the population had at least a 10% increase in bonotonin levels.

We could perform the same analysis specifying multiple values for the mediator. Here we wish to estimate both \( \text{CDE}(0) \) and \( \text{CDE}(1) \):

```
. estat cde, mvalue(0 1)
Controlled direct effect Number of obs = 2,000
Mediator variable: bbonotonin
Mediator values:
  1._at: bbonotonin = 0
  2._at: bbonotonin = 1

|                | Delta-method | CDE  | std. err. | z    | P>|z| | [95% conf. interval] |
|----------------|--------------|------|-----------|------|------|----------------------|
| exercise@_at   |              |      |           |      |      |                      |
| (Exercise vs Control) |       | 1    | .1605355  | .039731 | 4.04 | 0.000 | .0826641 .2384068   |
| (Exercise vs Control) |       | 2    | .2224479  | .0493025 | 4.51 | 0.000 | .1258166 .3190791  |
```

If we “switch on” the mediator, the CDE is higher by around 0.06 points. We could also estimate this difference directly by using the `contrast` option:

```
. estat cde, mvalue(0 1) contrast
Controlled direct effect Number of obs = 2,000
Mediator variable: bbonotonin
Mediator values:
  1._at: bbonotonin = 0
  2._at: bbonotonin = 1

|                | Delta-method | CDE  | std. err. | z    | P>|z| | [95% conf. interval] |
|----------------|--------------|------|-----------|------|------|----------------------|
| _at#exercise   |              |      |           |      |      |                      |
| (2 vs 1)       |              |      |           |      |      |                      |
| (Exercise vs Control) |       | 1    | .0619124  | .0630241 | 0.98 | 0.326 | -.0616126 .1854373  |
```

See [CAUSAL] `mediate postestimation` for further information about `estat cde`.

**Example 12: Estimating treatment effects on different scales**

The `mediate` command estimates treatment effects on the natural scale of the outcome variable. However, some researchers may want to present their estimated effects on a different scale such as on the odds-ratio or risk-ratio scale if the outcome variable is binary or on the incidence-rate–ratio scale if the outcome variable is a count. The postestimation commands `estat rr`, `estat or`, and `estat irr` transform estimated treatment effects onto these different scales.
To see how this works, we first fit the following model with a binary outcome variable. We could use a probit or logit model for the outcome variable; here we are using `probit`:

```stata
.mEDIATE (bwellbeing age gender i.hstatus, probit)
> (bbonotonin, probit)
> (exercise), all
```

Iteration 0: EE criterion = 4.326e-19
Iteration 1: EE criterion = 5.886e-32

Causal mediation analysis

Number of obs = 2,000

Outcome model: Probit
Mediator model: Probit
Mediator variable: bbonotonin
Treatment type: Binary

|                       | Coefficient | Std. err. | z     | P>|z|  | [95% conf. interval] |
|-----------------------|-------------|-----------|-------|-----|---------------------|
| **POmeans**           |             |           |       |     |                     |
| bwellbeing            |             |           |       |     |                     |
| Y0M0                  | .3014153    | .0146737  | 20.54 | 0.000 | .2726554 - .3301752 |
| Y1M0                  | .4686457    | .0328309  | 14.27 | 0.000 | .4042984 - .5329931 |
| Y0M1                  | .3536121    | .0381972  | 9.26  | 0.000 | .2787469 - .4284773 |
| Y1M1                  | .5649289    | .0154435  | 36.58 | 0.000 | .5346603 - .5951976 |
| **NIE**               |             |           |       |     |                     |
| exercise (Exercise vs Control) | .0962832    | .0288905  | 3.33  | 0.001 | .0396588 - .1529076 |
| **NDE**               |             |           |       |     |                     |
| exercise (Exercise vs Control) | .1672304    | .0358936  | 4.66  | 0.000 | .0968803 - .2375805 |
| **PNIE**              |             |           |       |     |                     |
| exercise (Exercise vs Control) | .0521968    | .0348642  | 1.50  | 0.134 | -.0161357 - .1205293 |
| **TNDE**              |             |           |       |     |                     |
| exercise (Exercise vs Control) | .2113169    | .041136   | 5.14  | 0.000 | .1306917 - .291942  |
| **TE**                |             |           |       |     |                     |
| exercise (Exercise vs Control) | .2635137    | .0212346  | 12.41 | 0.000 | .2218946 - .3051327 |

Note: Outcome equation includes treatment-mediator interaction.
Given that our outcome variable is binary and our outcome model is `probit`, the potential-outcome means are averaged probabilities, and because the treatment effects are differences between potential-outcome means, the estimated effects can be interpreted as risk differences. For example, the natural indirect effect is the difference between potential-outcome means $Y_{1M1}$ and $Y_{1M0}$:

\[ \text{display } \_b[\text{POmeans:}Y_{1M1}]-\_b[\text{POmeans:}Y_{1M0}] \]

\[ 0.09628322 \]

If, instead of interpreting the effect on the risk-difference scale, we wanted to interpret it on the risk-ratio scale, we could simply compute the ratio of the potential-outcome means:

\[ \text{display } \_b[\text{POmeans:}Y_{1M1}] / \_b[\text{POmeans:}Y_{1M0}] \]

\[ 1.2054499 \]

This is what `estat rr` is doing:

\[ \text{estat rr} \]

<table>
<thead>
<tr>
<th>Transformed treatment effects</th>
<th>Number of obs = 2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td>wellbeing</td>
<td></td>
</tr>
<tr>
<td>Risk ratio</td>
<td>Robust std. err.</td>
</tr>
<tr>
<td>NIE exercise (Exercise vs Control)</td>
<td>1.20545</td>
</tr>
<tr>
<td>NDE exercise (Exercise vs Control)</td>
<td>1.554817</td>
</tr>
<tr>
<td>PNIE exercise (Exercise vs Control)</td>
<td>1.173172</td>
</tr>
<tr>
<td>TNDE exercise (Exercise vs Control)</td>
<td>1.597595</td>
</tr>
<tr>
<td>TE exercise (Exercise vs Control)</td>
<td>1.874254</td>
</tr>
</tbody>
</table>

The total treatment effect is now decomposed into multiplicative components NIE and NDE as well as PNIE and TNDE. That is, taking their product, rather than their sum, will yield the total effect.
Similarly, we can express all effects on the odds-ratio scale by using `estat or`:

```
estat or
```

<table>
<thead>
<tr>
<th>Transformed treatment effects</th>
<th>Number of obs = 2,000</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>bwellbeing</code></td>
<td></td>
</tr>
<tr>
<td>NIE exercise (Exercise vs Control)</td>
<td>1.472222 .1708025 3.33 0.001 1.172788 1.848106</td>
</tr>
<tr>
<td>NDE exercise (Exercise vs Control)</td>
<td>2.044157 .3042049 4.80 0.000 1.527008 2.736449</td>
</tr>
<tr>
<td>PNIE exercise (Exercise vs Control)</td>
<td>1.267908 .1933291 1.56 0.120 .9403671 1.709534</td>
</tr>
<tr>
<td>TNDE exercise (Exercise vs Control)</td>
<td>2.373558 .4231302 4.85 0.000 1.673622 3.366217</td>
</tr>
<tr>
<td>TE exercise (Exercise vs Control)</td>
<td>3.009452 .281479 11.78 0.000 2.505377 3.614945</td>
</tr>
</tbody>
</table>

Here the total average treatment effect is 3 on the odds-ratio scale and is composed of odds ratios 1.47 and 2.04 in regard to NIE and NDE, respectively, and of odds ratios 1.27 and 2.37 in regard to PNIE and TNDE. Typically, the treatment effects can be interpreted more intuitively on the risk-difference scale, but there may be applications where transforming them to the risk-ratio or odds-ratio scale is desirable.

Notice that `estat rr`, `estat or`, and `estat irr` require estimation of potential-outcome means with `mediate`. If the fitted model does not contain potential-outcome mean estimates, these `estat` commands will refit the model. The reestimation does not affect the results, but computation takes longer. See [CAUSAL] `mediate postestimation` for further information about `estat rr`, `estat or`, and `estat irr`. 
Stored results

`mediate` stores the following in `e()`:

Scalars
- `e(N)` number of observations
- `e(N_clust)` number of clusters
- `e(k_eq)` number of equations in `e(b)`
- `e(k_levels)` number of levels in treatment variable
- `e(rank)` rank of `e(V)`
- `e(interact)` 1 if treatment–mediator interaction included, 0 otherwise
- `e(converged)` 1 if converged, 0 otherwise

Macros
- `e(cmd)` `mediate`
- `e(cmdline)` command as typed
- `e(depvar)` name of outcome variable
- `e(mvar)` name of mediator variable
- `e(tvar)` name of treatment variable
- `e(omodel)` linear, logit, probit, poisson, or expmean
- `e(mmodel)` linear, logit, probit, poisson, or expmean
- `e(wtype)` weight type
- `e(wexp)` weight expression
- `e(title)` title in estimation output
- `e(clustvar)` name of cluster variable
- `e(tlevels)` levels of treatment variable
- `e(tvartype)` binary, multivalued, or continuous
- `e(control)` control level
- `e(vce)` `vcetype` specified in `vce()`
- `e(vcetype)` title used to label Std. err.
- `e(properties)` `b V`
- `e(estat_cmd)` program used to implement `estat`
- `e(predict)` program used to implement `predict`
- `e(marginsnotok)` predictions disallowed by `margins`

Matrices
- `e(b)` coefficient vector
- `e(V)` variance–covariance matrix of the estimators

Functions
- `e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices
- `r(table)` matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.
Methods and formulas

mediate fits causal mediation models and estimates direct, indirect, and total treatment effects. Using the potential-outcomes framework, the estimated treatment effects are the result of contrasts between potential-outcome means. Without loss of generality, let \( T_i \) be a binary treatment, \( t \in \{0, 1\} \), for observations \( i = 1, \ldots, N \), and let \( Y_i \) be the outcome and \( M_i \) be the mediator variable. The potential-outcome means are

\[
POM_{t,t'} \equiv E[Y_i(t, M_i(t'))]
\]

The treatment effects are then defined as follows:

- \( \text{NIE} \equiv E[Y_i(1, M_i(1)) - Y_i(1, M_i(0))] \)
- \( \text{NDE} \equiv E[Y_i(1, M_i(0)) - Y_i(0, M_i(0))] \)
- \( \text{PNIE} \equiv E[Y_i(0, M_i(1)) - Y_i(0, M_i(0))] \)
- \( \text{TNDE} \equiv E[Y_i(1, M_i(1)) - Y_i(0, M_i(1))] \)
- \( \text{TE} \equiv E[Y_i(1, M_i(1)) - Y_i(0, M_i(0))] \)

Synonyms for \( \text{NIE} \), \( \text{NDE} \), \( \text{PNIE} \), \( \text{TNDE} \), and \( \text{TE} \) are \( \text{AITE} \), \( \text{ADTE} \), \( \text{AITEC} \), \( \text{ADTET} \), and \( \text{ATE} \), respectively.

The potential-outcome means are the result of an integral of the conditional expectation of the outcome with respect to the conditional distribution of the mediator (Imai, Keele, and Tingley 2010):

\[
E[Y_i(t, M_i(t'))|X_i = x] = \int E[Y_i|M_i = m, T_i = t, X_i = x] dF[m|T_i = t', X_i = x] \tag{1}
\]

They are estimated as the sample average

\[
\hat{POM}_{t,t'} = \frac{1}{N} \sum_{i=1}^{N} Y_i(t, M_i(t'))|X_i = x
\]

The estimated treatment effects are then the result of differences between estimated potential-outcome means.

mediate uses analytical solutions for the integral in (1) for a variety of parametric outcome and mediator model combinations. Let \( X_i = \{W_i, Z_i\} \), the index function of the outcome model is

\[
\eta_i^Y = \beta_0 + \beta_1 T_i + \beta_2 M_i + \beta_3 T_i M_i + W_i \gamma
\]

and that of the mediator model is

\[
\eta_i^M = \alpha_0 + \alpha_1 T_i + Z_i \zeta
\]

where \( W_i \) and \( Z_i \) are potentially overlapping sets of covariates. If the nointeraction option is used, \( \eta_i^Y \) reduces to the simpler function where \( \beta_3 = 0 \). Depending on which model is specified, the expected values of the outcome and mediator follow these functional forms:

<table>
<thead>
<tr>
<th>Model</th>
<th>Link function</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>( \eta_i )</td>
</tr>
<tr>
<td>exponential mean</td>
<td>( e^{\eta_i} )</td>
</tr>
<tr>
<td>Poisson</td>
<td>( e^{\eta_i} )</td>
</tr>
<tr>
<td>logit</td>
<td>( \Pi(\eta_i) )</td>
</tr>
<tr>
<td>probit</td>
<td>( \Phi(\eta_i) )</td>
</tr>
</tbody>
</table>
Π and Φ are the cumulative logistic and cumulative normal distribution functions, respectively. Between the outcome and mediator models, all combinations of the above functional forms are allowed with the exception of logit outcome models in combination with linear or exponential-mean mediator models.

mediate uses the estimated coefficients from (2) and (3) to estimate $\hat{POM}_{i,t,t'}$. Calculation of $\hat{POM}_{i,t,t'}$ depends on the combination of functional forms of the outcome and mediator models. We define the following terms, where $t$ represents counterfactual values for the treatment with respect to the outcome equation and $t'$ represents treatment counterfactuals in regard to the mediator equation:

$$
\nu_t = \beta_0 + \beta_1 t + W_i \gamma
$$

$$
\xi_{t'} = \alpha_0 + \alpha_1 t' + Z_i \zeta
$$

$$
\kappa_t = \beta_2 + \beta_3 t
$$

If the outcome model is linear, we have

$$
E[Y_i(t, M_i(t'))] = \nu_t + \Theta_m(\xi_{t'})\kappa_t
$$

where $\Theta_m(\cdot)$ denotes the identity function if the mediator model is linear, the cumulative normal distribution function if probit, the cumulative logistic distribution function if logit, and the exponential function if exponential mean or Poisson. In this case, exponential mean and Poisson are synonyms when specifying the mediator model; notice, though, that this is not necessarily the case for other model combinations.

If the outcome model is probit and the mediator model is linear or exponential mean, we have

$$
E[Y_i(t, M_i(t'))] = \Phi\left[\frac{\nu_t + \Gamma_m(\xi_{t'})\kappa_t}{\sqrt{1 + \kappa_t^2 \sigma_m^2}}\right]
$$

where $\Gamma_m(\cdot)$ denotes the identity function if the mediator model is linear and denotes the exponential function if it is exponential mean, and $\sigma_m^2$ is the error variance pertaining to the mediator model.

For probit and logit outcome models in combination with probit and logit mediator models, the potential outcomes are

$$
E[Y_i(t, M_i(t'))] = \Lambda_y(\nu_t + \kappa_t)\Lambda_m(\xi_{t'}) + \Lambda_y(\nu_t)\{1 - \Lambda_m(\xi_{t'})\}
$$

where $\Lambda_y(\cdot)$ is the cumulative normal distribution function if the outcome model is probit and the cumulative logistic distribution function if the outcome model is logit. $\Lambda_m(\cdot)$ denotes the cumulative normal distribution function if the mediator model is probit and the cumulative logistic distribution function if the mediator model is logit.

Regarding the outcome model, notice that, unlike the case of the mediator model, Poisson and exponential mean always refer to the same model. Thus, we can use the terms Poisson and exponential mean interchangeably in regard to the outcome model. The potential outcomes in the case of the exponential-mean outcome model and the linear or exponential-mean mediator model are

$$
E[Y_i(t, M_i(t'))] = e^{\nu_t + \kappa_t \Gamma_m(\xi_{t'}) + (\kappa_t^2 \sigma_m^2)/2}
$$

where $\Gamma_m(\cdot)$ is the identity function if the mediator model is linear and is the exponential function if the mediator model is exponential mean. For probit and logit mediator models, the potential outcomes are

$$
E[Y_i(t, M_i(t'))] = \Lambda_m(\xi_{t'})e^{\nu_t + \kappa_t} + \{1 - \Lambda_m(\xi_{t'})\}e^{\nu_t}
$$
If the outcome model is exponential mean, probit, or logit, and if the mediator model is Poisson, the potential outcomes are

$$E[Y_i(t, M_i(t'))] = \sum_{j=0}^{K} \Psi_y(\nu_t + \kappa_i j) e^{j\xi_t'} e^{-\xi_t'} \frac{j!}{j!}$$

where $j$ indexes the counts of the mediator variable, and $\Psi_y(\cdot)$ denotes the exponential function if the outcome model is exponential mean, denotes the cumulative normal distribution function if the outcome model is probit, and denotes the cumulative logistic distribution function if the outcome model is logit.

`mediate` uses a method of moments estimator, also known as an estimating equations estimator, to estimate all auxiliary and effect parameters as well as their variance–covariance matrix. For more information about the underlying `gmm` command, see [R] `gmm`.

The postestimation commands `estat` or, `estat rr`, and `estat irr` calculate treatment effects on different scales. If the outcome variable is binary and the model for the outcome variable is logit or probit, `estat` or computes marginal treatment effects on the odds-ratio scale, whereas `estat rr` computes marginal treatment effects on the risk-ratio scale. If the model for the outcome variable is poisson or expmean, `estat irr` computes marginal treatment effects on the incidence-rate–ratio scale. Let $Y_{tM_i}$ be a shorthand for $E[Y_i(t, M_i(t'))]$; the treatment effects on risk-ratio and incidence-rate–ratio scales are ratios of potential-outcome means:

$$\text{NIE}^{\text{RR}} \equiv Y_{1M_1}/Y_{1M_0}$$

$$\text{NDE}^{\text{RR}} \equiv Y_{1M_0}/Y_{0M_0}$$

$$\text{PNIE}^{\text{RR}} \equiv Y_{0M_1}/Y_{0M_0}$$

$$\text{TNDE}^{\text{RR}} \equiv Y_{1M_1}/Y_{0M_1}$$

$$\text{TE}^{\text{RR}} \equiv Y_{1M_1}/Y_{0M_0}$$

For logit and probit outcome models, $Y_{tM_i}$ are probabilities, and so the treatment effects on odds-ratio scale are

$$\text{NIE}^{\text{OR}} \equiv Y_{1M_1}/(1 - Y_{1M_1})/\{Y_{1M_0}/(1 - Y_{1M_0})\}$$

$$\text{NDE}^{\text{OR}} \equiv Y_{1M_0}/(1 - Y_{1M_0})/\{Y_{0M_0}/(1 - Y_{0M_0})\}$$

$$\text{PNIE}^{\text{OR}} \equiv Y_{0M_1}/(1 - Y_{0M_1})/\{Y_{0M_0}/(1 - Y_{0M_0})\}$$

$$\text{TNDE}^{\text{OR}} \equiv Y_{1M_1}/(1 - Y_{1M_1})/\{Y_{0M_1}/(1 - Y_{0M_1})\}$$

$$\text{TE}^{\text{OR}} \equiv Y_{1M_1}/(1 - Y_{1M_1})/\{Y_{0M_0}/(1 - Y_{0M_0})\}$$

Notice that, with all three of these scales, the total effect is the product of direct and indirect effects, rather than their sum.

`CDEs` use only the results of the outcome equation. Rather than having potential outcomes of the form $Y_i(t, M_i(t'))$, here we have potential outcomes $Y_i(t|M_i = m)$. That is, we have potential outcomes for each treatment level $t$ that are evaluated at value $m$ of the mediator. `CDE(m)` then is the average of the differences between potential outcomes. For binary treatment, `CDE(m)` is defined as $Y_i(1|M_i = m) - Y_i(0|M_i = m)$. Letting $Y_{tm}$ be a shorthand for $Y_i(t|M_i = m)$, we have that

$$\text{CDE(m)} \equiv Y_{1m} - Y_{0m}$$

$$\text{CDE(m)}^{\text{RR}} \equiv Y_{1m}/Y_{0m}$$

$$\text{CDE(m)}^{\text{IRR}} \equiv Y_{1m}/Y_{0m}$$

$$\text{CDE(m)}^{\text{OR}} \equiv Y_{1m}/(1 - Y_{1m})/\{Y_{0m}/(1 - Y_{0m})\}$$
Acknowledgments

Stata has an active research community adding features to the area of causal mediation. We would like to acknowledge their previous and ongoing contributions to the area: paramed by Hanhua Liu, Richard Emsley, Graham Dunn, Tyler VanderWeele, and Linda Valeri; medeff by Raymond Hicks and Dustin Tingley; rrmmed by Ariel Linden, Chuck Huber, and Geoffrey T. Wodtke; and many more. Type search casual mediation to see Stata’s official and community-contributed features for causal mediation.

References


Also see

[CAUSAL] mediate postestimation — Postestimation tools for mediate
[CAUSAL] teffects — Treatment-effects estimation for observational data
[CAUSAL] teffects ra — Regression adjustment
[SEM] sem — Structural equation model estimation command
[U] 20 Estimation and postestimation commands
mediate postestimation — Postestimation tools for mediate

Postestimation commands predict estat
Remarks and examples Stored results Also see

Postestimation commands

The following postestimation commands are of special interest after mediate:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>estat proportion</td>
<td>proportion mediated</td>
</tr>
<tr>
<td>estat cde</td>
<td>controlled direct effects</td>
</tr>
<tr>
<td>estat or</td>
<td>effects on the odds-ratio scale</td>
</tr>
<tr>
<td>estat rr</td>
<td>effects on the risk-ratio scale</td>
</tr>
<tr>
<td>estat irr</td>
<td>effects on the incidence-rate–ratio scale</td>
</tr>
<tr>
<td>estat effectsplot</td>
<td>effects plot</td>
</tr>
</tbody>
</table>

The following standard postestimation commands are also available:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>estat summarize</td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td>estat vce</td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td>estimates</td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td>etable</td>
<td>table of estimation results</td>
</tr>
<tr>
<td>lincom</td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td>nlcom</td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td>predict</td>
<td>treatment effects, conditional means, etc.</td>
</tr>
<tr>
<td>predictnl</td>
<td>point estimates, standard errors, testing, and inference for generalized predictions</td>
</tr>
<tr>
<td>test</td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td>testnl</td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>
predict

Description for predict

predict creates a new variable (or variables) containing predictions such as treatment effects, conditional means, linear predictions, and expected values.

Menu for predict

Statistics > Postestimation

Syntax for predict

\[
\text{predict} \ [type] \ \{ stub* | newvar | newvarlist \} \ [if] \ [in] \\
\quad [ , \ \text{effect_statistic} \ \text{tlevel}(\text{treat_level}) ]
\]

\[
\text{predict} \ [type] \ \{ stub* | newvar | newvarlist \} \ [if] \ [in] \\
\quad [ , \ \text{po_statistic} \ \text{plevels}(t,t') ]
\]

\[
\text{predict} \ [type] \ \text{newvar} \ [if] \ [in] \ [ , \ \text{fitted_statistic} ]
\]

effect_statistic          Description

Main
nie                      natural indirect effect; the default
nde                      natural direct effect
te                       total effect
pnie                     pure natural indirect effect
tndte                     total natural direct effect
ite                      indirect treatment effect; synonym for nie
dte                      direct treatment effect; synonym for nde
tte                      total treatment effect; synonym for te
itec                     indirect treatment effect with respect to controls; synonym for pnie
dtet                     direct treatment effect with respect to the treated; synonym for tndte

po_statistic            Description

Main
cmean                    conditional mean at treatment levels

fitted_statistic          Description

Main
xb                       linear prediction for outcome model
medxb                    linear prediction for mediator model
mu                       expected values for outcome model
medmu                    expected values for mediator model
If you do not specify `tlevel()` and only specify one new variable, then `effect_statistics` assume `tlevel()` specifies the first noncontrol treatment level. You specify one or \( t-1 \) new variables with `effect_statistic`, where \( t \) is the number of treatment levels.

If you do not specify `polevels()` and only specify one new variable, then `polevels(c,c)` is assumed, where \( c \) is the control group. You specify one or \( d \) new variables with `cmean`, where \( d \) is the number of potential outcomes. You specify one new variable with `fitted_statistic`.

### Options for predict

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>nie</code></td>
<td>the default, calculates the natural indirect effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>nde</code></td>
<td>calculates the natural direct effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>te</code></td>
<td>calculates the total effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>pnie</code></td>
<td>calculates the pure natural indirect effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>tnde</code></td>
<td>calculates the total natural direct effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>ite</code></td>
<td>calculates the indirect treatment effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>dte</code></td>
<td>calculates the direct treatment effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>tte</code></td>
<td>calculates the total treatment effect for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
<tr>
<td><code>itec</code></td>
<td>calculates the indirect treatment effect with respect to controls for each noncontrol treatment level or for the treatment level specified in <code>tlevel()</code>. If you specify the <code>tlevel()</code> option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).</td>
</tr>
</tbody>
</table>
mediate postestimation — Postestimation tools for mediate

\texttt{dtet} calculates the direct treatment effect with respect to the treated for each noncontrol treatment level or for the treatment level specified in \texttt{tlevel()}. If you specify the \texttt{tlevel()} option, you must specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

\texttt{tlevel(treat\_level)} specifies the treatment level for prediction.

\texttt{cmean} calculates the conditional mean for each potential outcome \(Y(t, M(t'))\) or the potential outcome specified in \texttt{polevels()}. If you specify the \texttt{polevels()} option, you must specify only one new variable; otherwise, you must specify a new variable for each potential outcome.

\texttt{polevels(t, t')} specifies the values of the treatment for which potential outcomes are to be calculated. The first value, \(t\), refers to the value that the treatment is set to in the outcome equation; the second value, \(t'\), refers to the value of the treatment in the mediator equation.

\texttt{xb} calculates the linear prediction for the outcome model.

\texttt{medxb} calculates the linear prediction for the mediator model.

\texttt{mu} calculates the expected values of the dependent variable of the outcome model.

\texttt{medmu} calculates the expected values of the dependent variable of the mediator model.
**estat**

**Description for estat**

- `estat proportion` calculates the indirect effect as a proportion of the total effect.
- `estat cde` calculates controlled direct effects.
- `estat or` calculates effects on the odds-ratio scale after `mediate` with the `logit` or `probit` outcome model.
- `estat rr` calculates effects on the risk-ratio scale after `mediate` with the `logit` or `probit` outcome model.
- `estat irr` calculates effects on the incidence-rate–ratio scale after `mediate` with the `poisson` or `expmean` outcome model.
- `estat effectsplot` plots the estimated effects. Typically, this is useful if there are more than two treatment groups in the case of a multivalued treatment or if a continuous treatment is evaluated at more than two points. By default, `estat effectsplot` plots the effects estimated in the previous `mediate` command.

**Menu for estat**

Statistics > Postestimation

**Syntax for estat**

*Proportion mediated*

- `estat proportion [ , prop_options ]`

*Controlled direct effects*

- `estat cde, mvalue(numlist) [ cde_options ]`

*Effects on the odds-ratio scale*

- `estat or [ , scale_options ]`

*Effects on the risk-ratio scale*

- `estat rr [ , scale_options ]`

*Effects on the incidence-rate–ratio scale*

- `estat irr [ , scale_options ]`

*Effects plot*

- `estat effectsplot [ , effectsplot_options ]`
mediate postestimation — Postestimation tools for mediate

**prop_options**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>_level(#)</td>
<td>set confidence level; default is _level(95)</td>
</tr>
<tr>
<td>percent</td>
<td>display percentage instead of proportion</td>
</tr>
<tr>
<td>force</td>
<td>force calculations to proceed in case of conflicting signs</td>
</tr>
<tr>
<td>nolegend</td>
<td>suppress table legend</td>
</tr>
</tbody>
</table>

**display_options**

control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

**cde_options**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>mvalue(numlist)</em></td>
<td>value of the mediator variable</td>
</tr>
<tr>
<td>rr</td>
<td>controlled direct effect on risk-ratio scale</td>
</tr>
<tr>
<td>or</td>
<td>controlled direct effect on odds-ratio scale</td>
</tr>
<tr>
<td>irr</td>
<td>controlled direct effect on incidence-rate–ratio scale</td>
</tr>
<tr>
<td>_level(#)</td>
<td>set confidence level; default is _level(95)</td>
</tr>
<tr>
<td>contrast</td>
<td>differences of controlled direct effects</td>
</tr>
<tr>
<td>nolegend</td>
<td>suppress table legend</td>
</tr>
<tr>
<td>atmeans</td>
<td>controlled direct effect at the means of covariates</td>
</tr>
</tbody>
</table>

**display_options**

control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

* *mvalue(numlist)* is required.

**scale_options**

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>_level(#)</td>
<td>set confidence level; default is _level(95)</td>
</tr>
<tr>
<td>nolegend</td>
<td>suppress table legend</td>
</tr>
</tbody>
</table>

**display_options**

control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

estat or, estat rr, and estat irr require estimation of potential-outcome means with mediate.

If no potential-outcome means were estimated, estat or, estat rr, and estat irr will refit the model in the background; the reestimation does not affect the results, but computation takes longer.
### mediate postestimation — Postestimation tools for mediate

<table>
<thead>
<tr>
<th>effectsplot_options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>When mediate had Pearl’s labeling of effects</strong></td>
<td></td>
</tr>
<tr>
<td>nie</td>
<td>plot natural indirect effects</td>
</tr>
<tr>
<td>nde</td>
<td>plot natural direct effects</td>
</tr>
<tr>
<td>te</td>
<td>plot total effects</td>
</tr>
<tr>
<td>pnie</td>
<td>plot pure natural indirect effects</td>
</tr>
<tr>
<td>tnde</td>
<td>plot total natural direct effects</td>
</tr>
<tr>
<td><strong>When mediate had ATE labeling of effects</strong></td>
<td></td>
</tr>
<tr>
<td>aite</td>
<td>plot average indirect treatment effects</td>
</tr>
<tr>
<td>adte</td>
<td>plot average direct treatment effects</td>
</tr>
<tr>
<td>ate</td>
<td>plot average treatment effects</td>
</tr>
<tr>
<td>aitec</td>
<td>plot average indirect treatment effects with respect to controls</td>
</tr>
<tr>
<td>adtet</td>
<td>plot average direct treatment effects with respect to the treated</td>
</tr>
<tr>
<td><strong>Main</strong></td>
<td></td>
</tr>
<tr>
<td>noci</td>
<td>do not plot confidence intervals</td>
</tr>
<tr>
<td><strong>Plot</strong></td>
<td>affect rendition of all effect plots</td>
</tr>
<tr>
<td>plot#opts(plot_options)</td>
<td>affect rendition of #th effect plot</td>
</tr>
<tr>
<td>recast(plottype)</td>
<td>plot effects using plottype</td>
</tr>
<tr>
<td><strong>CI plot</strong></td>
<td>affect rendition of confidence intervals</td>
</tr>
<tr>
<td>ciopts(rcap_options)</td>
<td>affect rendition of #th confidence interval plot</td>
</tr>
<tr>
<td>ci#opts(rcap_options)</td>
<td>plot confidence intervals using plottype</td>
</tr>
<tr>
<td>recastci(plottype)</td>
<td>set confidence level; default is level(95)</td>
</tr>
<tr>
<td><strong>Add plots</strong></td>
<td>add other plots to the graph</td>
</tr>
<tr>
<td>addplot(plot)</td>
<td>any options other than by() documented in [G-3] twoway_options</td>
</tr>
</tbody>
</table>

### Options for estat proportion

- **level(#)** specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.
- **percent** specifies to calculate percentages. By default, estat proportion calculates proportions.
- **force** forces calculations to proceed in case of conflicting signs. By default, estat proportion issues an error message if opposite signs among indirect, direct, and total effects are detected. In that case, the result is typically not interpretable in a meaningful way.
nolegend suppresses the display of the table legend.

display_options: noci, nopvalues, nofylabel, fwrap(#), fwrapon(style), cformat(%,fmt),
pformat(%,fmt), sformat(%,fmt), and nolstretch; see [R] Estimation options.

Options for estat cde

mvalue(numlist) specifies the value of the mediator variable at which to evaluate the controlled direct effect. If the causal mediation model contained a continuous treatment variable, only a single value may be specified. mvalue() is required.

rr specifies to calculate controlled direct effect on the risk-ratio scale after mediate with the logit or probit outcome model.
or specifies to calculate controlled direct effect on the odds-ratio scale after mediate with the logit or probit outcome model.

irr specifies to calculate controlled direct effect on the incidence-rate–ratio scale after mediate with the poisson or expmean outcome model.

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

contrast specifies to calculate differences of controlled direct effects between evaluations at different points of the mediator, where the base effect is the one defined by the first value in mvalue(); this option requires at least two evaluation points to be specified in mvalue().

nolegend suppresses the display of the table legend.
atmeans specifies to evaluate the controlled direct effect at the means of covariates. By default, the counterfactual predictions are averaged over the covariates.

display_options: noci, nopvalues, nofylabel, fwrap(#), fwrapon(style), cformat(%,fmt),
pformat(%,fmt), sformat(%,fmt), and nolstretch; see [R] Estimation options.

Options for estat or, estat rr, and estat irr

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

nolegend suppresses the display of the table legend.

display_options: noci, nopvalues, nofylabel, fwrap(#), fwrapon(style), cformat(%,fmt),
pformat(%,fmt), sformat(%,fmt), and nolstretch; see [R] Estimation options.

Options for estat effectsplot

nie, nde, te, pnie, tnde, aite, adte, ate, aitec, and adtet specify to plot the respective treatment effects. For these effects to be plotted, they must be part of the model estimates. By default, estat effectsplot plots the effects estimated in the previous mediate command.

Main

noci removes plots of the pointwise confidence intervals. The default is to plot the confidence intervals.
Plot options affect the rendition of all effect plots. The plot_options can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] marker_options, [G-3] marker_label_options, and [G-3] cline_options.

These settings may be overridden for specific plots by using the plot#opts() option.

plot#opts(plot_options) affects the rendition of the #th effect plot. The plot_options can affect the size and color of markers, whether and how the markers are labeled, and whether and how the points are connected; see [G-3] marker_options, [G-3] marker_label_options, and [G-3] cline_options.

recast(plottype) specifies that effects be plotted using plottype. plottype may be scatter, line, connected, bar, area, spike, dropline, or dot; see [G-2] graph twoway. When recast() is specified, the plot-rendition options appropriate to the specified plottype may be used in lieu of plot_options. For details on those options, follow the appropriate link from [G-2] graph twoway.

CI plot

ciopts(rcap_options) affects the rendition of confidence intervals; see [G-3] rcap_options.

These settings may be overridden for specific confidence interval plots with the ci#opts() option.

ci#opts(rcap_options) affects the rendition of the #th confidence interval; see [G-3] rcap_options.

recastci(plottype) specifies that confidence intervals be plotted using plottype. plottype may be rarea, rbar, rspike, rcap, rcapsym, rline, rconnected, or rscatter; see [G-2] graph twoway. When recastci() is specified, the plot-rendition options appropriate to the specified plottype may be used in lieu of rcap_options. For details on those options, follow the appropriate link from [G-2] graph twoway.

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.

Add plots

addplot(plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall
twoway_options are any of the options documented in [G-3] twoway_options, excluding by(). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).
Remarks and examples

Below we provide examples for predict. To see an example of estat proportion, see A simple causal mediation model in [CAUSAL] mediate. To see an example of estat cde, see Estimating controlled direct effects in [CAUSAL] mediate. To see an example of estat effectsplot, see Causal mediation model with continuous treatment in [CAUSAL] mediate. An example of estat rr and estat or is shown in Estimating treatment effects on different scales in [CAUSAL] mediate.

Example 1: Predicting individual-level direct, indirect, and total effects

We can use predict to make a variety of predictions from the fitted mediation model, such as individual-level direct, indirect, and total effects; potential outcomes; and linear predictions and expected values of the outcome and mediator. Suppose we have the following mediation model with binary outcome and binary mediator:

```
. use https://www.stata-press.com/data/r18/wellbeing
    (Fictional well-being data)
. mediate (bwellbeing age gender i.hstatus basewell, logit)
    > (bbonotonin, logit)
    > (exercise)
Iteration 0: EE criterion = 8.253e-18
Iteration 1: EE criterion = 6.067e-33
Causal mediation analysis
    Number of obs = 2,000
    Outcome model:     Logit
    Mediator model:    Logit
    Mediator variable: bbonotonin
    Treatment type:    Binary

| bwellbeing | Coefficient | std. err. | z     | P>|z| [95% conf. interval] |
|------------|-------------|-----------|-------|------------------------|
| NIE         |             |           |       |                        |
| exercise   |             |           |       |                        |
| (Exercise vs Control) | .1052971 | .0170666 | 6.17  | 0.000 .0718472 .1387471 |
| NDE         |             |           |       |                        |
| exercise   |             |           |       |                        |
| (Exercise vs Control) | .1524917 | .0208284 | 7.32  | 0.000 .1116689 .1933146 |
| TE          |             |           |       |                        |
| exercise   |             |           |       |                        |
| (Exercise vs Control) | .2577899 | .0143    | 18.03 | 0.000 .2297613 .2858164 |

Note: Outcome equation includes treatment-mediator interaction.

Using predict without options yields estimated individual-level natural indirect effects:

```
. predict nie
    (option nie assumed; natural indirect effect)
```
We could go ahead and predict individual-level direct and total effects by using options `nde` and `te`, respectively:

```
predict nde, nde
predict te, te
```

Here is an excerpt from the data showing the predicted effects for five individuals:

```
. list nie nde te in 1/5

<table>
<thead>
<tr>
<th>nie</th>
<th>nde</th>
<th>te</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. 0.0504899</td>
<td>0.2496191</td>
<td>0.3001091</td>
</tr>
<tr>
<td>2. 0.1693522</td>
<td>0.1037404</td>
<td>0.2730926</td>
</tr>
<tr>
<td>3. 0.2145208</td>
<td>0.3612216</td>
<td>0.5757424</td>
</tr>
<tr>
<td>4. 0.0265223</td>
<td>0.1576028</td>
<td>0.1841251</td>
</tr>
<tr>
<td>5. 0.2005004</td>
<td>0.3735286</td>
<td>0.574029</td>
</tr>
</tbody>
</table>
```

We can see that the indirect and direct effects sum to the total effect for each individual. The differences in effects between individuals are due to their differences in covariates. Had we fit the model without covariates, the predicted effects would be constant over the sample.

If we look at the sample means of the newly generated variables `nie`, `nde`, and `te`, we can see that their averages match the estimates from `mediate` for NIE, NDE, and TE, respectively:

```
. summarize nie nde te

<table>
<thead>
<tr>
<th>Variable</th>
<th>Obs</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>Min</th>
<th>Max</th>
</tr>
</thead>
<tbody>
<tr>
<td>nie</td>
<td>2000</td>
<td>0.1052971</td>
<td>0.0883299</td>
<td>0.000014</td>
<td>0.2529227</td>
</tr>
<tr>
<td>nde</td>
<td>2000</td>
<td>0.1524917</td>
<td>0.1354418</td>
<td>0.0001044</td>
<td>0.3877526</td>
</tr>
<tr>
<td>te</td>
<td>2000</td>
<td>0.2577889</td>
<td>0.2037248</td>
<td>0.0001184</td>
<td>0.5757925</td>
</tr>
</tbody>
</table>
```

Example 2: Predicting potential outcomes

In addition to individual-level effects, we can also predict individual-level potential outcomes by using the `cmean` option. By default, `predict` with `cmean` will compute the potential outcomes for the control level of the treatment variable. For example, if the treatment variable is binary and takes on the values 0 and 1, where 0 is the control level, we will predict potential outcomes $Y_i[0, M_i(0)]$:

```
predict po_y0m0, cmean
```

We can also target other potential outcomes by using the `polevels()` option. For instance, to compute potential outcomes $Y_i[1, M_i(0)]$, we specify option `polevels(1,0)`:

```
predict po_y1m0, cmean polevels(1,0)
```

If we wish to predict all potential outcomes at once, we can use the `stub*` notation:

```
predict po_*, cmean
```

In this case, there are four potential outcomes available, so Stata creates four new variables. Using `describe`, we can also see that the new variables are labeled according to the estimated potential outcome:
. describe po_?

<table>
<thead>
<tr>
<th>Variable</th>
<th>Storage type</th>
<th>Display format</th>
<th>Value label</th>
</tr>
</thead>
<tbody>
<tr>
<td>po_1</td>
<td>float</td>
<td>%9.0g</td>
<td>Conditional mean, Y[0,M(0)]</td>
</tr>
<tr>
<td>po_2</td>
<td>float</td>
<td>%9.0g</td>
<td>Conditional mean, Y[1,M(0)]</td>
</tr>
<tr>
<td>po_3</td>
<td>float</td>
<td>%9.0g</td>
<td>Conditional mean, Y[0,M(1)]</td>
</tr>
<tr>
<td>po_4</td>
<td>float</td>
<td>%9.0g</td>
<td>Conditional mean, Y[1,M(1)]</td>
</tr>
</tbody>
</table>

**Stored results**

**estat proportion** stores the following results in r():

- Scalars
  - r(N): number of observations
- Macros
  - r(title): title in estimation output
- Matrices
  - r(b): vector of estimated proportions or percentages
  - r(V): variance–covariance matrix of the estimates
  - r(table): matrix containing the estimates with their standard errors, test statistics, p-values, and confidence intervals

**estat cde** stores the following results in r():

- Scalars
  - r(N): number of observations
- Macros
  - r(title): title in estimation output
- Matrices
  - r(b): vector of estimated controlled direct effects or their contrasts
  - r(V): variance–covariance matrix of the estimates
  - r(table): matrix containing the estimates with their standard errors, test statistics, p-values, and confidence intervals

**estat or, estat rr, and estat irr** store the following results in r():

- Scalars
  - r(N): number of observations
  - r(level): confidence level
- Matrices
  - r(b): vector of transformed treatment effects (log scale)
  - r(V): variance–covariance matrix of the estimates
  - r(table): matrix containing the estimates with their standard errors, test statistics, p-values, and confidence intervals

**Also see**

[CAUSAL] mediate — Causal mediation analysis

[U] 20 Estimation and postestimation commands
**Description**

`stteffects` estimates average treatment effects, average treatment effects on the treated, and potential-outcome means using observational survival-time data. The available estimators are regression adjustment, inverse-probability weighting, and more efficient methods that combine regression adjustment and inverse-probability weighting.

For a brief description and example of each estimator, see *Remarks and examples* in [CAUSAL] `stteffects intro`.

**Syntax**

```
stteffects subcommand ... [ , options ]
```

<table>
<thead>
<tr>
<th><code>subcommand</code></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ra</code></td>
<td>regression adjustment</td>
</tr>
<tr>
<td><code>ipw</code></td>
<td>inverse-probability weighting</td>
</tr>
<tr>
<td><code>ipwra</code></td>
<td>inverse-probability-weighted regression adjustment</td>
</tr>
<tr>
<td><code>wra</code></td>
<td>weighted regression adjustment</td>
</tr>
</tbody>
</table>

**Also see**

[CAUSAL] `stteffects intro` — Introduction to treatment effects for observational survival-time data
Description

This entry provides an overview of the treatment-effects estimators that use observational survival-time data and are implemented in `stteffects`. It also provides an overview of the potential-outcomes framework and its application to survival-time data and to the interpretation of the treatment-effects parameters estimated.

The `stteffects` command estimates average treatment effects (ATEs), average treatment effects on the treated (ATETs), and potential-outcome means (POMs). Each of these effect parameters is discussed in this entry. `stteffects` implements a variety of estimators for the ATE, ATET, and POM. The treatment effects can be estimated using regression adjustment (RA), inverse-probability weights (IPW), inverse-probability-weighted regression adjustment (IPWRA), and weighted regression adjustment (WRA). This entry also provides some intuition for the estimators and discusses the tradeoffs between them.

Remarks and examples

Remarks are presented under the following headings:

- Introduction
- A quick tour of the estimators
  - Regression adjustment
  - Inverse-probability weighting
  - Combinations of RA and IPW
  - Weighted regression adjustment
- Average treatment effect on the treated
- Comparison of treatment-effects estimators
- Assumptions and tradeoffs
  - The conditional independence assumption
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  - The correct adjustment for censoring assumption
  - Assumptions for the ATET
- Specification diagnostics and tests
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Introduction

The `stteffects` command estimates treatment effects using observational survival-time data.

For some intuition about the methods implemented in the `stteffects` command, consider the following question: Does smoking decrease the time to a second heart attack in the population of women aged 45–55 who have had one heart attack? Three aspects of this question stand out.

1. For ethical reasons, these data will be observational.
2. This question is about the time to an event, and such data are commonly known as survival-time data or time-to-event data. These data are nonnegative and, frequently, right-censored.
3. Many researchers and practitioners want an effect estimate in easy-to-understand units of time.
Aspect 1 is one of the most common reasons for using observational data, and aspect 2 focuses interest on survival-time data.

We are most concerned with aspect 3 because it helps us define and understand the effect of interest. In particular, we would like to know the average change in time to a second heart attack that would occur in the population if all women smoked instead of if no women smoked. This effect is an ATE.

We must solve a missing-data problem to estimate the ATE. The ATE is the population average of the contrast in outcomes when everyone gets the treatment and when no one gets the treatment. Formally, we write this as

\[ \text{ATE} = E(t_1 - t_0) \]

where \( t_1 \) is the survival time when a subject gets the treatment and \( t_0 \) is the survival time when a subject does not get the treatment. For each treatment level, there is a potential outcome that would be observed if a subject received that treatment level: \( t_1 \) is the potential outcome that would occur if someone gets the treatment and \( t_0 \) is the potential outcome that would occur if someone does not get the treatment. The missing-data problem arises because each subject receives only one treatment level, and so we observe only one of the two potential outcomes.

Much of the survival-time literature uses a hazard ratio as the effect of interest. The ATE has three advantages over the hazard ratio as an effect measure.

1. The ATE measures the effect in the same time units as the outcome instead of in relative conditional probabilities.
2. The ATE is much easier to explain to nontechnical audiences.
3. The models used to estimate the ATE can be much more flexible. Hazard ratios are useful for population effects when they are constant, which occurs when the treatment enters linearly and the distribution of the outcome has a proportional-hazards form. Neither linearity in treatment nor proportional-hazards form is required for the ATE, and neither is imposed on the models fit by the estimators implemented in stteffects.

The estimators implemented in stteffects use the common missing-data techniques of regression modeling, weighting, and combinations thereof to account for data lost to censoring and to unobserved potential outcomes.

Here we note only a few contributions and entry points to the vast literature on estimating ATEs. The use of potential outcomes to define treatment effects has proved extraordinarily useful; see Holland (1986), Rubin (1974), and Heckman (1997). Cameron and Trivedi (2005, chap. 25), Wooldridge (2010, chap. 21), and Vittinghoff et al. (2012, chap. 9) provide excellent general introductions to estimating ATEs.

Technical note

Left-truncation would be another type of missing data. The estimators implemented in stteffects do not adjust for left-truncation, so stteffects cannot be used with delayed-entry data.

stteffects cannot be used with time-varying covariates or multiple-record data because these add a repeated-measure structure that significantly complicates the estimation problem.
A quick tour of the estimators

The `stteffects` command implements five estimators of treatment effects. We introduce each one by showing the basic syntax used to apply it to a common example dataset. See each command’s entry for detailed information.

We have some fictional data on the time to a second heart attack among women aged 45–55 years. The treatment, smoking, is stored in the 0/1 indicator `smoke`. These data also contain each woman’s age at the time of her first heart attack (`age`), and indices of her exercise level (`exercise`), diet quality (`diet`), and education attainment (`education`) prior to her first heart attack.

Like `streg` and other survival-time commands, `stteffects` uses the outcome variable and the failure indicator computed by `stset`. In this dataset, `atime` is the observed time in years to the second heart attack, and `fail` is the 0/1 indicator that a second heart attack was observed and recorded in `atime`. (When `fail` is 1, `atime` records the time to the second attack; when `fail` is 0, `atime` records a censored observation of the time to the second attack.)

We begin our examples by first reading in the data and then specifying the raw outcome and failure variables to `stset`.

```
. use https://www.stata-press.com/data/r18/sheart
   (Time to second heart attack (fictional))
. stset atime, failure(fail)
   Survival-time data settings
       Failure event: fail!=0 & fail<.
       Observed time interval: (0, atime]
       Exit on or before: failure

   2,000 total observations
      0 exclusions

   2,000 observations remaining, representing
   1,208 failures in single-record/single-failure data
   3,795.226 total analysis time at risk and under observation

   At risk from t =   0
   Earliest observed entry t =   0
   Last observed exit t = 34.17743
```

The output indicates that 1,208 of the 2,000 observations record actual time to a second heart attack. The remaining observations were censored. Now that we have `stset` the data, we can use `stteffects`.

Regression adjustment

Regression modeling of the outcome variable is a venerable approach to solving the missing-data problem in treatment-effects estimation. Known as the regression-adjustment (RA) estimator, this method uses averages of predicted outcomes to estimate the ATE. If the outcome model is well specified, this approach is surprisingly robust.
Example 1: RA estimation

We now use `stteffects ra` to estimate the ATE by RA. We model the outcome as a function of age, exercise, diet, and education, and we specify that smoke is the treatment variable.

```
. stteffects ra (age exercise diet education) (smoke)
```

```
   Failure `d': fail
   Analysis time `t': atime

Iteration 0: EE criterion = 1.006e-14
Iteration 1: EE criterion = 2.305e-25

Survival treatment-effects estimation
Number of obs = 2,000
Estimator : regression adjustment
Outcome model : Weibull
Treatment model: none
Censoring model: none

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
<th></th>
<th></th>
<th></th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>std. err.</td>
<td>z</td>
<td>P&gt;</td>
<td>z</td>
</tr>
<tr>
<td>ATE</td>
<td>smoke (Smoker vs Nonsmoker)</td>
<td>-1.956657</td>
<td>.3331787</td>
<td>-5.87</td>
<td>0.000</td>
</tr>
<tr>
<td>POmean</td>
<td>smoke Nonsmoker</td>
<td>4.243974</td>
<td>.2620538</td>
<td>16.20</td>
<td>0.000</td>
</tr>
</tbody>
</table>
```

When all women in the population smoke, the average time to a second heart attack is estimated to be 1.96 years less than when no women smoke. The estimated average time to a second heart attack when no women smoke is 4.24 years.

The output reports that a Weibull model was used for the outcome. The other outcome models available are exponential, gamma, and log normal. See example 2 in [CAUSAL] stteffects ra for an application of the gamma parameterization to this model.

The ratio of the ATE to control-level PO mean measures the importance of the effect. In this example, when all women smoke, the time to a second heart attack falls by an estimated 46% relative to the case in which none of them smoke. See example 3 in [CAUSAL] stteffects ra for an example that uses `nlcom` to compute a point estimate and a confidence interval for this ratio.

Unlike the IPW estimator discussed in the next section, RA does not model treatment assignment or the censoring process. Treatment assignment is handled by fitting separate models for each treatment level and averaging the predicted outcomes. As is standard in the survival-time literature, the censoring term in the log-likelihood function accounts for censoring; see Kalbfleisch and Prentice (2002, chap. 3), Cameron and Trivedi (2005, chap. 17), Cleves, Gould, and Marchenko (2016, chap. 13), and Wooldridge (2010, chap. 22).

See [CAUSAL] stteffects ra for further discussion of this command and the RA estimator.
Inverse-probability weighting

Sometimes researchers are more comfortable modeling treatment assignment than the outcome. Inverse-probability-weighted (IPW) estimators use weighted averages of the observed outcome to estimate the POMs and the ATE. The weights correct for the missing data. When there is no censoring, the missing potential outcome is the only missing data, and the weights are constructed from a model of treatment assignment. When the data may be censored, the weights must control for censoring and the missing potential outcome. In this case, IPW estimators construct the weights from two models, one for the censoring time and one for treatment assignment.

Example 2: IPW estimation

Here we use stteffects ipw to estimate the effect of smoking on the time to a second heart attack. The model of assignment to the treatment smoke depends on age, exercise, diet, and education. The time-to-censoring model also depends on age, exercise, diet, and education.

```
. stteffects ipw (smoke age exercise diet education)
> (age exercise diet education)

Failure _d: fail
Analysis time _t: atime
Iteration 0: EE criterion = 2.042e-18
Iteration 1: EE criterion = 3.283e-31
Survival treatment-effects estimation Number of obs = 2,000
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: logit
Censoring model: Weibull

|              | Coefficient | std. err. | z    | P>|z|   | [95% conf. interval] |
|--------------|-------------|-----------|------|-------|----------------------|
| ATE          |             |           |      |       |                      |
| smoke (Smoker vs Nonsmoker) | -2.187297 | .6319837 | -3.46 | .001  | -3.425962 -.9486314  |
| POmean       |             |           |      |       |                      |
| smoke Nonsmoker | 4.225331 | .517501   | 8.16 | .000  | 3.211047 5.239614    |
```

When all women in the population smoke, the average time to a second heart attack is estimated to be 2.19 years less than when no women smoke. The estimated average time to a second heart attack when no women smoke is 4.23 years. When all women smoke, the average time to a second heart attack falls by an estimated 52% relative to the case when no women smoke.

The estimates have changed; however, the interpretation is the same as for the RA estimator because the IPW and RA estimators are estimating the same population effects. Under correct model specification, the estimates will differ in finite samples, but the size of these differences will decrease as the sample size gets larger. For the case at hand, the estimated ATE and control-level POM are roughly similar to those produced by the RA estimator using the Weibull model for the outcome.

Recall that IPW estimators are weighted averages of observed outcomes and that the weights control for the missing outcomes. Weights in survival-time data have two components: one for the missing potential outcome and one for data lost to censoring. We used a logit model for treatment assignment, so the component of the weights that controls for the missing potential outcome comes from the
estimated logit parameters. We used a Weibull model for the time to censoring, so the component of the weights that controls for data lost to censoring comes from the estimated Weibull parameters.

Using weighting from an estimated treatment-assignment model to control for the missing potential outcome is standard in the treatment-effects literature; for example, see [CAUSAL] tteffects intro advanced, Wooldridge (2010, chap. 21), Vittinghoff et al. (2012, chap. 9), Hirano, Imbens, and Ridder (2003), Cattaneo (2010), and Cattaneo, Drukker, and Holland (2013). Modeling the time to censoring is specific to the survival-time treatment-effects literature; see Bai, Tsiatis, and O’Brien (2013) and Robins and Rotnitzky (2006). See Methods and formulas in [CAUSAL] stteffects ipwra for more details.

See [CAUSAL] stteffects ipw for further discussion of this command and the IPW estimator.

Combinations of RA and IPW

More efficient estimators are obtained by combining IPW and RA, due to Wooldridge (2007) and Wooldridge (2010, chap. 21) and denoted by IPWRA. Unlike the estimators discussed in Wooldridge (2010, chap. 21), both the treatment and the outcome models must be correctly specified to estimate the ATE.

The IPWRA estimator uses estimated weights that control for missing data to obtain missingness-adjusted regression coefficients that are used to compute averages of predicted outcomes to estimate the POMs. The estimated ATE is a contrast of the estimated POMs. These weights always involve a model for treatment assignment. You choose whether to account for censoring by including a term in the log-likelihood function or whether to use weights that also account for the data lost to censoring.

例 3: Likelihood-adjusted-censoring IPWRA estimation

We model the outcome (time to a second heart attack) as a function of age, exercise, diet, and education. We model assignment to the treatment smoke as a function of the same covariates.

```
. stteffects ipwra (age exercise diet education) 
  > (smoke age exercise diet education)
   Failure _d: fail 
   Analysis time _t: atime 
Iteration 0: EE criterion = 6.476e-15
Iteration 1: EE criterion = 1.116e-26
Survival treatment-effects estimation 
  Number of obs = 2,000 
  Estimator : IPW regression adjustment 
  Outcome model : Weibull 
  Treatment model: logit 
  Censoring model: none 

| T | Robust Coefficient std. err. z P>|z| [95% conf. interval] |
|---|-------------------|----------|---------|------------------|------------------|
| _t |                      |             |             |                   |                   |
| ATE smoke | (Smoker vs Nonsmoker) | -1.592494 | .4872777 | -3.27 | 0.001 | -2.54754 - .637447 |
| POmean smoke | Nonsmoker | 4.214523 | .2600165 | 16.21 | 0.000 | 3.7049 4.724146 |
```
The estimated ATE of $-1.59$ and control-level POM of $4.21$ are similar to the reported values of $-1.96$ and $4.24$ in example 1.

We did not specify a model for the time to censoring, so censoring is handled by including a term in the log-likelihood function in the Weibull outcome model. We denote this likelihood-adjusted-censoring (LAC) version of the IPWRA estimator by LAC-IPWRA.

### Example 4: Weighted-adjusted-censoring IPWRA estimation

Instead of including a term in the log-likelihood function, the weighted-adjusted-censoring IPWRA (WAC-IPWRA) estimator uses estimated weights to adjust for censoring. We model the time to a second heart attack as a function of age, exercise, diet, and education; we model assignment to the treatment smoke as a function of the same covariates; and we model the time to censoring as a function of age, exercise, and diet.

```
. stteffects ipwra (age exercise diet education)
  > (smoke age exercise diet education) (age exercise diet)
```

```
   Failure d: fail
   Analysis time t: atime
   Iteration 0: EE criterion = 2.797e-13
   Iteration 1: EE criterion = 2.032e-25

   Survival treatment-effects estimation
   Number of obs   = 2,000
   Estimator : IPW regression adjustment
   Outcome model : Weibull
   Treatment model: logit
   Censoring model: Weibull

                      Robust
       _t        Coefficient   std. err.     z    P>|z|   [95% conf. interval]
     ATE
     smoke
      (Smoker vs Nonsmoker)
              -2.037944    .6032549   -3.38   0.001    -3.220302   -.855586
     POmean
     smoke
       Nonsmoker       4.14284    .4811052    8.61   0.000     3.199891    5.085789
```

The estimated ATE of $-2.04$ and control-level POM of $4.14$ are similar to the reported values of $-1.96$ and $4.24$ in example 1.

The weights for censoring are constructed from the estimated parameters because we specified a time-to-censoring model.

Under correct specification, both versions of the IPWRA estimator estimate the same ATE and control-level POM as estimated by RA and IPW.

The addition of the time-to-censoring model makes the WAC-IPWRA somewhat less robust than the LAC-IPWRA estimator. Weighting methods to control for censoring also place more restrictive assumptions on the censoring process. For example, the censoring time must be random, otherwise it would be impossible to construct the weights. In Assumptions and tradeoffs below, we discuss the tradeoffs among the estimators and the assumptions that each requires. For the moment, we note that we believe the LAC-IPWRA estimator is more robust than the WAC-IPWRA estimator.

See [CAUSAL] stteffects ipwra for further discussion of this command and the IPWRA estimator.
Weighted regression adjustment

When estimating the parameters of an outcome model, the weighted regression-adjustment (WRA) estimator uses weights instead of a term in the log-likelihood function to adjust for censoring. These weights are constructed from a model for the censoring process. The estimated parameters are subsequently used to compute averages of predicted outcomes that estimate the POMs. A contrast of the estimated POMs estimates the ATE.

Example 5: WRA estimation

We model the time to a second heart attack as a function of age, exercise, diet, and education; we specify that smoke is the treatment; and we model the time to censoring as a function of age, exercise, and diet.

\[
.\text{stteffects wra (age exercise diet education) (smoke) (age exercise diet)}
\]

| Robust                  | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|-------------------------|-------------|-----------|------|------|----------------------|
| ATE                     |             |           |      |      |                      |
| smoke (Smoker vs Nonsmoker) | -2.152014   | .4986005  | -4.32| .000 | -3.129253 -1.174775  |
| POmean                  |             |           |      |      |                      |
| smoke Nonsmoker         | 4.079273    | .4379517  | 9.31 | .000 | 3.220903 4.937642    |

The estimated ATE of \(-2.15\) and control-level POM of 4.08 are similar to the reported values of \(-1.96\) and 4.24 in example 1. Like the other estimators discussed, the WRA estimators estimate the same effect parameters as the RA estimator, so the interpretation is the same.

In many survival-time applications, using weights to adjust for censoring is probably less robust than just including a term in the log-likelihood function for the outcome model. The model used to construct the weights is just as complicated as the outcome model, and including the term in the log-likelihood function places fewer restrictions on the censoring process, as discussed in The correct adjustment for censoring assumption below.

See [CAUSAL] stteffects wra for further discussion of this command and the WRA estimator.
Average treatment effect on the treated

Intuitively, the average treatment effect on the treated (ATET) is the effect in a well-defined, at-risk subpopulation. Sometimes the subpopulation that gets the treatment defines such an at-risk subpopulation. For example, we may want to know the average change in time to a second heart attack among female smokers aged 45–55 who have had a heart attack if they all became nonsmokers. This effect is the ATET.

Below, we use \texttt{stteffects ra} to estimate the ATET by RA.

\begin{verbatim}
. stteffects ra (age exercise diet education) (smoke), atet

Failure _d: fail
Analysis time _t: atime
Iteration 0: EE criterion = 1.006e-14
Iteration 1: EE criterion = 2.970e-26

Survival treatment-effects estimation
Number of obs = 2,000
Estimator : regression adjustment
Outcome model : Weibull
Treatment model: none
Censoring model: none

|               | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|---------------|-------------|-----------|------|------|---------------------|
| ATET          |             |           |      |      |                     |
| smoke         | -1.527476   | .2489203  | -6.14| 0.000| -2.015351 -1.039602 |
| (Smoker vs Non smoker) |
| POmean        | 3.436937    | .2217808  | 15.50| 0.000| 3.002255 3.87162   |
| smoke         |             |           |      |      |                     |
| Non smoker    |             |           |      |      |                     |

Now, all effects are calculated only for the subpopulation of women aged 45–55 years who smoke after their first heart attack. If no women in the subpopulation were to smoke, the average time to a second heart attack would be 3.44 years. When all women in the subpopulation smoke (the observed behavior), the average time to a second heart attack is estimated to be 1.53 years less than if no women in the subpopulation had smoked. In other words, if we could somehow turn all smokers in the subpopulation into nonsmokers, the average time to a second heart attack would be 3.44 years instead of 1.91 years ($3.44 - 1.53 = 1.91$).

These point estimates are a little different than those for the ATE and the control-level POM in the full population of women aged 45–55 years who have had one heart attack. The difference indicates that this particular health cost of smoking may be smaller among women who choose to smoke than in the full population.

Comparison of treatment-effects estimators

We can classify the estimators implemented in \texttt{stteffects} into five categories: 1) estimators based on a model for the outcome variable; 2) estimators based on models for the treatment assignment and the censoring time; 3) estimators based on models for the outcome variable and the treatment assignment; 4) estimators based on models for the outcome variable, the treatment assignment, and the censoring time; and 5) estimators based on models for the outcome variable and the censoring time.
Because there are several categories of estimators, the user must decide whether to model the outcome, the probability of treatment, the time to censoring, or some combination thereof.

Each category of estimator contains a variety of choices about the functional forms for the models. We now provide some intuition behind each category of estimator and discuss the relationships.

1. When modeling only the outcome, separate outcome models for each treatment level account for treatment assignment, and censoring is adjusted for in the log-likelihood function. This approach is used in the RA estimators.

2. Some researchers would rather avoid modeling the outcome. Some estimators use weighted averages of the observed outcome to estimate the effect. When estimating treatment effects from observational survival-time data, the weights used must account for treatment assignment and censoring. Models for treatment assignment and time to censoring are used to construct the weights. This approach is used in the IPW estimators.

3. When seeking a more efficient estimator, it is natural to model both the outcome and the treatment and to adjust for censoring in the outcome model. This approach is used in the LAC-IPWRRA estimators.

4. When seeking a more efficient estimator, another natural approach is to model both the outcome and the treatment and to adjust for censoring by weights that come from a time-to-censoring model. This approach is used in the WAC-IPWRRA estimators.

5. We could modify approach 1 to model the outcome and the time to censoring so that censoring is handled by weighting and its own model instead of by likelihood adjustment. This approach is used in the WRA estimators.

While researcher preferences over what to model largely dictate the approach selected, we quickly note two points that could affect which approach works best. First, we can adjust for censoring by weighting only when censoring time is random. Second, weighting estimators become unstable if the weights get too large.

In the next section, we elaborate on the assumptions needed and the tradeoffs among the approaches to estimation.

Assumptions and tradeoffs

The estimators implemented in stteffects require three assumptions: conditional independence, sufficient overlap, and correct adjustment for censoring.

The conditional independence assumption

All estimators implemented in stteffects require the potential outcomes to be independent of the treatment assignment after conditioning on the covariates. Randomized experiments and the Heckman selection model are two motivating frameworks for the conditional independence assumption.

When the treatment is assigned randomly, the randomization ensures that the potential outcomes are independent of the treatment assignment. In observational data, the treatment is not randomly assigned. However, many important questions can only be answered using observational data because it would be unethical to randomly allocate hazardous treatments, for example, smoking. The conditional independence assumption in observational data says that treatment assignment is as good as random after conditioning on the covariates.
We can also understand conditional independence from a modeling framework. The Heckman selection model specifies that each of the potential outcomes and the treatment assignment process are functions of observable covariates and unobservable errors. The potential outcomes are conditionally independent of the treatment assignment when the unobservable errors in the treatment-assignment process are independent of the unobservable errors in each of the potential-outcome processes. See *The CI assumption* in [*CAUSAL*]()`teffects intro advanced` for a detailed example.

Both frameworks lead to the same conclusion: we need to observe and to condition on a sufficient number of covariates.

Essentially, all the estimators in `stteffects` are equally susceptible to violations of the conditional independence assumption. No one estimator is any more robust to the conditional independence assumption than any other one.

Estimating the ATE among the subpopulation of those who get the treatment requires a significantly weaker version of the CI assumption; see *Assumptions for the ATET* below.

For more details about the conditional independence assumption, see *The CI assumption* in [*CAUSAL*]()`teffects intro advanced`, and see Rosenbaum and Rubin (1983), Heckman (1997), Imbens and Wooldridge (2009), Cameron and Trivedi (2005, sec. 25.2), Wooldridge (2010, chap. 21), and Vittinghoff et al. (2012, chap. 9).

### The sufficient overlap assumption

The sufficient overlap assumption requires that each individual have a sufficiently positive probability of being assigned to each treatment level. We believe that the RA estimator is more robust than the other estimators to near violations of the sufficient overlap condition, under correct model specification.

The overlap condition has no specification test, but using `teoverlap` and then summarizing the predicted treatment probabilities presents good diagnostics of overlap problems.

### The correct adjustment for censoring assumption

The correct adjustment for censoring assumption has two parts. First, either the censoring time must be fixed or the process must be conditionally-on-covariates independent of the potential outcomes and the treatment-assignment process. This assumption is standard in survival analysis; see, for example, Kalbfleisch and Prentice (2002, chap. 3).

Second, the method used to adjust to censoring must be correct. For the RA and LAC-IPWRA estimators, which use likelihood-adjusted censoring, the second assumption is no more restrictive than assuming correct specification of the outcome model. For the IPW, WAC-IPWRA, and WRA estimators, which adjust by weighting, the second assumption requires that the censoring be random and that the censoring process be correctly modeled.

Under correct specification, all the estimators in `stteffects` perform well. However, we believe that estimators that use likelihood adjustment instead of weighting are more robust for three reasons.

1. The estimators that use weighting to adjust for censoring cannot handle fixed censoring processes. If the censoring process is not random, the weights are not well defined.
2. The estimators that use weighting to adjust for censoring do not allow the random censoring process to vary by treatment level.
3. The estimators that use weighting to adjust for censoring require an additional sufficient overlap condition: the probability of not being censored must be sufficiently greater than 0 or else the weights that adjust for censoring get too large.
While the estimators that use WAC instead of LAC require a few more assumptions, some researchers are more comfortable modeling the treatment and censoring than the outcome. In this case, the IPW or WAC-IPWRA estimator would be the estimator of choice.

See *Specification diagnostics and tests* below for information about testing these assumptions.

### Assumptions for the ATET

We noted in *Average treatment effect on the treated* that the ATET is sometimes more interesting than the ATE. We can also estimate the ATET under less restrictive versions of the conditional independence assumption and the sufficient overlap assumption than those required for the ATE.

While ATE estimation requires that the potential outcomes for both the treated and the not treated be conditionally independent of treatment assignment, ATET estimation requires that only the not treated potential outcome be conditionally independent of treatment assignment.

This weaker version of conditional independence allows the gains from the treatment to be related to treatment assignment, after conditioning on the covariates. We can estimate the ATET, but not the ATE, if some unobserved factor increases (or decreases) the likelihood of assignment to the treatment, increases (or decreases) the time to event in the treatment group, and has no effect on the time to event when not in the treatment group.

For example, suppose that smoking is an acquired taste and that individuals who acquire the taste for smoking more easily are less adversely affected by smoking and otherwise similar to everyone else when not smoking. Taste for smoking is unobservable, and our data have no measure of this variable. In this case, we could estimate the ATET but not the ATE.

The weaker version of the sufficient overlap assumption only requires that each individual in the treated subpopulation have a positive probability of not getting treated. In contrast, ATE estimation requires that each individual in the population have a positive probability of getting each treatment level. In particular, we can estimate the ATET, but not the ATE, when some individuals in the population have zero chance of getting the treatment. For example, we could estimate the ATET, but not the ATE, if some women will never smoke for religious reasons.

Even when the conditions for ATE estimation hold, the ATE and ATET may differ. Finding that the ATET is significantly different from the ATE does not mean that the ATE is incorrectly estimated.

See Heckman (1997) and Wooldridge (2010, 911–912) for more information about the assumptions necessary to estimate the ATET.

### Specification diagnostics and tests

After *stteffects ipw* and *stteffects ipwra*, some specification checks for the treatment-assignment model and the overlap condition are available.

The checks for the treatment-assignment model are known as balance checks. When the covariate distributions are invariant to the treatment level, the covariates are said to be balanced. The concept of balanced covariates comes from the experimental literature, in which random treatment assignment ensures that the covariates are balanced.

In observational data, the covariates are almost never balanced in the raw data. Weighting methods can be viewed as using a treatment-assignment model to balance the covariates. If the treatment-assignment model is well specified, the weights constructed from this model will balance the covariates. One of the nice features of balance checks is that they do not depend on the outcome or its distribution. This fact is especially useful for survival-time outcomes because censoring of the outcome has no effect on the balance checks, so the balance checks implemented in *tebalance* work without modification.
Conditional on the treatment-assignment model being well specified, we can use the estimated probabilities of treatment, known as the propensity scores, to look for signs that the overlap condition is violated. These checks depend only on the estimated treatment probabilities and are not affected by any censoring of the outcome, so the methods implemented in `teoverlap` work without modification.

We begin examining our model by using `tebalance summarize` after refitting the models used by the LAC-IPWRA estimator.

```
. quietly stteffects ipwra (age exercise diet education)
> (smoke age exercise diet education)
. tebalance summarize
```

<table>
<thead>
<tr>
<th></th>
<th>Raw</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs</td>
<td>2,000</td>
<td>2,000.0</td>
</tr>
<tr>
<td>Treated obs</td>
<td>738</td>
<td>994.1</td>
</tr>
<tr>
<td>Control obs</td>
<td>1,262</td>
<td>1,005.9</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Raw Weighted</th>
<th>Raw Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>age</td>
<td>-.3122094</td>
<td>-.0184574</td>
</tr>
<tr>
<td>exercise</td>
<td>-.4975269</td>
<td>-.0458412</td>
</tr>
<tr>
<td>diet</td>
<td>-.2479756</td>
<td>.0021802</td>
</tr>
<tr>
<td>education</td>
<td>-.4801442</td>
<td>-.0216366</td>
</tr>
</tbody>
</table>

The weighted standardized differences are much closer to 0 than the raw standardized differences, and the weighted variance ratios are much closer to 1 than the raw variance ratios. These results indicate that the model-based treatment weights balanced the covariates; see [CAUSAL tebalance] and [CAUSAL tebalance summarize] for details.

The diagnostics presented by `tebalance summarize` are not a formal test. However, we can use `tebalance overid` to conduct a formal test of the hypothesis that the weights constructed from the treatment-assignment model balanced the covariates.

```
. tebalance overid
```

```
Iteration 0:   Criterion =    .22681884
Iteration 1:   Criterion =    .22692316  (backed up)
Iteration 2:   Criterion =    .23090158
Iteration 3:   Criterion =    .2311461
Iteration 4:   Criterion =    .23256285
Iteration 5:   Criterion =    .23286304
Iteration 6:   Criterion =    .23335858
Iteration 7:   Criterion =    .23355671
Iteration 8:   Criterion =    .23356711
Iteration 9:   Criterion =    .23356711

Overidentification test for covariate balance
H0: Covariates are balanced
    chi2(5) = 3.28142
    Prob > chi2 = 0.6567
```

There is no significant evidence against the null hypothesis. The interpretation is that we do not reject the null hypothesis that the treatment-assignment model is well specified; see [CAUSAL tebalance] and [CAUSAL tebalance overid] for details.

Given that we do not reject the treatment-assignment model, we can use this model to look for evidence that the overlap condition is violated. We begin by using `teoverlap`.

```
```
The densities of the propensity scores for the smokers and nonsmokers appear to have the same support, indicating that there is no violation of the overlap condition. The only indicator of a possible problem is that the support of the density for nonsmokers gets very close to 0. This problem would affect ATE estimation but not ATET estimation, as discussed in Assumptions and tradeoffs. To further investigate, we compute and summarize the predicted propensity score by treatment level.

```
. predict ps1, ps tlevel(Smoker)
. summarize ps1 if smoke == 0

Variable | Obs | Mean  | Std. dev. | Min    | Max     
-----------|-----|-------|-----------|--------|---------
ps1        | 1262| .3410001 | .1381673  | .014819| .6161401 |

. summarize ps1 if smoke == 1

Variable | Obs | Mean  | Std. dev. | Min    | Max     
-----------|-----|-------|-----------|--------|---------
ps1        | 738 | .4168805 | .1107557  | .0454891| .6216282 |
```

To interpret these results, recall that ATE estimation requires that the minimum propensity score for each treatment level be sufficiently greater than 0 and that the maximum propensity score for each treatment level be sufficiently less than 1. Also recall that ATET estimation only requires that the maximum propensity score for each treatment level be sufficiently less than 1.

For ATE estimation, only the minimum predicted propensity score for nonsmokers presents a challenge, and 0.015 is probably not too small. For ATET estimation, neither maximum causes concern.

For information about choosing among the stteffects estimators and their functional forms for the different models, see Model choice under Remarks and examples in [CAUSAL] teffects intro advanced.

**Multivalued treatments**

stteffects can estimate treatment effects for multivalued treatments; here we provide some examples. See [CAUSAL] teffects multivalued for an introduction to interpreting effects from multivalued treatments.
Example 6: Multivalued ATE estimation

We have another fictional dataset that records the time to a second heart attack among women aged 45–55 years. In this dataset, \texttt{atime} is the observed time in years to the second heart attack, and \texttt{fail} is the 0/1 indicator that a second heart attack was observed and recorded in \texttt{atime}. (When \texttt{fail} is 1, \texttt{atime} records the time to the second attack; when \texttt{fail} is 0, \texttt{atime} records a censored observation of the time to the second attack.)

These data also contain the age at the time of the first heart attack (\texttt{age}), and indices of each woman’s exercise level (\texttt{exercise}), diet quality (\texttt{diet}), and education attainment (\texttt{education}) prior to her first heart attack.

The treatment, smoking, is stored in the categorical variable \texttt{smoke}, which has the following value labels. The women who never smoked are labeled as \texttt{N}; the women who previously smoked but quit before their first heart attack are labeled as \texttt{B}; the women who previously smoked but quit after their first heart attack are labeled as \texttt{A}; and the women who continued to smoke after their first heart attack are labeled as \texttt{S}.

We begin by first reading in the data and then reviewing information previously stored using \texttt{stset}.

```
. use https://www.stata-press.com/data/r18/sheartm, clear
(Time to second heart attack (fictional))
. stset atime, failure(fail)
Survival-time data settings
  Failure event: fail!=0 & fail<.
  Observed time interval: (0, atime]  
  Exit on or before: failure
   10,000 total observations
   0 exclusions
   10,000 observations remaining, representing
   9,741 failures in single-record/single-failure data
   27,999.155 total analysis time at risk and under observation
   At risk from t = 0
   Earliest observed entry t = 0
   Last observed exit t = 17.40826
```

We continue by tabulating the treatment variable \texttt{smoke}.

```
. tabulate smoke

<table>
<thead>
<tr>
<th>Smoking level</th>
<th>Freq.</th>
<th>Percent</th>
<th>Cum.</th>
</tr>
</thead>
<tbody>
<tr>
<td>N</td>
<td>3,167</td>
<td>31.67</td>
<td>31.67</td>
</tr>
<tr>
<td>B</td>
<td>2,263</td>
<td>22.63</td>
<td>54.30</td>
</tr>
<tr>
<td>A</td>
<td>1,924</td>
<td>19.24</td>
<td>73.54</td>
</tr>
<tr>
<td>S</td>
<td>2,646</td>
<td>26.46</td>
<td>100.00</td>
</tr>
<tr>
<td>Total</td>
<td>10,000</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>
```

We see that 31.67% of the women never smoked, 22.63% of the women previously smoked but quit before their first heart attack, 19.24% of the women previously smoked but quit after their first heart attack, and 26.46% of the women continued to smoke after their first heart attack.
We now use `stteffects ra` to estimate the ATE by RA. We model the outcome as a function of age, exercise, diet, and education, and we specify that smoke is the treatment variable.

```
. stteffects ra (age exercise diet education) (smoke)
   Failure _d: fail
   Analysis time _t: atime
Iteration 0: EE criterion = 1.390e-20
Iteration 1: EE criterion = 2.220e-29
Survival treatment-effects estimation
   Number of obs = 10,000
   Estimator : regression adjustment
   Outcome model : Weibull
   Treatment model: none
   Censoring model: none

|     _t | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|--------|-------------|-----------|------|------|----------------------|
| ATE    |             |           |      |      |                      |
| smoke  |             |           |      |      |                      |
| (B vs N) | -.4129793   | .0317    | -13.03 | 0.000 | -.47511 -.3508485  |
| (A vs N) | -1.281031   | .032866  | -38.98 | 0.000 | -1.345447 -1.216614 |
| (S vs N) | -2.167359   | .0338994 | -63.93 | 0.000 | -2.233801 -2.100917 |
| POmean |             |           |      |      |                      |
| smoke  |             |           |      |      |                      |
| N     | 3.745919    | .0289014 | 129.61 | 0.000 | 3.689273 3.802565   |
```

The average time to a second heart attack is 0.41 years sooner when all the women smoked at some point but quit before their first heart attack than when all the women never smoked. The average time to a second heart attack is 1.28 years sooner when all the women smoked at some point but quit after their first heart attack than when all the women never smoked. The average time to a second heart attack is 2.17 years sooner when all the women continued to smoke after their first heart attack than when all the women never smoked.
Example 7: Multivalued ATET estimation

In the at-risk subpopulation of women who continued to smoke, we want to estimate the effect of continuing to smoke ($S$) versus quitting after the first heart attack ($A$). Below we estimate the ATETs by RA, specifying $A$ to be the control level and $S$ to be the treatment level.

```stata
. stteffects ra (age exercise diet education) (smoke), atet control(A) tlevel(S)
    Failure _d: fail
    Analysis time _t: atime

Iteration 0:  EE criterion = 1.390e-20
Iteration 1:  EE criterion = 2.049e-29

Survival treatment-effects estimation
Number of obs = 10,000
Estimator : regression adjustment
Outcome model : Weibull
Treatment model: none
Censoring model: none

|              | Robust Coefficient | std. err. | z   | P>|z| | [95% conf. interval] |
|--------------|--------------------|-----------|-----|-----|----------------------|
| ATET         |                    |           |     |     |                      |
| smoke        |                    |           |     |     |                      |
| (N vs A)     | 1.290123           | .0377552  | 34.17 | 0.000 | 1.216125 1.364122    |
| (B vs A)     | .8748349           | .0239595  | 36.51 | 0.000 | .8278751 .9217946    |
| (S vs A)     | -.8869257          | .0272301  | -32.57| 0.000 | -.9402958 -.8335557  |
| P0mean       |                    |           |     |     |                      |
| smoke        | 2.500108           | .0217833  | 114.77| 0.000 | 2.457413 2.542802    |
```

The parameter (S vs A) is the one of interest. The estimate implies that the average time to a second heart attack among women who continue to smoke is 0.89 years sooner when they all continue to smoke than when they all quit smoking after their first heart attack.

Acknowledgments

We thank Xioafei Bai of Servier, Matias Cattaneo of Princeton University, Barbara Sianesi of the Institute for Fiscal Studies, and Jeff Wooldridge of Michigan State University for helpful conversations about methods and the literature.

References


Also see

[CAUSAL] stteffects postestimation — Postestimation tools for stteffects
[CAUSAL] teffects intro advanced — Advanced introduction to treatment effects for observational data
[ST] streg — Parametric survival models
[ST] stset — Declare data to be survival-time data
[U] 20 Estimation and postestimation commands
**stteffects ipw — Survival-time inverse-probability weighting**

**Description**

`stteffects ipw` estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational survival-time data with random time to censoring. Estimation is by inverse-probability weighting (IPW). IPW estimators use weighted averages of the observed outcome. The estimated weights correct for missing data on the potential outcomes and for censored survival times. `stteffects ipw` offers several choices for the functional forms of the treatment model and the time-to-censoring model. Binary and multivalued treatments are accommodated.

See [CAUSAL] `stteffects intro` for an overview of estimating treatment effects from observational survival-time data.

**Quick start**

Specify time as observed failure time and fail as failure indicator

```
stset time, failure(fail)
```

ATE of binary `treat2` on time by IPW using a logistic model of `treat2` on `x` and `w` and using `x` and `w` in a Weibull model for the censoring time

```
stteffects ipw (treat2 x w) (x w)
```

Same as above, but estimate the ATET

```
stteffects ipw (treat2 x w) (x w), atet
```

ATE of `treat2` on time by IPW using a probit model of `treat2` on `x` and `w` and using `x` and `w` in a gamma model for the censoring time

```
stteffects ipw (treat2 x w, probit) (x w, gamma)
```

ATE for treatment levels 2 and 3 of three-valued treatment `treat3`

```
stteffects ipw (treat3 x w) (x w)
```

Same as above, and specify that `treat3 = 3` is the control level using the value label “MyControl” for 3

```
stteffects ipw (treat3 x w) (x w), control("MyControl")
```

**Menu**

Statistics > Causal inference/treatment effects > Survival outcomes > Inverse-probability weighting (IPW)
Syntax

```
stteffects ipw (tvar tmvarlist [, tmoptions]) (cmvarlist [, cmoptions])
   [if] [in] [, stat options]
```

tvar must contain integer values representing the treatment levels.
tmvarlist specifies the variables that predict treatment assignment in the treatment model.
cmvarlist specifies the variables that predict censoring in the censoring model.

tmoptions

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>logit</td>
<td>logistic treatment model; the default</td>
</tr>
<tr>
<td>probit</td>
<td>probit treatment model</td>
</tr>
<tr>
<td>hetprobit(varlist)</td>
<td>heteroskedastic probit treatment model</td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant from treatment model</td>
</tr>
</tbody>
</table>

cmoptions

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>weibull</td>
<td>Weibull; the default</td>
</tr>
<tr>
<td>exponential</td>
<td>exponential</td>
</tr>
<tr>
<td>gamma</td>
<td>two-parameter gamma</td>
</tr>
<tr>
<td>normal</td>
<td>lognormal</td>
</tr>
<tr>
<td>ancillary(avarlist [, noconstant])</td>
<td>specify variables used to model ancillary parameter</td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant from censoring model</td>
</tr>
</tbody>
</table>

stat

<table>
<thead>
<tr>
<th>Stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
</tr>
<tr>
<td>pomeans</td>
<td>estimate potential-outcome means</td>
</tr>
</tbody>
</table>
**Options**

<table>
<thead>
<tr>
<th>Model</th>
<th>Stat</th>
</tr>
</thead>
</table>

**ancillary**(_avarlist_ [, noconstant _]) specifies the variables used to model the ancillary parameter. By default, the ancillary parameter does not depend on covariates. Specifying **ancillary**(_avarlist_, noconstant) causes the constant to be suppressed in the model for the ancillary parameter. **noconstant**; see [R] Estimation options.

**stat** is one of three statistics: **ate**, **atet**, or **pomeans**. **ate** is the default.

- **ate** specifies that the average treatment effect be estimated.
- **atet** specifies that the average treatment effect on the treated be estimated.
- **pomeans** specifies that the potential-outcome means for each treatment level be estimated.

---

**options**

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>options</strong></td>
</tr>
<tr>
<td><strong>SE/Robust</strong></td>
</tr>
<tr>
<td><strong>vce(vcetype)</strong>, vcetype may be robust, cluster clustvar, bootstrap, or jackknife</td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
</tr>
<tr>
<td><strong>level(#)</strong> set confidence level; default is level(95)</td>
</tr>
<tr>
<td><strong>aequations</strong> display auxiliary-equation results</td>
</tr>
<tr>
<td><strong>nosh</strong> do not show st setting information</td>
</tr>
<tr>
<td><strong>display_options</strong> control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td><strong>Maximization</strong></td>
</tr>
<tr>
<td><strong>maximize_options</strong> control the maximization process; seldom used</td>
</tr>
<tr>
<td><strong>iterinit(#)</strong> specify starting-value iterations; seldom used</td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
</tr>
<tr>
<td><strong>pстolerance(#)</strong> set tolerance for the overlap assumption</td>
</tr>
<tr>
<td><strong>osample(newvar)</strong> identify observations that violate the overlap assumption</td>
</tr>
<tr>
<td>**control(#</td>
</tr>
<tr>
<td>**tlevel(#</td>
</tr>
<tr>
<td><strong>coeflegend</strong> display legend instead of statistics</td>
</tr>
</tbody>
</table>

You must stset your data before using stteffects; see [ST] stset.
tvarlist, cmvarlist, and avarlist may contain factor variables; see [U] 11.4.3 Factor variables.
bootstrap, by, collect, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.
Weights are not allowed with the bootstrap prefix; see [R] bootstrap.
fweights, iweights, and pweights may be specified using stset; see Weights under Remarks and examples in [ST] stset. However, weights may not be specified if you are using the bootstrap prefix.
coeflegend does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting

Reporting level(#); see [R] Estimation options.

aequations specifies that the results for the outcome-model or the treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

noshow prevents stteffects ipw from showing the key st variables. This option is rarely used because most people type stset, show or stset, noshow to permanently set whether they want to see these variables mentioned at the top of the output of every st command; see [ST] stset.

display_options: noci, nopvalues, nommitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and noemptycells; see [R] Estimation options.

Maximization

maximize_options: iterate(#), [no]log, and from(init_specs); see [R] Maximize. These options are seldom used.

init_specs is one of

matname [, skip copy]

# [, # ...], copy

iterinit(#) specifies the maximum number of iterations used to calculate the starting values. This option is seldom used.

Advanced

pstolerance(#) specifies the tolerance used to check the overlap assumption. The default value is pstolerance(1e-5). stteffects will exit with an error if an observation has an estimated propensity score smaller than that specified by pstolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that violate the overlap assumption.

control(#) | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with the statistic pomeans. control() and tlevel() may not specify the same treatment level.

tlevel(#) | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

The following option is available with stteffects but is not shown in the dialog box:

coeflegend; see [R] Estimation options.
Remarks and examples

If you are not familiar with the framework for treatment-effects estimation from observational survival-time data, please see [CAUSAL] stteffects intro.

IPW estimators use contrasts of weighted averages of observed outcomes to estimate treatment effects. The estimated weights correct for data that are missing because each subject is only observed after receiving one of the possible treatment levels and because some survival-time outcomes are censored.

The IPW estimators implemented in stteffects ipw use a three-step approach to estimating the ATE:

1. Estimate the parameters of a treatment-assignment model, and compute the component of the estimated weights that accounts for data missing because each subject is only observed after receiving one of the possible treatment levels.
2. Estimate the parameters of a time-to-censoring model, and compute the component of the estimated weights that accounts for data lost to censoring.
3. Use the estimated weights to compute weighted averages of the outcomes for each treatment level.

To estimate the ATET, we use different weights in step 2.

The time to censoring must be random to use stteffects ipw because the model in step 2 is not well defined if the time to censoring is fixed. See [CAUSAL] stteffects intro for more details. For information about estimators that accommodate a fixed time to censoring, see [CAUSAL] stteffects ra and [CAUSAL] stteffects ipwra.

Here we note only a few entry points to the vast literature on IPW estimators. Hirano, Imbens, and Ridder (2003), Imbens (2000, 2004), Imbens and Wooldridge (2009), Rosenbaum and Rubin (1983), Robins and Rotnitzky (2006), Wooldridge (2002, 2007), Cameron and Trivedi (2005, chap. 25), Wooldridge (2010, chap. 21), and Vittinghoff et al. (2012, chap. 9) provide excellent general introductions to estimating ATEs and to the IPW estimators in particular.

Like streg and other survival-time commands, stteffects ipw uses the outcome variable and the failure indicator computed by, and optionally weights specified with, stset. stteffects ipw is not appropriate for data with time-varying covariates, also known as multiple-record survival-time data, or for delayed-entry data.

Example 1: Estimating the ATE

Suppose we wish to study the effect of smoking on the time to a second heart attack among women aged 45–55 years. In our fictional sheart dataset, atime is the observed time in years to a second heart attack or censoring, and fail is the 0/1 indicator that a second heart attack was observed. (When fail is 1, atime records the time to the second heart attack; when fail is 0, atime records a censored observation of the time to a second heart attack.) We previously stset these data; see A quick tour of the estimators in [CAUSAL] stteffects intro.

The treatment, smoking, is stored in the 0/1 indicator smoke. These data also contain age at the time of the first heart attack (age), and indices of the level of exercise (exercise), diet quality (diet), and education (education) prior to the first heart attack.

We can use stteffects ipw to estimate the ATE. We model treatment assignment using the default logit model with covariates on age, exercise, and education. We model the time to censoring using the default Weibull model with covariates on age, exercise, diet, and education.
. use https://www.stata-press.com/data/r18/sheart
  (Time to second heart attack (fictional))
. stteffects ipw (smoke age exercise education) (age exercise diet education)

    Failure _d: fail
    Analysis time _t: atime
Iteration 0:  EE criterion = 2.042e-18
Iteration 1:  EE criterion = 5.191e-31
Survival treatment-effects estimation
  Number of obs   =    2,000
  Estimator : inverse-probability weights
  Outcome model : weighted mean
  Treatment model: logit
  Censoring model: Weibull

                       Robust
              _t    Coefficient   std. err.      z    P>|z|      [95% conf. interval]
ATE
  smoke (Smoker vs Nonsmoker)  -2.22226   .6307573   -3.52  0.000   -3.458522   -0.9859983
POmean
  smoke Nonsmoker            4.235569   .5210937    8.13  0.000    3.214244    5.256894

When every woman smoked in the population of women aged 45–55 years who have had a heart
attack, the average time to a second heart attack is estimated to be 2.22 years less than when no
women in the population of interest smoked. The estimated average time to a second heart attack
when no women in the population of interest smoked is 4.24 years.

The ratio of the ATE to the control-level POM measures the importance of the effect. In this example,
when every woman smoked, the average time to a second heart attack falls by an estimated 52%
relative to the case when none of them smoked. See example 3 in [CAUSAL] stteffects ra for an
example that uses nlcom to compute a point estimate and a confidence interval for this ratio.

Example 2: Different treatment and censoring models

Instead of a logit model for the treatment assignment, we could have used a probit or a heteroskedastic
probit model. Instead of a Weibull model for the censoring time, we could have used an exponential,
a gamma, or a lognormal model. For a quick comparison, we now estimate the ATE using a probit
model for the treatment assignment and using a gamma model for the censoring time.
. stteffects ipw (smoke age exercise education, probit)  
> (age exercise diet education, gamma)

Failure _d: fail  
Analysis time _t: atime  
Iteration 0: EE criterion = 3.534e-15  
Iteration 1: EE criterion = 5.263e-27  
Survival treatment-effects estimation Number of obs = 2,000  
Estimator : inverse-probability weights  
Outcome model : weighted mean  
Treatment model: probit  
Censoring model: gamma

| _t   | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|------|-------------|-----------|------|-----|----------------------|
| ATE  | smoke       |           |      |     |                      |
| (Smoker vs Nonsmoker) | -2.646808 | .8368254 | -3.16 | 0.002 | -4.286956 -1.006661 |
| POmean | smoke      |           |      |     |                      |
| Nonsmoker | 4.702301 | .7404567 | 6.35 | 0.000 | 3.251033 6.15357    |

The estimated ATE of $-2.65$ and control-level POmean of $4.70$ are similar to the values of $-2.22$ and $4.24$ reported in example 1.

Example 3: Estimating the ATET

Intuitively, the ATET measures the effect of the treatment on an at-risk subpopulation. Sometimes the subpopulation that gets the treatment defines such an at-risk subpopulation. The ATET has the added benefit that it can be estimated under weaker conditions than the ATE; see Assumptions and tradeoffs under Remarks and examples in [CAUSAL] stteffects intro.
. stteffects ipw (smoke age exercise education) (age exercise diet education), > atet
  
  Failure _d: fail
  Analysis time _t: atime
  
  Iteration 0: EE criterion = 2.042e-18
  Iteration 1: EE criterion = 1.248e-32
  
  Survival treatment-effects estimation Number of obs = 2,000
  Estimator : inverse-probability weights
  Outcome model : weighted mean
  Treatment model: logit
  Censoring model: Weibull

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
<th></th>
<th></th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
<td>std. err.</td>
<td>z</td>
<td>P&gt;</td>
</tr>
<tr>
<td>ATET</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>smoke</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(Smoker vs Nonsmoker)</td>
<td>-1.846136</td>
<td>.5076872</td>
<td>-3.64</td>
<td>0.000</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>POmean</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>smoke</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonsmoker</td>
<td>3.543788</td>
<td>.474395</td>
<td>7.47</td>
<td>0.000</td>
</tr>
</tbody>
</table>

When every woman in the subpopulation smoked, the average time to a second heart attack is estimated to be 1.85 years less than when no women in the subpopulation smoked. The estimated average time to a second heart attack when no women in the subpopulation smoked is 3.54 years.

Stored results

stteffects ipw stores the following in e():

Scalars
  e(N) number of observations
  e(n_j) number of observations for treatment level j
  e(N_clust) number of clusters
  e(k_eq) number of equations in e(b)
  e(k_levels) number of levels in treatment variable
  e(treated) level of treatment variable defined as treated
  e(control) level of treatment variable defined as control
  e(converged) 1 if converged, 0 otherwise

Macros
  e(cmd) stteffects
  e(cmdline) command as typed
  e(dead) _d
  e(depvar) _t
  e(tvar) name of treatment variable
  e(subcmd) ipw
  e(tmodel) treatment model: logit, probit, or hetprobit
  e(cmodel) censoring model: weibull, exponential, gamma, or lognormal
  e(stat) statistic estimated: ate, atet, or pomeans
  e(wtype) weight type
  e(wexp) weight expression
  e(title) title in estimation output
  e(clustvar) name of cluster variable
  e(tlevels) levels of treatment variable
In addition to the above, the following is stored in \texttt{r}():

Matrices\
\texttt{r(table)} matrix containing the coefficients with their standard errors, test statistics, \textit{p}-values, and confidence intervals

Note that results stored in \texttt{r}() are updated when the command is replayed and will be replaced when any \texttt{r}-class command is run after the estimation command.

\section*{Methods and formulas}

The methods and formulas for the IPW estimators implemented in \texttt{stteffects ipw} are given in \textit{Methods and formulas} of [CAUSAL] \texttt{stteffects ipwra}.

\section*{References}


Also see

[CAUSAL] *stteffects postestimation* — Postestimation tools for stteffects

[CAUSAL] *stteffects intro* — Introduction to treatment effects for observational survival-time data

[ST] *streg* — Parametric survival models

[ST] *stset* — Declare data to be survival-time data

[U] *20 Estimation and postestimation commands*
stteffects ipwra — Survival-time inverse-probability-weighted regression adjustment

Description

stteffects ipwra estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational survival-time data by inverse-probability-weighted regression adjustment (IPWRA). IPWRA estimators use missingness-adjusted regression coefficients to compute averages of treatment-level predicted outcomes. Contrasts of these averages estimate the treatment effects. stteffects ipwra offers several choices for the functional forms of the outcome model, of the treatment model, and of the optional time-to-censoring model. Binary and multivalued treatments are accommodated.

See [CAUSAL] stteffects intro for an overview of estimating treatment effects from observational survival-time data.

Quick start

Specify time as observed failure time and fail as failure indicator

    stset time, failure(fail)

ATE of binary treatment treat2 estimated by IPWRA using a Weibull model for time on x1 and x2 and a logistic model for treat2 on x1 and w

    stteffects ipwra (x1 x2) (treat2 x1 w)

Same as above, but estimate the ATET

    stteffects ipwra (x1 x2) (treat2 x1 w), atet

Gamma model for time and probit model for treat2

    stteffects ipwra (x1 x2, gamma) (treat2 x1 w, probit)

ATE for each level of three-valued treatment treat3

    stteffects ipwra (x1 x2) (treat3 x1 w)

Same as above, and specify that treat3 = 3 is the control level using the value label “MyControl” for 3

    stteffects ipwra (x1 x2) (treat3 x1 w), control("MyControl")

ATE of treat2 estimated by IPWRA using a Weibull model for time on x1 and x2, a logistic model for treat2 on x1 and w, and a Weibull model for the time to censoring with covariates x1 and x2

    stteffects ipwra (x1 x2) (treat2 x1 w) (x1 x2)

Gamma model for time, probit model for treat2, and gamma model for censoring

    stteffects ipwra (x1 x2, gamma) (treat2 x1 w, probit) (x1 x2, gamma)
Menu
Statistics > Causal inference/treatment effects > Survival outcomes > Regression adjustment with IPW

Syntax

```
stteffects ipwra (omvarlist [ , omoptions ]) (tvar tmvarlist [ , tmoptions ])
    [(cmvarlist [ , cmoptions ])) [ if ] [ in ] [ , stat options ]
```

*omvarlist* specifies the variables that predict the survival-time variable in the outcome model.
*tvar* must contain integer values representing the treatment levels.
*tmvarlist* specifies the variables that predict treatment assignment in the treatment model.
*cmvarlist* specifies the variables that predict censoring in the censoring model.

### omoptions

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>weibull</strong></td>
<td>Weibull; the default</td>
</tr>
<tr>
<td><strong>exponential</strong></td>
<td>exponential</td>
</tr>
<tr>
<td><strong>gamma</strong></td>
<td>two-parameter gamma</td>
</tr>
<tr>
<td><strong>lnormal</strong></td>
<td>lognormal</td>
</tr>
<tr>
<td><strong>ancillary(avarlist [ , noconstant ])</strong></td>
<td>specify variables used to model ancillary parameter</td>
</tr>
<tr>
<td><strong>noconstant</strong></td>
<td>suppress constant from outcome model</td>
</tr>
</tbody>
</table>

### tmoptions

<table>
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<th>Model</th>
<th>Description</th>
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</thead>
<tbody>
<tr>
<td><strong>logit</strong></td>
<td>logistic treatment model; the default</td>
</tr>
<tr>
<td><strong>probit</strong></td>
<td>probit treatment model</td>
</tr>
<tr>
<td><strong>hetprobit(varlist)</strong></td>
<td>heteroskedastic probit treatment model</td>
</tr>
<tr>
<td><strong>noconstant</strong></td>
<td>suppress constant from treatment model</td>
</tr>
</tbody>
</table>

### cmoptions

<table>
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<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>weibull</strong></td>
<td>Weibull; the default</td>
</tr>
<tr>
<td><strong>exponential</strong></td>
<td>exponential</td>
</tr>
<tr>
<td><strong>gamma</strong></td>
<td>two-parameter gamma</td>
</tr>
<tr>
<td><strong>lnormal</strong></td>
<td>lognormal</td>
</tr>
<tr>
<td><strong>ancillary(avarlist [ , noconstant ])</strong></td>
<td>specify variables used to model ancillary parameter</td>
</tr>
<tr>
<td><strong>noconstant</strong></td>
<td>suppress constant from censoring model</td>
</tr>
</tbody>
</table>
**stat**

Stat

<table>
<thead>
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<th>Description</th>
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</thead>
<tbody>
<tr>
<td><strong>ate</strong></td>
</tr>
<tr>
<td><strong>atet</strong></td>
</tr>
<tr>
<td><strong>pomeans</strong></td>
</tr>
</tbody>
</table>

**options**

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SE/Robust</strong></td>
</tr>
<tr>
<td><strong>vce(vcetype)</strong></td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
</tr>
<tr>
<td><strong>level(#)</strong></td>
</tr>
<tr>
<td><strong>aequations</strong></td>
</tr>
<tr>
<td><strong>noshow</strong></td>
</tr>
<tr>
<td><strong>display_options</strong></td>
</tr>
<tr>
<td><strong>Maximization</strong></td>
</tr>
<tr>
<td><strong>maximize_options</strong></td>
</tr>
<tr>
<td><strong>iterinit(#)</strong></td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
</tr>
<tr>
<td><strong>pstolerance(#)</strong></td>
</tr>
<tr>
<td><strong>osample(newvar)</strong></td>
</tr>
<tr>
<td>**control(#)</td>
</tr>
<tr>
<td>**tlevel(#)</td>
</tr>
<tr>
<td><strong>coeflegend</strong></td>
</tr>
</tbody>
</table>

You must stset your data before using stteffects; see [ST] stset.

omvarlist, tmvarlist, cmvarlist, and avarlist may contain factor variables; see [U] 11.4.3 Factor variables.

bootstrap, by, collect, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

fweights, iweights, and pweights may be specified using stset; see Weights under Remarks and examples in [ST] stset. However, weights may not be specified if you are using the bootstrap prefix.

coefflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

### Options

**ancillary(avarlist [, noconstant ])** specifies the variables used to model the ancillary parameter.

By default, the ancillary parameter does not depend on covariates. Specifying ancillary(avarlist, noconstant) causes the constant to be suppressed in the model for the ancillary parameter.
ancillary() may be specified for the model for survival-time outcome, for the model for the censoring variable, or for both. If ancillary() is specified for both, the varlist used for each model may be different.

noconstant; see [R] Estimation options.

**Stat**

/stat is one of three statistics: ate, atet, or pomeans. ate is the default.

ate specifies that the average treatment effect be estimated.

atet specifies that the average treatment effect on the treated be estimated.

pomeans specifies that the potential-outcome means for each treatment level be estimated.

**SE/Robust**

vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

**Reporting**

level(#) ; see [R] Estimation options.

aequations specifies that the results for the outcome-model or treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

noshow prevents stteffects ipwra from showing the key st variables. This option is rarely used because most people type stset, show or stset, noshow to permanently set whether they want to see these variables mentioned at the top of the output of every st command; see [ST] stset.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

**Maximization**

maximize_options: iterate(#), [no]log, and from(init_specs); see [R] Maximize. These options are seldom used.

-init_specs is one of

matname [, skip copy]

# [, # ...], copy

iterinit(#) specifies the maximum number of iterations used to calculate the starting values. This option is seldom used.

**Advanced**

pstolerance(#) specifies the tolerance used to check the overlap assumption. The default value is pstolerance(1e-5). stteffects will exit with an error if an observation has an estimated propensity score smaller than that specified by pstolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that violate the overlap assumption.
control(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with the statistic pmeans. control() and tlevel() may not specify the same treatment level.

tlevel(# | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

The following option is available with stteffects but is not shown in the dialog box: coeflegend; see [R] Estimation options.

Remarks and examples

If you are not familiar with the framework for treatment-effects estimation from observational survival-time data, please see [CAUSAL] stteffects intro.

IPWRA estimators use estimated weights to obtain missingness-adjusted outcome-regression parameters. The missingness-adjusted outcome-regression parameters are used to compute averages of treatment-level predicted outcomes. Contrasts of these averages estimate the treatment effects.

The estimated weights account for the missing potential outcome and, optionally, for data lost to censoring. The weights are estimated using a treatment-assignment model and, optionally, a model for the censoring time. A term in the estimator for the outcome-regression parameters accounts for data lost to censoring when estimated weights are not used.

There are two versions of the IPWRA estimator because there are two methods of accounting for the data lost to censoring.

1. IPWRA estimators that adjust for censoring by including a term in the likelihood function for the outcome-model parameters are known as likelihood-adjusted-censoring IPWRA (LAC-IPWRA) estimators.

2. IPWRA estimators that adjust for censoring by weighting the likelihood function for the outcome-model parameters by estimated inverse-probability-of-censoring weights are known as weighted-adjusted-censoring IPWRA (WAC-IPWRA) estimators.

The LAC-IPWRA estimators require fewer assumptions than the WAC-IPWRA estimators. Outlining the steps performed by LAC-IPWRA and WAC-IPWRA estimators allows us to be more specific about the tradeoffs between the estimators.

LAC-IPWRA estimators use a three-step approach to estimating treatment effects:

1. Estimate the parameters of a treatment-assignment model and compute inverse-probability-of-treatment weights.

2. Obtain the treatment-specific predicted mean outcomes for each subject by using the weighted maximum likelihood estimators. Estimated inverse-probability-of-treatment weights are used to weight the maximum likelihood estimator. A term in the likelihood function adjusts for right-censored survival times.

3. Compute the means of the treatment-specific predicted mean outcomes. Contrasts of these averages provide the estimates of the ATEs. By restricting the computations of the means to the subset of treated subjects, we can obtain the ATETs.
WAC-IPWRA estimators use a four-step approach to estimating treatment effects:

1. Estimate the parameters of a treatment-assignment model and compute inverse-probability-of-treatment weights.
2. Estimate the parameters of a time-to-censoring model and compute inverse-probability-of-censoring weights.
3. Obtain the treatment-specific predicted mean outcomes for each subject by using the weighted maximum likelihood estimators. Estimated inverse-probability-of-treatment weights and inverse-probability-of-censoring weights are used to weight the maximum likelihood estimator. The inverse-probability-of-censoring weights account for right-censored survival times.
4. Compute the means of the treatment-specific predicted mean outcomes. Contrasts of these averages provide the estimates of the ATEs. By restricting the computations of the means to the subset of treated subjects, we can obtain the ATETs.

The WAC-IPWRA estimators require that the censoring time be random and that the time-to-censoring model be well specified. The implemented WAC-IPWRA estimators also require that the time-to-censoring process not vary by treatment level. The LAC-IPWRA estimators do not require these extra assumptions because they use a likelihood term instead of weights to adjust for the data lost to censoring.


Like streg and other survival-time commands, stteffects ipwra uses the outcome variable and the failure indicator computed by, and optionally weights specified with, stset. stteffects ipwra is not appropriate for data with time-varying covariates, also known as multiple-record survival-time data, or for delayed-entry data.

Example 1: Estimating the ATE by LAC-IPWRA

Suppose we wish to study the effect of smoking on the time to a second heart attack among women aged 45–55 years. In our fictional sheart dataset, atime is the observed time in years to a second heart attack or censoring, and fail is the 0/1 indicator that a second heart attack was observed. (When fail is 1, atime records the time to the second heart attack; when fail is 0, atime records a censored observation of the time to a second heart attack.) We previously stset these data; see A quick tour of the estimators in [CAUSAL] stteffects intro.

The treatment, smoking, is stored in the 0/1 indicator smoke. These data also contain age at the time of the first heart attack (age), and indices of the level of exercise (exercise), diet quality (diet), and education (education) prior to the first heart attack.

We can use stteffects ipwra to estimate the ATE. We model the mean survival time using the default Weibull model, controlling for age, exercise, diet, and education. We model treatment assignment using the default logit model with covariates age, exercise, and education. We do not specify a time-to-censoring model so that we obtain the LAC estimator.
. use https://www.stata-press.com/data/r18/sheart
   (Time to second heart attack (fictional))
. stteffects ipwra (age exercise diet education) (smoke age exercise education)
    Failure _d:  fail
    Analysis time _t:  atime
Iteration 0:  EE criterion = 7.439e-15
Iteration 1:  EE criterion = 1.756e-26
Survival treatment-effects estimation Number of obs = 2,000
Estimator : IPW regression adjustment
Outcome model : Weibull
Treatment model: logit
Censoring model: none

| _t | Coefficient | std. err. | z       | P>|z|   | [95% conf. interval] |
|----|-------------|-----------|---------|-------|----------------------|
| ATE | smoke       |           |         |       |                      |
|     | (Smoker vs Nonsmoker) | | -1.591874 | .4837332 | -3.29 0.001 | -2.539973 -.643774 |
| POmean | smoke | Nonsmoker | | 4.214263 | .2598689 | 16.22 0.000 | 3.704929 4.723597 |

When every woman smoked in the population of women aged 45–55 years who have had a heart attack, the average time to a second heart attack is estimated to be 1.59 years less than when no women in the population of interest smoked. The estimated average time to a second heart attack when no women in the population of interest smoked is 4.21 years.

The ratio of the ATE to the control-level potential-outcome mean (POM) measures the importance of the effect. In this example, when all women smoked, the time to the second heart attack falls by an estimated 38% relative to the case in which no women smoked. See example 3 in [CAUSAL] stteffects ra for an example that uses nlcom to compute a point estimate and a confidence interval for this ratio.
Example 2: Different outcome and treatment models

Instead of a Weibull model for the outcome model, we could have used an exponential, a gamma, or a lognormal model. Instead of a logit model for the treatment assignment, we could have used a probit or a heteroskedastic probit model. This example uses a gamma model for the outcome and a probit model for the treatment assignment.

```
. stteffects ipwra (age exercise diet education, gamma)
> (smoke age exercise education, probit)

Failure _d: fail
Analysis time _t: atime
Iteration 0: EE criterion = 2.644e-13
Iteration 1: EE criterion = 2.153e-23
Survival treatment-effects estimation Number of obs = 2,000
Estimator : IPW regression adjustment
Outcome model : gamma
Treatment model: probit
Censoring model: none

|         | Coefficient | std. err. | z    | P>|z|    | [95% conf. interval] |
|---------|-------------|-----------|------|--------|--------------------|
| ATE     |             |           |      |        |                    |
| smoke   |             |           |      |        |                    |
| (Smoker |             |           |      |        |                    |
| vs Nonsmoker) | -1.387303  | .4786032  | -2.90| 0.004  | -2.325348           | -.4492583       |
| POmean  |             |           |      |        |                    |
| smoke   |             |           |      |        |                    |
| Nonsmoker | 3.97986   | .2258474  | 17.62| 0.000  | 3.537207           | 4.422512        |
```

The estimated $\text{ATE}$ of $-1.39$ and control-level $\text{POM}$ of $3.98$ are similar to the values of $-1.59$ and $4.21$ that we obtained in example 1.
Example 3: Estimating the ATE by WAC-IPWRA

Rather than using LAC, we may want to specify a time-to-censoring model. We now use \texttt{stteffects ipwra} to estimate the ATE by WAC-IPWRA. We use the same specification of the outcome and treatment models that we used in \textit{example 1}. However, now we specify a time-to-censoring model, using the default Weibull model with covariates \texttt{age}, \texttt{exercise}, \texttt{diet}, and \texttt{education}.

\begin{verbatim}
. stteffects ipwra (age exercise diet education) (smoke age exercise education)
    > (age exercise diet education)
    Failure _d: fail
    Analysis time _t: atime
Iteration 0:   EE criterion = 3.673e-18
Iteration 1:   EE criterion = 3.383e-31
Survival treatment-effects estimation
Estimator : IPW regression adjustment
Outcome model : Weibull
Treatment model: logit
Censoring model: Weibull

|     | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|-----|-------------|-----------|------|------|---------------------|
| ATE | smoke       |           |      |      |                     |
|     | (Smoker vs Nonsmoker) | -2.285057 | .7318456 | -3.12 | 0.002 | -3.719448 -.8506656 |
| P0mean | smoke | Nonsmoker | 4.385841 | .6427521 | 6.82 | 0.000 | 3.12607 5.645612 |

The estimated ATE of $-2.29$ differs from the ATE of $-1.59$ estimated by LAC-IPWRA, but the estimates of the control-level POM are similar between the two models: $4.39$ for the WAC compared with $4.21$ for the LAC.

\end{verbatim}
Example 4: Estimating the ATET by LAC-IPWRA

Intuitively, the ATET measures the effect of the treatment on an at-risk subpopulation. Sometimes the subpopulation that gets the treatment defines such an at-risk subpopulation. The ATET has the added benefit that it can be estimated under weaker conditions than the ATE; see Assumptions and tradeoffs under Remarks and examples in [CAUSAL] stteffects intro.

```
. stteffects ipwra (age exercise diet education) > (smoke age exercise education), atet
   Failure _d: fail
   Analysis time _t: atime
Iteration 0: EE criterion = 2.815e-19
Iteration 1: EE criterion = 6.331e-31
Survival treatment-effects estimation Number of obs = 2,000
Estimator : IPW regression adjustment
Outcome model : Weibull
Treatment model: logit
Censoring model: none

_t             Coefficient   std. err.     z   P>|z|    [95% conf. interval]

ATET
  smoke (Smoker vs Nonsmoker)   -1.775107    .3437506   -5.16   0.000   -2.448846   -1.101368

POmean
  smoke Nonsmoker               4.062424    .2779877   14.61   0.000    3.517578    4.60727
```

When all women in the subpopulation smoked, the average time to a second heart attack is estimated to be 1.78 years less than when no women in the subpopulation of interest smoked. If no women in the subpopulation of interest smoked, the average time to a second heart attack is 4.06 years.

Stored results

stteffects ipwra stores the following in e():

Scalars
- e(N): number of observations
- e(nj): number of observations for treatment level j
- e(N_clust): number of clusters
- e(k_eq): number of equations in e(b)
- e(k_levels): number of levels in treatment variable
- e(treated): level of treatment variable defined as treated
- e(control): level of treatment variable defined as control
- e(converged): 1 if converged, 0 otherwise

Macros
- e(cmd): stteffects
- e(cmdline): command as typed
- e(dead): _d
- e(depvar): _t
- e(tvar): name of treatment variable
Methods and formulas

Methods and formulas are presented under the following headings:

- Introduction
- Regression-adjusted estimators
- Weighted-adjusted-censoring assumptions
- Weighted regression-adjusted estimators
- Inverse-probability-weighted estimators
  - Uncensored data
- Inverse-probability-weighted regression-adjustment estimators
  - Weighted-adjusted-censoring IPWRA
  - Likelihood-adjusted-censoring IPWRA
- Functional-form details

Introduction

This section presents the methods and formulas used by the estimators implemented in `stteffects ra`, `stteffects wra`, `stteffects ipw`, and `stteffects ipwra`. This section assumes that you are familiar with the concepts and intuition from the estimators discussed in [CAUSAL] `teffects intro advanced`.

Each of the estimators implemented in `stteffects` has a multistep logic but is implemented as one step by simultaneously solving the estimating equations that define each step. This one-step estimating-equation approach provides consistent point estimates and a consistent variance–covariance of the estimator (VCE); see Newey (1984), Wooldridge (2010), and Drukker (2014).
Survival-time treatment-effects estimators handle two types of missing data. First, only one of the potential outcomes is observed, as is standard in causal inference. Second, the potential outcome for the received treatment may be censored. The data missing because of censoring may be handled by an outcome model, a censoring model, or both, just like the data missing due to observing only one potential outcome.

Technical note

Delayed entry would be a third type of missing data. The left-truncation process caused by delayed entry would also need to be modeled to estimate ATE parameters. The estimators implement in stteffects do not allow for delayed entry because they do not have a method for modeling how the left-truncation process selects the sample, conditional on the covariates.

All the implemented estimators are combinations of regression-adjustment (RA) and inverse-probability-weighted (IPW) techniques. RA estimators use an outcome model to account for the missing potential outcome and for censoring. IPW estimators use models for treatment assignment and censoring to construct weights that account for the missing potential outcome and for censoring.

The remainder of this section provides technical details about how the estimators in stteffects were implemented. We provide details only for the two-treatment-level case to simplify the formulas. We provide outlines for how the extensions to the multiple-treatment-level case were implemented.

Regression-adjusted estimators

We begin with the RA estimators implemented in stteffects ra. The RA estimators have the following logic:

RA1. For each treatment level \( \tau \in \{0, 1\} \), estimate by maximum likelihood (ML) the parameters \( \beta_\tau \) of a parametric model for the survival-time outcome \( t \) in which \( F(t|x, \tau, \beta_\tau) \) is the distribution of \( t \) conditional on covariates \( x \) and treatment level \( \tau \). Denote the estimates \( \beta_\tau \) by \( \hat{\beta}_{ra,\tau} \).

RA2. Use the estimated \( \hat{\beta}_{ra,\tau} \) and the functional form implied by \( F(t|x, \tau, \beta_\tau) \) to estimate the mean survival time, conditional on \( x \) and treatment level \( \tau \), for each sample observation, denoted by \( \hat{E}(t_i|x_i, \tau, \hat{\beta}_{ra,\tau}) \). Conditional independence of the treatment and the survival-time potential outcomes ensures that \( E(t|x, \tau, \beta_\tau) = E(t_\tau|x, \beta_\tau) \), where \( t_\tau \) is the potential survival-time outcome corresponding to treatment level \( \tau \). Under correct model specification, sample averages of \( \hat{E}(t_i|x_i, \tau, \hat{\beta}_{ra,\tau}) \) consistently estimate the POM for treatment level \( \tau \), denoted by \( \text{POM}_\tau \).

RA3. A contrast of the estimated POMs estimates the ATE.

If estimating an ATET, step RA2 is modified to use only the treated observations when estimating the POMs. A contrast of these POMs then estimates the ATET.

The contribution of the \( i \)th observation to the log likelihood that is maximized in step RA1 is

\[
L_{ra}(t_i, x_i, \tau, \hat{\beta}_{ra,\tau}) = \omega_i(\tau_i == \tau) \left[ (1 - c_i) \ln \left\{ f(t_i|x_i, \tau, \hat{\beta}_{ra,\tau}) \right\} \right. \\
\left. + c_i \ln \left\{ 1 - F(t_i|x_i, \tau, \hat{\beta}_{ra,\tau}) \right\} \right] 
\]  

(1)
where \( \varpi_i \) is the observation-level weight, \( c_i \) is the 0/1 indicator for whether the survival-time observation on person \( i \) was censored, and \( f(t_i|x_i, \tau, \widehat{\beta}_{ra,\tau}) \) is the density corresponding to distribution \( F(t_i|x_i, \tau, \widehat{\beta}_{ra,\tau}) \). The first term inside the curly braces in (1) accounts for the noncensored observations, and the second term inside the curly braces accounts for the censored observations.

The RA estimators for the POMs simultaneously solve estimating equations (2a) through (2d) for \( \widehat{\beta}_{ra,0}, \widehat{\beta}_{ra,1}, \widehat{POM}_{ra,0}, \) and \( \widehat{POM}_{ra,1} \).

\[
1/N \sum_{i=1}^{N} s_{ra}(t_i, x_i, 0, \widehat{\beta}_{ra,0}, F) = 0 \quad (2a)
\]

\[
1/N \sum_{i=1}^{N} s_{ra}(t_i, x_i, 1, \widehat{\beta}_{ra,1}, F) = 0 \quad (2b)
\]

\[
1/N \sum_{i=1}^{N} \varpi_i \left\{ \widehat{E}(t_i|x_i, \tau = 0, \widehat{\beta}_{ra,0}) - \widehat{POM}_{ra,0} \right\} = 0 \quad (2c)
\]

\[
1/N \sum_{i=1}^{N} \varpi_i \left\{ \widehat{E}(t_i|x_i, \tau = 1, \widehat{\beta}_{ra,1}) - \widehat{POM}_{ra,1} \right\} = 0 \quad (2d)
\]

where

\[
s_{ra}(t_i, x_i, 0, \widehat{\beta}_{ra,0}, F) = \frac{\partial L_{ra}(t_i, x_i, 0, \widehat{\beta}_{ra,0})}{\partial \beta_{ra,0}}
\]

is the vector of score equations from the ML estimator for \( \widehat{\beta}_{ra,0} \) based on survival-time model \( F \),

\[
s_{ra}(t_i, x_i, 1, \widehat{\beta}_{ra,1}, F) = \frac{\partial L_{ra}(t_i, x_i, 1, \widehat{\beta}_{ra,1})}{\partial \beta_{ra,1}}
\]

is the vector of score equations from the ML estimator for \( \widehat{\beta}_{ra,1} \) based on survival-time model \( F \),

\( \widehat{E}(t_i|x_i, \tau = 0, \widehat{\beta}_{ra,0}) \) is the predicted mean survival time assuming treatment level 0 for observation \( i \) conditional on \( x_i \), and

\( \widehat{E}(t_i|x_i, \tau = 1, \widehat{\beta}_{ra,1}) \) is the predicted mean survival time assuming treatment level 1 for observation \( i \) conditional on \( x_i \).

The ATE is estimated by replacing (2d) with

\[
1/N \sum_{i=1}^{N} \varpi_i \left\{ \widehat{E}(t_i|x_i, \tau = 1, \widehat{\beta}_{ra,1}) - \widehat{POM}_{ra,0} - \text{ATE}_{ra} \right\} = 0 \quad (3)
\]

and the ATET is estimated by replacing (2c) and (3) with

\[
1/N_1 \sum_{i=1}^{N} \varpi_i(\tau_i == 1) \left\{ \widehat{E}(t_i|x_i, \tau = 0, \widehat{\beta}_{ra,0}) - \widehat{POM}_{ra,cot,0} \right\} = 0
\]

\[
1/N_1 \sum_{i=1}^{N} \varpi_i(\tau_i == 1) \left\{ \widehat{E}(t_i|x_i, \tau = 1, \widehat{\beta}_{ra,1}) - \widehat{POM}_{ra,cot,0} - \text{ATET}_{ra} \right\} = 0
\]
where \( N_1 = \sum_{i=1}^{N} (t_i == 1) \) and \( \hat{POM}_{ra,cot,0} \) is the estimated conditional-on-treatment POM for treatment level 0.

Asymptotic standard errors for estimating equation estimators, also known as exactly identified generalized method of moments estimators, are standard in the literature; see Newey (1984), Newey and McFadden (1994), Tsiatis (2006), and Wooldridge (2010). These standard errors always have a robust structure and have been generalized to cluster–robust standard errors (see Wooldridge (2010)).

The score equations and the functional form for the predicted mean survival time depend on the model for survival-time outcome \( F \). We provide these details below, under Functional-form details.

**Weighted-adjusted-censoring assumptions**

All estimators that permit you to model the time to censoring are subject to three assumptions:

1. The censoring time must be random.
2. The censoring time must be from a known distribution.
3. The distribution of the censoring time cannot vary by treatment level.

We call these three requirements the WAC assumptions. If the WAC assumptions are violated, you can use either an RA estimator or the LAC version of the IPWRA estimator.

💻 Technical note

We now describe how the observed survival-time outcome \( t \) is generated from the random censoring time \( t_c \), the received treatment \( \tau \), and the potential-outcome survival times \( t_0 \) and \( t_1 \) under the WAC assumptions. First, each potential outcome is either censored or not censored.

\[
\tilde{t}_0 = t_c(t_0 \geq t_c) + t_0\{1 - (t_0 \geq t_c)\}
\]

\[
\tilde{t}_1 = t_c(t_1 \geq t_c) + t_1\{1 - (t_1 \geq t_c)\}
\]

Under the WAC assumptions, \( t_c \) is a random variable from a known distribution, and \( t_c \) does not vary by treatment level.

Next, the received treatment \( \tau \in \{0, 1\} \) determines which, possibly censored, potential outcome is observed.

\[
t = (1 - \tau)\tilde{t}_0 + \tau\tilde{t}_1
\]

The 0/1 indicator for whether the observed \( t \) was censored, denoted by \( c \), is given by

\[
c = (1 - \tau)(t_0 \geq t_c) + \tau(t_1 \geq t_c)
\]

**Weighted regression-adjusted estimators**

As is standard in the survival literature, the RA estimators account for censored survival times by adding a term to the log-likelihood function for censored observations [see (1)]. In contrast, weighted regression-adjustment (WRA) estimators use weights to account for censored observations and are subject to the WAC assumptions.
Wooldridge (2007) and Lin (2000) derived estimators for the regression parameters that maximize a weighted objective function of the uncensored observations. Each observation-level weight is the inverse of the probability of not being censored. Like the RA estimators, the WRA estimators use averages of the predicted mean survival times to estimate treatment-effect parameters.

The WRA estimators have the following logic.

WRA1. Estimate by ML the parameters $\gamma$ of a parametric survival-time model for the time to censoring $t_c$, in which $F_c(t_c|w, \gamma)$ is the distribution of $t_c$ conditional on covariates $w$. Note that the censoring process does not vary by treatment level and that we only observe $t_c$ when the observed potential outcome was censored. Denote the estimates of $\gamma$ by $\hat{\gamma}$.

WRA2. For each treatment level $\tau \in \{0, 1\}$, estimate by weighted maximum likelihood (WML) the $\beta_\tau$ parameters of a parametric survival-time model, denoted by $F(t|x, \tau, \beta_\tau)$, where $t$ is the survival-time outcome and $x$ are the covariates. The weights are the inverse of the estimated probabilities of not being censored, $1/\{1 - F_c(t_c|w, \hat{\gamma})\}$, and only the uncensored observations are used. Denote the estimates of $\beta_\tau$ by $\hat{\beta}_{wra, \tau}$.

WRA3. Use the estimated $\hat{\beta}_{wra, \tau}$ and the functional form implied by $F(t|x, \tau, \beta_\tau)$ to estimate the mean survival time, conditional on $x$ and treatment level $\tau$, for each sample observation, denoted by $\hat{E}(t_i|x_i, \tau, \hat{\beta}_{wra, \tau})$. Conditional independence of the treatment and the survival-time potential outcomes ensures that $E(t|x, \tau, \beta_\tau) = E(t|t_c, x, \beta_\tau)$, where $t_\tau$ is the potential survival-time outcome corresponding to treatment level $\tau$. Under correct model specification, sample averages of $\hat{E}(t_i|x_i, \tau, \hat{\beta}_{wra, \tau})$ consistently estimate the POM for treatment level $\tau$, denoted by $\text{POM}_\tau$.

WRA4. A contrast of the estimated POMs estimates the ATE.

If estimating an ATET, step WRA3 is modified to use only the treated observations when estimating the POMs. A contrast of these POMs then estimates the ATET.

The contribution of the $i$th observation to the log likelihood that is maximized in step WRA1 is

$$L_{c, wra}(t_i, w_i, \hat{\gamma}) = \omega_i \{ c_i \ln\{ f_c(t_i|w_i, \hat{\gamma}) \} + (1 - c_i) \ln\{ 1 - F_c(t_i|w_i, \hat{\gamma}) \} \}$$

(4)

where $\omega_i$ is the observation-level weight, $c_i$ is the 0/1 indicator for whether the survival-time observation on person $i$ was censored, $t_i$ is the observed failure time, and $f_c(t_i|w_i, \hat{\gamma})$ is the density corresponding to conditional time-to-censoring distribution $F_c(t_i|w_i, \hat{\gamma})$. When $c_i = 1$, $t_i$ is the time to censoring. When $c_i = 0$, the censoring time is not observed; we only know that it is greater than the observed $t_i$. The first term accounts for the observations in which $t_i$ is observed to be the censoring time, and the second term accounts for the observations in which the censoring time is greater than the observed $t_i$.

The contribution of the $i$th observation to the log likelihood that is maximized in step WRA2 is

$$L_{wra}(t_i, x_i, \tau, \hat{\beta}_{wra, \tau}) = \omega_i(\tau_i == \tau) \left[ \frac{(1 - c_i)}{1 - F_c(t_i|w_i, \hat{\gamma})} \right] \ln\{ f(t_i|x_i, \tau, \hat{\beta}_{wra, \tau}) \}$$

(5)

where $f(t_i|x_i, \tau, \hat{\beta}_{wra, \tau})$ is the density corresponding to distribution $F(t_i|x_i, \tau, \hat{\beta}_{wra, \tau})$. Equation (5) does not contain a term that adjusts for censoring; see (1) for a comparison. Rather, it uses inverse-probability weights to account for both the censored and the uncensored observations.

The WRA estimators for the POMs simultaneously solve estimating equations (6a) through (6e) for $\hat{\gamma}$, $\hat{\beta}_{wra, 0}$, $\hat{\beta}_{wra, 1}$, $\text{POM}_{wra, 0}$, and $\text{POM}_{wra, 1}$.
\[ \frac{1}{N} \sum_{i=1}^{N} s_{\text{wra}}(t_i, w_i, \gamma, F_c) = 0 \]  
\[ \frac{1}{N} \sum_{i=1}^{N} s_{\text{wra}}(t_i, x_i, 0, \hat{\beta}_{\text{wra},0}, F) = 0 \]  
\[ \frac{1}{N} \sum_{i=1}^{N} s_{\text{wra}}(t_i, x_i, 1, \hat{\beta}_{\text{wra},1}, F) = 0 \]  
\[ \frac{1}{N} \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 0, \hat{\beta}_{\text{wra},0}) - \hat{\text{POM}}_{\text{wra},0} \right\} = 0 \]  
\[ \frac{1}{N} \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{wra},1}) - \hat{\text{POM}}_{\text{wra},1} \right\} = 0 \]

where

\[ s_{\text{wra}}(t_i, w_i, \gamma, F_c) = \frac{\partial L_{\text{wra}}(t_i, w_i, \gamma)}{\partial \gamma} \] is the vector of score equations from the ML estimator for \( \gamma \) based on survival-time model \( F_c \).

\[ s_{\text{wra}}(t_i, x_i, 0, \hat{\beta}_{\text{wra},0}, F) = \frac{\partial L(t_i, x_i, 0, \hat{\beta}_{\text{wra},0})}{\partial \hat{\beta}_{\text{wra},0}} \] is the vector of score equations from the WML estimator for \( \hat{\beta}_{\text{wra},0} \) based on survival-time model \( F \).

\[ s_{\text{wra}}(t_i, x_i, 1, \hat{\beta}_{\text{wra},1}, F) = \frac{\partial L(t_i, x_i, 1, \hat{\beta}_{\text{wra},1})}{\partial \hat{\beta}_{\text{wra},1}} \] is the vector of score equations from the WML estimator for \( \hat{\beta}_{\text{wra},1} \) based on survival-time model \( F \).

\( \hat{E}(t_i | x_i, \tau = 0, \hat{\beta}_{\text{wra},0}) \) is the predicted mean survival time assuming treatment level 0 for observation \( i \) conditional on \( x_i \), and

\( \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{wra},1}) \) is the predicted mean survival time assuming treatment level 1 for observation \( i \) conditional on \( x_i \).

The observation-level scores \( s_{\text{wra}}(t_i, x_i, 0, \hat{\beta}_{\text{wra},0}, F) \) and \( s_{\text{wra}}(t_i, x_i, 1, \hat{\beta}_{\text{wra},1}, F) \) also depend on \( c_i, w_i, \gamma, \) and \( F_c \), but we ignored this dependence to simplify the notation.

The ATE is estimated by replacing (6e) with

\[ \frac{1}{N} \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{wra},1}) - \hat{\text{POM}}_{\text{wra},0} - \hat{\text{ATE}}_{\text{wra}} \right\} = 0 \]

and the ATET is estimated by replacing (6e) and (7) with

\[ \frac{1}{N_1} \sum_{i=1}^{N} \omega_i(\tau_i == 1) \left\{ \hat{E}(t_i | x_i, \tau = 0, \hat{\beta}_{\text{wra},0}) - \hat{\text{POM}}_{\text{wra},\text{cot},0} \right\} = 0 \]

\[ \frac{1}{N_1} \sum_{i=1}^{N} \omega_i(\tau_i == 1) \left\{ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{wra},1}) - \hat{\text{POM}}_{\text{wra},\text{cot},0} - \hat{\text{ATE}}_{\text{wra}} \right\} = 0 \]

where \( \hat{\text{POM}}_{\text{wra},\text{cot},0} \) is the estimated conditional-on-treatment POM.
Inverse-probability-weighted estimators

IPW estimators are weighted averages of the observed outcome. The weights correct for missing data due to unobserved potential outcomes and censoring. Each weight is the inverse of the probability that a given value is observed. Observed values that were not likely to be observed have higher weights.

When the outcome variable is never censored, the missing data are the unobserved potential outcome and an observation’s weight is the inverse of a treatment probability. When the outcome may be censored, the censoring is an additional source of missing data. In this case, an observation’s weight is the inverse of the joint probability that an observation is uncensored and has a particular treatment level.

To define this joint probability, the censoring time must be random. In practice, we make the WAC assumptions.

As is standard in the survival-time literature, we assume that the censoring-time process is independent of treatment assignment after conditioning on the covariates. This conditional independence assumption implies that the probability that observation \(i\) receives treatment level 1 and is not censored is the product of the probability that \(i\) gets treatment level 1 and the probability that \(i\) is not censored at time \(t_i\), which we denote by

\[
p(z_i, \alpha)\{1 - F_c(t_i|w_i, \gamma)\}
\]

where

\[p(z_i, \alpha)\] is the modeled probability that \(i\) gets treatment level 1, conditional on covariates \(z_i\) with parameters \(\alpha\), and

\[F_c(t_i|w_i, \gamma)\] is the survival-time model for the censoring time, conditional on covariates \(w_i\) with parameters \(\gamma\), and evaluated at time \(t_i\).

Bai, Tsiatis, and O’Brien (2013) formally derive these weights to control jointly for the missing potential outcome and censoring.

The IPW estimators have the following logic.

**IPW1.** Estimate by ML the parameters \(\gamma\) of a parametric survival-time model for the time to censoring, in which \(F_c(t_c|w, \gamma)\) is the distribution of censoring time, conditional on covariates \(w\). Denote the estimates of \(\gamma\) by \(\hat{\gamma}\).

**IPW2.** Estimate by ML the parameters \(\alpha\) of a parametric model for the probability of treatment model \(p(z_i, \alpha)\). Denote the estimates of \(\alpha\) by \(\hat{\alpha}\).

**IPW3.** Use the \(\hat{\gamma}\) estimated in IPW1 and the \(\hat{\alpha}\) estimated in IPW2 to construct inverse-probability weights by (8a) for treatment level 1 and by (8b) for treatment level 0.

\[
\omega_{i,1} = \frac{(\tau_i == 1)(c_i == 0)}{[p(z_i, \hat{\alpha})\{1 - F_c(t_i|w_i, \hat{\gamma})\}]}
\]

(8a)

\[
\omega_{i,0} = \frac{(\tau_i == 0)(c_i == 0)}{[\{1 - p(z_i, \hat{\alpha})\}\{1 - F_c(t_i|w_i, \hat{\gamma})\}]}
\]

(8b)

**IPW4.** Use the estimated weights to estimate each POM\(_\tau\) by a weighted average of the uncensored observations on the observed potential outcome.
The contribution of the \(i\)th observation to the log likelihood that is maximized in step IPW1 is

\[
L_{c, \text{ipw}}(t_i, w_i, \gamma) = \omega_i \left[ c_i \ln \{ f_c(t_i \mid w_i, \gamma) \} + (1 - c_i) \ln \{ 1 - F_c(t_i \mid w_i, \gamma) \} \right]
\]

where the definitions and intuition are as described after (4).

The contribution of the \(i\)th observation to the log likelihood that is maximized in step IPW2 is

\[
L_{p, \text{ipw}}(\tau_i, z_i, \alpha) = \omega_i \left[ (\tau_i == 1) \ln \{ p(z_i, \alpha) \} + \{ 1 - (\tau_i == 1) \} \ln \{ 1 - p(z_i, \alpha) \} \right]
\]

where \(p(z_i, \alpha)\) is the model for the probability that \(i\) gets treatment level 1.

The IPW estimators for the POMs simultaneously solve estimating equations (9a) through (9d) for \(\hat{\gamma}, \hat{\alpha}, \hat{\text{POM}}_{\text{ipw}, 0}, \) and \(\hat{\text{POM}}_{\text{ipw}, 1}\).

\[
1/N \sum_{i=1}^{N} s_{\text{ipw}}(t_i, w_i, \hat{\gamma}, F_c) = 0 \quad (9a)
\]

\[
1/N \sum_{i=1}^{N} s_{\text{ipw}}(\tau_i, z_i, \hat{\alpha}, p) = 0 \quad (9b)
\]

\[
1/N \sum_{i=1}^{N} \omega_i \omega_i,0 \left( t_i - \hat{\text{POM}}_{\text{ipw}, 0} \right) = 0 \quad (9c)
\]

\[
1/N \sum_{i=1}^{N} \omega_i \omega_i,1 \left( t_i - \hat{\text{POM}}_{\text{ipw}, 1} \right) = 0 \quad (9d)
\]

where

\[
s_{\text{ipw}}(t_i, w_i, \hat{\gamma}, F_c) = \frac{\partial L_{c, \text{ipw}}(t_i, w_i, \hat{\gamma})}{\partial \hat{\gamma}}
\]

is the vector of score equations from the ML estimator for \(\hat{\gamma}\) based on survival-time model \(F_c\), and

\[
s_{\text{ipw}}(\tau_i, z_i, \hat{\alpha}, p) = \frac{\partial L_{p, \text{ipw}}(\tau_i, z_i, \hat{\alpha})}{\partial \hat{\alpha}}
\]

is the vector of score equations from the ML estimator for \(\hat{\alpha}\) based on probability model \(p\).

The literature on IPW estimators discusses using normalized versus unnormalized weights, with normalized weights doing better in simulation studies; see Busso, DiNardo, and McCrary (2014) for example. The way that weights enter moment equations (9c) and (9d) implies that they are normalized, because the scale of the weights does not affect the estimates.

The estimated ATE is computed as

\[
\hat{\text{POM}}_{\text{ipw}, 1} - \hat{\text{POM}}_{\text{ipw}, 0} = \hat{\text{ATE}}_{\text{ipw}}
\]

The estimated ATET uses weights

\[
\omega_{i, \text{cot}, 1} = \frac{(\tau_i == 1)(c_i == 0)}{\{1 - F_c(t_i \mid w_i, \hat{\gamma})\}} \quad (10a)
\]

for treatment level 1 and

\[
\omega_{i, \text{cot}, 0} = \frac{p(z_i, \hat{\alpha})(\tau_i == 0)(c_i == 0)}{\{1 - p(z_i, \hat{\alpha})\}{1 - F_c(t_i \mid w_i, \hat{\gamma})}} \quad (10b)
\]
for treatment level 0, and replaces (9c) and (9d) with

\[ \frac{1}{N_1} \sum_{i=1}^{N} \overline{w}_{i,\text{cot},0} \left( t_i - \widehat{POM}_{\text{ipw},\text{cot},0} \right) = 0 \] (11a)

\[ \frac{1}{N_1} \sum_{i=1}^{N} \overline{w}_{i,\text{cot},1} \left( t_i - \widehat{POM}_{\text{ipw},\text{cot},1} \right) = 0 \] (11b)

and then computes

\[ \widehat{POM}_{\text{ipw},\text{cot},1} - \widehat{POM}_{\text{ipw},\text{cot},0} = ATET_{\text{ipw}} \]

These IPW estimators can be viewed as weighted IPW estimators and are thus related to those in Hirano, Imbens, and Ridder (2003).

**Uncensored data**

As mentioned, when the outcome variable is never censored, the missing data are the unobserved potential outcome and an observation’s weight is the inverse of a treatment probability. In the never-censored case, the IPW estimators are identical to those implemented in *teffects ipw*; see IPW estimators under Methods and formulas in [CAUSAL] *teffects aipw*.

*stteffects ipw* computes the estimator for never-censored data when a censoring model is not specified and there are no censored observations in the sample. In the never-censored case, the following changes are made to the IPW estimator for the POMs and the ATE.

1. **Step IPW1** is not performed.
2. The weights in (8a) and (8b) for the POMs and the ATE are replaced with (12a) for treatment level 1 and (12b) for treatment level 0.

   \[ \omega_{i,1} = \frac{(\tau_i = 1)}{p(z_i, \hat{\alpha})} \] (12a)

   \[ \omega_{i,0} = \frac{(\tau_i = 0)}{\{1 - p(z_i, \hat{\alpha})\}} \] (12b)

3. Only moment conditions (9b), (9c), and (9d) are used.

The following changes also are made to the IPW estimator for the ATET.

1. **Step IPW1** is not performed.
2. The weights in (10a) and (10b) are replaced with (13a) for treatment level 1 and (13b) for treatment level 0.

   \[ \omega_{i,\text{cot},1} = (\tau_i = 1) \] (13a)

   \[ \omega_{i,\text{cot},0} = \frac{p(z_i, \hat{\alpha})(\tau_i = 0)}{\{1 - p(z_i, \hat{\alpha})\}} \] (13b)

3. Only moment conditions (9b), (11a), and (11b) are used.
Inverse-probability-weighted regression-adjustment estimators

IPWRA estimators are averages of treatment-specific predicted conditional means that were made using missingness-adjusted regression parameters. These estimators are Wooldridge’s IPWRA for survival-time outcomes; see Wooldridge (2010, chap. 21) and Wooldridge (2007).

The censored observations can be handled either by weighting under the WAC assumptions to obtain the WAC-IPWRA estimator or by adding a term to the log-likelihood function (which we call likelihood-adjusted censoring) to obtain the LAC-IPWRA estimator. Correspondingly, there are two versions of formulas for the IPWRA estimator.

1. When a censoring model is specified, stteffects ipwra uses the formulas for the WAC-IPWRA estimator given in Weighted-adjusted-censoring IPWRA.

2. When a censoring model is not specified, stteffects ipwra uses the formulas for the LAC-IPWRA given in Likelihood-adjusted-censoring IPWRA, below.

The WAC-IPWRA estimator requires that some observations be censored and that the WAC assumptions hold; see Weighted-adjusted-censoring assumptions, above. The LAC-IPWRA estimator handles the case in which no observations are censored and requires the weaker independent censoring assumptions, which allows for fixed censoring times.

Weighted-adjusted-censoring IPWRA

When a censoring model is specified, stteffects ipwra uses the formulas for the WAC-IPWRA estimator to obtain the model-based weights that account for censoring. For notational conciseness and to reinforce its dependence on random censoring, we denote the WAC-IPWRA estimator by IPWRAR in lists and formulas. The WAC-IPWRA estimators have the following logic.

IPWRAR1. Estimate by ML the parameters \( \gamma \) of a parametric survival-time model for the time to censoring, in which \( F_c(t_c|w, \gamma) \) is the censoring-time distribution, conditional on covariates \( w \). We denote the estimates of \( \gamma \) by \( \hat{\gamma} \).

IPWRAR2. Estimate by ML the parameters \( \alpha \) of a parametric model for the probability of treatment model \( p(x_i, \alpha) \). We denote the estimates of \( \alpha \) by \( \hat{\alpha} \).

IPWRAR3. For each treatment level \( \tau \in \{0, 1\} \), estimate by WML the parameters \( \beta_\tau \) of a parametric model for the survival-time outcome \( t \), in which \( F(t|x, \tau, \beta_\tau) \) is the distribution of \( t \) conditional on covariates \( x \) and treatment level \( \tau \). For the ATET, the weights are those in equations (8a) and (8b). For the ATE, the weights are those in equations (10a) and (10b). We denote the estimates of \( \beta_{\text{ipwrar}, \tau} \) by \( \hat{\beta}_{\tau} \).

IPWRAR4. Use the estimated \( \hat{\beta}_{\text{ipwrar}, \tau} \) and the functional form implied by \( F(t|x, \tau, \beta_\tau) \) to estimate the mean survival time, conditional on \( x \) and treatment level \( \tau \), for each sample observation, denoted by \( \hat{E}(t_i|x_i, \tau, \hat{\beta}_{\text{ipwrar}, \tau}) \). Conditional independence of the treatment and the survival-time potential outcomes ensures that \( E(t|x, \tau, \beta_\tau) = E(t_\tau|x, \beta_\tau) \), where \( t_\tau \) is the potential survival-time outcome corresponding to treatment level \( \tau \). Under correct model specification, sample averages of \( \hat{E}(t_i|x_i, \tau, \hat{\beta}_{\text{ipwrar}, \tau}) \) consistently estimate the POM for treatment level \( \tau \), denoted by \( \text{POM}_\tau \).

The contribution of the \( i \)th observation to the log likelihood that is maximized in step IPWRAR1 is

\[
L_{c, \text{ipwrar}}(t_i, w_i, \hat{\gamma}) = \omega_i [c_i \ln \{f_c(t_i|w_i, \hat{\gamma})\} + (1 - c_i) \ln \{1 - F_c(t_i|w_i, \hat{\gamma})\}]
\]

where the definitions and intuition are as described after (4).
The contribution of the $i$th observation to the log likelihood that is maximized in step IPWRAR2 is

$$L_{p,\text{ipwrar}}(\tau_i, z_i, \alpha) = \omega_i \left[ (\tau_i = 1) \ln(p(z_i, \alpha)) + (1 - (\tau_i = 1)) \ln(1 - p(z_i, \alpha)) \right]$$

where $p(z_i, \alpha)$ is the model for the probability that $i$ gets treatment level 1.

The weights and the parameters in step IPWRAR3 used to estimate the ATE differ from those used to estimate the ATET. For the ATE, the contribution of the $i$th observation to the log likelihood that is maximized in step IPWRAR3 is

$$L_{\text{ipwrar}}(t_i, x_i, \tau, \beta_{\text{ipwrar}, \text{ate}, \tau}) = \omega_i \omega_i, \tau \ln \{ f(t_i | x_i, \tau, \beta_{\text{ipwrar}, \text{ate}, \tau}) \}$$

where $\omega_{i,1}$ is given in (8a), $\omega_{i,0}$ is given in (8b), and $f(t_i | x_i, \tau, \beta_{\text{ipwrar}, \text{ate}, \tau})$ is the density corresponding to distribution $F(t_i | x_i, \tau, \beta_{\text{ipwrar}, \text{ate}, \tau})$. Like WRA, only the uncensored observations are used because the weights account for censoring.

The IPWRAR estimators for the POMs simultaneously solve estimating equations (14a) through (14f) for $\hat{\gamma}, \hat{\alpha}, \hat{\beta}_{\text{ipwrar}, \text{ate}, 0}, \hat{\beta}_{\text{ipwrar}, \text{ate}, 1}, \text{POM}_{\text{ipwrar}, 0}, \text{and POM}_{\text{ipwrar}, 1}$.

$$1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(t_i, w_i, \hat{\gamma}, F_c) = 0 \quad (14a)$$

$$1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(\tau_i, z_i, \hat{\alpha}, p) = 0 \quad (14b)$$

$$1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwrar}, \text{ate}, 0}, F) = 0 \quad (14c)$$

$$1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(t_i, x_i, 1, \hat{\beta}_{\text{ipwrar}, \text{ate}, 1}, F) = 0 \quad (14d)$$

$$1/N \sum_{i=1}^{N} \omega_i \left\{ E(t_i | x_i, \tau = 0, \hat{\beta}_{\text{ipwrar}, \text{ate}, 0}) - \text{POM}_{\text{ipwrar}, 0} \right\} = 0 \quad (14e)$$

$$1/N \sum_{i=1}^{N} \omega_i \left\{ E(t_i | x_i, \tau = 1, \hat{\beta}_{\text{ipwrar}, \text{ate}, 1}) - \text{POM}_{\text{ipwrar}, 1} \right\} = 0 \quad (14f)$$

where

$$s_{\text{ipwrar}}(t_i, w_i, \hat{\gamma}, F_c) = \frac{\partial L_{c,\text{ipwrar}}(t_i, w_i, \hat{\gamma})}{\partial \hat{\gamma}}$$

is the vector of score equations from the ML estimator for $\hat{\gamma}$ based on survival-time model $F_c$,

$$s_{\text{ipwrar}}(\tau_i, z_i, \hat{\alpha}, p) = \frac{\partial L_{p,\text{ipwrar}}(\tau_i, z_i, \hat{\alpha})}{\partial \hat{\alpha}}$$

is the vector of score equations from the ML estimator for $\hat{\alpha}$ based on probability model $p$,

$$s_{\text{ipwrar}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwrar}, \text{ate}, 0}, F) = \frac{\partial L_{\text{ipwrar}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwrar}, \text{ate}, 0})}{\partial \hat{\beta}_{\text{ipwrar}, \text{ate}, 0}}$$

is the vector of score equations from the ML estimator for $\hat{\beta}_{\text{ipwrar}, \text{ate}, 0}$ based on survival-time model $F$. 

\( s_{\text{ipwrar}}(t_i, x_{i, 1}, \beta_{\text{ipwrar, ate}, 1}, F) = \frac{\partial L_{\text{ipwrar}}(t_i, x_{i, 1}, \beta_{\text{ipwrar, ate}, 1})}{\partial \beta_{\text{ipwrar, ate}, 1}} \) is the vector of score equations from the ML estimator for \( \beta_{\text{ipwrar, ate}, 1} \) based on survival-time model \( F \),

\[
\hat{E}(t_i | x_i, \tau = 0, \beta_{\text{ipwrar, ate}, 0}) \]

is the predicted mean survival time assuming treatment level 0 for observation \( i \) conditional on \( x_i \), and

\[
\hat{E}(t_i | x_i, \tau = 1, \beta_{\text{ipwrar, ate}, 1}) \]

is the predicted mean survival time assuming treatment level 1 for observation \( i \) conditional on \( x_i \).

The ATE is estimated by replacing (14f) with

\[
1/N \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 1, \beta_{\text{ipwrar, ate}, 1}) - POM_{\text{ipwrar, 0}} - \hat{ATE}_{\text{ipwrar}} \right\} = 0
\]

For the ATET, the contribution of the \( i \)th observation to the weighted log likelihood that is maximized in step IPWRAR3 is

\[
L_{\text{ipwrar}}(t_i, x_i, \tau, \beta_{\text{ipwrar, ate}, \tau}) = \omega_i \omega_{i, \text{cot}, \tau} (\tau_i = \tau) \ln \{ f(t_i | x_i, \tau, \beta_{\text{ipwrar, ate}, \tau}) \}
\]

where \( \omega_{i, \text{cot}, 1} \) is given in (10a), \( \omega_{i, \text{cot}, 0} \) is given in (10b), and \( f(t_i | x_i, \tau, \beta_{\text{ipwrar, ate}, \tau}) \) is the density corresponding to distribution \( F(t_i | x_i, \tau, \beta_{\text{ipwrar, ate}, \tau}) \).

The WAC-IPWRA estimators for the conditional-on-treatment POMs simultaneously solve estimating equations (15a) through (15f) for \( \beta_{\text{ipwrar, ate}, 0}, \beta_{\text{ipwrar, ate}, 0}, \gamma, \alpha, POM_{\text{ipwrar, cot}, 0}, \) and \( POM_{\text{ipwrar, cot}, 1} \).

\[
1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(t_i, w_i, \gamma, F_c) = 0 \quad (15a)
\]

\[
1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(\tau_i, z_i, \alpha, p) = 0 \quad (15b)
\]

\[
1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(t_i, x_i, 0, \beta_{\text{ipwrar, ate}, 0}, F) = 0 \quad (15c)
\]

\[
1/N \sum_{i=1}^{N} s_{\text{ipwrar}}(t_i, x_i, 1, \beta_{\text{ipwrar, ate}, 1}, F) = 0 \quad (15d)
\]

\[
1/N \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 0, \beta_{\text{ipwrar, ate}, 0}) - POM_{\text{ipwrar, cot}, 0} \right\} = 0 \quad (15e)
\]

\[
1/N \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 1, \beta_{\text{ipwrar, ate}, 1}) - POM_{\text{ipwrar, cot}, 1} \right\} = 0 \quad (15f)
\]

where

\[
s_{\text{ipwrar}}(t_i, w_i, \gamma, F_c) = \frac{\partial L_{\text{c, ipwrar}}(t_i, w_i, \gamma)}{\partial \gamma}
\]

is the vector of score equations from the ML estimator for \( \gamma \) based on survival-time model \( F_c \),
The vector of score equations from the WML estimator for IPWRA

\[ s_{ipwrar}(t_i, x_i, 0, \hat{\beta}_{ipwrar, atet, 0}, F) = \frac{\partial L_{ipwrar}(t_i, x_i, 0, \hat{\beta}_{ipwrar, atet, 0})}{\partial \hat{\beta}_{ipwrar, atet, 0}} \]

is the vector of score equations from the WML estimator for \( \hat{\beta}_{ipwrar, atet, 0} \) based on survival-time model \( F \),

\[ s_{ipwrar}(t_i, x_i, 1, \hat{\beta}_{ipwrar, atet, 1}, F) = \frac{\partial L_{ipwrar}(t_i, x_i, 1, \hat{\beta}_{ipwrar, atet, 1})}{\partial \hat{\beta}_{ipwrar, atet, 1}} \]

is the vector of score equations from the WML estimator for \( \hat{\beta}_{ipwrar, atet, 1} \) based on survival-time model \( F \),

\[ \hat{E}(t_i | x_i, \tau = 0, \hat{\beta}_{ipwrar, atet, 0}) \]

is the predicted mean survival time assuming treatment level 0 for observation \( i \) conditional on \( x_i \), and

\[ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{ipwrar, atet, 1}) \]

is the predicted mean survival time assuming treatment level 1 for observation \( i \) conditional on \( x_i \).

The \( \text{ATET} \) is estimated by replacing (15f) with

\[ 1/N \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{ipwrar, atet, 1}) - \hat{\text{POM}}_{ipwrar, cot, 0} - \hat{\text{ATET}}_{ipwrar} \right\} = 0 \]

**Likelihood-adjusted-censoring IPWRA**

When a censoring model is not specified, stteffects ipwra uses the formulas for the LAC-IPWRA estimator that add a term to the log-likelihood function. For notational conciseness and to reinforce its use of an additional term in the log likelihood, we denote the LAC-IPWRA estimator by IPWRAL in lists and formulas.

The methods and formulas for the LAC-IPWRA estimator differ in three ways from those for the WAC-IPWRA estimator.

1. No censoring model is specified, so LAC-IPWRA does not perform a version of step IPWRAR1 and it does not use the moment equations (14a).

2. The weights only depend on the treatment level and treatment assignment probabilities, not on the censoring.

3. The WML estimator for \( \beta_\tau \) includes a term for censored observations and censored observations are used. Recall that for the WAC-IPWRA estimator, the weights used in the WML estimator for \( \beta_\tau \) account for the censoring, and the censored observations are not used in the WML estimator.

The LAC-IPWRA estimators have the following logic.

**IPWRAL1.** Estimate by ML the parameters \( \alpha \) of a parametric model for the probability of treatment model \( p(z_i, \alpha) \).

**IPWRAL2.** For each treatment level \( \tau \in \{0, 1\} \), estimate by WML the parameters \( \beta_\tau \) of a parametric model for the survival-time outcome \( t \) in which \( F(t | x, \tau, \beta_\tau) \) is the distribution of \( t \) conditional on covariates \( x \) and treatment level \( \tau \). The weights depend only on the treatment level and the treatment-assignment probabilities. For the \( \text{ATE} \), the weights are those in (12a) and (12b). For the \( \text{ATET} \), the weights are those in (13a) and (13b). We denote the estimates of \( \beta_\tau \) by \( \hat{\beta}_{ipwral, \tau} \).
IPWRAL3. Use the estimated $\widehat{\beta}_{\text{ipwral}, \tau}$ and the functional form implied by $F(t|x, \tau, \beta_{\tau})$ to estimate the mean survival time, conditional on $x$ and treatment level $\tau$, for each sample observation, denoted by $\widehat{E}(t_i|x_i, \tau, \widehat{\beta}_{\text{ipwral}, \tau})$. Conditional independence of the treatment and the survival-time potential outcomes ensures that $E(t|x, \tau, \beta_{\tau}) = E(t_{\tau}|x, \beta_{\tau})$, where $t_{\tau}$ is the potential survival-time outcome corresponding to treatment level $\tau$. Under correct model specification, sample averages of $\widehat{E}(t_i|x_i, \tau, \widehat{\beta}_{\text{ipwral}, \tau})$ consistently estimate the POM for treatment level $\tau$, denoted by $\text{POM}_\tau$.

The contribution of the $i$th observation to the log likelihood that is maximized in step IPWRAL1 is

$$L_{p, \text{ipwral}}(\tau_i, z_i, \widehat{\alpha}) = \omega_i \left[ (\tau_i = 1) \ln\{p(z_i, \widehat{\alpha})\} + \{1 - (\tau_i = 1)\} \ln\{1 - p(z_i, \widehat{\alpha})\} \right]$$

where $p(z_i, \widehat{\alpha})$ is the model for the probability that $i$ gets treatment level 1.

The weights and the parameters in step IPWRAL2 used to estimate the ATE differ from those used to estimate the ATET. For the ATE, the contribution of the $i$th observation to the log likelihood that is maximized in step IPWRAL2 is

$$L_{\text{ipwral}}(t_i, x_i, \tau, \widehat{\beta}_{\text{ipwral, ate}, \tau}) = (\tau_i = \tau) \omega_i \omega_{i, \tau} \left. \left\{ (1 - c_i) \ln\{f(t_i|x_i, \tau, \widehat{\beta}_{\text{ipwral, ate}, \tau})\} \right\} \right\}$$

where $\omega_{i, 1}$ is given in (12a), $\omega_{i, 0}$ is given in (12b), and $f(t_i|x_i, \tau, \widehat{\beta}_{\text{ipwral, ate}, \tau})$ is the density corresponding to distribution $F(t_i|x_i, \tau, \widehat{\beta}_{\text{ipwral, ate}, \tau})$. Unlike the WRA estimator, the censored observations are used, and there is a term in the likelihood function to account for censoring.

The LAC-IPWRA estimators for the POMs simultaneously solve estimating equations (16a) through (16e) for $\widehat{\alpha}$, $\widehat{\beta}_{\text{ipwral, ate}, 0}$, $\widehat{\beta}_{\text{ipwral, ate}, 1}$, $\text{POM}_{\text{ipwral}, 0}$, and $\text{POM}_{\text{ipwral}, 1}$.

$$\frac{1}{N} \sum_{i=1}^{N} s_{\text{ipwral}}(\tau_i, z_i, \widehat{\alpha}, p) = 0 \quad (16a)$$

$$\frac{1}{N} \sum_{i=1}^{N} s_{\text{ipwral}}(t_i, x_i, 0, \widehat{\beta}_{\text{ipwral, ate}, 0}, F) = 0 \quad (16b)$$

$$\frac{1}{N} \sum_{i=1}^{N} s_{\text{ipwral}}(t_i, x_i, 1, \widehat{\beta}_{\text{ipwral, ate}, 1}, F) = 0 \quad (16c)$$

$$\frac{1}{N} \sum_{i=1}^{N} \omega_i \left\{ \widehat{E}(t_i|x_i, \tau = 0, \widehat{\beta}_{\text{ipwral, ate}, 0}) - \text{POM}_{\text{ipwral}, 0} \right\} = 0 \quad (16d)$$

$$\frac{1}{N} \sum_{i=1}^{N} \omega_i \left\{ \widehat{E}(t_i|x_i, \tau = 1, \widehat{\beta}_{\text{ipwral, ate}, 1}) - \text{POM}_{\text{ipwral}, 1} \right\} = 0 \quad (16e)$$

where

$$s_{\text{ipwral}}(\tau_i, z_i, \widehat{\alpha}, p) = \frac{\partial L_{p, \text{ipwral}}(\tau_i, z_i, \widehat{\alpha})}{\partial \widehat{\alpha}}$$

is the vector of score equations from the ML estimator for $\widehat{\alpha}$ based on probability model $p$. 
\( s_{\text{ipwral}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwral,ate},0}, F) = \frac{\partial L_{\text{ipwral}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwral,ate},0})}{\partial \beta_{\text{ipwral,ate},0}} \) is the vector of score equations from the WML estimator for \( \hat{\beta}_{\text{ipwral,ate},0} \) based on survival-time model \( F \),

\[ s_{\text{ipwral}}(t_i, x_i, 1, \hat{\beta}_{\text{ipwral,ate},1}, F) = \frac{\partial L_{\text{ipwral}}(t_i, x_i, 1, \hat{\beta}_{\text{ipwral,ate},1})}{\partial \beta_{\text{ipwral,ate},1}} \]

is the vector of score equations from the WML estimator for \( \hat{\beta}_{\text{ipwral,ate},1} \) based on survival-time model \( F \),

\( \hat{E}(t_i | x_i, \tau = 0, \hat{\beta}_{\text{ipwral,ate},0}) \) is the predicted mean survival time assuming treatment level 0 for observation \( i \) conditional on \( x_i \), and

\( \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{ipwral,ate},1}) \) is the predicted mean survival time assuming treatment level 1 for observation \( i \) conditional on \( x_i \).

The \textit{ATE} is estimated by replacing \( (16e) \) with

\[ 1/N \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{ipwral,ate},1}) - \hat{\text{POM}}_{\text{ipwral},0} - \text{ATE}_{\text{ipwral}} \right\} = 0 \]

For the ATET, the contribution of the \( i \)th observation to the WML function that is maximized in \textit{step IPWRA2} is

\[ L_{\text{ipwral}}(t_i, x_i, \tau, \hat{\beta}_{\text{ipwral,atet},\tau}) = (\tau_i == \tau) \omega_i \omega_{i,\text{cot},\tau} \left\{ (1 - c_i) \ln \{ f(t_i | x_i, \tau, \hat{\beta}_{\text{ipwral,atet},\tau}) \} - c_i \ln \{ 1 - F(t_i | x_i, \tau, \hat{\beta}_{\text{ipwral,atet},\tau}) \} \right\} \]

where \( \omega_{i,\text{cot},\tau} \) is given in \( (13a) \), \( \omega_{i,\text{cot},0} \) is given in \( (13b) \), and \( f(t_i | x_i, \tau, \hat{\beta}_{\text{ipwral,atet},\tau}) \) is the density corresponding to distribution \( F(t_i | x_i, \tau, \hat{\beta}_{\text{ipwral,atet},\tau}) \). Again unlike the WRA, the censored observations are used, and there is a term in the likelihood function to account for censoring.

The LAC-IPWRA estimators for the conditional-on-treatment POMs simultaneously solve estimating equations \( (17a) \) through \( (17e) \) for \( \hat{\alpha}, \hat{\beta}_{\text{ipwral,atet},0}, \hat{\beta}_{\text{ipwral,atet},1}, \hat{\text{POM}}_{\text{ipwral,cot},0}, \) and \( \hat{\text{POM}}_{\text{ipwral,cot},1} \).

\[ 1/N \sum_{i=1}^{N} s_{\text{ipwral}}(\tau_i, z_i, \hat{\alpha}, p) = 0 \]  \hspace{1cm} \text{(17a)}

\[ 1/N \sum_{i=1}^{N} s_{\text{ipwral}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwral,atet},0}, F) = 0 \]  \hspace{1cm} \text{(17b)}

\[ 1/N \sum_{i=1}^{N} s_{\text{ipwral}}(t_i, x_i, 1, \hat{\beta}_{\text{ipwral,atet},1}, F) = 0 \]  \hspace{1cm} \text{(17c)}

\[ 1/N \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 0, \hat{\beta}_{\text{ipwral,atet},0}) - \hat{\text{POM}}_{\text{ipwral,cot},0} \right\} = 0 \]  \hspace{1cm} \text{(17d)}

\[ 1/N \sum_{i=1}^{N} \omega_i \left\{ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{ipwral,atet},1}) - \hat{\text{POM}}_{\text{ipwral,cot},1} \right\} = 0 \]  \hspace{1cm} \text{(17e)}
where

\[ s_{\text{ipwral}}(\tau_i, x_i, \tilde{\alpha}, p) = \frac{\partial L_{0, \text{ipwral}}(\tau_i, x_i, \tilde{\alpha})}{\partial \tilde{\alpha}} \] is the vector of score equations from the ML estimator for \( \tilde{\alpha} \) based on probability model \( p \),

\[ s_{\text{ipwral}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwral,0}}, F) = \frac{\partial L_{\text{ipwral}}(t_i, x_i, 0, \hat{\beta}_{\text{ipwral,0}}, F)}{\partial \hat{\beta}_{\text{ipwral,0}}, F} \] is the vector of score equations from the WML estimator for \( \hat{\beta}_{\text{ipwral,0}} \) based on survival-time model \( F \),

\[ s_{\text{ipwral}}(t_i, x_i, 1, \hat{\beta}_{\text{ipwral,1}}, F) = \frac{\partial L_{\text{ipwral}}(t_i, x_i, 1, \hat{\beta}_{\text{ipwral,1}}, F)}{\partial \hat{\beta}_{\text{ipwral,1}}, F} \] is the vector of score equations from the WML estimator for \( \hat{\beta}_{\text{ipwral,1}} \) based on survival-time model \( F \),

\[ \hat{E}(t_i | x_i, \tau = 0, \hat{\beta}_{\text{ipwral,0}}) \] is the predicted mean survival time assuming treatment level 0 for observation \( i \) conditional on \( x_i \), and

\[ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{ipwral,1}}) \] is the predicted mean survival time assuming treatment level 1 for observation \( i \) conditional on \( x_i \).

The ATET is estimated by replacing (17c) with

\[
1/N \sum_{i=1}^{N} \{ \hat{E}(t_i | x_i, \tau = 1, \hat{\beta}_{\text{ipwral,1}}) - \hat{\text{pOM}}_{\text{ipwral,cot,0}} - \hat{\text{ATET}}_{\text{ipwral}} \} = 0
\]

### Functional-form details

In this section, we specify the functional forms for the conditional distribution function used in the survival-time outcome model \( F \), the conditional distribution function used in the survival-time censoring model \( F_c \), and the conditional distribution used to model the treatment probabilities \( p \).

You may choose among the same set of conditional distribution functions for either \( F \) or \( F_c \): exponential, weibull, lognormal, or gamma.

<table>
<thead>
<tr>
<th>Name</th>
<th>Cumulative</th>
<th>Density</th>
<th>Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponential</td>
<td>( 1 - \exp(-\lambda_i t_i) )</td>
<td>( \lambda_i \exp(-\lambda_i t_i) )</td>
<td>( \lambda_i )</td>
</tr>
<tr>
<td>Weibull</td>
<td>( 1 - \exp(-\lambda_i t_i^{s_i}) )</td>
<td>( s_i t_i^{s_i - 1} \lambda_i^{s_i} \exp{-\lambda_i t_i^{s_i} } )</td>
<td>( \frac{1}{\lambda_i} \Gamma{(s_i + 1)/s_i } )</td>
</tr>
<tr>
<td>log normal</td>
<td>( \Phi{(\ln(t_i) - \lambda_i)/s_i } )</td>
<td>( \frac{1}{s_i t_i^{s_i - 1}} \phi{(\ln(t_i) - \lambda_i)/s_i } )</td>
<td>( \exp(\lambda_i + s_i^2/2) )</td>
</tr>
<tr>
<td>gamma</td>
<td>( \text{gammad}{s_i, (s_i t_i/\lambda_i)} )</td>
<td>( s_i^{s_i} t_i^{s_i - 1}/{\lambda_i^{s_i} \Gamma(s_i)} \exp{-s_i t_i/\lambda_i} \lambda_i )</td>
<td></td>
</tr>
</tbody>
</table>

where the following table specifies how \( \lambda_i \) and \( s_i \) are parameterized in terms of the covariates \( x_i \) and the ancillary covariates \( \tilde{x}_i \), respectively.

<table>
<thead>
<tr>
<th>Name</th>
<th>( \lambda_i )</th>
<th>( s_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>exponential</td>
<td>( \exp(-x_i \beta) )</td>
<td></td>
</tr>
<tr>
<td>Weibull</td>
<td>( \exp(-x_i \beta) )</td>
<td>( \exp(\tilde{x}_i \beta) )</td>
</tr>
<tr>
<td>log normal</td>
<td>( x_i \beta )</td>
<td>( \exp(\tilde{x}_i \beta) )</td>
</tr>
<tr>
<td>gamma</td>
<td>( \exp(x_i \beta) )</td>
<td>( \exp(-2\tilde{x}_i \beta) )</td>
</tr>
</tbody>
</table>
For the treatment-assignment models, the {\texttt{probit}} model uses the standard normal distribution, the {\texttt{logit}} uses the standard logistic distribution, the {\texttt{hetprobit}} model uses 
\[
\Phi\{z_1\alpha_1/\exp(z_2\alpha_2)\}
\]
and the multinomial logit uses 
\[
p(z, t) = \exp(z\alpha_t)/\{1 + \sum_{k=1}^{q} \exp(z\alpha_k)\}
\]
where the notation is defined below.

In the {\texttt{hetprobit}} model, $z_1$ are the covariates specified in the treatment-assignment specification, $z_2$ are the covariates specified in the {\texttt{hetprobit}}() option, and $\alpha_1$ and $\alpha_2$ are the corresponding coefficients.

In the multinomial logit model, $z$ are the covariates specified in the treatment-assignment specification and $\alpha_k$ are the coefficients; see [\texttt{R}] {\texttt{mlogit}} for further details.

References


Also see

[CAUSAL] **stteffects postestimation** — Postestimation tools for stteffects

[CAUSAL] **stteffects intro** — Introduction to treatment effects for observational survival-time data

[ST] **streg** — Parametric survival models

[ST] **stset** — Declare data to be survival-time data

[U] **20 Estimation and postestimation commands**
Postestimation commands

The following postestimation commands are of special interest after \texttt{stteffects}:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{teoverlap}</td>
<td>overlap plots</td>
</tr>
<tr>
<td>\texttt{tebalance}</td>
<td>check balance of covariates</td>
</tr>
</tbody>
</table>

The following standard postestimation commands are also available:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{estat summarize}</td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td>\texttt{estat vce}</td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td>\texttt{estimates}</td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td>\texttt{etable}</td>
<td>table of estimation results</td>
</tr>
<tr>
<td>\texttt{hausman}</td>
<td>Hausman’s specification test</td>
</tr>
<tr>
<td>\texttt{lincom}</td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td>\texttt{nlcom}</td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td>\texttt{predict}</td>
<td>propensity scores, censored survival probability, etc.</td>
</tr>
<tr>
<td>\texttt{predictnl}</td>
<td>point estimates, standard errors, testing, and inference for generalized predictions</td>
</tr>
<tr>
<td>\texttt{test}</td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td>\texttt{testnl}</td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>
predict

Description for predict

`predict` creates a new variable containing predictions such as treatment effects, conditional means, propensity scores, linear predictions, and log square roots of latent variances.

Menu for predict

Statistics > Postestimation

Syntaxes for predict

Syntaxes are presented under the following headings:

- Syntax for predict after stteffects ipw
- Syntax for predict after stteffects ipwra
- Syntax for predict after stteffects ra
- Syntax for predict after stteffects wra

Syntax for predict after stteffects ipw

```
predict [type] { stub* | newvar | newvarlist } [if] [in]
[ , statistic tlevel(treat_level) ]
```

 predict `type` `stub*` [if] [in], scores

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ps</code></td>
<td>propensity score; the default</td>
</tr>
<tr>
<td><code>censurv</code></td>
<td>censored survival probability</td>
</tr>
<tr>
<td><code>xb</code></td>
<td>linear prediction for propensity score</td>
</tr>
<tr>
<td><code>cxb</code></td>
<td>linear prediction for censoring model</td>
</tr>
<tr>
<td><code>lnsigma</code></td>
<td>log square root of latent variance (for treatment model <code>hetprobit()</code>)</td>
</tr>
<tr>
<td><code>clnshape</code></td>
<td>log of conditional latent shape (for censoring distribution Weibull, log normal, or gamma)</td>
</tr>
</tbody>
</table>

If you do not specify `tlevel()` and only specify one new variable, `ps` assumes `tlevel()` specifies the first treatment level.

If you do not specify `tlevel()` and only specify one new variable, `xb` and `lnsigma` assume `tlevel()` specifies the first noncontrol treatment level.

You specify one or `t` new variables with `ps`, where `t` is the number of treatment levels.

You specify one or `t−1` new variables with `xb` and `lnsigma`.

You specify one new variable with `censurv`, `cxb`, and `clnshape`. 
Syntax for predict after stteffects ipwra

predict [type] { stub* | newvar | newvarlist } [if] [in]
               [, statistic tlevel(treat_level)]

predict [type] stub* [if] [in], scores

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>te</strong></td>
<td>treatment effect; the default</td>
</tr>
<tr>
<td><strong>cmean</strong></td>
<td>conditional mean at treatment level</td>
</tr>
<tr>
<td><strong>ps</strong></td>
<td>propensity score</td>
</tr>
<tr>
<td><strong>censurv</strong></td>
<td>censored survival probability</td>
</tr>
<tr>
<td><strong>xb</strong></td>
<td>linear prediction for outcome model</td>
</tr>
<tr>
<td><strong>cxb</strong></td>
<td>linear prediction for censoring model</td>
</tr>
<tr>
<td><strong>psxb</strong></td>
<td>linear prediction for propensity score</td>
</tr>
<tr>
<td><strong>lnshape</strong></td>
<td>log of conditional latent shape (for outcome distribution Weibull, log normal, or gamma) at treatment level</td>
</tr>
<tr>
<td><strong>clnshape</strong></td>
<td>log of conditional latent shape (for censoring distribution Weibull, log normal, or gamma)</td>
</tr>
<tr>
<td><strong>pslnsigma</strong></td>
<td>log square root of latent variance (for treatment model hetprobit()) for propensity score</td>
</tr>
</tbody>
</table>

If you do not specify tlevel() and only specify one new variable, te and psxb assume tlevel() specifies the first noncontrol treatment level.

If you do not specify tlevel() and only specify one new variable, cmean, ps, xb, and pslnsigma assume tlevel() specifies the first treatment level.

You specify one or t new variables with cmean, ps, xb, and lnshape, where t is the number of treatment levels.

You specify one or t−1 new variables with te, psxb, and pslnsigma.

You specify one new variable with censurv, cxb, and clnshape.
Syntax for predict after stteffects ra

```
predict [type] { stub* | newvar | newvarlist } [ if ] [ in ]
   [ , statistic tlevel(treat_level) ]
```

```
predict [type] stub* [ if ] [ in ], scores
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>te</td>
<td>treatment effect; the default</td>
</tr>
<tr>
<td>cmean</td>
<td>conditional mean at treatment level</td>
</tr>
<tr>
<td>xb</td>
<td>linear prediction for outcome model</td>
</tr>
<tr>
<td>lnshape</td>
<td>log of conditional latent shape (for outcome distribution Weibull, log normal, or gamma) at treatment level</td>
</tr>
</tbody>
</table>

If you do not specify tlevel() and only specify one new variable, te assumes tlevel() specifies the first noncontrol treatment level.

If you do not specify tlevel() and only specify one new variable, cmean, xb, and lnshape assume tlevel() specifies the first treatment level.

You specify one or t new variables with cmean, xb, and lnshape, where t is the number of treatment levels.

You specify one or t−1 new variables with te.

Syntax for predict after stteffects wra

```
predict [type] { stub* | newvar | newvarlist } [ if ] [ in ]
   [ , statistic tlevel(treat_level) ]
```

```
predict [type] stub* [ if ] [ in ], scores
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>te</td>
<td>treatment effect; the default</td>
</tr>
<tr>
<td>cmean</td>
<td>conditional mean at treatment level</td>
</tr>
<tr>
<td>censurv</td>
<td>censored survival probability</td>
</tr>
<tr>
<td>xb</td>
<td>linear prediction for outcome model</td>
</tr>
<tr>
<td>cbx</td>
<td>linear prediction for censoring model</td>
</tr>
<tr>
<td>lnshape</td>
<td>log of conditional latent shape (for outcome distribution Weibull, log normal, or gamma) at treatment level</td>
</tr>
<tr>
<td>clnshape</td>
<td>log of conditional latent shape (for censoring distribution Weibull, log normal, or gamma)</td>
</tr>
</tbody>
</table>

If you do not specify tlevel() and only specify one new variable, te assumes tlevel() specifies the first noncontrol treatment level.

If you do not specify tlevel() and only specify one new variable, cmean, xb, and lnshape assume tlevel() specifies the first treatment level.

You specify one or t new variables with cmean, xb, and lnshape, where t is the number of treatment levels.

You specify one or t−1 new variables with te.

You specify one new variable with censurv, cbx, and clnshape.
Options for predict

Options are presented under the following headings:

- Options for predict after stteffects ipw
- Options for predict after stteffects ipwra
- Options for predict after stteffects ra
- Options for predict after stteffects wra

Options for predict after stteffects ipw

- **ps** is the default, calculates the propensity score of each treatment level or the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

- **censurv** calculates the survivor probability from the time-to-censoring model. (In other words, it calculates the probability that an outcome is not censored.) This option is allowed only if a censoring model is specified at estimation time. You need to specify only one new variable.

- **xb** calculates the propensity score linear prediction at each noncontrol level of the treatment or the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

- **cxb** calculates the linear prediction of the censoring model. This option is allowed only if a censoring model is specified at estimation time. You need to specify only one new variable.

- **lnsigma** calculates the log square root of the latent variance. This option is valid only when treatment model `hetprobit()` is used. You need to specify only one new variable.

- **clnshape** calculates the log of the conditional latent shape parameter of the censoring distribution. This option is valid when censoring distribution Weibull, log normal, or gamma is used. You need to specify only one new variable.

- **tlevel(treat_level)** specifies the treatment level for prediction.

- **scores** calculates the score variables. Parameter-level scores are computed for the treatment mean and average treatment-effect equations. Equation-level scores are computed for the censoring and propensity-score equations.

  - The `j`th new variable will contain the scores for the `j`th parameter in the coefficient table if `j ≤ t`, where `t` is the number of treatment levels. Otherwise, it will contain the scores for fitted equation `j – t` following the first `t` parameters in the coefficient table.

Options for predict after stteffects ipwra

- **te**, the default, calculates the treatment effect for each noncontrol treatment level or the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

- **cmean** calculates the conditional mean for each treatment level or the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.
ps calculates the propensity score of each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

censurv calculates the survivor probability from the time-to-censoring model. (In other words, it calculates the probability that an outcome is not censored.) This option is allowed only if a censoring model is specified at estimation time. You need to specify only one new variable.

xb calculates the outcome model linear prediction at each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

cxb calculates the linear prediction of the censoring model. This option is allowed only if a censoring model is specified at estimation time. You need to specify only one new variable.

psxb calculates the propensity score linear prediction at each noncontrol level of the treatment or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

lnshape calculates the log of the conditional latent shape parameter for each treatment level or the treatment level specified in tlevel(). This option is valid when outcome distribution Weibull, log normal, or gamma is used. If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

clnshape calculates the log of the conditional latent shape parameter for the censoring distribution. This option is valid when censoring distribution Weibull, log normal, or gamma is used. You need to specify only one new variable.

pslnsigma calculates the log square root of the latent variance for the propensity score. This option is valid only when treatment model hetprobit() is used. You need to specify only one new variable.

tlevel(treat_level) specifies the treatment level for prediction.

scores calculates the score variables. Parameter-level scores are computed for the treatment mean and average treatment-effect equations. Equation-level scores are computed for the outcome, censoring, and propensity-score equations.

The jth new variable will contain the scores for the jth parameter in the coefficient table if j ≤ t, where t is the number of treatment levels. Otherwise, it will contain the scores for fitted equation j − t following the first t parameters in the coefficient table.

Options for predict after stteffects ra

<table>
<thead>
<tr>
<th>Options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>te</td>
<td>The default, calculates the treatment effect for each noncontrol treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.</td>
</tr>
<tr>
<td>cmean</td>
<td>Calculates the conditional mean for each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.</td>
</tr>
<tr>
<td>xb</td>
<td>Calculates the outcome model linear prediction at each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.</td>
</tr>
</tbody>
</table>
lnshape calculates the log of the conditional latent shape parameter for each treatment level or the treatment level specified in tlevel(). This option is valid when the outcome distribution Weibull, log normal, or gamma is used. If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

tlevel(treat_level) specifies the treatment level for prediction.

scores calculates the score variables. Parameter-level scores are computed for the treatment mean and average treatment-effect equations. Equation-level scores are computed for the outcome equations.

The jth new variable will contain the scores for the jth parameter in the coefficient table if $j \leq t$, where $t$ is the number of treatment levels. Otherwise, it will contain the scores for fitted equation $j - t$ following the first $t$ parameters in the coefficient table.

**Options for predict after stteffects wra**

Main, the default, calculates the treatment effect for each noncontrol treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

Cmean calculates the conditional mean for each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

censurv calculates the survivor probability from the time-to-censoring model. (In other words, it calculates the probability that an outcome is not censored.) This option is allowed only if a censoring model is specified at estimation time. You need to specify only one new variable.

xb calculates the outcome model linear prediction at each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

lnshape calculates the log of the conditional latent shape parameter for each treatment level or the treatment level specified in tlevel(). This option is valid when the outcome distribution Weibull, log normal, or gamma is used. If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

clnshape calculates the log of the conditional latent shape parameter of the censoring distribution. This option is valid when the censoring distribution Weibull, log normal, or gamma is used. You need to specify only one new variable.

tlevel(treat_level) specifies the treatment level for prediction.

scores calculates the score variables. Parameter-level scores are computed for the treatment mean and average treatment-effect equations. Equation-level scores are computed for the outcome and censoring equations.

The jth new variable will contain the scores for the jth parameter in the coefficient table if $j \leq t$, where $t$ is the number of treatment levels. Otherwise, it will contain the scores for fitted equation $j - t$ following the first $t$ parameters in the coefficient table.
Remarks and examples

Checking model specification is the most frequent reason for postestimation computation after \texttt{stteffects teoverlap} provides a graphical method for checking the overlap assumption; see \cite{causal} \texttt{teoverlap}. Summarizing the estimated probabilities provides another check. Recall that the reciprocals of these estimated probabilities are used as weights by some of the estimators. If the estimated probabilities are too small, the weights get too large and the estimators become unstable.

We estimate the average treatment effect of smoking on the time to a second heart attack by inverse-probability weighting; see example 1 of \cite{causal} \texttt{stteffects ipw} for background.

\begin{verbatim}
. use https://www.stata-press.com/data/r18/sheart
(Time to second heart attack (fictional))
. stteffects ipw (smoke age exercise education) (age exercise diet education)
   \hspace{1em} Failure \texttt{d}: fail\hspace{1em} Analysis time \texttt{t}: atime
  Iteration 0: EE criterion = 2.042e-18
  Iteration 1: EE criterion = 5.191e-31
Survival treatment-effects estimation Number of obs = 2,000
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: logit
Censoring model: Weibull

| \_t | Coefficient | std. err. | z | P>|z| | [95% conf. interval] |
|-----|-------------|-----------|---|-----|----------------------|
| ATE | smoke \hspace{1em} (Smoker vs Nonsmoker) | -2.22226 | .6307573 | -3.52 | 0.000 | -3.458522 \hspace{1em} -0.9859983 |
| POmean | smoke \hspace{1em} Nonsmoker | 4.235569 | .5210937 | 8.13 | 0.000 | 3.214244 \hspace{1em} 5.256894 |
\end{verbatim}

Below, we compute the estimated probabilities of being a Nonsmoker and store them in \texttt{ps0}. Likewise, the estimated probabilities of being a Smoker are stored in \texttt{ps1}.

\begin{verbatim}
. predict ps0 ps1, ps
\end{verbatim}

The overlap condition requires that each of these probabilities be sufficiently greater than 0 and less than 1 for every individual; see \cite{causal} \texttt{Assumptions and tradeoffs} under Remarks and examples in \cite{causal} \texttt{stteffects intro}.

In practice, we know that weighting estimators perform poorly when the weights become too large. This approach requires that the probability of being a Nonsmoker not be too small among Nonsmokers and that the probability of being a Smoker not be too small among Smokers. Below, we summarize these probabilities.
The minimum probability of being a Non smoker among Non smokers is 0.39. The minimum probability of being a Smoker among Smokers is 0.09. Neither minimum seems too small.

Estimating survival-time treatment effects also uses weights to adjust for censored outcomes; see \[\text{CAUSAL}\] \textit{stteffects intro}. Thus we require that the probability of an uncensored failure also be sufficiently greater than 0. Below, we compute the estimated probabilities of failure and summarize them among those that fail.

```
. predict fprob2, censurv
. summarize fprob if fail==1
```

The minimum probability of 0.04 does not appear too small.

\[\text{Technical note}\]

The previous discussion builds on the intuition that the weights used in a weighting estimator should not be too large.

This technical note goes a little further by explicitly computing the weights and using them to replicate the inverse-probability-weighted point estimate for the Non smoker potential-outcome mean.

We now compute the weights using the predicted probabilities computed in the examples above and then use \texttt{mean} to compute the weighted average that estimates the potential-outcome mean for Non smokers.

```
. generate double ipw0 = 1/(ps0*fprob)
. mean _t [pw=ipw0] if smoke==0 & fail==1
```

The weights account for data lost to the Smoker potential outcome or to censoring by increasing the importance of observations that were observed to be Non smoker failure times even though they were not likely to be observed.

The point estimate matches that reported by \texttt{stteffects ipw}; the standard errors differ because \texttt{mean} takes the estimated weights as given. See \textit{Inverse-probability-weighted estimators} under \textit{Methods and formulas} in \[\text{CAUSAL}\] \textit{stteffects ipwra}.  

\[\text{CAUSAL}\]
References


Also see

[Causal] tebalance — Check balance after tfeffects or stteffects estimation

[Causal] teoverlap — Overlap plots

[Causal] stteffects ipw — Survival-time inverse-probability weighting

[Causal] stteffects ipwra — Survival-time inverse-probability-weighted regression adjustment

[Causal] stteffects ra — Survival-time regression adjustment

[Causal] stteffects wra — Survival-time weighted regression adjustment

[U] 20 Estimation and postestimation commands
stteffects ra — Survival-time regression adjustment

Description

stteffects ra estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational survival-time data by regression adjustment (RA). RA uses averages of treatment-specific predicted mean survival times to estimate mean survival times for each potential outcome. Contrasts of these predicted mean survival times estimate the treatment effects. stteffects ra offers several choices for the model used to predict mean survival time. Binary and multivalued treatments are accommodated.

See [CAUSAL] stteffects intro for an overview of estimating treatment effects from observational survival-time data.

Quick start

Specify time as observed failure time and fail as failure indicator

    stset time, failure(fail)

ATE from a Weibull model for time on x1 and x2 with binary treatment treat2

    stteffects ra (x1 x2) (treat2)

Same as above, but estimate the ATET

    stteffects ra (x1 x2) (treat2), atet

Same as above, but estimate the potential-outcome means

    stteffects ra (x1 x2) (treat2), pomeans

ATE of treat2 using a gamma model for time

    stteffects ra (x1 x2, gamma) (treat2)

ATE for each level of three-valued treatment treat3

    stteffects ra (x1 x2) (treat3)

Same as above, and specify that treat3 = 3 is the control level using the value label “MyControl” for 3

    stteffects ra (x1 x2) (treat3), control("MyControl")

Menu

Statistics > Causal inference/treatment effects > Survival outcomes > Regression adjustment
## Syntax

```
stteffects ra (omvarlist [, omoptions]) (tvar) [if] [in] [, stat options]
```

`omvarlist` specifies the variables that predict the survival-time variable in the outcome model. `tvar` must contain integer values representing the treatment levels.

### `omoptions` Description

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>weibull</code></td>
<td>Weibull; the default</td>
</tr>
<tr>
<td><code>exponential</code></td>
<td>exponential</td>
</tr>
<tr>
<td><code>gamma</code></td>
<td>two-parameter gamma</td>
</tr>
<tr>
<td><code>lnormal</code></td>
<td>lognormal</td>
</tr>
<tr>
<td><code>ancillary(avarlist [, noconstant])</code></td>
<td>specify variables used to model ancillary parameter</td>
</tr>
<tr>
<td><code>noconstant</code></td>
<td>suppress constant from outcome model</td>
</tr>
</tbody>
</table>

### `stat` Description

<table>
<thead>
<tr>
<th>Stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ate</code></td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td><code>atet</code></td>
<td>estimate average treatment effect on the treated</td>
</tr>
<tr>
<td><code>pomeans</code></td>
<td>estimate potential-outcome means</td>
</tr>
</tbody>
</table>

### `options` Description

<table>
<thead>
<tr>
<th>SE/Robust</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>vce(vcetype)</code></td>
<td><code>vcetype</code> may be robust, cluster clustvar, bootstrap, or jackknife</td>
</tr>
</tbody>
</table>

### Reporting

<table>
<thead>
<tr>
<th><code>level(#)</code></th>
<th>set confidence level; default is level(95)</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>aequations</code></td>
<td>display auxiliary-equation results</td>
</tr>
<tr>
<td><code>noshow</code></td>
<td>do not show st setting information</td>
</tr>
<tr>
<td><code>display_options</code></td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
</tbody>
</table>

### Maximization

<table>
<thead>
<tr>
<th><code>maximize_options</code></th>
<th>control the maximization process; seldom used</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>iterinit(#)</code></td>
<td>specify starting-value iterations; seldom used</td>
</tr>
</tbody>
</table>

### Advanced

| `control(# | label)` | specify the level of `tvar` that is the control |
|--------------|--------------------------------------------------|
| `tlevel(# | label)` | specify the level of `tvar` that is the treatment |
| `coeflegend`  | display legend instead of statistics             |
You must `stset` your data before using `stteffects`; see [ST] `stset`.

`omvarlist` and `avarlist` may contain factor variables; see [U] 11.4.3 Factor variables.

`bootstrap`, `by`, `collect`, `jackknife`, and `statsby` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] `bootstrap`.

`fweight`, `iweight`, and `pweight` may be specified using `stset`; see Weights under Remarks and examples in [ST] `stset`. However, weights may not be specified if you are using the `bootstrap` prefix.

`coeflegend` does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

**Ancillary**

`ancillary(avarlist [, noconstant])` specifies the variables used to model the ancillary parameter.

By default, the ancillary parameter does not depend on covariates. Specifying `ancillary(avarlist, noconstant)` causes the constant to be suppressed in the model for the ancillary parameter.

`noconstant`; see [R] Estimation options.

**Stat**

`stat` is one of three statistics: `ate`, `atet`, or `pomeans`. `ate` is the default.

- `ate` specifies that the average treatment effect be estimated.
- `atet` specifies that the average treatment effect on the treated be estimated.
- `pomeans` specifies that the potential-outcome means for each treatment level be estimated.

**SE/Robust**

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster `clustvar`), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] `vce_option`.

**Reporting**

`level(#)`; see [R] Estimation options.

`aequations` specifies that the results for the outcome-model or treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

`noshow` prevents `stteffects ra` from showing the key `st` variables. This option is rarely used because most people type `stset`, `show` or `stset`, `noshow` to permanently set whether they want to see these variables mentioned at the top of the output of every `st` command; see [ST] `stset`.

`display_options`: `noci`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapping(style)`, `cformat(%)`, `pformat(%)`, `sformat(%)`, and `nolstretch`; see [R] Estimation options.

**Maximization**

`maximize_options`: `iterate(#)`, `[no]log`, and `from(init_specs)`; see [R] Maximize. These options are seldom used.

`init_specs` is one of

- `matname [ , skip copy ]`
- `# [ , # ... ]`, `copy`
iterinit(#) specifies the maximum number of iterations used to calculate the starting values. This option is seldom used.

control(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with the statistic pomeans. control() and tlevel() may not specify the same treatment level.

tlevel(# | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

The following option is available with stteffects but is not shown in the dialog box:

coefflegend; see [R] Estimation options.

Remarks and examples

If you are not familiar with the framework for treatment-effects estimation from observational survival-time data, please see [CAUSAL] stteffects intro.

RA estimators use contrasts of the averages of treatment-specific predicted mean outcomes to estimate treatment effects. RA estimators use a two-step approach to estimating treatment effects:

1. For each treatment level, fit a model of the survival-time outcome on the same set of covariates.
2. Compute the averages of the predicted outcomes for each subject within each treatment level.

These averages estimate the potential-outcome means (POMs). Contrasts of these averages estimate the ATEs. By restricting the computations of the averages to the subset of treated subjects, we obtain estimates of the ATETs.

Here we note only a few entry points to the vast literature on RA estimators. Imbens (2004), Imbens and Wooldridge (2009), Cameron and Trivedi (2005, chap. 25), Wooldridge (2010, chap. 21), and Vittinghoff et al. (2012, chap. 9) provide excellent general introductions to estimating ATEs and to RA estimators in particular.

Like streg and other survival-time commands, stteffects ra uses the outcome variable and the failure indicator computed by, and optionally weights specified with, stset. stteffects ra is not appropriate for data with time-varying covariates, also known as multiple-record survival-time data, or for delayed-entry data.

Example 1: Estimating the ATE

Suppose we wish to study the effect of smoking on the time to a second heart attack among women aged 45–55 years. In our fictional sheart dataset, atime is the observed time in years to a second heart attack or censoring, and fail is the 0/1 indicator that a second heart attack was observed. (When fail is 1, atime records the time to the second heart attack; when fail is 0, atime records a censored observation of the time to a second heart attack.) We previously stset these data; see A quick tour of the estimators in [CAUSAL] stteffects intro.
The treatment, smoking, is stored in the 0/1 indicator smoke. These data also contain age at the time of the first heart attack (age), and indices of the level of exercise (exercise), diet quality (diet), and education (education) prior to the first heart attack.

We can use stteffects ra to estimate the ATE by RA. We model the mean survival time using the default Weibull model, controlling for age, exercise, diet, and education, and we specify that smoke is the treatment variable.

```
. use https://www.stata-press.com/data/r18/sheart
   (Time to second heart attack (fictional))
. stteffects ra (age exercise diet education) (smoke)
   Failure _d:  fail
   Analysis time _t:  atime
 iteration 0:  EE criterion = 1.006e-14
 iteration 1:  EE criterion = 2.305e-25
 Survival treatment-effects estimation  Number of obs = 2,000
 Estimator : regression adjustment
 Outcome model : Weibull
 Treatment model: none
 Censoring model: none

          _t |  Coefficient  std. err.     z    P>|z|     [95% conf. interval]
-----------------|--------------------------------------------------------------
 ATE             |                        |
 smoke          |                        |
 (Smoker vs Nonsmoker) | -1.956657    .3331787   -5.87    0.000   -2.609676   -1.303639

 P0mean          |                        |
 smoke          |                        |
 Nonsmoker      |   4.243974    .2620538   16.20    0.000   3.730358   4.75759
```

When every woman smoked in the population of women aged 45–55 years who have had a heart attack, the average time to a second heart attack is estimated to be 1.96 years less than when no women in the population of interest smoked. The estimated average time to a second heart attack when no women in the population of interest smoked is 4.24 years. In other words, if every woman in the population of interest smoked, then the average time to a second heart attack would fall by an estimated 46% relative to the case when no women smoked.

Example 2: Changing the outcome model

Instead of a Weibull model for the outcome model, we could have used an exponential, a gamma, or a lognormal model. By way of comparison, we use a gamma model and the same covariates to estimate the ATE.
. stteffects ra (age exercise diet education, gamma) (smoke)
   Failure _d: fail
   Analysis time _t: atime
Iteration 0:  EE criterion = 6.212e-25
Iteration 1:  EE criterion = 7.256e-32
Survival treatment-effects estimation  Number of obs = 2,000
Estimator : regression adjustment
Outcome model : gamma
Treatment model: none
Censoring model: none

|          | Coefficient | std. err. |     z |   P>|z| |      [95% conf. interval] |
|----------|-------------|-----------|-------|------|--------------------------|
| ATE      |             |           |       |     |                          |
| smoke    |             |           |       |     |                          |
| (Smoker  |             |           |       |     |                          |
| vs      |             |           |       |     |                          |
| Non smoker) | -1.801787   |  .2924388 | -6.16 | 0.000 | -2.374956 -1.228617    |
| POmean   |             |           |       |     |                          |
| smoke    |             |           |       |     |                          |
| Non smoker) | 3.994327    |  .2258257 | 17.69 | 0.000 | 3.551717 4.436937   |

The estimated ATE of −1.80 and control-level POM of 3.99 are similar to those of −1.96 and 4.24 obtained from the Weibull model in example 1. The ratio of the estimated ATE to the control-level POM indicates a 45% reduction instead of the 46% reduction obtained from the Weibull model.

Example 3: Estimating the ratio of the ATE to the control-level POM

The ratio of the ATE to the control-level POM measures the importance of the effect. In example 1, we computed the point estimate of this ratio from the output, but we were left without a confidence interval. In this example, we use \texttt{nlcom} to compute a point estimate and a confidence interval.

Below, we refit the model from example 1, specifying the \texttt{coeflegend} option to learn the parameter names. We use the parameter names in \texttt{nlcom} to estimate the ratio of the ATE to the control-level POM.
The output shows that when every woman smoked, the average time to a second heart attack falls by an estimated 46% relative to the case when no women smoked, as we computed earlier. We also obtain a 95% confidence interval of 34% to 58% for this estimate.

Example 4: Estimating the ATET

Intuitively, the ATET measures the effect of the treatment on an at-risk subpopulation. Sometimes the subpopulation that gets the treatment defines such an at-risk subpopulation. The ATET has the added benefit that it can be estimated under weaker conditions than the ATE; see Assumptions and tradeoffs in [CAUSAL] stteffects intro.
. stteffects ra (age exercise diet education) (smoke), atet
  Failure _d: fail
  Analysis time _t: atime
Iteration 0:  EE criterion = 1.006e-14
Iteration 1:  EE criterion = 2.970e-26
Survival treatment-effects estimation
Number of obs      =   2,000
Estimator : regression adjustment
Outcome model : Weibull
Treatment model: none
Censoring model: none

|             | Coefficient | std. err. | z     | P>|z|  | [95% conf. interval] |
|-------------|-------------|-----------|-------|------|---------------------|
| _t ATET     |             |           |       |      |                     |
| smoke       | -1.527476   | .2489203  | -6.14 | 0.000| -2.015351 -1.039602 |
| (Smoker vs  |             |           |       |      |                     |
| Nonsmoker)  |             |           |       |      |                     |
| POmean      | 3.436937    | .2217808  | 15.50 | 0.000| 3.002255 3.87162   |
| smoke       |             |           |       |      |                     |
| Nonsmoker   |             |           |       |      |                     |

When every woman in the subpopulation smoked, the average time to a second heart attack is estimated to be 1.53 years less than when no women in the subpopulation smoked. The estimated average time to a second heart attack when no women in the subpopulation smoked is 3.44 years.

Example 5: Fixed or random censoring time

The time to censoring in survival-time data can be random or deterministic, although it must be independent of treatment assignment and the potential outcomes; see Kalbfleisch and Prentice (2002, chap. 3) for the standard case and see The correct adjustment for censoring assumption under Assumptions and tradeoffs in [CAUSAL] stteffects intro for the treatment-effects case.

The RA estimator and the likelihood-adjusted-censoring version of the inverse-probability-weighted RA estimator can accommodate a fixed time to censoring; see The correct adjustment for censoring assumption in [CAUSAL] stteffects intro. (The estimators that handle censoring by weighting cannot accommodate a fixed time to censoring because the weights are not well defined with a fixed time to censoring.)

We have fictional data on the time to rearrest among men aged 25–35 who were previously in prison for a felony conviction (rtime). The time to censoring is fixed in these data because individuals were followed for a maximum of five years.

Some of the young men chose to enter a vocational training program before release from prison; train is 1 for participants and 0 for nonparticipants. The dataset also contains fail (which is 1 if the observed time is a failure time and 0 if it is time to censoring), age at the time of the first arrest (age), an index of the parents’ socioeconomic level (parental), and the number of years behind in school at the time of the first arrest (edeficit).

We estimate the ATET because we wish to allow the gains from the training program to be related to an unobservable characteristic that affects who self-selects into the program; see Average treatment effect on the treated in [CAUSAL] stteffects intro.
We model the outcome as a function of age, parental, and edeficit.

. use https://www.stata-press.com/data/r18/recid2, clear
(Time to rearrest (fictional))
. stteffects ra (age parental edeficit) (train), atet
    Failure _d: fail
    Analysis time _t: rtime
Iteration 0: EE criterion = 1.875e-23
Iteration 1: EE criterion = 4.785e-32
Survival treatment-effects estimation
Number of obs = 2,000
Estimator : regression adjustment
Outcome model : Weibull
Treatment model: none
Censoring model: none

|                | Coefficient | std. err. | z     | P>|z| | [95% conf. interval] |
|----------------|-------------|-----------|-------|-----|----------------------|
| ATET           |             |           |       |     |                      |
| train (Student vs Nonstudent) | 2.440919    | 0.4689057 | 5.21  | 0.000 | 1.52188              | 3.359957 |
| POmean         |             |           |       |     |                      |
| train Nonstudent | 2.062029    | 0.1231492 | 16.74 | 0.000 | 1.820661             | 2.303397 |

When everyone who selected the training got the training, the average time to rearrest is 2.44 years later than the average rearrest time if none of those who selected the training got the training. The average rearrest time if none of those who selected the training got the training is 2.06 years. In other words, the average time to rearrest increases from about 2.06 years to about 4.50 years for the subpopulation of young men who self-selected into the prerelease vocational training program.

Stored results

stteffects ra stores the following in e():

Scalars
- e(N) number of observations
- e(n_j) number of observations for treatment level j
- e(N_clust) number of clusters
- e(k_eq) number of equations in e(b)
- e(k_levels) number of levels in treatment variable
- e(treated) level of treatment variable defined as treated
- e(control) level of treatment variable defined as control
- e(converged) 1 if converged, 0 otherwise

Macros
- e(cmd) stteffects
- e(cmdline) command as typed
- e(dead) _d
- e(depvar) _t
- e(tvar) name of treatment variable
- e(subcmd) ra
- e(omodel) outcome model: weibull, exponential, gamma, or lognormal
- e(stat) statistic estimated: ate, atet, or pomeans
weight type
weight expression
title in estimation output
name of cluster variable
levels of treatment variable
vcetype specified in vce()
title used to label Std. err.
program used to implement estat
program used to implement predict
predictions disallowed by margins
factor variables fvset as asbalanced
factor variables fvset as asobserved
coefficient vector
variance–covariance matrix of the estimators
marks estimation sample
matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

In addition to the above, the following is stored in r():

Methods and formulas

The methods and formulas for the RA estimators implemented in stteffects ra are given in Methods and formulas of [CAUSAL] stteffects ipwra.

References


Also see

[CAUSAL] **stteffects postestimation** — Postestimation tools for stteffects

[CAUSAL] **stteffects intro** — Introduction to treatment effects for observational survival-time data

[ST] **streg** — Parametric survival models

[ST] **stset** — Declare data to be survival-time data

[U] **20 Estimation and postestimation commands**
**stteffects wra — Survival-time weighted regression adjustment**

**Description**

`stteffects wra` estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational survival-time data with random time to censoring. Estimation is by weighted regression adjustment (WRA). WRA estimators use inverse-probability-of-censoring adjusted regression coefficients to compute averages of treatment-level predicted outcomes. Contrasts of these averages estimate the treatment effects. WRA uses estimated weights from a time-to-censoring model to account for censored survival times instead of including a term in the likelihood function. `stteffects wra` offers several choices for the functional forms of the outcome model and the time-to-censoring model. Binary and multivalued treatments are accommodated.

See [CAUSAL] `stteffects intro` for an overview of estimating treatment effects from observational survival-time data.

**Quick start**

Specify time as observed failure time and fail as failure indicator

```
stset time, failure(fail)
```

ATE from a Weibull model for `time` on `x1` and `x2` with binary treatment `treat2` and a Weibull model on `x1` and `x2` for censoring

```
stteffects wra (x1 x2) (treat2) (x1 x2)
```

Same as above, but estimate the ATET

```
stteffects wra (x1 x2) (treat2) (x1 x2), atet
```

ATE of `treat2` using a gamma model for `time` and a gamma censoring model

```
stteffects wra (x1 x2, gamma) (treat2) (x1 x2, gamma)
```

ATE for each level of three-valued treatment `treat3`

```
stteffects wra (x1 x2) (treat3) (x1 x2)
```

Same as above, and specify that `treat3 = 3` is the control level using the value label “MyControl” for 3

```
stteffects wra (x1 x2) (treat3) (x1 x2), control("MyControl")
```

**Menu**

Statistics > Causal inference/treatment effects > Survival outcomes > Weighted regression adjustment
**Syntax**

```stata
stteffects wra (omvarlist [, omoptions]) (tvar) (cmvarlist [, cmoptions])
                [if] [in] [, stat options]
```

*omvarlist* specifies the variables that predict the survival-time variable in the outcome model.  
*tvar* must contain integer values representing the treatment levels.  
*cmvarlist* specifies the variables that predict censoring in the censoring model.

**omoptions**

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>weibull</td>
<td>Weibull; the default</td>
</tr>
<tr>
<td>exponential</td>
<td>exponential</td>
</tr>
<tr>
<td>gamma</td>
<td>two-parameter gamma</td>
</tr>
<tr>
<td>lnormal</td>
<td>lognormal</td>
</tr>
<tr>
<td>ancillary(avarlist [, noconstant])</td>
<td>specify variables used to model ancillary parameter</td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant from outcome model</td>
</tr>
</tbody>
</table>

**cmoptions**

<table>
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<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>weibull</td>
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<td>lnormal</td>
<td>lognormal</td>
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<tr>
<td>ancillary(avarlist [, noconstant])</td>
<td>specify variables used to model ancillary parameter</td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant from censoring model</td>
</tr>
</tbody>
</table>

**stat**

<table>
<thead>
<tr>
<th>Stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
</tr>
<tr>
<td>pomeans</td>
<td>estimate potential-outcome means</td>
</tr>
</tbody>
</table>
### options

<table>
<thead>
<tr>
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<tr>
<td><strong>SE/Robust</strong></td>
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<td><code>vce(vcetype)</code></td>
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<tr>
<td><strong>Maximization</strong></td>
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<td><code>maximize_options</code></td>
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<tr>
<td><code>iterinit(#)</code></td>
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<tr>
<td><strong>Advanced</strong></td>
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<td><code>pstolerance(#)</code></td>
</tr>
<tr>
<td><code>osample(newvar)</code></td>
</tr>
<tr>
<td>`control(#</td>
</tr>
<tr>
<td>`tlevel(#</td>
</tr>
<tr>
<td><code>coeflegend</code></td>
</tr>
</tbody>
</table>

You must `stset` your data before using `stteffects`; see `[ST] stset`. `omvarlist`, `cmvarlist`, and `avarlist` may contain factor variables; see `[U] 11.4.3 Factor variables`. `bootstrap`, `by`, `collect`, `jackknife`, and `statsby` are allowed; see `[U] 11.1.10 Prefix commands`. Weights are not allowed with the bootstrap prefix; see `[R] bootstrap`. `fweights`, `iweights`, and `pweights` may be specified using `stset`; see `Weights` under `Remarks and examples` in `[ST] stset`. However, weights may not be specified if you are using the bootstrap prefix. `coeflegend` does not appear in the dialog box.

See `[U] 20 Estimation and postestimation commands` for more capabilities of estimation commands.

### Options

**ancillary(avarlist [, noconstant ])** specifies the variables used to model the ancillary parameter. By default, the ancillary parameter does not depend on covariates. Specifying `ancillary(avarlist, noconstant)` causes the constant to be suppressed in the model for the ancillary parameter. `ancillary()` may be specified for the model for survival-time outcome, for the model for the censoring variable, or for both. If `ancillary()` is specified for both, the varlist used for each model may be different. `noconstant`; see `[R] Estimation options`. 

---

**Note:** This response is based on the provided text and does not include any visual elements or additional context that might be present in the page. The natural text is formatted to match the original as closely as possible, considering the limitations of the text extraction process.
Stat is one of three statistics: ate, atet, or pomeans. ate is the default.

ate specifies that the average treatment effect be estimated.
atet specifies that the average treatment effect on the treated be estimated.
pomeans specifies that the potential-outcome means for each treatment level be estimated.

vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

level(#); see [R] Estimation options.

eaquations specifies that the results for the outcome-model or treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

noshow prevents stteffects wra from showing the key st variables. This option is rarely used because most people type stset, show or stset, noshow to permanently set whether they want to see these variables mentioned at the top of the output of every st command; see [ST] stset.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

maximize_options: iterate(#), [no] log, and from(init_spec); see [R] Maximize. These options are seldom used.

init_spec is one of

matname [, skip copy]

# [, # ...], copy

iterinit(#) specifies the maximum number of iterations used to calculate the starting values. This option is seldom used.

pstolerance(#) specifies the tolerance used to check the overlap assumption. The default value is pstolerance(1e-5). stteffects will exit with an error if an observation has an estimated propensity score smaller than that specified by pstolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that violate the overlap assumption.

control(#) | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with the statistic pomeans. control() and tlevel() may not specify the same treatment level.
tlevel(# | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

The following option is available with stteffects but is not shown in the dialog box: coeflegend; see [R] Estimation options.

Remarks and examples

If you are not familiar with the framework for treatment-effects estimation from observational survival-time data, please see [CAUSAL] stteffects intro.

Weighted regression-adjustment (WRA) estimators use estimated weights to account for censoring when estimating outcome-regression parameters. The estimated outcome-regression parameters are used to compute averages of treatment-level predicted outcomes. Contrasts of these averages estimate the treatment effects.

WRA estimators use a three-step approach to estimating treatment effects:

1. They estimate the parameters of a time-to-censoring model and compute inverse-probability-of-censoring weights.
2. Using the estimated inverse-probability-of-censoring weights, they use weighted maximum likelihood estimators for the outcome for each treatment level and obtain the treatment-specific predicted mean outcomes for each subject. The inverse-probability-of-censoring weights account for right-censored survival times.
3. They compute the means of the treatment-specific predicted mean outcomes. Contrasts of these averages provide the estimates of the ATEs. By restricting the computations of the means to the subset of treated subjects, we can obtain the ATETs.

WRA estimators differ from RA estimators in that WRA estimators use weights to account for observations lost to censoring while RA estimators use an additional term in the likelihood function. A model for the time to censoring is used to estimate the weights.

WRA estimators require more assumptions than RA estimators. Specifically, they require that the censoring time be random and that the time-to-censoring model be well specified. The implemented WRA estimators also require that the time-to-censoring process not vary by treatment level. The RA estimator and the likelihood-adjusted-censoring version of the inverse-probability-weighted RA estimator do not require these extra assumptions, because they use a likelihood term instead of weights to adjust for the data lost to censoring; see [CAUSAL] stteffects ra and [CAUSAL] stteffects ipwra.

Here we note only a few entry points to the vast literature on weighted estimators. Imbens (2004), Imbens and Wooldridge (2009), Robins and Rotnitzky (2006), Wooldridge (2002, 2007), Cameron and Trivedi (2005, chap. 25), Wooldridge (2010, chap. 21), and Vittinghoff et al. (2012, chap. 9) provide excellent general introductions to estimating ATEs and to WRA estimators in particular.

Like streg and other survival-time commands, stteffects wra uses the outcome variable and the failure indicator computed by, and optionally weights specified with, stset. stteffects wra is not appropriate for data with time-varying covariates, also known as multiple-record survival-time data, or for delayed-entry data.
Example 1: Estimating the ATE

Suppose we wish to study the effect of smoking on the time to a second heart attack among women aged 45–55 years. In our fictional `sheart` dataset, `atime` is the observed time in years to a second heart attack or censoring, and `fail` is the 0/1 indicator that a second heart attack was observed. (When `fail` is 1, `atime` records the time to the second heart attack; when `fail` is 0, `atime` records a censored observation of the time to a second heart attack.) We previously `stset` these data; see A quick tour of the estimators in [CAUSAL] stteffects intro.

The treatment, smoking, is stored in the 0/1 indicator `smoke`. These data also contain age at the time of the first heart attack (`age`), and indices of the level of exercise (`exercise`), diet quality (`diet`), and education (`education`) prior to the first heart attack.

We can use `stteffects wra` to estimate the ATE by WRA. We model the mean survival time using the default Weibull outcome model with `age`, `exercise`, `diet`, and `education` as covariates, and we specify that `smoke` is the treatment variable. We also specify the default Weibull time-to-censoring model and include `age`, square of `age`, `exercise`, and `education`.

```
. use https://www.stata-press.com/data/r18/sheart
   (Time to second heart attack (fictional))
. stteffects wra (age exercise diet education)
   > (smoke)
   > (age c.age#c.age exercise diet education)
```

When every woman smoked in the population of women aged 45–55 years who have had a heart attack, the average time to a second heart attack is estimated to be 2.37 years less than when no women in the subpopulation of interest smoked. The estimated average time to a second heart attack when no women in the subpopulation of interest smoked is 4.30 years.
Stored results

\textbf{stteffects wra} stores the following in \texttt{e():}

\begin{description}
\item[Scalars]
- \texttt{e(N)}: number of observations
- \texttt{e(n_j)}: number of observations for treatment level \texttt{j}
- \texttt{e(N_clust)}: number of clusters
- \texttt{e(k_eq)}: number of equations in \texttt{e(b)}
- \texttt{e(k_levels)}: number of levels in treatment variable
- \texttt{e(treated)}: level of treatment variable defined as treated
- \texttt{e(control)}: level of treatment variable defined as control
- \texttt{e(converged)}: 1 if converged, 0 otherwise
\item[Macros]
- \texttt{e(cmd)}: \texttt{stteffects}
- \texttt{e(cmdline)}: command as typed
- \texttt{e(dead)}: \texttt{_d}
- \texttt{e(depvar)}: name of treatment variable
- \texttt{e(tvar)}: name of treatment variable
- \texttt{e(subcmd)}: \texttt{wra}
- \texttt{e(omodel)}: outcome model: \texttt{weibull}, \texttt{exponential}, \texttt{gamma}, or \texttt{lognormal}
- \texttt{e(cmodel)}: censoring model: \texttt{weibull}, \texttt{exponential}, \texttt{gamma}, or \texttt{lognormal}
- \texttt{e(stat)}: statistic estimated: \texttt{ate}, \texttt{atet}, or \texttt{pomeans}
- \texttt{e(wtype)}: weight type
- \texttt{e(wexp)}: weight expression
- \texttt{e(title)}: title in estimation output
- \texttt{e(clustvar)}: name of cluster variable
- \texttt{e(tlevels)}: levels of treatment variable
- \texttt{e(vce)}: \texttt{vcetype} specified in \texttt{vce()}
- \texttt{e(vcetype)}: title used to label Std. err.
- \texttt{e(properties)}: \texttt{b V}
- \texttt{e(estat_cmd)}: program used to implement \texttt{estat}
- \texttt{e(predict)}: program used to implement \texttt{predict}
- \texttt{e(marginsnotok)}: predictions disallowed by \texttt{margins}
- \texttt{e(asbalanced)}: factor variables \texttt{fvset} as \texttt{asbalanced}
- \texttt{e(asobserved)}: factor variables \texttt{fvset} as \texttt{asobserved}
\item[Matrices]
- \texttt{e(b)}: coefficient vector
- \texttt{e(V)}: variance–covariance matrix of the estimators
\item[Functions]
- \texttt{e(sample)}: marks estimation sample
\end{description}

In addition to the above, the following is stored in \texttt{r()}:\n
\begin{description}
\item[Matrices]
- \texttt{r(table)}: matrix containing the coefficients with their standard errors, test statistics, \textit{p}-values, and confidence intervals
\end{description}

Note that results stored in \texttt{r()} are updated when the command is replayed and will be replaced when any \texttt{r-class} command is run after the estimation command.

\section*{Methods and formulas}

The methods and formulas for the WRA estimators implemented in \texttt{stteffects wra} are given in \textit{Methods and formulas} of [CAUSAL] \texttt{stteffects ipwra}.
References


Also see

[CAUSAL] stteffects postestimation — Postestimation tools for stteffects

[CAUSAL] stteffects intro — Introduction to treatment effects for observational survival-time data

[ST] streg — Parametric survival models

[ST] stset — Declare data to be survival-time data

[U] 20 Estimation and postestimation commands
Description

The tebalance postestimation commands produce diagnostic statistics, test statistics, and diagnostic plots to assess whether a teffects or an stteffects command balanced the covariates over treatment levels.

Syntax

```
tebalance subcommand ... [ , options ]
```

<table>
<thead>
<tr>
<th>subcommand</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>summarize</td>
<td>compare means and variances in raw and balanced data</td>
</tr>
<tr>
<td>overid</td>
<td>overidentification test</td>
</tr>
<tr>
<td>density</td>
<td>kernel density plots for raw and balanced data</td>
</tr>
<tr>
<td>box</td>
<td>box plots for each treatment level for balanced data</td>
</tr>
</tbody>
</table>

Remarks and examples

This entry provides an overview of the commands in tebalance. We recommend that you read this entry before proceeding to [CAUSAL] tebalance summarize, [CAUSAL] tebalance overid, [CAUSAL] tebalance density, or [CAUSAL] tebalance box for command-specific syntax and details.

A covariate is said to be balanced when its distribution does not vary over treatment levels.

Covariates are balanced in experimental data because treatment assignment is independent of the covariates because of the study design. In contrast, covariates must be balanced by weighting or matching in observational data because treatment assignment is related to the covariates that also affect the outcome of interest.

The estimators implemented in teffects and stteffects use a model or matching method to make the outcome conditionally independent of the treatment by conditioning on covariates. If this model or matching method is well specified, it should balance the covariates. Balance diagnostic techniques and tests check the specification of the conditioning method used by a teffects or an stteffects estimator; see [CAUSAL] teffects intro advanced for an introduction to teffects, and [CAUSAL] stteffects intro for an introduction to stteffects.

tebalance implements four methods to check for balance after teffects and stteffects. Which tebalance methods are available depends on the teffects estimation method, as summarized in the table below.
tebalance is a command that checks balance after tefeets or stteffects estimation. It works after tefeets and stteffects.

<table>
<thead>
<tr>
<th>Method</th>
<th>Description</th>
<th>ipw</th>
<th>aipw</th>
<th>ipwra</th>
<th>nnmatch</th>
<th>psmatch</th>
<th>ipw</th>
<th>ipwra</th>
</tr>
</thead>
<tbody>
<tr>
<td>summarize</td>
<td>standardized differences and variance ratios</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>overid</td>
<td>$\chi^2$ test for balance</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>density</td>
<td>diagnostic kernel-density plots</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td>box</td>
<td>diagnostic box plots</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

tebalance overid implements a formal test, while the other three methods are exploratory diagnostic techniques. There is no balance check after tefeets ra or stteffects ra, because they use neither a treatment model nor a matching method.

Austin (2009, 2011) and Guo and Fraser (2015, sec. 5.52) provide introductions to covariate balance. Imai and Ratkovic (2014) derived a test for balance implemented in tebalance overid.

The remainder of this entry provides a quick introduction to using tebalance to check for balance after tefeets. See [CAUSAL] stteffects intro for examples after stteffects.

**Example 1: Balance after estimators that use weighting**

Inverse-probability-weighted (IPW) estimators use a model for the treatment to make the outcome conditionally independent of the treatment. If this model is well specified, it will also balance the covariates.

Using an extract from Cattaneo (2010), we use tefeets ipw to estimate the effect of a mother’s smoking behavior (mbsmoke) on the birthweight of her child (bweight), controlling for marital status (mmarried), the mother’s age (mage), whether the mother had a prenatal doctor’s visit in the baby’s first trimester (prenatal1), and whether this baby is the mother’s first child (fbaby).

```
. use https://www.stata-press.com/data/r18/cattaneo2
. tefeets ipw (bweight) (mbsmoke mmarried mage prenatal1 fbaby)
```

```
Iteration 0: EE criterion = 1.873e-22
Iteration 1: EE criterion = 3.315e-26
```

```
Treatment-effects estimation
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: logit
Number of obs = 4,642

| bweight | Coefficient | Robust std. err. | z | P>|z| | [95% conf. interval] |
|---------|-------------|------------------|---|-------|----------------------|
| ATE     |             |                  |   |       |                      |
| mbsmoke (Smoker vs Nonsmoker) | -236.1038 | 23.86187 | -9.89 | 0.000 | -282.8722 -189.3354 |
| POmean  |             |                  |   |       |                      |
| mbsmoke Nonsmoker | 3402.552 | 9.539555 | 356.68 | 0.000 | 3383.855 3421.249    |
Rubin (2008) recommends finding a model that balances the covariates before looking at results for the estimated treatment effect. Thus we do not interpret the above results, and we note that we could pay closer heed to Rubin’s recommendation by preceding the teffects command with quietly to suppress the output.

Imai and Ratkovic (2014) derived a test for balance by viewing the restrictions imposed by balance as overidentifying conditions. This test is implemented in tebalance overid, and we use it to test whether the above treatment model balanced the covariates.

```
. tebalance overid
Iteration 0:  Criterion =  .02146858
Iteration 1:  Criterion =  .02159149  (backed up)
Iteration 2:  Criterion =  .02260102
Iteration 4:  Criterion =  .0227956
Iteration 5:  Criterion =  .02292367
Iteration 6:  Criterion =  .02431655
Iteration 7:  Criterion =  .02457028
Iteration 8:  Criterion =  .02488569
Iteration 9:  Criterion =  .02529483
Iteration 10: Criterion =  .0254588
Iteration 11: Criterion =  .02550245
Iteration 12: Criterion =  .02552864
Iteration 13: Criterion =  .02554462
Iteration 14: Criterion =  .02554512
Iteration 15: Criterion =  .02554514
Overidentification test for covariate balance
H0: Covariates are balanced
    chi2(5)    = 38.1464
    Prob > chi2 = 0.0000
```

We reject the null hypothesis that the treatment model balanced the covariates.

Let’s use tebalance summarize to see where the problem lies. To get an idea of the extent to which the covariates are unbalanced, we begin by summarizing the covariates by group in the raw data by specifying the baseline option.

```
. tebalance summarize, baseline
Covariate balance summary

<table>
<thead>
<tr>
<th></th>
<th>Raw</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs</td>
<td>4,642</td>
<td>4,642.0</td>
</tr>
<tr>
<td>Treated obs</td>
<td>864</td>
<td>2,315.3</td>
</tr>
<tr>
<td>Control obs</td>
<td>3,778</td>
<td>2,326.7</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Means</th>
<th>Variances</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control</td>
<td>Treated</td>
</tr>
<tr>
<td>mmmarried</td>
<td>.7514558</td>
<td>.4733796</td>
</tr>
<tr>
<td>mage</td>
<td>26.81048</td>
<td>25.16667</td>
</tr>
<tr>
<td>prenatal1</td>
<td>.8268925</td>
<td>.6898148</td>
</tr>
<tr>
<td>fbaby</td>
<td>.4531498</td>
<td>.3715278</td>
</tr>
</tbody>
</table>
```

The output indicates that the covariates may not be balanced in the raw data. For example, the distribution of the mother’s age may differ over the treatment groups. We can investigate the differences further with standardized differences and variance ratios. A perfectly balanced covariate has a standardized difference of zero and variance ratio of one. There are no standard errors on these
statistics, so inference is informal. Austin (2009) provides a recent introduction to these diagnostics, although they have been used at least since Rosenbaum and Rubin (1985).

By omitting the baseline option, we obtain these diagnostic statistics for the raw data and the weighted data.

```
tebalance summarize
Covariate balance summary

<table>
<thead>
<tr>
<th></th>
<th>Raw</th>
<th>Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs</td>
<td>4,642</td>
<td>4,642.0</td>
</tr>
<tr>
<td>Treated obs</td>
<td>864</td>
<td>2,315.3</td>
</tr>
<tr>
<td>Control obs</td>
<td>3,778</td>
<td>2,326.7</td>
</tr>
</tbody>
</table>
```

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Standardized differences Raw</th>
<th>Standardized differences Weighted</th>
<th>Variance ratio Raw</th>
<th>Variance ratio Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>married</td>
<td>-.5953009</td>
<td>-.0105562</td>
<td>1.335944</td>
<td>1.009079</td>
</tr>
<tr>
<td>age</td>
<td>-.300179</td>
<td>-.0672115</td>
<td>.8818025</td>
<td>.8536401</td>
</tr>
<tr>
<td>prenatal1</td>
<td>-.3242695</td>
<td>-.0156339</td>
<td>1.496155</td>
<td>1.023424</td>
</tr>
<tr>
<td>baby</td>
<td>-.1663271</td>
<td>.0257705</td>
<td>.9430944</td>
<td>1.005698</td>
</tr>
</tbody>
</table>

Reviewing the output, we see that for married, prenatal1, and baby, our model improved the level of balance. The weighted standardized differences are all close to zero and the variance ratios are all close to one. However, output indicates that age may not be balanced by our model. The weighted standardized difference is close to zero, but the weighted variance ratio still appears to be considerably less than one.

Now, let’s use tebalance density to look at how the densities of age for treated and control groups differ.

```
tebalance density age
```

The plots also indicate a lack of balance in age between the treatment groups.
To try to achieve better balance, we specify a richer model with interactions between `mage` and the other covariates and look at the resulting standardized differences.

```
. quietly teffects ipw (bweight) (mbsmoke mmarried mage prenatal1 fbaby > c.mage#(c.mage i.mmarried prenatal1))
. tebalance summarize
```

Covariate balance summary

<table>
<thead>
<tr>
<th>Raw Weighted</th>
<th>Raw Weighted</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs = 4,642 4,642.0</td>
<td>Treated obs = 864 2,329.1</td>
</tr>
<tr>
<td>Control obs = 3,778 2,312.9</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Covariate</th>
<th>Standardized differences</th>
<th>Variance ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw</td>
<td>Weighted</td>
</tr>
<tr>
<td>mmarried</td>
<td>-.5953009</td>
<td>.0053497</td>
</tr>
<tr>
<td>mage</td>
<td>-.300179</td>
<td>.0410889</td>
</tr>
<tr>
<td>prenatal1</td>
<td>-.3242695</td>
<td>.0009807</td>
</tr>
<tr>
<td>fbaby</td>
<td>-.1663271</td>
<td>-.0130638</td>
</tr>
<tr>
<td>mage#mage</td>
<td>-.3028275</td>
<td>.0477465</td>
</tr>
<tr>
<td>mmarried#mage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Married</td>
<td>-.6329701</td>
<td>.0197209</td>
</tr>
<tr>
<td>prenatal1#mage</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Yes</td>
<td>-.4053969</td>
<td>.0182109</td>
</tr>
</tbody>
</table>
The standardized difference and variance ratio results for `mage` look closer to the expected values of zero and one, so we proceed to the formal test.

```
    . tebalance overid
    Iteration 0:  Criterion =  .0602349
    Iteration 1:  Criterion =  .06172749 (backed up)
    Iteration 2:  Criterion =  .06428588 (backed up)
    Iteration 3:  Criterion =  .06489623 (backed up)
    Iteration 4:  Criterion =  .06527284 (backed up)
    Iteration 5:  Criterion =  .06643426
    Iteration 6:  Criterion =  .07134338
    Iteration 7:  Criterion =  .07638414
    Iteration 8:  Criterion =  .07673211
    Iteration 9:  Criterion =  .07681959
    Iteration 10: Criterion =  .077044
    Iteration 11: Criterion =  .07759547
    Iteration 12: Criterion =  .07771973
    Iteration 13: Criterion =  .0777271
    Iteration 14: Criterion =  .07773395
    Iteration 15: Criterion =  .07774839
    Iteration 16: Criterion =  .07775314
    Iteration 17: Criterion =  .07775324

Overidentification test for covariate balance
H0: Covariates are balanced
    chi2(8) = 11.8612
    Prob > chi2 = 0.1575
```

We do not reject the null hypothesis that the specified treatment model balances the covariates.

---

**Example 2: Balance after estimators that use matching**

Instead of weighting, we might want to use a matching estimator. We can select `teffects nnmatch` or `teffects psmatch` for balance and estimation; in this example, we use `teffects nnmatch`.

```
    . teffects nnmatch (bweight mmarried mage prenatal1 fbaby)
    > (mbsmoke), bias(mage) ematch(mmarried prenatal1 fbaby)
```

```
Treatment-effects estimation  Number of obs =  4,642
Estimator : nearest-neighbor matching  Matches: requested =  1
Outcome model : matching  min =  1
Distance metric: Mahalanobis  max =  139

  AI robust
Coefficient  std. err.  z  P>|z|  [95% conf. interval]

ATE
mbsmoke
(Smoker vs Nonsmoker)  -240.4589  28.43008  -8.46  0.000  -296.1808  -184.7369
```
Again we ignore the estimated effect and first check for balance. We begin by reviewing whether the summary statistics indicate good balance.

```
  . tebalance summarize
  (refitting the model using the generate() option)
  Covariate balance summary

<table>
<thead>
<tr>
<th>Raw</th>
<th>Matched</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of obs = 4,642</td>
<td>9,284</td>
</tr>
<tr>
<td>Treated obs = 864</td>
<td>4,642</td>
</tr>
<tr>
<td>Control obs = 3,778</td>
<td>4,642</td>
</tr>
</tbody>
</table>
```

<table>
<thead>
<tr>
<th></th>
<th>Raw</th>
<th>Matched</th>
<th>Variance ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw</td>
<td>Matched</td>
<td>Raw</td>
</tr>
<tr>
<td>mmarried</td>
<td>-.5953009</td>
<td>-2.42e-16</td>
<td>1.335944</td>
</tr>
<tr>
<td>mage</td>
<td>-.300179</td>
<td>-.0040826</td>
<td>.8818025</td>
</tr>
<tr>
<td>prenatal1</td>
<td>-.3242695</td>
<td>-2.78e-16</td>
<td>1.496155</td>
</tr>
<tr>
<td>fbaby</td>
<td>-.1663271</td>
<td>2.24e-16</td>
<td>.9430944</td>
</tr>
</tbody>
</table>

We do not have standard errors on these statistics, so we cannot make any formal conclusions, but the summary statistics appear to indicate much better balance than the IPW results. `tebalance summarize` has to refit the model to recover the matched sample because the `generate()` option was not specified on the `teffects nnmatch` command. The reestimation does not affect the results, although the computation takes longer; see example 3 for details.

Because it is a matching estimator, and not an IPW estimator, we cannot use `tebalance overid` after `teffects nnmatch`. The matching estimators, however, provide diagnostic box plots using `tebalance box` that are not available after the IPW estimators.

```
  . tebalance box mage
  (refitting the model using the generate() option)
```

The box plots of the matched data also indicate covariate balance.
Let’s look at the kernel density plots using the matched data.

```
. tebalance density mage
(refitting the model using the generate() option)
```

![Balance plot](image)

The plots using the matched data appear to be balanced.

---

**Technical note**

tffects implements matching estimators, IPW estimators, regression-adjustment (RA) estimators, and estimators that combine IPW and RA. Matching estimators define a matched sample, and IPW estimators define a weighted sample, each of which can be used to compute covariate balance statistics. RA estimators do not define an adjusted sample that can be used to compute covariate balance statistics, and tebalance does not work after tffects ra. Only the IPW component of the estimators that combine RA and IPW defines a weighted sample that can be used to compute balance statistics. So, tebalance produces the same results after tffects aipw or tffects ipwra as it does after tffects ipw.

---

**Example 3: Faster results after a matching estimator**

The tebalance commands run in example 2 executed more slowly than necessary. tebalance issued the note

```
refitting the model using the generate() option
```

after the commands

```
. tebalance summarize
. tebalance box mage
```

and

```
. tebalance density mage
```
After `teffects nnmatch` or `teffects psmatch`, `tebalance` computes the balance statistics on the matched sample defined by the matching estimator. `teffects nnmatch` and `teffects psmatch` leave behind only variables that identify the matched sample when the `generate()` option is specified. Unless the `generate()` option is specified with `teffects nnmatch` or `teffects psmatch`, each `tebalance` command must rerun the estimation command to recover the matched sample.

Typing

```
. teffects nnmatch (bweight mmarried mage fbaby prenatal1)
  > (mbsmoke), bias(mage) ematch(mmarried fbaby prenatal1)
  > generate(matchv)
```

would generate variables that identify the matched sample that the `tebalance` commands could use. See Remarks and examples in [CAUSAL] `tebalance` box, [CAUSAL] `tebalance` density, and [CAUSAL] `tebalance` summarize for examples using the option `generate()` on `teffects psmatch` to speed up the postestimation computations.

### Methods and formulas

Methods and formulas are presented under the following headings:

- Introduction
- Matched samples
- IPW samples
- Testing the propensity-score model specification

#### Introduction

For covariate $z$, we observe values \{${z}_1, {z}_2, \ldots, {z}_N$\}. Define a treatment indicator variable for $J$ treatment levels as $t_i \in \{1, 2, \ldots, J\}$, for $i = 1, \ldots, N$, and frequency weights as $\{w_1, w_2, \ldots, w_N\}$. The sample mean and variance of $z$ for level $t$ are

\[
\hat{\mu}_z(t) = \frac{\sum_i I(t_i = t)w_iz_i}{N_t} \quad \text{and} \quad \hat{\sigma}^2_z(t) = \frac{\sum_i I(t_i = t)w_i(z_i - \hat{\mu}_z(t))^2}{N_t - 1}
\]

where $N_t = \sum_i w_iI(t_i = t)$, and

\[
I(t_i = t) = \begin{cases} 
  1 & \text{if } t_i = t \\
  0 & \text{otherwise} 
\end{cases}
\]

As shown in Austin (2011), the standardized differences for covariate $z$ between level $t$ and the control $t_0$ are computed as

\[
\delta_z(t) = \frac{\hat{\mu}_z(t) - \hat{\mu}_z(t_0)}{\sqrt{\hat{\sigma}^2_z(t) + \hat{\sigma}^2_z(t_0)}} \tag{1}
\]

The variance ratio is $\rho_z(t) = \{\hat{\sigma}^2_z(t)\}/\{\hat{\sigma}^2_z(t_0)\}$.  

Matched samples

We now turn our attention to the matched samples for the potential-outcome mean (POM), average treatment effect (ATE), and average treatment effect on the treated (ATET) estimators. We estimate the covariate for the counter-factual treatment by taking the mean of the matched observations

$$
\tilde{z}_i = \frac{\sum_{j \in \Omega(i)} w_j z_j}{\sum_{j \in \Omega(i)} w_j}
$$

where $\Omega(i) = (k_1, k_2, \ldots, k_m)$ is the set of observation indices that are matched to observation $i$ of the opposite treatment condition. The observed covariate and matched covariate pairs, $(z_i, \tilde{z}_i)$, $i = 1, \ldots, N$, are used in the box plot (see [G-2] graph box) and the kernel density plot (see [R] kdensity). The ATET sample is limited to those observations from the conditional treatment, $\tilde{t}$, and their matched covariate means.

In Methods and formulas of [CAUSAL] teffects nnmatch, we define $K_m(i)$ as the number of times observation $i$ is used in a match with observation $j$ of the opposite treatment condition, $i \in \Omega(j)$, weighted by the total number of matches for observation $j$. Specifically,

$$
K_m(i) = \sum_{j=1}^{N} I\{i \in \Omega(j)\} \frac{w_j}{\sum_{k \in \Omega(j)} w_k}
$$

These weights are used in the estimation of the mean and variance for the matched dataset. For the POM and ATE models, the estimated mean and variance are computed as

$$
\hat{\mu}_{\tilde{z}}(t) = \frac{\sum_{i}^{N} I(t_i = t) w_i z_i \{1 + K_m(i)\}}{M_t}
$$

and

$$
\hat{\sigma}^2_{\tilde{z}}(t) = \frac{\sum_{i}^{N} I(t_i = t) w_i \{1 + K_m(i)\} \{z_i - \hat{\mu}_{\tilde{z}}(t)\}^2}{M_t - 1}
$$

where $M_t = \sum_{i}^{N} I(t_i = t) w_i \{1 + K_m(i)\}$.

The standardized differences between the control level and all other levels for the matched covariate distribution are computed as in (1), but $\hat{\mu}_{\tilde{z}}(t)$ is substituted for $\hat{\mu}_{\tilde{z}}(t)$ and $\hat{\sigma}^2_{\tilde{z}}(t)$ for $\hat{\sigma}^2_{\tilde{z}}(t)$.

For the ATET model, there is no matched sample for the treatment levels other than the conditional treatment $\tilde{t}$. The covariate mean and variance for the conditional treatment are the same as that of the raw data, $\mu_{\tilde{z}}(\tilde{t})$ and $\sigma^2_{\tilde{z}}(\tilde{t})$. However, the covariate mean and variance for the sample matched to the conditional treatment, $t \neq \tilde{t}$, are computed as

$$
\tilde{\mu}_{\tilde{z}}(t) = \frac{\sum_{i}^{N} I(t_i = t) w_i z_i K_m(i)}{M_t}
$$

and

$$
\tilde{\sigma}^2_{\tilde{z}}(t) = \frac{\sum_{i}^{N} I(t_i = t) w_i K_m(i) \{z_i - \tilde{\mu}_{\tilde{z}}(t)\}^2}{M_t - 1}
$$

where $M_t = \sum_{i}^{N} I(t_i = t) w_i K_m(i)$. 
IPW samples

Computation of the inverse-probability weights is discussed in Methods and formulas of [CAUSAL] teffects aipw and in Methods and formulas of [CAUSAL] stteffects ipwra. For the POM and ATE estimators, we defined the normalized IPW weights as \( \bar{d}_i(t) = N_i d_i(t) / \sum_i N_i d_i(t) \), where
\[
d_i(t) = I(t_i = t) / p(z_i, t, \hat{\gamma})
\]
for treatment level \( t \) and individual \( i \).

For the ATET estimator, we use the normalized weights \( \bar{f}_i = N_i f_i / \sum_i N_i f_i \), where \( f_i = p(z_i, \tilde{t}, \hat{\gamma}) / p(z_i, t_i, \hat{\gamma}) \) are the treatment-adjusted inverse-probability weights, and \( \tilde{t} \) is the conditional treatment.

We will simplify notation by defining a single weight
\[
\bar{w}_i(t) = \begin{cases} 
\bar{d}_i(t) & \text{if model is ATE or POM} \\
\bar{f}_i(t) & \text{if model is ATET}
\end{cases}
\]

The covariate mean and variance for treatment level \( t \) are
\[
\tilde{\mu}_z(t) = \frac{\sum_i N_i I(t_i = t) \bar{w}_i x_i}{M_t} \quad \text{and} \quad \tilde{\sigma}^2_z(t) = \frac{\sum_i N_i I(t_i = t) \bar{w}_i \{ z_i - \tilde{\mu}_z(t) \}^2}{M_t - 1}
\]
where \( M_t = \sum_i N_i I(t_i = t) \bar{w}_i \).

The kernel density is computed by \texttt{kdensity} for each covariate conditioned on each treatment level using the raw covariate with \texttt{iweights} equal to \( \bar{w}_i \).

Testing the propensity-score model specification

We estimate the probability of treatment conditioned on a set of covariates with a propensity-score model. Imai and Ratkovic (2014) derive a test for whether the estimated propensity score balances the covariates. The score equations for parameters of the propensity-score model define an exactly identified generalized method of moments (GMM) estimator. Imai and Ratkovic (2014) use the conditions imposed by mean balance as overidentifying conditions. A standard GMM test for the validity of the overidentifying conditions is then a test for covariate balance. See \texttt{R gmm} for a discussion of this overidentifying test, which is known as Hansen’s \( J \) test in the econometrics literature.

Here are the details about the score equations and the overidentifying balance conditions. Recall from Methods and formulas of [CAUSAL] teffects aipw and Methods and formulas of [CAUSAL] stteffects ipwra, we have the first-order condition of the treatment model
\[
\frac{1}{N} \sum_{i=1}^N s_{tm,i}(z_i, \hat{\gamma}) = 0
\]

For a two-level treatment-effects model with conditional treatment \( \tilde{t} \) and control \( t_0 \), the score is
\[
s_{tm,i}(z_i, \gamma) = \frac{I(t_i = \tilde{t})}{p(z_i, \tilde{t}, \gamma)} \frac{\partial p(z_i, t, \gamma)}{\partial \gamma} - \left\{ \frac{I(t_i = t_0)}{1 - p(z_i, \tilde{t}, \gamma)} \right\} \frac{\partial p(z_i, \tilde{t}, \gamma)}{\partial \gamma} \bigg|_{\gamma = \hat{\gamma}}
\]
The score reduces to
\[ s_{tm,i}(z_i, \tilde{\gamma}) = \left[ \frac{I(t_i = \tilde{t}) - p(z_i, \tilde{t}, \gamma)}{p(z_i, \tilde{t}, \gamma) \{1 - p(z_i, \tilde{t}, \gamma)\}} \right] \frac{\partial p(z_i, \tilde{t}, \gamma)}{\partial \gamma} \bigg|_{\gamma = \tilde{\gamma}} \]

The corresponding covariate balancing moment conditions are
\[ w_{tm,i}(z_i, \gamma) = \left[ \frac{I(t_i = \tilde{t}) - p(z_i, \tilde{t}, \gamma)}{p(z_i, \tilde{t}, \gamma) \{1 - p(z_i, \tilde{t}, \gamma)\}} \right] z_i \]

for the POM and ATE models. For the ATET model with conditional treatment \( \tilde{t} \), we multiply by \( p(z_i, \tilde{t}, \gamma) \) and scale by \( N/N_{\tilde{t}} \):
\[ w_{tm,i}(z_i, \gamma) = \frac{N}{N_{\tilde{t}}} \left\{ \frac{I(t_i = \tilde{t}) - p(z_i, \tilde{t}, \gamma)}{1 - p(z_i, \tilde{t}, \gamma)} \right\} z_i \]

We stack the moment conditions
\[ g_{tm}(Z, \gamma) = \frac{1}{N} \sum_{i=1}^{N} \left\{ s_{tm,i}(z_i, \gamma) \right\} \]
\[ = \frac{1}{N} \sum_{i=1}^{N} g_{tm,i}(z_i, \gamma) \]

The overidentified GMM estimator is then
\[ \tilde{\gamma} = \arg\min_{\gamma} N g_{tm}(Z, \gamma)' W_{tm}(Z, \gamma)^{-1} g_{tm}(Z, \gamma) \] (2)
where
\[ W_{tm}(Z, \gamma) = \frac{1}{N} \sum_{i=1}^{N} E_T \left\{ g_{tm,i}(z, \gamma) g_{tm,i}(z, \gamma)' \right\} \]
and the expectation is taken with respect to treatment distribution. The weight matrix \( W_{tm}(Z, \gamma) \) is computed explicitly (Imai and Ratkovic 2014), and (2), written as a maximization problem, is solved using ml.

Finally, Hansen’s \( J \) statistic is evaluated at its minimum,
\[ J = N g_{tm}(Z, \tilde{\gamma})' W_{tm}(Z, \tilde{\gamma})^{-1} g_{tm}(Z, \tilde{\gamma}) \]
and is asymptotically distributed \( \chi^2 \) with degrees of freedom \( d \),
\[ d = \text{rank} \{ W_{tm}(Z, \tilde{\gamma}) \} - \text{rank} \left[ \frac{1}{N} \sum_{i=1}^{N} E_T \left\{ s_{tm,i}(z_i, \tilde{\gamma}) s_{tm,i}(z_i, \tilde{\gamma})' \right\} \right] \]
References


Also see

[CAUSAL] stteffects intro — Introduction to treatment effects for observational survival-time data

[CAUSAL] tebalance box — Covariate balance box

[CAUSAL] tebalance density — Covariate balance density

[CAUSAL] tebalance overid — Test for covariate balance

[CAUSAL] tebalance summarize — Covariate-balance summary statistics

[CAUSAL] teffects — Treatment-effects estimation for observational data


tebalance box produces box plots that are used to check for balance in matched samples after 
\texttt{teffects nmatch} and \texttt{teffects pmatch}.

**Quick start**

Box plot of the propensity score from the last \texttt{teffects pmatch} command

\begin{verbatim}
tebalance box
\end{verbatim}

Box plot of values of $x_1$ in the treatment and control groups from raw data and the matched sample
from the last \texttt{teffects nmatch} or \texttt{teffects pmatch} command

\begin{verbatim}
tebalance box x1
\end{verbatim}

**Menu**

Statistics $>$ Causal inference/treatment effects $>$ Balance $>$ Graphs
Syntax

Box plots for the propensity score

    tebalance box [, options]

Box plots for a covariate

    tebalance box varname [, options]

Options

<table>
<thead>
<tr>
<th>Options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>boxlook_options</td>
<td>graph box options controlling how the box looks</td>
</tr>
<tr>
<td>legending_options</td>
<td>graph box options controlling how the variables are labeled</td>
</tr>
<tr>
<td>axis_options</td>
<td>graph box options controlling how numerical y axis is labeled</td>
</tr>
<tr>
<td>title_and_other_options</td>
<td>graph box options controlling titles, added text, aspect ratio, etc.</td>
</tr>
<tr>
<td>by_options</td>
<td>suboptions inside by() controlling plots by raw and matched samples</td>
</tr>
</tbody>
</table>

Options

- `boxlook_options` are any of the options documented in `boxlook_options` in [G-2] `graph box`.
- `legending_options` are any of the options documented in `legending_options` in [G-2] `graph box`.
- `axis_options` are any of the options documented in `axis_options` in [G-2] `graph box`.
- `title_and_other_options` are any of the options, except by(), documented in `title_and_other_options` in [G-2] `graph box`. `tebalance box` uses by() to differentiate between raw and matched samples, and some `twoway_options` will be repeated for by `graph` and might be better specified as `byopts()`.
- `by_options` are any of the `byopts` documented in [G-3] `by_option`. `byopts()` generally affects the entire graph, and some `by_options` may be better specified as `twoway_options`; see [G-3] `twoway_options`.

Remarks and examples

When the distribution of a covariate does not vary over the treatment levels, the covariate is said to be balanced. `tebalance box` produces box plots of a covariate over treatment levels for the raw data and for the matched sample produced by `teffects`. If the matched-sample box plots are the same over the treatment levels, the covariate is balanced in the matched sample.

After `teffects nnmatch` and `teffects psmatch`,

    . tebalance box varname [, options]

produces box plots to check whether `varname` is balanced.
After `teffects psmatch`,

```stata
  . tebalance box [, options]
```

produces box plots to check whether the propensity score estimated by `teffects` is balanced.

We recommend that you read `[CAUSAL] tebalance` before proceeding; it provides an introduction to covariate balance and an overview of the implemented methods.

### Example 1: Checking covariate balance after psmatch

Using an extract from the data used by Cattaneo (2010), we use `teffects psmatch` to estimate the effect of a mother’s smoking behavior (`mbsmoke`) on the birthweight of her child (`bweight`), controlling for marital status (`mmarried`), the mother’s age (`mage`), whether the mother had a prenatal doctor’s visit in the baby’s first trimester (`prenatal1`), and whether this baby is the mother’s first child (`fbaby`).

```stata
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects psmatch (bweight) (mbsmoke mmarried mage prenatal1 fbaby),
> generate(matchv)
```

![Treatment-effects estimation](https://www.stata-press.com/data/r18/cattaneo2)

| AI robust | bweight  | Coefficient | std. err. | z     | P>|z| | [95% conf. interval] |
|-----------|---------|-------------|-----------|-------|------|---------------------|
| ATE       | mbsmoke | -235.1714   | 27.74409  | -8.48 | 0.000 | -289.5488 -180.794  |

We specified the option `generate(matchv)` to speed up the postestimation command that produces density plots, as discussed in example 3 under Remarks and examples of `[CAUSAL] tebalance`. We do not interpret the estimated effect produced by this preliminary model but rather check the specification. Now we look at the box plots.
The box plots for the matched sample are very similar. The medians, the 25th percentiles, and the 75th percentiles appear to be the same, although there may be some differences in the tails, the upper adjacent values, the lower adjacent values, and the outliers. Matching on the estimated propensity score appears to have balanced `mage`, except for the tails.

To get an idea of whether `teffects psmatch` balanced all the covariates, we look at the box plots for the estimated propensity score.

The box plots indicate that `teffects psmatch` balanced the estimated propensity scores.

---

**Reference**

Also see

[CAUSAL] tebalance — Check balance after teffects or stteffects estimation

[CAUSAL] teffects nnmatch — Nearest-neighbor matching

[CAUSAL] teffects psmatch — Propensity-score matching

[CAUSAL] teoverlap — Overlap plots


tebalance density — Covariate balance density

Description

tebalance density produces kernel density plots that are used to check for covariate balance after estimation by a teffects inverse-probability-weighted estimator, a teffects matching estimator, or an stteffects inverse-probability-weighted estimator.

Quick start

Kernel density plot of the propensity score after teffects psmatch

```plaintext
tebalance density
```

Kernel density plot of x1 after a teffects command or an stteffects command

```plaintext
tebalance density x1
```

Same as above, but rescale the kernel bandwidth by a factor of 2

```plaintext
tebalance density x1, bwidth(*2)
```

Menu

Statistics > Causal inference/treatment effects > Balance > Graphs
Syntax

Density plots for the propensity score

tebalance density [, options]

Density plots for a covariate

tebalance density varname [, options]

options

<table>
<thead>
<tr>
<th>Main</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>kernel(kernel)</td>
<td>specify the kernel function; default is kernel(epanechnikov)</td>
</tr>
<tr>
<td>bwidth(*#)</td>
<td>rescale default bandwidth</td>
</tr>
<tr>
<td>line#opts(line_options)</td>
<td>twoway line options for density line number #</td>
</tr>
<tr>
<td>twoway_options</td>
<td>any options other than by() documented in [G-3] twoway_options</td>
</tr>
<tr>
<td>byopts(byopts)</td>
<td>how subgraphs are combined, labeled, etc.</td>
</tr>
</tbody>
</table>

collect is allowed; see [U] 11.1.10 Prefix commands.

kernel

<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>triangle</td>
</tr>
<tr>
<td>ep</td>
</tr>
<tr>
<td>epan2</td>
</tr>
<tr>
<td>biweight</td>
</tr>
<tr>
<td>cosine</td>
</tr>
<tr>
<td>gaussian</td>
</tr>
<tr>
<td>parzen</td>
</tr>
<tr>
<td>rectangle</td>
</tr>
</tbody>
</table>

Options

kernel(kernel) specifies the kernel function for use in calculating the kernel density estimates. The default kernel is the kernel(epanechnikov).

bwidth(*#) specifies the factor by which the default bandwidths are to be rescaled. A bandwidth is the half-width of the kernel, the width of the density window around each point. Each kernel density plot has its own bandwidth, and by default, each kernel density plot uses its own optimal bandwidth; see [R] kdensity. bwidth() rescales each plot’s optimal bandwidth by the specified amount.

line#opts(line_options) specifies the line pattern, width, color, and overall style of density line number #. The line numbers are in the same order as the treatment levels specified in e(tlevels).

twoway_options are any of the options documented in [G-3] twoway_options, excluding by(). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option). tebalance density uses by() to differentiate between raw and weighted or matched samples, and some twoway_options will be repeated for by graph and might be better specified as byopts().
byopts(by_option) is as documented in [G-3] by_option. byopts() affects how the subgraphs are combined, labeled, etc. byopts() generally affects the entire graph, and some by_option may be better specified as twoway_options; see [G-3] twoway_options.

Remarks and examples

When the distribution of a covariate does not vary over the treatment levels, the covariate is said to be balanced. tebalance density produces kernel density plots of a covariate over treatment levels for the raw data and the weighted or matched sample produced by teffects or stteffects. If the weighted-sample or matched-sample kernel density plots of the covariate are the same over the treatment levels, the covariate is balanced in the weighted or matched sample.

After telasso and all teeffects and stteffects commands except teffects ra, stteffects ra, and stteffects wra,

```
. tebalance density varname [, options]
```

produces kernel density plots to check whether varname is balanced.

After teffects psmatch,

```
. tebalance density [, options]
```

produces kernel density plots to check whether the propensity score estimated by teffects psmatch is balanced. Our discussion of the use of tebalance density and interpretation of its results for a covariate below also apply to a propensity score.

We recommend that you read [CAUSAL] tebalance before proceeding; it provides an introduction to covariate balance and an overview of the implemented methods. See [CAUSAL] stteffects intro for a discussion of survival-time features.

Example 1: Checking covariate balance after psmatch

Using an extract from the data used by Cattaneo (2010), we use teffects psmatch to estimate the effect of a mother’s smoking behavior (mbsmoke) on the birthweight of her child (bweight), controlling for marital status (mmarried), the mother’s age (mage), whether the mother had a prenatal doctor’s visit in the baby’s first trimester (prenatal1), and whether this baby is the mother’s first child (fbaby).

```
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects psmatch (bweight) (mbsmoke mmarried mage prenatal1 fbaby), > generate(matchv)
```

<table>
<thead>
<tr>
<th>Treatment-effects estimation</th>
<th>Number of obs = 4,642</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimator</td>
<td>propensity-score matching</td>
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<tr>
<td>Outcome model</td>
<td>matching</td>
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<td>Treatment model</td>
<td>logit</td>
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<tr>
<td>min</td>
<td>1</td>
</tr>
<tr>
<td>max</td>
<td>139</td>
</tr>
<tr>
<td>bweight</td>
<td>Al robust</td>
</tr>
<tr>
<td>Coefficient</td>
<td>std. err.</td>
</tr>
<tr>
<td>ATE</td>
<td>mbsmoke (Smoker vs Nonsmoker)</td>
</tr>
<tr>
<td>-235.1714</td>
<td>27.74409</td>
</tr>
</tbody>
</table>
We specified the option `generate(matchv)` to speed up the postestimation command that produces density plots, as discussed in example 3 under Remarks and examples in [CAUSAL] `tebalance` entry. We do not interpret the estimated effect produced by this preliminary model but rather check the specification. We begin by looking at the default density plots.

```
. tebalance density mage
```

The density plots for the matched sample are nearly indistinguishable, implying that matching on the estimated propensity score balanced the covariates. The density plots are too jagged for presentation, so we oversmooth them by scaling up the bandwidth used for each plot.

```
. tebalance density mage, bwidth(*1.5)
```

Option `bwidth()` rescales the default optimal bandwidths by the specified scale factor. Each of the four density plots has its own sample size and optimal bandwidth. Rescaling each of the four bandwidths by 1.5 produces smoother plots.
Stored results

After `teffects` or `stteffects` fits a binary treatment, `tebalance density` stores the following in `r()`:

Scalars
- `r(bwc_adj)` bandwidth for control in weighted or matched-adjusted sample
- `r(Nc_adj)` observations on control in weighted or matched-adjusted sample
- `r(bwt_adj)` bandwidth for treated in weighted or matched-adjusted sample
- `r(Nt_adj)` observations on treated in weighted or matched-adjusted sample
- `r(bwc_raw)` bandwidth for control in raw sample
- `r(Nc_raw)` observations on control in raw sample
- `r(bwt_raw)` bandwidth for treated in raw sample
- `r(Nt_raw)` observations on treated in raw sample

Macros
- `r(kernel)` name of kernel

After `teffects` or `stteffects` fits a multivalued treatment, `tebalance density` stores the following in `r()`:

Scalars
- `r(bw#_adj)` bandwidth for treatment level # in weighted or matched-adjusted sample
- `r(N#_adj)` observations on treatment level # in weighted or matched-adjusted sample
- `r(bw#_raw)` bandwidth for treatment level # in raw sample
- `r(N#_raw)` observations on treatment level # in raw sample

Macros
- `r(kernel)` name of kernel

Reference


Also see

- [CAUSAL] `stteffects intro` — Introduction to treatment effects for observational survival-time data
- [CAUSAL] `tebalance` — Check balance after `teffects` or `stteffects` estimation
- [CAUSAL] `teffects aipw` — Augmented inverse-probability weighting
- [CAUSAL] `teffects ipw` — Inverse-probability weighting
- [CAUSAL] `teffects ipwra` — Inverse-probability-weighted regression adjustment
- [CAUSAL] `teffects nnmatch` — Nearest-neighbor matching
- [CAUSAL] `teffects psmatch` — Propensity-score matching
- [CAUSAL] `teoverlap` — Overlap plots
Description

tebalance overid performs a test for covariate balance after estimation by a teffects inverse-probability-weighted (IPW) estimator or an stteffects IPW estimator.

Quick start

Test for covariate balance after a teffects or an stteffects IPW estimator
tebalance overid

Same as above, but test for balance only in base covariates and exclude interaction terms
tebalance overid, bonly

Menu

Statistics > Causal inference/treatment effects > Balance > Overidentification test
Syntax

tebalance overid [, bconly nolog iterate(#)]

collect is allowed; see [U] 11.1.10 Prefix commands.

Options

**bconly** specifies that only the base covariates be included in the test for balance. By default, the powers and interactions specified by factor-variable notation in the **teffects** or **stteffects** model are also included in the test for balance.

**nolog** suppresses the display of the optimization search log.

**iterate(#)** sets the maximum number of iterations to # in the generalized method of moments estimator used to compute the test statistic.

Remarks and examples

When the distribution of a covariate is the same for all treatment levels, the covariate is said to be balanced. **tebalance overid** performs a test to see whether the covariates are balanced after **teffects** or **stteffects**. **tebalance overid** can be executed after **teffects ipw**, **teffects aipw**, **teffects ipwra**, **stteffects ipw**, or **stteffects ipwra**, which use the inverse-probability weights predicted by a treatment model to account for how treatment assignment depends on observed covariates. If the treatment model is well specified, IPW functions of the covariates from the model are balanced.

We recommend that you read [CAUSAL] **tebalance** before proceeding; it provides an introduction to covariate balance and an overview of the implemented methods. See [CAUSAL] **stteffects intro** for survival-time discussion and examples.

➢ Example 1: Base covariates and interactions

This example illustrates the interpretation of the **bconly** option, which excludes powers and interactions when factor variables are included in the propensity-score model.

We frequently use factor variables to include powers of, and interactions between, base covariates in our specification of the propensity-score model. In example 1 under Remarks and examples in [CAUSAL] **tebalance**, we rejected the null hypothesis of balance in a model using only base covariates but not in the richer model that included power and interaction terms. By default, **tebalance overid** tests whether the model balances the base covariates and the power-and-interaction covariates. When option **bconly** is specified, **tebalance overid** tests whether the model balances the base covariates only.

Using an extract from the data used by Cattaneo (2010), we use **teffects ipw** to estimate the effect of a mother’s smoking behavior (mbsmoke) on the birthweight of her child (bweight), controlling for marital status (mmarried), the mother’s age (mage), whether the mother had a prenatal doctor’s visit in the baby’s first trimester (prenatal1), and whether this baby is the mother’s first child (fbaby). In addition to the base covariates, we include the square of **mage**, an interaction between **mage** and **mmarried**, and an interaction between **mage** and **prenatal1** in the model for the propensity score.
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ipw (bweight) (mbsmoke mmarried mage prenatal1 fbaby > c.mage#(c.mage i.mmarried prenatal1)), aequations

Iteration 0:  EE criterion = 9.365e-20
Iteration 1:  EE criterion = 2.612e-26

Treatment-effects estimation  Number of obs = 4,642
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: logit

|          | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|----------|-------------|-----------|------|------|----------------------|
| ATE      |             |           |      |      |                      |
| mbsmoke  |             |           |      |      |                      |
| Smoker   | -239.6875   | 26.43427  | -9.07| 0.000| [-291.4977, -187.8773]|
| vs       |             |           |      |      |                      |
| Non smoker|           |           |      |      |                      |
| P0mean   |             |           |      |      |                      |
| mbsmoke  |             |           |      |      |                      |
| Nonsmoker| 3403.638    | 9.56792   | 355.73| 0.000| [3384.885, 3422.39]   |
| TME1     |             |           |      |      |                      |
| mmarried |             |           |      |      |                      |
| mage     | .8522468    | .462536   | 1.84 | 0.065| [-.0543072, 1.758801] |
| prenatal1| .1742823    | .0651039  | 2.68 | 0.007| [.0466811, .3018836]  |
| fbaby    | .4018114    | .4341762  | 0.93 | 0.355| [-.4491584, 1.252781] |
| c.mage   | -.4824413   | .0868982  | -5.55| 0.000| [-.6527587, -.3121239]|
| c.mage#  |             |           |      |      |                      |
| c.mage   | -.002515    | .0012585  | -2.00| 0.046| [-.0049817, -.0000483]|
| mmarrried|             |           |      |      |                      |
| c.mage   | -.0787984   | .0175508  | -4.49| 0.000| [-.1131973, -.0443996]|
| Married  |             |           |      |      |                      |
| prenatal1|             |           |      |      |                      |
| c.mage   | -.0286228   | .0176391  | -1.62| 0.105| [-.0631948, .0059492] |
| Yes      |             |           |      |      |                      |
| _cons    | -2.928851   | .8409119  | -3.48| 0.000| [-4.577008, -1.280694]|

We specified option aequations to see the parameter estimates for the coefficients in the propensity-score model. There are eight coefficients, five on the base covariates (mmarrried, mage, fbaby, prenatal1, and _cons) and three on the power-and-interaction covariates (c.mage#c.mage, c.mage#1.mmarried, and c.mage#1.prenatal1). Below we test whether the model balances all eight covariates.
. tebalance overid
Iteration 0:  Criterion =  .0602349
Iteration 1:  Criterion =  .06172749 (backed up)
Iteration 2:  Criterion =  .06428588 (backed up)
Iteration 3:  Criterion =  .06489623 (backed up)
Iteration 4:  Criterion =  .06527284 (backed up)
Iteration 5:  Criterion =  .06643426
Iteration 6:  Criterion =  .07134338
Iteration 7:  Criterion =  .07638414
Iteration 8:  Criterion =  .07673211
Iteration 9:  Criterion =  .07681959
Iteration 10: Criterion =  .077044
Iteration 11: Criterion =  .07759547
Iteration 12: Criterion =  .07771973
Iteration 13: Criterion =  .0777271
Iteration 14: Criterion =  .07773395
Iteration 15: Criterion =  .07774839
Iteration 16: Criterion =  .07775314
Iteration 17: Criterion =  .07775324
Overidentification test for covariate balance
H0: Covariates are balanced
    chi2(8)   =  11.8612
    Prob > chi2 =  0.1575

We cannot reject the null hypothesis that the IPW model balanced all eight covariates.

Below we specify option bconly to test whether the IPW model balanced the five base covariates only.

. tebalance overid, bconly
Iteration 0:  Criterion =  .1079977
Iteration 1:  Criterion =  .10800825 (backed up)
Iteration 2:  Criterion =  .10844177 (backed up)
Iteration 3:  Criterion =  .10851228 (backed up)
Iteration 4:  Criterion =  .10860856 (backed up)
Iteration 5:  Criterion =  .10907494
Iteration 6:  Criterion =  .11009792
Iteration 7:  Criterion =  .11164102
Iteration 8:  Criterion =  .11260711
Iteration 9:  Criterion =  .11286467
Iteration 10: Criterion =  .11331528
Iteration 11: Criterion =  .11333981
Iteration 12: Criterion =  .1133561
Iteration 13: Criterion =  .11335696
Iteration 14: Criterion =  .11335696
Overidentification test for covariate balance
H0: Covariates are balanced
    chi2(5)   =  7.82169
    Prob > chi2 =  0.1663

We cannot reject the null hypothesis that the IPW model balanced the five base covariates.

Each test has a justification.
In a model-based approach, the Imai and Ratkovic (2014) test checks whether the propensity score is correctly specified. We include all eight covariates because they must all be balanced, if the propensity-score model is correctly specified.

A conditional-independence approach can be used to justify only including the base covariates in the test. In this approach, the propensity-score model need only balance the base covariates. Powers and interactions of the base covariates are included to get a propensity-score model that balances the base covariates, but balance of these higher-order terms is more than what needs to be checked.

In large samples, both tests should have nominal coverage under the null hypothesis that the propensity-score model is correctly specified. Under the alternative that the propensity-score model is misspecified, including all the covariates should yield a test with higher power.

The test that includes all the covariates is the default.

Stored results
tebalance overid stores the following in r():

Scalars
   r(p) p-value
   r(df) overidentifying constraints, test degrees of freedom
   r(chi2) $\chi^2$ statistic

References


Also see

[Causal] stteffects intro — Introduction to treatment effects for observational survival-time data
[Causal] tebalance — Check balance after tteffects or stteffects estimation
[Causal] tteffects aipw — Augmented inverse-probability weighting
[Causal] tteffects ipw — Inverse-probability weighting
[Causal] tteffects ipwra — Inverse-probability-weighted regression adjustment
[Causal] teoverlap — Overlap plots
tebalance summarize — Covariate-balance summary statistics

Description

tebalance summarize reports diagnostic statistics that are used to check for covariate balance over treatment groups after estimation by a teffects inverse-probability-weighted (IPW) estimator, a teffects matching estimator, or an stteffects IPW estimator.

Quick start

Raw and weighted standardized differences and variance ratios of all covariates from the most recently estimated teffects model or stteffects model

tebalance summarize

Same as above, but report statistics only for covariates x1 and x2

tebalance summarize x1 x2

Baseline means and variances for treated and control groups

tebalance summarize, baseline

Menu

Statistics > Causal inference/treatment effects > Balance > Summaries
Syntax

tebalance summarize [varlist] [ , baseline]

varlist may contain factor variables; see [U] 11.4.3 Factor variables.
collect is allowed; see [U] 11.1.10 Prefix commands.

Option

Main baseline specifies that tebalance summarize report means and variances by treatment level.

Remarks and examples

When the distribution of a covariate is the same for all treatment levels, the covariate is said to be balanced. tebalance summarize reports diagnostic statistics to check for covariate balance after teffects or stteffects. tebalance summarize can be executed after all teffects estimators with the exception of teffects ra and executed after stteffects ipw and stteffects ipwra.

We recommend that you read [CAUSAL] tebalance before proceeding; it provides an introduction to covariate balance and an overview of the implemented methods. See [CAUSAL] stteffects intro for survival-time discussion and examples.

Example 1: Checking covariate balance after psmatch

Using an extract from the data used by Cattaneo (2010), we use teffects psmatch to estimate the effect of a mother’s smoking behavior (mbsmoke) on the birthweight of her child (bweight), controlling for marital status (mmarried), the mother’s age (mage), whether the mother had a prenatal doctor’s visit in the baby’s first trimester (prenatal1), and whether this baby is the mother’s first child (fbaby).

. use https://www.stata-press.com/data/r18/cattaneo2
. teffects psmatch (bweight) (mbsmoke mmarried mage prenatal1 fbaby),
  > generate(matchv)

Treatment-effects estimation
Number of obs = 4,642
Estimator : propensity-score matching
Matches: requested = 1
Outcome model : matching
  min = 1
Treatment model: logit
  max = 139

|   | Coefficient | std. err. | z    | P>|z|   | [95% conf. interval] |
|---|-------------|-----------|------|-------|---------------------|
| ATE mbsmoke (Smoker vs Nonsmoker) | -235.1714 | 27.74409 | -8.48 | 0.000 | -289.5488 -180.794 |
We specified the option `generate(matchv)` to speed up the postestimation commands that compute balance statistics, as discussed in example 3 under Remarks and examples in [CAUSAL] `tebalance`. We do not interpret the estimated effect produced by this preliminary model but rather check the specification.

We begin by looking at the standardized differences and variance ratios for the raw data and the matched sample.

```
. tebalance summarize
Covariate balance summary

Number of obs = 4,642 9,284
Treated obs = 864 4,642
Control obs = 3,778 4,642

```

<table>
<thead>
<tr>
<th>Standardized differences</th>
<th>Variance ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Matched Raw Matched</td>
<td></td>
</tr>
<tr>
<td>mmmarried</td>
<td>-.5953009 .0014107 1.335944 .9987659</td>
</tr>
<tr>
<td>mage</td>
<td>-.300179 -.0120277 .8818025 .9952916</td>
</tr>
<tr>
<td>prenatal1</td>
<td>-.3242695 .0333609 1.496155 .9491524</td>
</tr>
<tr>
<td>fbaby</td>
<td>-.1663271 -.0117326 .9430944 .9969095</td>
</tr>
</tbody>
</table>

The matched sample results indicate that matching on the estimated propensity score balanced the covariates. The standardized differences are all close to zero, and the variance ratios are all close to one. This inference is informal because we do not have standard errors for these statistics.

We may also wish to see the baseline summary statistics.

```
. tebalance summarize, baseline
Covariate balance summary

Number of obs = 4,642 9,284
Treated obs = 864 4,642
Control obs = 3,778 4,642

<table>
<thead>
<tr>
<th>Means</th>
<th>Variances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Raw Control Treated</td>
<td>Raw Control Treated</td>
</tr>
<tr>
<td>mmmarried</td>
<td>.7514558 .4733796</td>
</tr>
<tr>
<td>mage</td>
<td>26.81048 25.16667</td>
</tr>
<tr>
<td>prenatal1</td>
<td>.8268925 .6898148</td>
</tr>
<tr>
<td>fbaby</td>
<td>.4531498 .3715278</td>
</tr>
</tbody>
</table>

While we rely on the standardized differences for conclusions about balance in the unmatched sample from this output, the baseline means and variances give us some idea of the scale of the differences.
Example 2: Multivalued treatments

In the multivalued-treatment case, `tebalance summarize` produces output grouped by treatment level. In the Cattaneo (2010) extract, the variable `msmoke` is an ordered categorical variable specifying the number of cigarettes smoked. We begin by tabulating `msmoke`.

```
.tabulate msmoke

<table>
<thead>
<tr>
<th>Cigarettes smoked during pregnancy</th>
<th>Freq.</th>
<th>Percent</th>
<th>Cum.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 daily</td>
<td>3,778</td>
<td>81.39</td>
<td>81.39</td>
</tr>
<tr>
<td>1-5 daily</td>
<td>200</td>
<td>4.31</td>
<td>85.70</td>
</tr>
<tr>
<td>6-10 daily</td>
<td>337</td>
<td>7.26</td>
<td>92.96</td>
</tr>
<tr>
<td>11+ daily</td>
<td>327</td>
<td>7.04</td>
<td>100.00</td>
</tr>
<tr>
<td>Total</td>
<td>4,642</td>
<td>100.00</td>
<td></td>
</tr>
</tbody>
</table>
```

All the treatment groups have significantly smaller numbers of observations than the control group of not smoking. Still, each group has at least 200 observations. We continue by quietly fitting a candidate IPW model and reporting the baseline summaries.

```
.quietly teffects ipw (bweight) (msmoke mmarried mage prenatal1 fbaby)
.tebalance summarize, baseline
```

Covariate balance summary

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Observations</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>0 daily</td>
<td>3,778</td>
<td>1,164</td>
<td></td>
</tr>
<tr>
<td>1-5 daily</td>
<td>200</td>
<td>1,164</td>
<td>1,164</td>
</tr>
<tr>
<td>6-10 daily</td>
<td>337</td>
<td>1,157</td>
<td>1,157</td>
</tr>
<tr>
<td>11+ daily</td>
<td>327</td>
<td>1,154</td>
<td>1,154</td>
</tr>
<tr>
<td>Total</td>
<td>4,642</td>
<td>4,642</td>
<td>4,642</td>
</tr>
</tbody>
</table>

```
<table>
<thead>
<tr>
<th>Treatment</th>
<th>Means</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Control</td>
<td>Treated</td>
<td>Control</td>
<td>Treated</td>
</tr>
<tr>
<td>----------------</td>
<td>----------</td>
<td>---------</td>
<td>----------</td>
<td>---------</td>
</tr>
<tr>
<td>1-5 daily</td>
<td>mmarried</td>
<td>.7514558</td>
<td>.455</td>
<td>.1868194</td>
</tr>
<tr>
<td></td>
<td>mage</td>
<td>26.81048</td>
<td>24.64</td>
<td>31.87141</td>
</tr>
<tr>
<td></td>
<td>prenatal1</td>
<td>.8268925</td>
<td>.695</td>
<td>.1431792</td>
</tr>
<tr>
<td></td>
<td>fbaby</td>
<td>.4531498</td>
<td>.48</td>
<td>.2478707</td>
</tr>
<tr>
<td>6-10 daily</td>
<td>mmarried</td>
<td>.7514558</td>
<td>.4480712</td>
<td>.1868194</td>
</tr>
<tr>
<td></td>
<td>mage</td>
<td>26.81048</td>
<td>25.06231</td>
<td>31.87141</td>
</tr>
<tr>
<td></td>
<td>prenatal1</td>
<td>.8268925</td>
<td>.6795252</td>
<td>.1431792</td>
</tr>
<tr>
<td></td>
<td>fbaby</td>
<td>.4531498</td>
<td>.3827893</td>
<td>.2478707</td>
</tr>
<tr>
<td>11+ daily</td>
<td>mmarried</td>
<td>.7514558</td>
<td>.5107034</td>
<td>.1868194</td>
</tr>
<tr>
<td></td>
<td>prenatal1</td>
<td>.8268925</td>
<td>.6972477</td>
<td>.1431792</td>
</tr>
<tr>
<td></td>
<td>fbaby</td>
<td>.4531498</td>
<td>.293578</td>
<td>.2478707</td>
</tr>
</tbody>
</table>
```

The results for the control level of 0 daily are repeated for the treatment group. These results give a sense of the scale of imbalance in the raw data. Now we compute the balance statistics.
. tebalance summarize

Covariate balance summary

<table>
<thead>
<tr>
<th>Treatment</th>
<th>Observations</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw</td>
<td>Weighted</td>
<td></td>
</tr>
<tr>
<td>0 daily</td>
<td>3,778</td>
<td>1,164.8</td>
<td></td>
</tr>
<tr>
<td>1-5 daily</td>
<td>200</td>
<td>1,164.4</td>
<td></td>
</tr>
<tr>
<td>6-10 daily</td>
<td>337</td>
<td>1,157.9</td>
<td></td>
</tr>
<tr>
<td>11+ daily</td>
<td>327</td>
<td>1,154.9</td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td>4,642</td>
<td>4,642.0</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>Standardized differences</th>
<th>Variance ratio</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Raw</td>
<td>Weighted</td>
<td>Raw</td>
<td>Weighted</td>
</tr>
<tr>
<td>1-5 daily</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mmarried</td>
<td>-.634909</td>
<td>-.0016208</td>
<td>1.334021</td>
<td>1.001406</td>
</tr>
<tr>
<td>mage</td>
<td>-.3857482</td>
<td>-.0219656</td>
<td>.9867038</td>
<td>.9905584</td>
</tr>
<tr>
<td>prenatal1</td>
<td>-.312519</td>
<td>-.0012611</td>
<td>1.487927</td>
<td>1.001898</td>
</tr>
<tr>
<td>fbaby</td>
<td>.053769</td>
<td>.0422102</td>
<td>1.012037</td>
<td>1.008631</td>
</tr>
<tr>
<td>6-10 daily</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mmarried</td>
<td>-.6506304</td>
<td>-.0108454</td>
<td>1.327696</td>
<td>1.009331</td>
</tr>
<tr>
<td>mage</td>
<td>-.3220222</td>
<td>-.0836571</td>
<td>.8493666</td>
<td>.7984901</td>
</tr>
<tr>
<td>prenatal1</td>
<td>-.3465797</td>
<td>-.0100232</td>
<td>1.525493</td>
<td>1.015051</td>
</tr>
<tr>
<td>fbaby</td>
<td>.1429048</td>
<td>.0268118</td>
<td>.9560018</td>
<td>1.005899</td>
</tr>
<tr>
<td>11+ daily</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>mmarried</td>
<td>-.5147672</td>
<td>-.0212969</td>
<td>1.34168</td>
<td>1.018136</td>
</tr>
<tr>
<td>mage</td>
<td>-.2239116</td>
<td>-.0636951</td>
<td>.8451058</td>
<td>.848934</td>
</tr>
<tr>
<td>prenatal1</td>
<td>-.3077549</td>
<td>-.0380744</td>
<td>1.478852</td>
<td>1.056645</td>
</tr>
<tr>
<td>fbaby</td>
<td>.3342243</td>
<td>.0155427</td>
<td>.8392526</td>
<td>1.003598</td>
</tr>
</tbody>
</table>

These results indicate that the IPW estimator probably did not fully balance the covariates (the variance ratios for \textit{mage} at the daily levels of 6–10 cigarettes and 11-plus cigarettes are not close to 1). At this point, we would use a richer model and see whether it balanced the covariates.

Note that we cannot use \texttt{tebalance overid}, because it has not been implemented for multivalued treatments.

**Stored results**

\texttt{tebalance summarize} stores the following in \texttt{r()}:  

- Matrices:  
  - \texttt{r(size)}: number of observations in the raw and matched or weighted samples  
  - \texttt{r(table)}: table of covariate statistics  

**Reference**

Also see

[CAUSAL] stteffects intro — Introduction to treatment effects for observational survival-time data

[CAUSAL] stteffects ipw — Survival-time inverse-probability weighting

[CAUSAL] stteffects ipwra — Survival-time inverse-probability-weighted regression adjustment

[CAUSAL] tebalance — Check balance after teffects or stteffects estimation

[CAUSAL] teffects aipw — Augmented inverse-probability weighting

[CAUSAL] teffects ipw — Inverse-probability weighting

[CAUSAL] teffects ipwra — Inverse-probability-weighted regression adjustment

[CAUSAL] teffects nnmatch — Nearest-neighbor matching

[CAUSAL] teffects psmatch — Propensity-score matching

[CAUSAL] teoverlap — Overlap plots
**teffects** — Treatment-effects estimation for observational data

### Description

**teffects** estimates potential-outcome means (POMs), average treatment effects (ATEs), and average treatment effects on the treated (ATETs) using observational data. Regression-adjustment, inverse-probability-weighting, and matching estimators are provided, as are doubly robust methods that combine regression adjustment and inverse-probability weighting.

The outcomes can be continuous, binary, count, fractional, or nonnegative. The treatment model can be binary, or it can be multinomial, allowing for multivalued treatments.

For a brief description and example of each estimator, see *Remarks and examples* in [CAUSAL] `teffects intro`.

### Syntax

```
tffects subcommand ... [, options]
```

<table>
<thead>
<tr>
<th>subcommand</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>aipw</td>
<td>augmented inverse-probability weighting</td>
</tr>
<tr>
<td>ipw</td>
<td>inverse-probability weighting</td>
</tr>
<tr>
<td>ipwra</td>
<td>inverse-probability-weighted regression adjustment</td>
</tr>
<tr>
<td>nnmatch</td>
<td>nearest-neighbor matching</td>
</tr>
<tr>
<td>psmatch</td>
<td>propensity-score matching</td>
</tr>
<tr>
<td>ra</td>
<td>regression adjustment</td>
</tr>
</tbody>
</table>

### Also see

[CAUSAL] `teffects intro` — Introduction to treatment effects for observational data

[CAUSAL] `teffects intro advanced` — Advanced introduction to treatment effects for observational data

[CAUSAL] `teffects multivalued` — Multivalued treatment effects
Description

This entry provides a nontechnical introduction to treatment-effects estimators and the `teffects` command in Stata. Advanced users may want to instead read [CAUSAL] `teffects intro advanced` or skip to the individual commands’ entries.

The `teffects` command estimates average treatment effects (ATEs), average treatment effects among treated subjects (ATETs), and potential-outcome means (POMs) using observational data.

Treatment effects can be estimated using regression adjustment (RA), inverse-probability weights (IPW), and “doubly robust” methods, including inverse-probability-weighted regression adjustment (IPWRA) and augmented inverse-probability weights (AIPW), and via matching on the propensity score or nearest neighbors.

The outcome can be continuous, binary, count, fractional, or nonnegative. Treatments can be binary or multivalued.

Remarks and examples

This entry presents a nontechnical overview of treatment-effects estimators for those who are new to the subject of treatment-effects estimation or are at least new to Stata’s facilities for estimating treatment effects. More advanced users may want to instead read [CAUSAL] `teffects intro advanced` or skip to the individual commands’ entries.

Remarks are presented under the following headings:

- Introduction
- Defining treatment effects
- Estimating treatment effects
  - Regression adjustment
  - Inverse-probability weighting
  - Doubly robust combinations of RA and IPW
- Matching
- Caveats and assumptions
- A quick tour of the estimators
  - RA
  - IPW
  - IPWRA
  - AIPW
  - Nearest-neighbor matching
  - Propensity-score matching
- Video examples
Introduction

Suppose we have observed a sample of subjects, some of whom received a treatment and the rest of whom did not. As the name suggests, in most applications, the “subjects” are indeed people. A “treatment” could indeed be a medical treatment such as a new drug regimen or surgical procedure. In social science applications, a treatment could be participation in a job-training program or inclusion in a classroom or school in which a new pedagogical method is being used. However, not all applications use individuals as the subjects. For example, a policy analyst might be interested in examining the impact of an experimental program in which a national agency held a lottery to award only some local governments the resources needed to implement the program. Here the subjects are the local governments, and treatment refers to whether a local government received the resources needed to implement the program.

We would like to know if a treatment has an effect on an outcome $Y$. The outcome could be the cholesterol level of a patient taking either an existing statin or a new experimental drug, or the outcome could be the wage offered to a person who either did or did not participate in a job-training program. In an ideal world, we would observe $Y$ when a subject is treated (which we denote as $Y_1$), and we would observe $Y$ when the same subject is not treated (which we denote as $Y_0$). We would be careful to make both observations under identical conditions so that the only difference is the presence or absence of the treatment. We could then average the difference between $Y_1$ and $Y_0$ across all the subjects in our dataset to obtain a measure of the average impact of the treatment.

Unfortunately, this ideal experiment is almost never available in observational data because it is not possible to observe a specific subject having received the treatment and having not received the treatment. When the outcome is the birthweight of a specific baby and the treatment is the mother smoking while pregnant, it is impossible to observe the baby’s birthweight under both treatments of the mother smoking and the mother not smoking.

A classic solution to this problem is to randomize the treatment. High costs or ethical issues rule out this solution in many observational datasets. For example, we could not ask a random selection of pregnant women to smoke.

The defining characteristic of observational data is that treatment status is not randomized. Moreover, that implies that the outcome and treatment are not necessarily independent. The goal of the estimators implemented by `teffects` is to utilize covariates to make treatment and outcome independent once we condition on those covariates.

The treatment-effect estimators implemented by `teffects` allow us to estimate the efficacy of treatments using observational data. The rest of this entry discusses these treatment-effect estimators at an introductory level. For a more technical introduction, see [CAUSAL] `teffects intro advanced`.

Defining treatment effects

We introduce treatment effects more formally by using the potential-outcomes framework, which is also known as the counterfactual framework. What is a potential outcome? Consider a subject that did not receive treatment so that we observe $Y_0$. What would $Y_1$ be for that same subject if it were exposed to treatment? We call $Y_1$ the potential outcome or counterfactual for that subject. For a subject that did receive treatment, we observe $Y_1$, so $Y_0$ would be the counterfactual outcome for that subject. We can view this as a missing-data problem, and treatment-effect methods can account for that problem.

Treatment-effect estimators allow us to estimate three parameters. The potential-outcome means (POMs) are the means of $Y_1$ and $Y_0$ in the population. The average treatment effect (ATE) is the mean of the difference $(Y_1 - Y_0)$. Finally, the average treatment effect on the treated (ATET) is the mean of the difference $(Y_1 - Y_0)$ among the subjects that actually receive the treatment.
To develop our intuition, suppose we have observed a sample of patients, some of whom received a medication to reduce their blood pressure. Figure 1 plots each of our patient’s systolic blood pressures as a function of weight. We use the color red to indicate patients who did not receive the drug and blue to indicate patients who did receive the drug.

A remarkable feature of our data is that the average blood pressure of patients not taking the drug is 160, and the average blood pressure of patients taking the drug is also 160. Can we therefore conclude that taking the drug has no impact on blood pressure? The answer is no.

Because this is observational data, we could not randomly assign who would receive the drug and who would not. As a result, treatment status could be related to covariates that also affect blood pressure. Heavier patients were more likely to be prescribed the medication, and blood pressure is correlated with weight. The difference in sample means does not estimate the true average treatment effect, because blood pressure depends on weight and weight is correlated with the treatment.

Suppose that we did in fact observe both potential outcomes for all patients. In figure 2, we continue to use solid dots for our observed data points, and we introduce hollow dots to represent the counterfactual outcomes. That is, the red hollow dots represent the blood pressures we would measure if only our treated patients had not taken the drug, and the blue hollow dots represent the blood pressures we would measure if only our untreated patients had taken the drug. The red and blue dashed lines represent the untreated and treated POMs, respectively. That is, the red line represents the mean of all the red dots, and the blue line represents the mean of all the blue dots.
Estimating treatment effects

We cannot estimate the ATE by simply taking the difference between the sample means for the treated and untreated subjects, because there are covariates that are related to the potential outcomes and the treatment. The estimators implemented by teffects require us to specify enough of these covariates so that after we condition on these covariates, any remaining influences on the treatment are not related to the potential outcomes. teffects implements several different estimators to accomplish this, including regression adjustment (RA), inverse-probability weighting (IPW), “doubly robust” methods that combine elements of RA and IPW, and matching methods. Here we introduce the methods by using intuition and simple examples.

See [CAUSAL] teffects intro advanced for a more technical introduction, and see the individual commands’ entries for estimator-specific details.
Regression adjustment

The RA method extends the idea of using sample means to estimate treatment effects by using a regression model to predict potential outcomes adjusted for covariates. In the examples here, we use linear regression, but the `teffects ra` command provides you with the flexibility to use logistic, probit, and heteroskedastic probit regression models for binary outcomes as well as Poisson regression for nonnegative outcomes; see [CAUSAL] `teffects ra` for more information.

`bweightex.dta` is a hypothetical dataset based on Cattaneo (2010) that we have created to illustrate treatment-effects estimators using graphs. The subjects in this dataset are women who were pregnant, some of whom smoked during the pregnancy. The outcome variable is the birthweight of the baby, and we want to know whether smoking during pregnancy affects the birthweight. The dataset also contains other demographic variables that we will use later.

Figure 3 illustrates the relationship between birthweight and smoking status as a function of the mother’s age:

We see that smokers tend to be older than nonsmokers and that birthweight depends on smoking. Therefore, the difference between the sample means of birthweights of babies born to smokers and nonsmokers will not estimate the true average treatment effect.

We also still have the same problem as in the previous section: we do not observe the counterfactual birthweights of babies. Suppose, however, that we did. In figure 4, we use solid points to represent observed birthweights and the colors red to represent nonsmokers and blue to represent smokers. The hollow points represent the counterfactual birthweights. The hollow blue points represent the birthweights of babies that we would observe if only our young nonsmoking mothers had instead smoked during their pregnancies. Similarly, the hollow red points represent the birthweights of babies that we would observe if only our older smoking mothers had instead not smoked during their pregnancies.
Figure 4 suggests a way to estimate the potential outcomes for each mother:

We could fit a linear regression of birthweight on mother’s age by using the observed birthweights for nonsmokers, and we could do likewise for smokers. The following graph includes these two regression lines:

Figure 5 illustrates the principle behind the RA method. We use the red regression line to predict each baby’s birthweight assuming the mother did not smoke, and we use the blue regression line to predict each baby’s birthweight assuming the mother did smoke. The treatment effect of smoking for a mother of a particular age is the vertical difference between the red and blue regression lines.

The three parameters we mentioned in the introduction are now easy to estimate. For each mother, we obtain two values, say, $bw_0$ and $bw_1$, representing our predictions of her baby’s birthweight assuming the mother did not or did smoke, respectively. The means of these variables represent the untreated and treated POMs. The ATE is the sample mean of the difference $(bw_1 - bw_0)$, and the ATET is the sample mean of that difference computed using only the mothers who in fact did smoke during pregnancy.
Adding the circles highlights the fact that the average age is higher for smokers than for nonsmokers. Even though the blue and red lines have different slopes, if the average age was the same for smokers and nonsmokers, a difference in the sample means of birthweights could still estimate the true ATE.

Figure 5 lets us address one more issue. Users who are versed in regression analysis may be inclined to estimate the effect of smoking using a regression model for birthweight as a function of smoking and the mother’s age. We clearly see in figure 5 that regression lines for smokers and nonsmokers have different slopes—the effect of age on birthweight is not the same for smokers and nonsmokers. In regression analysis, we would therefore include an interaction term between smoking and age. The RA method fits separate regression lines for smokers and nonsmokers, which also handles these differential effects of age on smoking.

**Inverse-probability weighting**

As we remarked in our discussion of the RA method, we cannot simply use the sample mean birthweights of babies born to smokers and nonsmokers to estimate the effect of smoking. If we did that, we would conflate the negative effect of smoking with the positive effect of age and the positive relationship between age and smoking. IPW is a treatment-effects estimator that uses weighted means rather than simple unweighted means to disentangle the effects of treatment and other confounders like age.

The concept underlying IPW can be gleaned from figure 2, where, as you will recall, the hollow points represent counterfactual outcomes. As we demonstrated in *Defining treatment effects*, we could estimate the average treatment effect if we knew the means of all the nonsmoking outcomes and the means of all the smoking outcomes. In the context of figure 4, we need the mean of all the red points, both solid and hollow, and the mean of all the blue.

If we could observe all of these points, then the ATE would be the difference between those two means. However, the outcomes illustrated by the hollow circles are unobserved. IPW estimators view the hollow circles as missing data and use weights to correct the estimates of the treated and untreated sample means for the missing data. If we calculate the mean nonsmoking birthweight using just the solid red points, that mean is biased downward because we are ignoring the hollow red points, which correspond to higher birthweights.

In IPW, we apply more weight to the solid red points corresponding to older mothers and less weight to those corresponding to younger mothers. Using this weighting scheme will pull up the estimated mean birthweight of babies born to nonsmoking mothers to estimate the true mean of all nonsmoking outcomes. The method for obtaining the mean smoking birthweight is virtually the same: we need to apply more weight to the younger smoking mothers than to the older smoking mothers to better approximate the true mean of all smoking outcomes.

Where do these weights for the weighted means come from? As the name implies, IPW uses the inverse (reciprocal) of the probability of being in the observed treatment group. These probabilities are obtained by modeling the observed treatment as a function of subject characteristics that determine treatment group. In our exposition of the RA method, we focused solely on the mother’s age and smoking status as determinants of each baby’s birthweight. To make the results comparable, we will use the same model in this example.
We first fit a logistic model of the mother’s smoking status, `mbsmoke`, as a function of the mother’s age (`mage`):

```stata
use https://www.stata-press.com/data/r18/bweightex
(Hypothetical birthweight data)
.logistic mbsmoke mage
Logistic regression
Number of obs = 60
LR chi2(1) = 30.45
Prob > chi2 = 0.0000
Log likelihood = -26.362201 Pseudo R2 = 0.3661

mbsmoke
| Odds ratio Std. err.  z  P>|z|      [95% conf. interval]  
|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
mage            | 1.631606 .21316 3.75 0.000 1.263022 2.107754 |
_cons           | 7.76e-06 .000243 -3.76 0.000 1.69e-08 .0035718 |
```

Note: `_cons` estimates baseline odds.

Next, we compute the inverse-probability weights, which we will store in a variable called `ps`. In the IPW method, for subjects who did receive treatment, the weight is equal to the reciprocal of the predicted probability of treatment. For subjects who did not receive treatment, the weight is equal to the reciprocal of the predicted probability of not receiving treatment; the probability of not receiving treatment is just one minus the probability of receiving treatment:

```stata
predict ps
(option `pr' assumed; Pr(mbsmoke))
replace ps = 1/ps if mbsmoke==1
(30 real changes made)
replace ps = 1/(1-ps) if mbsmoke==0
(30 real changes made)
```

Figure 6 replicates figure 3 with one twist. Rather than making all the points the same size, we have made the size of the points proportional to the IPW variable `ps`. Notice that the largest blue points correspond to the youngest smoking mothers in our sample, so they will receive the most weight when we compute the weighted mean birthweight of babies born to smoking mothers, just as we explained we wanted to do. Similarly, the red points corresponding to older nonsmoking mothers are larger, representing larger weights.
There is a caveat to the IPW estimator. When we fit our logistic or probit model to obtain the predicted probabilities, it is possible that some of the predictions will be close to zero. Because the IPW is the reciprocal of that probability, the weight becomes arbitrarily large as the probability goes to zero. In those cases, the IPW can become unstable. We can improve the estimated IPW by developing a more accurate treatment model. For example, in our dataset, we have other variables such as marital status and the education level of the baby’s father that may also help predict whether the mother smoked during pregnancy. We excluded these variables for simplicity, but in a real analysis, we would want to use all relevant data.

This phenomenon of unstable IPWs is related to the concept of overlap, which means that every subject must have a strictly positive probability of obtaining treatment. We remarked that in our sample, we had few young mothers who smoked. As should be clear from figure 6, the overlap assumption is likely to be violated—young mothers do not appear to have a positive probability of being smokers. We would want to check this assumption before proceeding with an IPW analysis. See [CAUSAL] teoverlap and [CAUSAL] teffects intro advanced for more information about overlap.

Another limitation of the IPW estimator is that we are using weighted means to estimate the POMs and ATE. Thus, unlike the RA estimator, we cannot obtain subject-level predictions of the treatment effects or potential outcomes, because we do not have the two regression lines that we can use to predict outcomes for each subject.

**Doubly robust combinations of RA and IPW**

You may have noticed a clear distinction between the RA and IPW estimators. In the case of RA, we built linear regression models to predict the outcomes (birthweights) of each subject but said nothing about how treatment (smoking) arises. In the case of IPW, we built a logistic regression model to predict treatment status but did not build a formal model of the outcome. Doubly robust estimators combine the outcome modeling strategy of RA and the treatment modeling strategy of IPW. These estimators have a remarkable property: although they require us to build two models, we only need to specify one of the two models correctly. If we misspecify the treatment model but correctly specify the outcome model, we still obtain correct estimates of the treatment effect. If we correctly specify the treatment model but misspecify the outcome model, we again will obtain correct estimates of the treatment effect.

Stata’s teffects command implements two doubly robust estimators, the augmented inverse-probability-weighted (AIPW) estimator and the inverse-probability-weighted regression-adjustment (IPWRA) estimator. These estimators combine elements of RA and IPW to be more robust to misspecification.

The AIPW estimator is an IPW estimator that includes an augmentation term that corrects the estimator when the treatment model is misspecified. When the treatment model is correctly specified, the augmentation term vanishes as the sample size becomes large. Like the IPW, the AIPW does not perform well when the predicted treatment probabilities are too close to zero or one.

The IPWRA estimator is an RA estimator that uses estimated inverse-probability weights to correct the estimator when the regression function is misspecified. When the regression function is correctly specified, the weights do not affect the consistency of the estimator.
Matching

Matching estimators are based on the idea of comparing the outcomes of subjects that are as similar as possible with the sole exception of their treatment status. In our birthweight and smoking example, we could select a mother who smokes and select a mother of the same age who does not smoke and compare the birthweights of their infants. The data of each mother serve as the potential outcome for the other mother.

For a single covariate such as age, identifying a pair of comparable mothers is not difficult. If we have a second covariate that is categorical, such as race, we might still be able to identify pairs of mothers who are the same age and of the same race assuming our dataset is large enough. However, once we consider covariates that are measured on continuous scales or allow for more than a few discrete ones, then finding identical matches is a challenge. The solution is to use what is called a similarity measure, which is a statistic that measures how “close” two observations are. teffects offers two methods to find comparable observations based on similarity measures: nearest-neighbor matching and propensity-score matching.

Nearest-neighbor matching (NNM) is accomplished by calculating the “distance” between pairs of observations with regard to a set of covariates and then “matching” each subject to comparable observations that are closest to it. For example, suppose we have a variable that records each subject’s annual income to the penny. Say one subject who received treatment had an income of $69,234.21. The likelihood that our dataset has an untreated subject who also earned $69,234.21 is nil. However, we can determine the difference between each untreated subject’s income and our treated subject’s income, then match our treated subject with the untreated subjects whose income differences are smallest. Measuring the distance between subjects when we have multiple covariates is no challenge. By default, teffects uses what is known as the Mahalanobis distance, which is really nothing more than the Pythagorean theorem adapted to handle the fact that covariates may be correlated and measured on different scales.

NNM does not use a formal model for either the outcome or the treatment status, but this flexibility comes at a price. When matching on more than one continuous covariate, the NNM estimator must be augmented with a bias-correction term. teffects.nnmatch uses a linear function of the covariates specified in the biasadj() option to remove the large-sample bias.

Propensity-score matching (PSM) is an alternative to NNM. PSM matches on the estimated predicted probabilities of treatment, known as the propensity scores. PSM does not require bias correction, because it uses a model for the treatment. If the treatment model is reasonably well specified, PSM will perform at least as well as NNM; see [CAUSAL] teffects intro advanced.

Caveats and assumptions

To use the estimators implemented in teffects, we must make several assumptions about the process that generated our data. Different estimators and statistics may require slightly more or slightly less restrictive assumptions and may exhibit varying degrees of robustness to departures from these assumptions, but in general, all the estimators require some form of the following three assumptions.

The independent and identically distributed (i.i.d.) sampling assumption ensures that the outcome and treatment status of each individual are unrelated to the outcome and treatment status of all the other individuals in the population. Correlated data arising from hierarchical or longitudinal study designs do not meet this assumption.

The conditional-independence (CI) assumption means once we control for all observable variables, the potential outcomes are independent of treatment assignment. The easiest way to understand the CI assumption is to understand when it is violated. In our birthweight example, suppose mothers
who did not smoke were more health conscious and consumed better prenatal diets than those who did smoke. Unless we explicitly controlled for health awareness or diet, our model would violate the CI assumption: the mother’s decision to smoke or not smoke would not be independent of the baby’s birthweight. If we did not control for health awareness, we would overstate the negative impact of smoking on birthweight. Babies born to mothers who smoke weigh less than babies born to nonsmoking mothers not just because of the effects of cigarettes but also because of poorer prenatal diets.

In a study examining the effect of a job-training program, the CI assumption requires that there not be any unobserved factors such as ambition or work ethic that influence both whether a person enrolls in the program and the wage received upon completion. To use the methods implemented by the \texttt{teffects} estimators, we must have variables in our dataset that allow us to control for those types of factors.

We mentioned the third assumption, overlap, in our discussions of IPW. More formally, the overlap assumption states that each individual have a positive probability of receiving treatment. In our birthweight example, we noted that there were no observations on young smokers and older nonsmokers. Perhaps we just have an unlucky sample, but to accurately assess the impact of treatment using these methods, we must have overlap to accurately estimate the counterfactual birthweights. In the context of matching estimators, overlap essentially means that we can actually match treated subjects with similar nontreated subjects.

A quick tour of the estimators

The \texttt{teffects} command implements six estimators of treatment effects. We introduce each one by showing the basic syntax one would use to apply them to our birthweight example. See each command’s entry for more information.

RA

\texttt{teffects ra} implements the RA estimator. We estimate the effect of a mother’s smoking behavior (\texttt{mbsmoke}) on the birthweight of her child (\texttt{bweight}), controlling for marital status (\texttt{mmarried}), the mother’s age (\texttt{mage}), whether the mother had a prenatal doctor’s visit in the baby’s first trimester (\texttt{prenatal1}), and whether this baby is the mother’s first child (\texttt{fbaby}). We use linear regression (the default) to model \texttt{bweight}:

\begin{verbatim}
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ra (bweight mmarried mage prenatal1 fbaby) (mbsmoke)
\end{verbatim}

IPW

\texttt{teffects ipw} implements the IPW estimator. Here we estimate the effect of smoking by using a probit model to predict the mother’s smoking behavior as a function of marital status, the mother’s age, and indicators for first-trimester doctor’s visits and firstborn status:

\begin{verbatim}
. teffects ipw (bweight) (mbsmoke mmarried mage prenatal1 fbaby, probit)
\end{verbatim}
**IPWRA**

`teffects ipwra` implements the IPWRA estimator. We model the outcome, birthweight, as a linear function of marital status, the mother’s age, and indicators for first-trimester doctor’s visits and firstborn status. We use a logistic model (the default) to predict the mother’s smoking behavior, using the same covariates as explanatory variables:

```
. teffects ipwra (bweight mmarried mage prenatal1 fbaby) ///
   (mbsmoke mmarried mage prenatal1 fbaby)
```

**AIPW**

`teffects aipw` implements the AIPW estimator. Here we use the same outcome- and treatment-model specifications as we did with the IPWRA estimator:

```
. teffects aipw (bweight mmarried mage prenatal1 fbaby) ///
   (mbsmoke mmarried mage prenatal1 fbaby)
```

**Nearest-neighbor matching**

`teffects nnmatch` implements the NNM estimator. In this example, we match treated and untreated subjects based on marital status, the mother’s age, the father’s age, and indicators for first-trimester doctor’s visits and firstborn status. We use the Mahalanobis distance based on the mother’s and father’s ages to find matches. We use exact matching on the other three variables to enforce the requirement that treated subjects are matched with untreated subjects who have the same marital status and indicators for first-trimester doctor’s visits and firstborn statuses. Because we are matching on two continuous covariates, we request that `teffects nnmatch` include a bias-correction term based on those two covariates:

```
. teffects nnmatch (bweight mage fage) (mbsmoke), ///
   ematch(prenatal1 mmarried fbaby) biasadj(mage fage)
```

**Propensity-score matching**

`teffects psmatch` implements the PSM estimator. Here we model the propensity score using a probit model, incorporating marital status, the mother’s age, and indicators for first-trimester doctor’s visits and firstborn status as covariates:

```
. teffects psmatch (bweight) (mbsmoke mmarried mage prenatal1 fbaby, probit)
```

**Video examples**

- Introduction to treatment effects in Stata, part 1
- Introduction to treatment effects in Stata, part 2


Also see

[CAUSAL] **teffects** — Treatment-effects estimation for observational data

[CAUSAL] **teffects intro advanced** — Advanced introduction to treatment effects for observational data

[CAUSAL] **teffects multivalued** — Multivalued treatment effects
Description

This entry provides a technical overview of treatment-effects estimators and their implementation in Stata. Those who are new to treatment-effects estimation may want to instead see [CAUSAL] `teffects intro`.

The `teffects` command estimates average treatment effects (ATEs), average treatment effects among treated subjects (ATETs), and potential-outcome means (POMs) using observational data.

Treatment effects can be estimated using regression adjustment (RA), inverse-probability weights (IPW), and “doubly robust” methods, including inverse-probability-weighted regression adjustment (IPWRA) and augmented inverse-probability weights (AIPW), and via matching on the propensity score or nearest neighbors.

The outcome can be continuous, binary, count, fractional, or nonnegative. Treatments can be binary or multivalued.

Remarks and examples

This entry presents a technical overview of treatment-effects estimators and their implementation in Stata. Users who are new to treatment-effects estimators for observational data should instead read [CAUSAL] `teffects intro`.

Remarks are presented under the following headings:

- Introduction
- Defining treatment effects
- The potential-outcome model
- Assumptions needed for estimation
  - The CI assumption
  - The overlap assumption
  - The i.i.d. assumption
- Comparing the ATE and ATET
- Overview of treatment-effect estimators
- RA estimators
- IPW estimators
- AIPW estimators
- IPWRA estimators
- Nearest-neighbor matching estimators
- Propensity-score matching estimators
- Choosing among estimators
- Model choice
Introduction

The `teffects` commands estimate treatment effects from observed data. A treatment effect is the change in an outcome caused by a subject, often an individual, getting one treatment instead of another. We cannot estimate individual-level treatment effects, because we only observe each individual getting one or another treatment.

Potential-outcome models provide a solution to this missing-data problem and allow us to estimate the distribution of individual-level treatment effects. A potential-outcome model specifies the potential outcomes that each individual would obtain under each treatment level, the treatment assignment process, and the dependence of the potential outcomes on the treatment assignment process.

When the potential outcomes do not depend on the treatment levels, after conditioning on covariates, regression estimators, inverse-probability-weighted estimators, and matching estimators are commonly used.

What we call the potential-outcome model is also known as the Rubin causal model and the counterfactual model. See Rubin (1974); Holland (1986); Robins (1986); Heckman (1997); Heckman and Navarro-Lozano (2004); Imbens (2004); Cameron and Trivedi (2005, chap. 2.7); Imbens and Wooldridge (2009); and Wooldridge (2010, chap. 21) for more detailed discussions.

Cameron and Trivedi (2022) provide an introduction to treatment-effects estimation and to commands within Stata. For a discussion related to health econometrics, see Deb, Norton, and Manning (2017).

Defining treatment effects

Three parameters are often used to measure treatment effects: the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs). In this section, we define each of these terms and introduce the notation and parameters used in the rest of our discussion.

In the binary-treatment case, the two potential outcomes for each individual are $y_{0i}$ and $y_{1i}$; $y_{0i}$ is the outcome that would be obtained if $i$ does not get the treatment, and $y_{1i}$ is the outcome that would be obtained if $i$ gets the treatment. $y_{0i}$ and $y_{1i}$ are realizations of the random variables $y_0$ and $y_1$. Throughout this entry, $i$ subscripts denote realizations of the corresponding unsubscripted random variables. We do not discuss multivalued treatments here, because doing so only increases the number of parameters and notation required and detracts from the essential points; see [CAUSAL] teffects multivalued for information about multivalued treatments.

The parameters of interest summarize the distribution of the unobservable individual-level treatment effect $y_1 - y_0$. In defining the parameters, $t$ denotes a random treatment, $t_i$ denotes the treatment received by individual $i$, $t = 1$ is the treatment level, and $t = 0$ is the control level. Given this notation, we can now define our parameters of interest.

**ATE** The ATE is the average effect of the treatment in the population:

\[ \text{ATE} = E(y_1 - y_0) \]

**POM** The POM for treatment level $t$ is the average potential outcome for that treatment level:

\[ \text{POM}_t = E(y_t) \]

**ATET** The ATET is the average treatment effect among those that receive the treatment:

\[ \text{ATET} = E(y_1 - y_0 | t = 1) \]

For an illustration of these concepts, see Defining treatment effects in [CAUSAL] teffects intro.
The potential-outcome model

Next we specify a potential-outcome model that serves as a touchstone for the rest of our discussion. The model described here generates data in which \( y_i \) is the observed outcome variable, \( t_i \) is the treatment variable, \( x_i \) is a vector of covariates that affect the outcome, and \( w_i \) is a vector of covariates that affect the treatment assignment. \( x_i \) and \( w_i \) may have elements in common.

This potential-outcome model specifies that the observed outcome variable \( y \) is \( y_0 \) when \( t = 0 \) and that \( y \) is \( y_1 \) when \( t = 1 \). Algebraically, we say that

\[
y = (1 - t)y_0 + ty_1
\]

The functional forms for \( y_0 \) and \( y_1 \) are

\[
y_0 = x'\beta_0 + \epsilon_0 \quad (1) \\
y_1 = x'\beta_1 + \epsilon_1 \quad (2)
\]

where \( \beta_0 \) and \( \beta_1 \) are coefficients to be estimated, and \( \epsilon_0 \) and \( \epsilon_1 \) are error terms that are not related to \( x \) or \( w \). This potential-outcome model separates each potential outcome into a predictable component, \( x\beta_t \), and an unobservable error term, \( \epsilon_t \).

The treatment assignment process is

\[
t = \begin{cases} 
1 & \text{if } w'\gamma + \eta > 0 \\
0 & \text{otherwise}
\end{cases} \quad (3)
\]

where \( \gamma \) is a coefficient vector, and \( \eta \) is an unobservable error term that is not related to either \( x \) or \( w \). The treatment assignment process is also separated into a predictable component, \( w'\gamma \), and an unobservable error term, \( \eta \).

We emphasize six points about this model:

1. The observed data from this model contain \( y_i, t_i, w_i, \) and \( x_i \). The data do not reveal both \( y_{0i} \) and \( y_{1i} \) for any given \( i \).
2. The model for \( t \) determines how the data on \( y_0 \) and \( y_1 \) are missing.
3. The model separates the potential outcomes and treatment assignment into observable and unobservable components.
4. Whether \( \eta \) is independent of the vector \((\epsilon_0, \epsilon_1)\) is essential in specifying the set of available estimators.
5. The coefficient vectors \( \beta_0, \beta_1, \) and \( \gamma \) are auxiliary parameters. We use estimates of these coefficient vectors to estimate the ATE, the POMs, and the ATET.
6. For notational simplicity, we represented \( y_0 \) and \( y_1 \) as linear functions. In practice, we can use other functional forms.

In specifying this potential-outcome model, we explicitly showed the functional forms for the potential outcomes and the treatment assignment process. To ease subsequent discussions, we refer to the set of functional forms for the potential outcomes as the “outcome model”, and we refer to the treatment assignment process as the “treatment model”.
Assumptions needed for estimation

As with any type of estimator, we must make some assumptions to use treatment-effects estimators. The particular assumptions we need for each estimator implemented by `teffects` and for each effect parameter vary, but some version of each of the following is required.

**CI** The conditional-independence CI assumption restricts the dependence between the treatment model and the potential outcomes.

**Overlap** The overlap assumption ensures that each individual could receive any treatment level.

**i.i.d.** The independent and identically distributed (i.i.d.) sampling assumption ensures that the potential outcomes and the treatment status of each individual are unrelated to the potential outcomes and treatment statuses of all other individuals in the population.

We now discuss each assumption in detail.

### The CI assumption

After conditioning on covariates, when no unobservable variable affects both treatment assignment and the potential outcomes, the potential outcomes are conditionally independent of the treatment. In epidemiological jargon, there are no unmeasured confounders. In econometric jargon, we have selection on observables. If we observe enough covariates, the potential outcomes may indeed be conditionally independent of the treatment.

Intuitively, the CI assumption says that only the covariates $x$ affect both the treatment and the potential outcomes. Any other factors that affect the treatment must be independent of the potential outcomes, and any other factors that affect the potential outcomes must be independent of the treatment. Formally, the CI assumption states that, conditional on covariates $x$, the treatment $t$ is independent of the vector of potential outcomes $(y_0, y_1)'$.

The CI assumption allows us to estimate the effects by regression-adjustment (RA) methods, inverse-probability-weighting (IPW) methods, methods that combine RA and IPW concepts, and matching methods. The data only reveal information about $E(y_0|x, w, t=0)$ and $E(y_1|x, w, t=1)$, but we are interested in an average of $E(y_0|x, w)$ and $E(y_1|x, w)$, where $x$ represents the outcome covariates and $w$ the treatment-assignment covariates. The CI assumption allows us to estimate $E(y_0|x, w)$ and $E(y_1|x, w)$ directly from the observations for which $E(y_0|x, w, t=0)$ and $E(y_1|x, w, t=1)$, respectively.

For our potential-outcome model presented in (1) through (3), the CI assumption can be viewed as a set of restrictions on the covariance matrix of the error terms. Suppose that the vector of unobservables $(\epsilon_0, \epsilon_1, \eta)$ is normally distributed

$$
\begin{pmatrix}
\epsilon_0 \\
\epsilon_1 \\
\eta
\end{pmatrix}
\sim N
\begin{pmatrix}
0 \\
0 \\
0
\end{pmatrix},
\begin{pmatrix}
\sigma_0^2 & \rho_{01} \sigma_0 \sigma_1 & \rho_{00} \sigma_0 \\
\rho_{01} \sigma_0 \sigma_1 & \sigma_1^2 & \rho_{11} \sigma_1 \\
\rho_{00} \sigma_0 & \rho_{11} \sigma_1 & 1
\end{pmatrix}
$$

(4)

where $\sigma_0$ is the standard deviation of $\epsilon_0$, $\rho_{01}$ is the correlation between $\epsilon_0$ and $\epsilon_1$, $\sigma_1$ is the standard deviation of $\epsilon_1$, $\rho_{00}$ is the correlation between $\eta$ and $\epsilon_0$, and $\rho_{11}$ is the correlation between $\eta$ and $\epsilon_1$. As is standard in the normally distributed latent-variable specification of a binary-dependent variable, we normalize the variance of $\eta$ to 1.
CI specifies that $\rho_{\eta 0} = \rho_{\eta 1} = 0$ so that we can write (4) as

$$\begin{pmatrix}
\epsilon_0 \\
\epsilon_1 \\
\eta
\end{pmatrix} \sim N \begin{pmatrix}
(0) & \sigma_0^2 & \rho_{01}\sigma_0\sigma_1 & 0 \\
(0) & 0 & \sigma_1^2 & 0 \\
(0) & 0 & 0 & 1
\end{pmatrix} \begin{pmatrix}
0 \\
\rho_{01}\sigma_0\sigma_1 \\
\sigma_1^2 \\
0
\end{pmatrix}$$

Writing the covariance matrix this way makes clear what we mean by conditional independence: unobserved shocks that affect whether a subject is treated do not have any effect on the potential outcomes, and unobserved shocks that affect a potential outcome do not affect treatment.

teffects implements estimators that require the CI assumption. See [CAUSAL] etregress and [CAUSAL] etpoisson for commands that handle two cases in which the CI assumption is replaced by precise specifications of the joint dependence among the unobservables. Brown and Mergoupis (2011) discuss the itreatreg command that extends [CAUSAL] etregress.

The CI assumption is also known as unconfoundedness and selection-on-observables in the literature. See Rosenbaum and Rubin (1983); Heckman (1997); Heckman and Navarro-Lozano (2004); Cameron and Trivedi (2005, sec. 25.2.1); Tsiatis (2006, sec. 13.3); Angrist and Pischke (2009, chap. 3); Imbens and Wooldridge (2009); and Wooldridge (2010, sec. 21.3). Some discussions with Stata commands can be found in Becker and Caliendo (2007), Nichols (2007), and Daniel, De Stavola, and Cousens (2011).

**Technical note**

In fact, full CI is stronger than what we need to estimate the ATE, the ATET, or the POMs. For the estimators implemented in teffects, we only need a conditional mean independence (CMI) assumption. Intuitively, the CMI assumption says that after accounting for the covariates $x_i$, the treatment does not affect the conditional mean of each potential outcome. Formally, the CMI requires that $E(y_0 | x, t) = E(y_0 | x)$ and that $E(y_1 | x, t) = E(y_1 | x)$. The CMI assumption allows the conditional variance to depend on the treatment, while the CI assumption does not.

The CI assumption implies the CMI assumption, but not vice versa.

See Wooldridge (2010, sec. 21.2 and 21.3) for an excellent introduction to this topic, and see Cattaneo, Drukker, and Holland (2013) for some discussion of the multiple treatment case.

**The overlap assumption**

The overlap assumption requires that each individual have a positive probability of receiving each treatment level. Formally, the overlap assumption requires that for each possible $x$ in the population and each treatment level $\tilde{t}$, $0 < \Pr(t = \tilde{t} | x) < 1$. Rosenbaum and Rubin (1983) call the combination of the CI and overlap assumptions strong ignorability; see also Abadie and Imbens (2006, 237–238) and Imbens and Wooldridge (2009).

**The i.i.d. assumption**

The third of the three assumptions listed above is the i.i.d. assumption; it is the standard assumption of having an i.i.d. sample from the population. In potential-outcome models, i.i.d. sampling implies that the potential outcomes and treatment status of each individual are unrelated to the potential outcomes and treatment statuses of all the other individuals in the population. I.i.d. sampling rules out interactions among the individuals. For instance, models of vaccinations in epidemiology and general
equilibrium effects in economics violate the independence assumption. This third assumption is a part of what is known as the stable unit treatment value assumption (SUTVA); see Wooldridge (2010, 905) and Imbens and Wooldridge (2009).

Comparing the ATE and ATET

When comparing the ATE and the ATET, two points should be mentioned.

First, the assumptions required to estimate the ATET are less restrictive than the assumptions required to estimate the ATE. Estimating the ATET requires a weaker form of the CI assumption and a weaker version of the overlap assumption.

To estimate the ATE under CI, we require that the unobservables in the treatment model be conditionally independent of the unobservables in both potential outcomes. In contrast, we can estimate the ATET under CI when the unobservables in the treatment model are conditionally independent of just the control-level potential outcome; see Wooldridge (2010, 906–912).

Although the ATE version of overlap requires that all covariate patterns have a positive probability of being allocated to each treatment state, we can estimate the ATET when only the covariate patterns for which someone is treated have a positive probability of being allocated to each treatment state. This weaker overlap assumption can be important in some studies. For example, Heckman (1997) discusses how the ATET makes sense in job-training programs for which many types of individuals have zero chance of signing up. See also Wooldridge (2010, 911–913).

Second, the ATET reduces to the ATE when the mean of the covariates among the treated is the same as the mean of the covariates in the population and when the average contribution from the unobservables for the participants is zero.

Overview of treatment-effect estimators

We can classify the estimators implemented by t efects into five categories: 1) estimators based on a model for the outcome variable; 2) estimators based on a model for treatment assignment; 3) estimators based on models for both the outcome variable and the treatment assignment; 4) estimators that match on covariates; and 5) estimators that match on predicted probabilities of treatment. Within each category of estimator, there is a variety of choices about the functional forms for the models.

Because there are several categories of estimators, the user must decide whether to model the outcomes, the probability of treatment, or both. Under correct model specification, using an outcome model and a model for the probability of treatment will produce more efficient estimates. Surprisingly, some of the estimators that use both models only require that one of the two be correctly specified to consistently estimate the effects of interest, a property known as the double-robust property.

With the exception of using a matching estimator with a single continuous covariate, some choice of functional forms is required. There are two aspects one must consider when choosing the functional form for the outcome or treatment assignment. First, one must select the functional form for the conditional mean or conditional probability; depending on the variable being modeled, a linear, a binary choice, or an exponential model may be appropriate. Second, one must determine the appropriate polynomials of the covariates to include in the model. t efects offers a wide variety of options to specify different functional form choices for the conditional mean and conditional probability models. The factor-variable notation in Stata allows us to easily specify the desired polynomial in the covariates.

We now provide some intuition behind each type of estimator.
**RA estimators**

RA estimators use means of predicted outcomes for each treatment level to estimate each POM. ATEs and ATETs are differences in estimated POMs.

The CI assumption implies that we can estimate $E(y_0|x)$ and $E(y_1|x)$ directly from the observations for which $t = 0$ and $t = 1$, respectively. Regression adjustment fits separate regressions for each treatment level and uses averages of the predicted outcomes over all the data to estimate the POMs. The estimated ATEs are differences in the estimated POMs. The estimated ATETs are averages of the predicted outcomes over the treated observations.

RA is a venerable estimator. See Lane and Nelder (1982); Cameron and Trivedi (2005, chap. 25); Wooldridge (2010, chap. 21); and Vittinghoff et al. (2012, chap. 9). The usefulness of RA has been periodically questioned in the literature because it relies on specifying functional forms for the conditional means and because it requires having sufficient observations of each covariate pattern in each treatment level; see Rubin (1973) for an early salvo. Our experience is that RA is an exceptionally useful base-case estimator. We describe its relative advantages and disadvantages in the course of covering other estimators.

**IPW estimators**

IPW estimators use weighted averages of the observed outcome variable to estimate means of the potential outcomes. The weights account for the missing data inherent in the potential-outcome framework. Each weight is the inverse of the estimated probability that an individual receives a treatment level. Outcomes of individuals who receive a likely treatment get a weight close to one. Outcomes of individuals who receive an unlikely treatment get a weight larger than one, potentially much larger.

IPW estimators model the probability of treatment without any assumptions about the functional form for the outcome model. In contrast, RA estimators model the outcome without any assumptions about the functional form for the probability of treatment model.

IPW estimators become extremely unstable as the overlap assumption gets close to being violated. When the overlap assumption is nearly violated, some of the inverse-probability weights become very large, IPW estimators produce erratic estimates, and the large-sample distribution provides a poor approximation to the finite-sample distribution of IPW estimators. This instability occurs even though the functional form for the treatment model is correctly specified.

In contrast, when the overlap assumption is nearly violated, there are very few observations in a treatment level for some covariate patterns, so RA estimators use the model to predict in regions in which there are very little data. If the model is well specified and there are “enough” observations, an RA estimator will not become unstable as quickly as an IPW estimator, and the large-sample distribution of the RA estimator still provides a good approximation to the finite-sample distribution. However, in real situations in which “all models are approximate”, relying on a correctly specified outcome model with little data is extremely risky.

IPW estimators are a general approach to missing-data problems that obey some CI assumptions. While IPW is an old idea in statistics that dates back to Horvitz and Thompson (1952), biostatisticians and econometricians have been actively working on extending it to handle modern problems and estimation methods. See Robins and Rotnitzky (1995); Robins, Rotnitzky, and Zhao (1994, 1995); and Wooldridge (2002, 2007). IPW has been used extensively in the modern treatment-effect estimation literature. See Imbens (2000); Hirano, Imbens, and Ridder (2003); Tan (2010); Wooldridge (2010, chap. 19); van der Laan and Robins (2003); and Tsiatis (2006, chap. 6).

To see the intuition behind IPW, consider a study with observed outcome variable $y$, treatment variable $t \in \{0, 1\}$, and potential outcomes $y_0$ and $y_1$. As part of this process, we need to estimate...
the POM for treatment $t = 1$, $E(y_1)$. Using the observed data, $y_{it} t_i$ is $y_{1i}$ when $t = 1$, but $y_{1i}$ is unobserved when $t = 0$. An IPW estimator for $E(y_1)$ is $1/N \sum_{i=1}^{N} y_{it} t_i / p(x_i)$, where $p(x_i)$ is the probability that $t_i = 1$ and is a function of the covariates $x_i$. If $y_{1i}$ were always observed, the weights would all equal 1. This IPW estimator places a larger weight on those observations for which $y_{1i}$ is observed even though its observation was not likely.

**AIPW estimators**

Instead of modeling either the outcome, like RA, or the treatment probability, like IPW, augmented inverse-probability-weighted (AIPW) estimators model both the outcome and the treatment probability. A surprising fact is that only one of the two models must be correctly specified to consistently estimate the treatment effects, a property of the AIPW estimators known as being “doubly robust”. Given that two models instead of one are used, it is less surprising that the AIPW estimators can be more efficient than either the RA or the IPW estimators, though deriving this result is rather technical and relies on the theory of semiparametric estimators.

Intuitively, the AIPW estimator is an IPW that includes an augmentation term that corrects the estimator when the treatment model is misspecified. When the treatment is correctly specified, the augmentation term vanishes as the sample size becomes large. Like the IPW, the AIPW does not perform well when the predicted treatment probabilities are too close to zero or one.

AIPW estimators emerge naturally from a technique of producing more efficient estimators from estimators that have a few main parameters of interest and some auxiliary, or nuisance, parameters used in estimating the few main parameters. This method constructs efficient estimating equations for the main parameters that are orthogonal to the auxiliary parameters. The estimators produced by this method are known as efficient-influence function (EIF) estimators.

To gain some intuition, consider finding an EIF estimator from an IPW estimator for two POMs. Note that we only care about the two POM parameters and not about the many auxiliary parameters used to estimate the treatment probabilities. EIF estimators project the equations that yield the POM parameters onto the equations that yield the auxiliary treatment-model parameters and then use the residuals from this projection to estimate the POM parameters.

We refer to these estimators as “AIPW estimators” instead of “EIF estimators” because the former is commonly used in the biostatistical literature for some of the binary-treatment estimators and because the term “augmented inverse-probability-weighted” tells more about how these estimators relate to the other implemented estimators; see Tsiatis (2006) and Tan (2010). The estimators implemented in *teffects aipw* with the *wnls* option are based on those of Rubin and van der Laan (2008), which did well in simulations reported by Tan (2010), and denoted as $\hat{\alpha}_{RV}(\hat{\pi})$ in Tan (2010, 663).

When either the outcome model or the treatment model is well specified, the AIPW estimators implemented in *teffects aipw* are more robust than either the RA or the IPW estimators because the AIPW estimators are doubly robust but the RA and IPW estimators are not. When both the outcome and the treatment model are misspecified, which estimator is more robust is a matter of debate in the literature; see Kang and Schafer (2007) and Robins et al. (2007) for some debate, and see Tan (2010) for a more recent discussion.

To the best of our knowledge, there is no general solution to the question of which estimator performs best when both the outcome and the treatment models are misspecified. We suspect that the answer depends on the true models, the implemented specifications, and the polynomials in the covariates used. To help users through this process, the estimators implemented in *teffects* offer many functional forms to approximate either the outcome process or the treatment process. In addition, Stata’s factor-variable notation makes it easy to include polynomials in the covariates. Both of these approximation methods rely on having enough data. *teffects* also makes it easy to compare the results produced by different estimators.
The literature on these methods is vast and growing. For double-robust results and explanations, see Robins and Rotnitzky (1995); Robins, Rotnitzky, and Zhao (1995); van der Laan and Robins (2003, chap. 6); Bang and Robins (2005); Tsiatis (2006, chap. 13); Wooldridge (2007; 2010, sec. 21.3.4); and Tan (2010).

**IPWRA estimators**

Like AIPW estimators, inverse-probability-weighted regression-adjustment (IPWRA) estimators combine models for the outcome and treatment status; also like AIPW estimators, IPWRA estimators are doubly robust. IPWRA estimators emerge naturally from a robust approach to missing-data methods. IPWRA estimators use the inverse of the estimated treatment-probability weights to estimate missing-data-corrected regression coefficients that are subsequently used to compute the POMs.

As far as we know, there is no literature that compares the relative efficiency of AIPW estimators, which emerge from a general approach to creating efficient estimators, and the IPWRA estimators, which emerge from a robust-correction approach to missing-data analysis.

The IPWRA estimators are also known as “Wooldridge’s double-robust” estimators because they were derived in Wooldridge (2007) and discussed at length in Wooldridge (2010, section 21.3.4).

**Nearest-neighbor matching estimators**

Matching estimators use an average of the outcomes of the nearest individuals to impute the missing potential outcome for each sampled individual. The difference between the observed outcome and the imputed potential outcome is an estimate of the individual-level treatment effect. These estimated individual-level treatment effects are averaged to estimate the ATE or the ATET.

teffects nnmatch determines the “nearest” by using a weighted function of the covariates for each observation. This type of matching is known as nearest-neighbor matching (NNM). teffects psmatch determines the “nearest” by using the estimated treatment probabilities, which are known as the propensity scores. This second type of matching is known as propensity-score matching (PSM).

NNM is nonparametric in that no explicit functional form for either the outcome model or the treatment model is specified. This flexibility comes at a price: the estimator needs more data to get to the true value than an estimator that imposes a functional form. More formally, the NNM estimator converges to the true value at a rate slower than the parametric rate, which is the square root of the sample size, when matching on more than one continuous covariate. teffects nnmatch uses bias correction to fix this problem. PSM provides an alternative to bias correction because it matches on a single continuous covariate, the estimated treatment probabilities.

Abadie and Imbens (2006, 2011) derived the rate of convergence of the NNM estimator and the bias-corrected NNM estimator and the large-sample distributions of the NNM and the bias-corrected NNM estimators. These articles provided the formal results that built on methods suggested in Rubin (1973, 1977).

teffects nnmatch is based on the results in Abadie and Imbens (2006, 2011) and a previous implementation in Abadie et al. (2004).
Propensity-score matching estimators

Instead of performing bias correction to handle the case of more than one continuous covariate, a common solution is to combine all the covariate information into estimated treatment probabilities, known as propensity scores, and use this single continuous covariate as the matching variable.

The term “propensity score” is widely used, but we continue to refer to it as the “treatment probability” to be consistent with the other estimators. We call the estimator that matches on the estimated treatment probabilities the “propensity-score matching (PSM) estimator” because the latter term is ubiquitous.

In effect, the PSM estimator parameterizes the bias-correction term in the treatment probability model. One advantage of matching on the estimated treatment probabilities over the bias-correction method is that one can explore the fit of different treatment probability models using standard methods before performing the nonparametric matching. For example, one can select the treatment model by minimizing an information criterion under i.i.d. sampling. We know of no counterpart for selecting the proper order of the bias-correction term for the NNM estimator.

Matching on estimated treatment probability models has been very popular since Rosenbaum and Rubin (1983) showed that if adjusting for covariates \( x_i \) is sufficient to estimate the effects, then one can use the probability of treatment to perform the adjustment. Abadie and Imbens (2012) derived a method to estimate the standard errors of the estimator that matches on estimated treatment probabilities, and this method is implemented in \textit{teffects psmatch}.

Choosing among estimators

There is no definitive way to select one of the estimators implemented in \textit{teffects} over the others. We offer three observations about the tradeoffs among the estimators.

First, if the outcome model is correctly specified, the RA estimator will break down more slowly than the IPW, AIPW, IPWRA, or PSM estimators as the overlap assumption begins to fail. This result depends critically on the ability of the RA estimator to predict into regions in which there are little data.

Second, if the overlap assumption holds, the AIPW and IPWRA estimators have the double-robust property for some functional form combinations. The double-robust property says that if either the outcome model or the treatment model is correctly specified, we can consistently estimate the effects. The properties of double-robust estimators when both models are misspecified are not known, although there is some discussion in the literature about the properties of the AIPW estimators; see Kang and Schafer (2007), Robins et al. (2007), and Tan (2010).

Third, all the estimators require the same assumptions, so if each is correctly specified, they should all produce similar results. Of course, just because they produce similar results does not mean that they are correctly specified; it is possible that they are just behaving similarly in response to some underlying problem.

Model choice

\textit{teffects} offers a broad selection of functional form combinations so that you can choose a combination that fits your data. Picking a functional form that respects the values of the observed outcomes is usually best. Select \textit{linear} for continuous outcomes over the real line; \textit{logit}, \textit{probit}, or \textit{hetprobit} for binary outcomes; and \textit{poisson} for counts or nonnegative outcomes.

For binary treatments, you can select among \textit{logit}, \textit{probit}, or \textit{hetprobit} models. For multivalued treatments, only the multinomial logit is available to model the treatment probabilities.
Selecting a functional form of a given set of covariates is a famously difficult problem in statistics. In the treatment-effects context, Cattaneo, Drukker, and Holland (2013) found that model selection by minimizing an information criterion worked well. Cattaneo, Drukker, and Holland (2013) discuss a method and a community-contributed command to facilitate the process.

Acknowledgments

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We thank Matias Cattaneo for his collaboration on Cattaneo, Drukker, and Holland (2013) and for many conversations about Cattaneo (2010), a part of which is implemented in teffects ipw and teffects aipw.

We thank Ashley Holland for her collaboration on Cattaneo, Drukker, and Holland (2013).

We thank Alberto Abadie, Jane Herr, and Guido Imbens for collaboration on Abadie et al. (2004), which is implemented in teffects nnmatch.

We thank Matias Cattaneo for sharing an extract of the dataset that he used in Cattaneo (2010) and Barbara Sianesi for sharing an extract of the dataset that she used in Blundell, Dearden, and Sianesi (2005).

We thank Alberto Abadie, Guido Imbens, and László Sándor for sharing their implementation of the standard errors derived in Abadie and Imbens (2012), which are implemented in teffects psmatch.

We thank Edwin Leuven and Barbara Sianesi for their inspirational command psmatch2, which computes several of the estimators in teffects and a few more that we have not implemented.

References


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Also see

[CAUSAL] **teffects** — Treatment-effects estimation for observational data

[CAUSAL] **teffects intro** — Introduction to treatment effects for observational data

[CAUSAL] **teffects multivalued** — Multivalued treatment effects
teffects aipw — Augmented inverse-probability weighting

Description

teffects aipw estimates the average treatment effect (ATE) and the potential-outcome means (POMs) from observational data by augmented inverse-probability weighting (AIPW). AIPW estimators combine aspects of regression-adjustment and inverse-probability-weighted methods. AIPW estimators have the double-robust property. teffects aipw accepts a continuous, binary, count, fractional, or nonnegative outcome and allows a multivalued treatment.

See [CAUSAL] teffects intro or [CAUSAL] teffects intro advanced for more information about estimating treatment effects from observational data.

Quick start

ATE of binary treatment treat2 by AIPW using a linear model for outcome y1 on x1 and x2 and a logistic model for treat2 on x1 and w

teffects aipw (y1 x1 x2) (treat2 x1 w)

Same as above, but use a fractional logistic model for fractional outcome y2

teffects aipw (y2 x1 x2, flogit) (treat2 x1 w)

Same as above, but use a heteroskedastic probit model for binary outcome y3 and a probit model for treat2

teffects aipw (y3 x1 x2, hetprobit(x1 x2)) (treat2 x1 w, probit)

ATE for each level of three-valued treatment treat3 on y1

teffects aipw (y1 x1 x2) (treat3 x1 w)

Same as above, and specify that treat3 = 3 is the control level

teffects aipw (y1 x1 x2) (treat3 x1 w), control(3)

Same as above, specified using the label “MyControl” corresponding to treat3 = 3

teffects aipw (y1 x1 x2) (treat3 x1 w), control(MyControl)

Menu

Statistics > Causal inference/treatment effects > Continuous outcomes > Augmented inverse-probability weighting
Statistics > Causal inference/treatment effects > Binary outcomes > Augmented inverse-probability weighting
Statistics > Causal inference/treatment effects > Count outcomes > Augmented inverse-probability weighting
Statistics > Causal inference/treatment effects > Fractional outcomes > Augmented inverse-probability weighting
Statistics > Causal inference/treatment effects > Nonnegative outcomes > Augmented inverse-probability weighting
Syntax

teffects aipw (ovar omvarlist [ , omodel noconstant])
 (tvar tmvarlist [ , tmodel noconstant]) [ if ] [ in ] [ weight]
 [ , stat options ]

ovar is a binary, count, continuous, fractional, or nonnegative outcome of interest.

omvarlist specifies the covariates in the outcome model.

tvar must contain integer values representing the treatment levels.

tmvarlist specifies the covariates in the treatment-assignment model.

<table>
<thead>
<tr>
<th>omodel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>linear outcome model; the default</td>
</tr>
<tr>
<td>logit</td>
<td>logistic outcome model</td>
</tr>
<tr>
<td>probit</td>
<td>probit outcome model</td>
</tr>
<tr>
<td>hetprobit(varlist)</td>
<td>heteroskedastic probit outcome model</td>
</tr>
<tr>
<td>poisson</td>
<td>exponential outcome model</td>
</tr>
<tr>
<td>flogit</td>
<td>fractional logistic outcome model</td>
</tr>
<tr>
<td>fprobit</td>
<td>fractional probit outcome model</td>
</tr>
<tr>
<td>fhetprobit(varlist)</td>
<td>fractional heteroskedastic probit outcome model</td>
</tr>
</tbody>
</table>

omodel specifies the model for the outcome variable.

<table>
<thead>
<tr>
<th>tmodel</th>
<th>Description</th>
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</thead>
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<td>logistic treatment model; the default</td>
</tr>
<tr>
<td>probit</td>
<td>probit treatment model</td>
</tr>
<tr>
<td>hetprobit(varlist)</td>
<td>heteroskedastic probit treatment model</td>
</tr>
</tbody>
</table>

tmodel specifies the model for the treatment variable.

For multivalued treatments, only logit is available and multinomial logit is used.

<table>
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<th>stat</th>
<th>Description</th>
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<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>pomeans</td>
<td>estimate potential-outcome means</td>
</tr>
</tbody>
</table>
Options

Model

noconstant; see [R] Estimation options.

tls specifies that the parameters of the outcome model be estimated by nonlinear least squares instead of the default maximum likelihood.

wnls specifies that the parameters of the outcome model be estimated by weighted nonlinear least squares instead of the default maximum likelihood. The weights make the estimator of the effect parameters more robust to a misspecified outcome model.

Stat

stat is one of two statistics: ate or pomeans. ate is the default.

ate specifies that the average treatment effect be estimated.

pomeans specifies that the potential-outcome means for each treatment level be estimated.
**SE/Robust**

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] `vce_option`.

**Reporting**

`level(#)`; see [R] `Estimation options`.

`aequations` specifies that the results for the outcome-model or the treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

`display_options`: `noci`, `nopvalues`, `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%)`, `pformat(%)`, `sformat(%)`, and `nolstretch`; see [R] `Estimation options`.

**Maximization**

`maximize_options`: `iterate(#)`, `[no] log`, and `from(init_specs)`; see [R] `Maximize`. These options are seldom used.

`init_specs` is one of

```
matname [, skip copy]
# [, # ...], copy
```

**Advanced**

`pstolerance(#)` specifies the tolerance used to check the overlap assumption. The default value is `pstolerance(1e-5)`. `teffects` will exit with an error if an observation has an estimated propensity score smaller than that specified by `pstolerance()`.

`osample(newvar)` specifies that indicator variable `newvar` be created to identify observations that violate the overlap assumption.

`control(# | label)` specifies the level of `tvar` that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. `control()` may not be specified with statistic `pomeans`.

The following option is available with `teffects aipw` but is not shown in the dialog box: `coeflegend`; see [R] `Estimation options`.

### Remarks and examples

Remarks are presented under the following headings:

- **Overview**
- **Video example**

#### Overview

AIPW estimators use inverse-probability weights to correct for the missing-data problem arising from the fact that each subject is observed in only one of the potential outcomes; these estimators also use an augmentation term in the outcome model to correct the estimator in case the treatment model is misspecified. If the treatment model is correctly specified, the augmentation term goes to zero in large samples.
AIPW estimators compute averages of the augmented inverse-probability-weighted outcomes for each treatment level. Contrasts of these averages provide estimates of the treatment effects.

AIPW estimators use a model to predict treatment status, and they use another model to predict outcomes. Because of the double-robust property, only one of these two models must be correctly specified for the AIPW estimator to be consistent.

AIPW estimators use a three-step approach to estimating treatment effects:

1. They estimate the parameters of the treatment model and compute inverse-probability weights.
2. They estimate separate regression models of the outcome for each treatment level and obtain the treatment-specific predicted outcomes for each subject.
3. They compute the weighted means of the treatment-specific predicted outcomes, where the weights are the inverse-probability weights computed in step 1. The contrasts of these weighted averages provide the estimates of the ATEs.

These steps produce consistent estimates of the effect parameters because the treatment is assumed to be independent of the potential outcomes after conditioning on the covariates. The overlap assumption ensures that predicted inverse-probability weights do not get too large. The standard errors reported by `teffects aipw` correct for the three-step process. See [CAUSAL] `teffects intro` or [CAUSAL] `teffects intro advanced` for more information about this estimator.

We will illustrate the use of `teffects aipw` by using data from a study of the effect of a mother’s smoking status during pregnancy (`mbsmoke`) on infant birthweight (`bweight`) as reported by Cattaneo (2010). This dataset also contains information about each mother’s age (`mage`), education level (`medu`), marital status (`mmarried`), whether the first prenatal exam occurred in the first trimester (`prenatal1`), and whether this baby was the mother’s first birth (`fbaby`).

> Example 1: Estimating the ATE

We begin by using `teffects aipw` to estimate the average treatment effect of `mbsmoke` on `bweight`. We use a probit model to predict treatment status as a function of `mmarried`, `mage`, and `fbaby`; to maximize the predictive power of this model, we use factor-variable notation to incorporate quadratic effects of the mother’s age, the only continuous covariate in our model. We use linear regression to model birthweight, using `prenatal1`, `mmarried`, `mage`, and `fbaby` as explanatory variables. We type
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects aipw (bweight prenatal1 mmarried mage fbaby)
> (mbsmoke mmarried c.mage##c.mage fbaby medu, probit)
Iteration 0: EE criterion = 4.629e-21
Iteration 1: EE criterion = 1.944e-25
Treatment-effects estimation Number of obs = 4,642
Estimator : augmented IPW
Outcome model : linear by ML
Treatment model: probit

|      | Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|------|-------------|-----------|------|-----|---------------------|
| ATE  |             |           |      |     |                     |
| mbsmoke (Smoker vs Nonsmoker) | -230.9892 | 26.21056 | -8.81 | 0.000 | -282.361 -179.6174 |
| POMean | mbsmoke | Nonsmoker | 3403.355 | 9.568472 | 355.68 | 0.000 | 3384.601 3422.109 |

The average birthweight if all mothers were to smoke would be 231 grams less than the average of 3,403 grams that would occur if none of the mothers had smoked.

By default, teffects aipw reports the ATE and the POM for the base (untreated) subjects. The pomeans option allows us to view the treated subjects’ POM as well; the aequations option displays the regression model coefficients used to predict the POMs as well as the coefficients from the model used to predict treatment.

Example 2: Displaying the POMs and equations

Here we use the pomeans and aequations options to obtain estimates of both POMs and view all the fitted equations underlying our estimates:
The coefficient table indicates that the treated POM is 3,172 grams, 231 grams less than the untreated POM. The sections of the table labeled OME0 and OME1 represent the linear regression coefficients for the untreated and treated potential-outcome equations, respectively. The coefficients of the TME1 equation are used in the probit model to predict treatment status.

As is well known, the standard probit model assumes that the error terms in the latent-utility framework are homoskedastic; the model is not robust to departures from this assumption. An alternative is to use the heteroskedastic probit model, which explicitly models the error variance as a function of a set of variables.

Example 3: Heteroskedastic probit treatment model

Here we refit our model as in the previous examples, but we instead use heteroskedastic probit to model the treatment variable. We posit that the heteroskedasticity is a function of the mother’s age. We type
. teffects aipw (bweight prenatal1 mmarried fbaby)
> (mbsmoke mmarried c.mage##c.mage fbaby medu, hetprobit(c.mage)), aequations
Iteration 0:  EE criterion = 1.746e-19
Iteration 1:  EE criterion = 1.746e-19 (backed up)
Treatment-effects estimation  Number of obs = 4,642
Estimator : augmented IPW
Outcome model : linear by ML
Treatment model: heteroskedastic probit

| bweight | Robust Coefficient | std. err. | z  | P>|z| | [95% conf. interval] |
|---------|--------------------|-----------|----|-------|----------------------|
| ATE     |                    |           |    |       |                      |
| mbsmoke | (Smoker vs Nonsmoker) |           |    |       |                      |
| -230.2699 | 27.49461            | -8.38    | 0.000 | -284.1584 | -176.3815 |
| POmean  |                    |           |    |       |                      |
| mbsmoke | Nonsmoker          |           |    |       |                      |
| 3403.657 | 9.540713            | 356.75   | 0.000 | 3384.957  | 3422.356  |
| OME0    |                    |           |    |       |                      |
| prenatal1 |                  |           |    |       |                      |
| 69.5048  | 27.04642            | 2.57     | 0.010 | 16.49479  | 122.5148  |
| mmarried |                  |           |    |       |                      |
| 173.74   | 24.63865            | 7.05     | 0.000 | 125.4491  | 222.0308  |
| fbaby    |                  |           |    |       |                      |
| -79.19473 | 18.62584            | -4.25    | 0.000 | -115.7007 | -42.68875 |
| _cons    |                  |           |    |       |                      |
| 3260.768 | 28.29282            | 115.25   | 0.000 | 3205.315  | 3316.221  |
| OME1    |                    |           |    |       |                      |
| prenatal1 |                  |           |    |       |                      |
| 12.86437 | 39.83916            | 0.32     | 0.747 | -65.21894 | 90.94768  |
| mmarried |                  |           |    |       |                      |
| 113.3491 | 39.47422            | 2.87     | 0.004 | 35.9811   | 190.7172  |
| fbaby    |                  |           |    |       |                      |
| 64.22326 | 38.42042            | 1.67     | 0.095 | -11.07939 | 139.5259  |
| _cons    |                  |           |    |       |                      |
| 3051.268 | 37.30413            | 81.79    | 0.000 | 2978.153  | 3124.383  |
| TME1    |                    |           |    |       |                      |
| mmarried |                  |           |    |       |                      |
| -.3551755 | .1044199            | -3.40    | 0.001 | -.5598347 | -.1505162 |
| mage     |                  |           |    |       |                      |
| .0831898 | .0349088            | 2.38     | 0.017 | .0147699  | .1516097  |
| c.mage   |                  |           |    |       |                      |
| -.0013458 | .0006659            | -2.02    | 0.043 | -.002651  | -.0000406 |
| c.mage#  |                  |           |    |       |                      |
| -.1170697 | .044998             | -2.60    | 0.009 | -.2052643 | -.0288752 |
| fbaby    |                  |           |    |       |                      |
| -.0435067 | .0147852            | -2.94    | 0.003 | -.0724842 | -.0145272 |
| medu     |                  |           |    |       |                      |
| -.8757021 | .347814             | -2.52    | 0.012 | -1.557405 | -.1939993 |
| _cons    |                  |           |    |       |                      |
| -.0236336 | .0107134            | -2.21    | 0.027 | -.0446315 | -.0026357 |

The equation labeled TME1\_lnsigma represents the heteroskedasticity function used to model the logarithm of the variance. Because the coefficient on the single variable we specified is significant below the 5% level, we conclude that allowing for heteroskedasticity was indeed necessary.

Rather than using maximum likelihood to fit the outcome model, you can instruct teffects aipw to use a weighted nonlinear least-squares (WNLS) estimator instead. The WNLS estimator may be more robust to outcome model misspecification.
Example 4: Using the WNLS estimator

Here we use WNLS to fit our outcome model:

```
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects aipw (bweight prenatal1 mmarried mage fbaby)
> (mbsmoke mmarried c.mage##c.mage fbaby medu, probit), wnls
```

Iteration 0:  EE criterion = 2.742e-20
Iteration 1:  EE criterion = 3.436e-24

```
Treatment-effects estimation  Number of obs =  4,642
Estimator : augmented IPW
Outcome model : linear by WNLS
Treatment model: probit
```

| bweight | Robust Coefficient | std. err. | z     | P>|z|  | [95% conf. interval] |
|---------|--------------------|-----------|-------|------|---------------------|
|        | ATE                |           |       |      |                     |
|        | mbsmoke            |           |       |      |                     |
|        | (Smoker vs Non smoker) |         |       |      |                     |
|        | -227.1956          | 27.34794  | -8.31 | 0.000| -280.7966 -173.5946 |
|        | POmean             |           |       |      |                     |
|        | mbsmoke            |           |       |      |                     |
|        | Non smoker         |           |       |      |                     |
|        | 3403.251           | 9.596622  | 354.63 | 0.000| 3384.442 3422.06   |

The ATE of $-227$ is slightly greater than the ATE of $-231$ estimated in example 1. The estimated POMs are nearly indistinguishable.

Video example

Treatment effects: Augmented inverse-probability weighting
Stored results

teffects aipw stores the following in e():

Scalars
- e(N): number of observations
- e(nj): number of observations for treatment level j
- e(N_clust): number of clusters
- e(k_eq): number of equations in e(b)
- e(k_levels): number of levels in treatment variable
- e(treated): level of treatment variable defined as treated
- e(control): level of treatment variable defined as control
- e(converged): 1 if converged, 0 otherwise

Macros
- e(cmd): teffects
- e(cmdline): command as typed
- e(depvar): name of outcome variable
- e(tvar): name of treatment variable
- e(subcmd): aipw
- e(tmodel): logit, probit, or hetprobit
- e(omodel): linear, logit, probit, hetprobit, poisson, flogit, fprobit, or fhetprobit
- e(stat): statistic estimated, ate or pomeans
- e(vtype): weight type
- e(wexp): weight expression
- e(title): title in estimation output
- e(clustvar): name of cluster variable
- e(tlevels): levels of treatment variable
- e(cme): ml, nls, or wnls
- e(vce): vcetype specified in vce()
- e(vcetype): title used to label Std. err.
- e(properties): b V
- e(estat_cmd): program used to implement estat
- e(predict): program used to implement predict
- e(marginsnotok): predictions disallowed by margins
- e(asbalanced): factor variables fvset as asbalanced
- e(asobserved): factor variables fvset as asobserved

Matrices
- e(b): coefficient vector
- e(V): variance–covariance matrix of the estimators

Functions
- e(sample): marks estimation sample

In addition to the above, the following is stored in r():

Matrices
- r(table): matrix containing the coefficients with their standard errors, test statistics, \( p \)-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

The methods and formulas presented here provide the technical details underlying the estimators implemented in teffects ra, teffects ipw, teffects aipw, and teffects ipwra. See Methods and formulas of [CAUSAL] teffects nmmatch for the methods and formulas used by teffects nmmatch and teffects psmatch.
Methods and formulas are presented under the following headings:

Parameters and notation
Overview of EE estimators
VCE for EE estimators
TM and OM estimating functions
  TM estimating functions
  OM estimating functions
Effect estimating functions
  RA estimators
  IPW estimators
  AIPW estimators
  IPWRA estimators

Parameters and notation

We begin by reviewing the effect parameters estimated by `teffects` and some essential notation.

The potential outcome that an individual would obtain if given treatment level \( t \in \{0, 1, \ldots, q\} \) is \( y_t \). Each \( y_t \) is a random variable, the realizations of which are \( y_{ti} \). Throughout this document, \( i \) subscripts denote realizations of the corresponding, unsubscripted random variables.

The three parameters of interest are

1. the potential-outcome mean (POM) \( \alpha_t = E(y_t) \);
2. the average treatment effect (ATE) \( \tau_t = E(y_t - y_0) \); and
3. the average treatment effect on the treated (ATET) \( \delta_t = E(y_t - y_0 | t = \tilde{t}) \).

The no-treatment level is 0.

The estimators implemented in `teffects` use three assumptions to justify the equations used for estimation and inference about the effect parameters of interest:

1. Conditional mean independence (CMI) allows us to estimate potential-outcome means from the observed outcomes in the sample.
2. Overlap ensures that we have data on each type of individual in each treatment level.
3. Independent observations ensure that the outcome and treatment for one individual has no effect on the outcome or treatment for any other individual.

`teffects ra` implements some regression-adjustment (RA) estimators; `teffects ipw` implements some inverse-probability-weighted (IPW) estimators; `teffects ipwra` implements some inverse-probability-weighted regression-adjustment (IPWRA) estimators; and `teffects aipw` implements some augmented inverse-probability-weighted (AIPW) estimators. All are implemented as estimating-equation (EE) estimators. The estimators are consistent and asymptotically normally distributed under the CMI, overlap, and independence assumptions.

Overview of EE estimators

EE estimators compute estimates by solving sample estimating equations. The sample estimating equations are the sample equivalents of population expectation equations.

Each EE estimator specifies a set of estimating equations for the effect parameters of interest and a set of estimating equations for the auxiliary parameters in the outcome model (OM) or the treatment model (TM). The next few sections provide tremendous detail about the estimating equations that define the RA, IPW, AIPW, and IPWRA estimators.
Ignoring the details for a moment, EE estimators solve systems of equations to compute estimates. A standard robust estimator is consistent for the variance of the estimator (VCE). All the details involve the equations specified by choices of estimator and functional forms for the OM or TM.

When used, the OM is a model for the conditional mean of the outcome variable. We let μ(x, t, βt) denote a conditional-mean model for the outcome y conditional on covariates x and treatment level t. Mathematically, \( E(y|x, t) = \mu(x, t, \beta_t) \), where \( \beta_t \) are the parameters of the conditional-mean model given treatment level t. The table below provides details about the available functional forms.

<table>
<thead>
<tr>
<th>Outcome model</th>
<th>Functional form for ( \mu(x, t, \beta_t) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>( x\beta_t )</td>
</tr>
<tr>
<td>logit, flogit</td>
<td>( \exp(x\beta_t)/(1 + \exp(x\beta_t)) )</td>
</tr>
<tr>
<td>probit, fprobit</td>
<td>( \Phi(x\beta_t) )</td>
</tr>
<tr>
<td>poisson</td>
<td>( \exp(x\beta_t) )</td>
</tr>
<tr>
<td>hetprobit, fhetprobit</td>
<td>( \Phi{x\beta_t/\exp(x\beta_t)} )</td>
</tr>
</tbody>
</table>

In the cases of hetprobit and fhetprobit, we use \( \hat{x} \) and \( \hat{\beta}_t \) to denote the variables and parameters in the index function, and we use \( \ddot{x} \) and \( \ddot{\beta}_t \) to denote the variables and parameters in the variance equation. We define \( x' = (\dot{x}', \ddot{x}') \) and \( \beta'_t = (\dot{\beta}_t', \ddot{\beta}_t') \).

We write the vector of parameters for the outcome model over all treatment levels as \( \beta' = (\beta'_0, \beta'_1, \ldots, \beta'_q) \).

Next we provide details about the estimating equations implied by each functional form choice.

When used, the TM is a model for the conditional probability of treatment. We let \( p(z, t, \gamma) \) denote the conditional probability model for the probability that a person receives treatment t, conditional on covariates \( z \). The table below provides details about the functional form options allowed in the case of a binary treatment.

<table>
<thead>
<tr>
<th>Treatment model</th>
<th>Functional form for ( p(z, t, \gamma) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>logit</td>
<td>( \exp(z\gamma)/(1 + \exp(z\gamma)) )</td>
</tr>
<tr>
<td>probit</td>
<td>( \Phi(z\gamma) )</td>
</tr>
<tr>
<td>hetprobit</td>
<td>( \Phi{z'\gamma/\exp(z'\gamma)} )</td>
</tr>
</tbody>
</table>

In the case of hetprobit, we use \( \hat{z} \) and \( \hat{\gamma} \) to denote the variables and parameters in the index function, and we use \( \ddot{z} \) and \( \ddot{\gamma} \) to represent the variables and parameters in the variance equation. We define \( z' = (\dot{z}', \ddot{z}') \), and \( \gamma' = (\dot{\gamma}', \ddot{\gamma}') \).

In the multivalued-treatment case, \( p(z, t, \gamma) \) is specified as a multinomial logit with \( p(z, t, \gamma) = \exp(z\gamma_t)/(1 + \sum_{k=1}^q \exp(z\gamma_k)) \) and \( \gamma' = (\gamma'_1, \gamma'_2, \ldots, \gamma'_q) \). (We present formulas for the case with treatment level 0 as the base with \( \gamma'_0 = 0' \); see [R] mlogit for background.) In teffects, the logit option in the treatment-model specification means binary logit for the binary-treatment case and multinomial logit for the multivalued-treatment case: this simplifies the use of the command and makes statistical sense.

Below we provide details about the estimating equations implied by each functional form choice. The effect parameters of interest are

1. the POMs denoted by \( \alpha' = (\alpha_0, \alpha_1, \ldots, \alpha_q) \);
2. the ATEs denoted by \( \tau' = (\tau_1, \tau_2, \ldots, \tau_q) \); and
3. the ATETs denoted by \( \delta' = (\delta_1, \delta_2, \ldots, \delta_q) \).
We denote the effect parameters by $\vartheta$ and all the parameters in any particular case by $\theta$. More formally, $\theta$ is the concatenation of the effect parameters, the OM parameters, and the TM parameters; $\theta' = (\vartheta', \beta', \gamma')$, where $\vartheta$ is $\alpha$, $\tau$, or $\delta$, and $\beta$ or $\gamma$ may not be present, depending on the case at hand.

In the subsections below, we discuss estimators for the elements in $\theta$ in detail and note how these elements change over the cases defined by effect parameters and estimators. The parameter vector $\theta$ denotes all the parameters, no matter which particular case is under consideration.

The EE estimators described in this section are defined by a set of equations,

$$E\{s(x, z, \theta)\} = 0$$

where $s(x, z, \theta)$ is a vector of estimating functions. Note the notation: estimating equations equate the expected value of a vector of estimating functions to zero.

Because each of the estimating functions has mean zero, we can construct estimators that find the estimates $\hat{\theta}$ by solving a system of equations,

$$\frac{1}{N}\sum_{i} s_i(x_i, z_i, \hat{\theta}) = 0$$

where $s_i(x_i, z_i, \hat{\theta})$ are the sample realizations of the estimating functions. In other words, the parameter estimates set the average of the realizations of each estimating function to zero. Almost all the details below involve specifying the sample realizations $s_i(x_i, z_i, \hat{\theta})$.

Estimators that set the expected value of estimating functions to zero are known as estimating-equations (EE) estimators, M estimators, or Z estimators in the statistics literature and as generalized method of moments (GMM) estimators in the econometrics literature. See van der Vaart (1998, 41), Stefanski and Boos (2002), and Tsiatis (2006, sec. 3.2) for statistics; and see Wooldridge (2010, chap. 14), Cameron and Trivedi (2005, chap. 6), and Newey and McFadden (1994) for econometrics.

We refer to them as EE estimators because this name is closest to being independent of any discipline. The estimators are implemented using gmm because they are exactly identified generalized method of moments (GMM) estimators when weights are specified by the user, they are applied to the estimating equations just as gmm applies user-specified weights.

Each estimator has a set of estimating equations for the effect parameters and either an OM or a TM, or both. The OM parameters or the TM parameters are auxiliary parameters used to estimate the effect parameters of interest.

Each set of parameters has its own set of sample estimating equations:

$$\frac{1}{N}\sum_{i} s_{e,i}(x_i, z_i, \hat{\theta}) = 0$$

are the sample estimating equations for the effect parameters. These equations determine the effect parameter estimates $\hat{\vartheta}$ as functions of the data and the other estimated parameters.

$$\frac{1}{N}\sum_{i} s_{om,i}(x_i, w_i, \hat{\beta}) = 0$$

are the sample estimating equations for OM parameters that use the weights $w_i$, which are functions of the TM parameters.

$$\frac{1}{N}\sum_{i} s_{tm,i}(z_i, \hat{\gamma}) = 0$$

are the sample estimating equations for TM parameters.

The whole set of sample estimating functions is $s_i(x_i, z_i, \hat{\theta})$ with

$$s_i(x_i, z_i, \hat{\theta})' = (s_{e,i}(x_i, z_i, \hat{\theta})', s_{om,i}(x_i, w_i(t), \hat{\beta})', s_{tm,i}(z_i, \hat{\gamma}))'$$

although not all the estimators have each of three components.
VCE for EE estimators

The Huber/White/robust sandwich estimator is consistent for the variance–covariance of the estimator (VCE). See van der Vaart (1998, 41), Stefanski and Boos (2002), and Tsiatis (2006, sec. 3.2) for statistics; and see Wooldridge (2010, chap. 14), Cameron and Trivedi (2005, chap. 6), and Newey and McFadden (1994) for econometrics.

The formula is

$$\hat{V} = (1/N) \bar{G} \bar{S} \bar{G}'$$

where

$$\bar{G} = \left\{ (1/N) \sum_i \frac{\partial s_i(x_i, z_i, \hat{\theta})}{\partial \hat{\theta}} \right\}^{-1}$$

and

$$\bar{S} = (1/N) \sum_i s_i(x_i, z_i, \hat{\theta})'$$

The matrix $\bar{G}$ is not symmetric because our EE estimators come from stacking moment conditions instead of optimizing a single objective function. The implication is that the robust formula should always be used because, even under correct specification, the nonsymmetric $\bar{G}$ and the symmetric $\bar{S}$ converge to different matrices.

TM and OM estimating functions

Although the sample estimating functions for the effect parameters, the $s_{e,i}(x_i, z_i, \hat{\theta})$, are estimator specific, the sample estimating functions for the TM parameters, the $s_{tm,i}(z_i, \hat{\gamma})$, and the sample estimating functions for the OM parameters, the $s_{om,i}(x_i, w_i(t), \hat{\beta})'$, are used in multiple estimators. We provide details about the TM and the OM sample estimating functions here.

TM estimating functions

The sample estimating functions used to estimate the parameters of the TM $p(z, t, \gamma)$ are the sample score equations from the quasimaximum likelihood (QML) estimator.

In the binary-treatment case, $p(z, t, \gamma)$ may be logit, probit, or heteroskedastic probit. In the multivalued-treatment case, $p(z, t, \gamma)$ is a multinomial logit. We now give formulas for the $s_{tm,i}(z_i, \hat{\gamma})$ for each case.

logit and probit

In the logit and probit cases,

$$s_{tm,i}(z_i, \hat{\gamma}) = \left[ g(z_i \hat{\gamma}) \left\{ t_i - G(z_i \hat{\gamma}) \right\} \right] \frac{G(z_i \hat{\gamma}) \left\{ 1 - G(z_i \hat{\gamma}) \right\}}{G(z_i \hat{\gamma})}$

where $G(z)$ is the logistic cumulative distribution function for the logit, $G(z)$ is the normal cumulative distribution function for the probit, and $g(\cdot) = \{ \partial G(z) \}/(\partial z)$ is the corresponding density function.
hetprobit

In the hetprobit case, there are two sets of sample score equations, \( s_{tm,1,i}(z_i, \hat{\gamma}) \) and \( s_{tm,2,i}(z_i, \hat{\gamma}) \):

\[
\begin{aligned}
    s_{tm,1,i}(z_i, \hat{\gamma}) &= \left( \frac{\phi (z_i, \hat{\gamma}) [t_i - \Phi (z_i, \hat{\gamma})]}{\Phi (z_i, \hat{\gamma}) [1 - \Phi (z_i, \hat{\gamma})] \exp(\tilde{z}_i \hat{\gamma})} \right) \tilde{z}_i' \\
    s_{tm,2,i}(z_i, \hat{\gamma}) &= \left( \frac{\phi (z_i, \hat{\gamma}) \hat{z}_i \hat{\gamma}' [\Phi (z_i, \hat{\gamma}) - t_i]}{\Phi (z_i, \hat{\gamma}) [1 - \Phi (z_i, \hat{\gamma})] \exp(\tilde{z}_i \hat{\gamma})} \right) \tilde{z}_i'
\end{aligned}
\]

where \( \phi(\cdot) \) is the standard normal density function, and \( q(z_i, \hat{\gamma}) = \left( \hat{z}_i \hat{\gamma}' / \exp(\tilde{z}_i \hat{\gamma}) \right) \).

mlogit

In the mlogit case, \( p(z, t, \gamma) = \exp(z \gamma_i) / \{1 + \sum_{k=1}^{q} \exp(z \gamma_k)\} \). We present formulas for the case with treatment level 0 as the base with \( \gamma_0 = 0 \); see \([R]\) mlogit for background.

There are \( q \) vectors of sample estimating functions for the mlogit case, \( s_{tm,k,i}(z_i, \hat{\gamma}) \) for each \( k \in \{1, \ldots, q\} \), 1 for each vector \( \hat{\gamma}_k \), \( k \in \{1, \ldots, q\} \). These sample estimating functions are

\[
s_{tm,k,i}(z_i, \hat{\gamma}) = \begin{cases} 
    \{1 - p(z_i, k, \hat{\gamma})\} z_i' & t_i = k \\
    -p(z_i, k, \hat{\gamma}) z_i' & \text{otherwise}
\end{cases}
\]

OM estimating functions

The parameters of the OM \( \mu(x, t, \beta_t) \) are estimated either by weighted QML or by weighted nonlinear least squares. The estimating functions used to estimate the parameters of the OM are either the score equations from the weighted QML estimator or the moment conditions for the weighted nonlinear least-squares estimator.

The estimating functions for the OM parameters in \( \mu(x, t, \beta_t) \) vary over the models for the conditional mean because \( \mu(x, t, \beta_t) \) may be linear, logit, probit, heteroskedastic probit, or poisson.

Let \( N_t \) be the number of observations in treatment level \( t \), and let \( t_i(t) = 1 \) if \( t_i = t \), with \( t_i(t) = 0 \) if \( t_i \neq t \).

There are two sets of sample estimating functions for the OM parameters with weights \( w_i(t) \):

1. \( s_{ml,om,i}\{x_i, w_i(t), \hat{\beta}_t\} \) are the sample estimating functions for the weighted QML estimator.
2. \( s_{nls,om,i}\{x_i, w_i(t), \hat{\beta}_t\} \) are the sample estimating functions for the weighted nonlinear least-squares estimator.

OM QML

Here are the formulas for the \( s_{ml,om,i}\{x_i, w_i(t), \hat{\beta}_t\} \) for each functional form choice.
linear

In the linear case,

\[ s_{\text{ml,om},i}\{x_i, w_i(t), \hat{\beta}_t \} = w_i(t) t_i(t) (y_i - x_i \hat{\beta}_t) x'_i \]

logit, flogit, probit, and fprobit

In the logit, flogit, probit, and fprobit cases,

\[ s_{\text{ml,om},i}\{x_i, w_i(t), \hat{\beta}_t \} = w_i(t) t_i(t) \left[ \frac{g(x_i \hat{\beta}_t)}{G(x_i \hat{\beta}_t)} \left\{ \frac{y_i - G(x_i \hat{\beta}_t)}{1 - G(x_i \hat{\beta}_t)} \right\} \right] x_i \]

where \( G(z) \) is the logistic cumulative distribution function for the logit and flogit, \( G(z) \) is the normal cumulative distribution function for the probit and fprobit, and \( g(\cdot) = \{\partial G(\cdot)\}/(\partial z) \) is the corresponding density function.

hetprobit and fhetprobit

In the hetprobit and fhetprobit cases, there are two sets of sample score equations, \( s_{\text{ml,om},1,i}\{x_i, w_i(t), \hat{\beta}_t \} \) and \( s_{\text{ml,om},2,i}\{x_i, w_i(t), \hat{\beta}_t \} \):

\[ s_{\text{ml,om},1,i}\{x_i, w_i(t), \hat{\beta}_t \} = w_i(t) t_i(t) \left( \frac{\phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \left[ y_i - \Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \right]}{\Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \left[ 1 - \Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \right]} \right) x'_i \]

and

\[ s_{\text{ml,om},2,i}\{x_i, w_i(t), \hat{\beta}_t \} = w_i(t) t_i(t) \left( \frac{\phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \left[ x_i \hat{\beta}_t \left[ \Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} - y_i \right] \right]}{\Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \left[ 1 - \Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \right]} \right) x'_i \]

where \( \phi(\cdot) \) is the standard normal density function, \( s_{\text{ml,om},i}\{x_i, w_i(t), \hat{\beta}_t \}' = [s_{\text{ml,om},1,i}\{x_i, w_i(t), \hat{\beta}_t \}', s_{\text{ml,om},2,i}\{x_i, w_i(t), \hat{\beta}_t \}'] \), and \( q \left( x_i, \hat{\beta}_t \right) \) is \( \frac{\hat{\beta}_t / \exp(\hat{\beta}_t)}{\exp(\hat{\beta}_t)} \).

poisson

In the poisson case,

\[ s_{\text{ml,om},i}\{x_i, w_i(t), \hat{\beta}_t \} = w_i(t) t_i(t) (y_i - \exp(x_i \hat{\beta}_t)) x'_i \]

OM WNL

Here are the formulas for the \( s_{\text{nls,om},i}\{x_i, w_i(t), \hat{\beta}_t \} \) for each functional form choice.
linear

In the linear case,

\[ s_{\text{nls,om},i}\{x_i, w_i(t), \hat{\beta}_t\} = w_i(t)t_i(t)(y_i - x_i\hat{\beta}_t)x'_i \]

logit, flogit, probit, and fprobit

In the logit, flogit, probit, and fprobit cases,

\[ s_{\text{nls,om},i}\{x_i, w_i(t), \hat{\beta}_t\} = w_i(t)t_i(t) \left[ g(x_i\hat{\beta}_t) \left\{ y_i - G(x_i\hat{\beta}_t) \right\} \right] x_i \]

where \( G(z) \) is the logistic cumulative distribution function for the logit and flogit, \( G(z) \) is the normal cumulative distribution function for the probit and fprobit, and \( g(\cdot) = \{\partial G(z)\}/(\partial z) \) is the corresponding density function.

hetprobit and fhetprobit

In the hetprobit and fhetprobit cases, there are two sets of sample score equations, \( s_{\text{nls,om},1,i}\{x_i, w_i(t), \hat{\beta}_t\} \) and \( s_{\text{nls,om},2,i}\{x_i, w_i(t), \hat{\beta}_t\} \):

\[ s_{\text{nls,om},1,i}\{x_i, w_i(t), \hat{\beta}_t\} = w_i(t)t_i(t) \left( \frac{\phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} }{\exp(\hat{x}_i\hat{\beta}_t)} \left[ y_i - \Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} \right] \right) \hat{x}_i' \\
and \]

\[ s_{\text{nls,om},2,i}\{x_i, w_i(t), \hat{\beta}_t\} = w_i(t)t_i(t) \left( \frac{\phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} }{\exp(\hat{x}_i\hat{\beta}_t)} \hat{x}_i\hat{\beta}_t' \left[ \Phi \left\{ q \left( x_i, \hat{\beta}_t \right) \right\} - y_i \right] \right) \hat{x}_i' \]

where \( \phi(\cdot) \) is the standard normal density function, \( s_{\text{nls,om},1}\{x_i, w_i(t), \hat{\beta}_t\}' = [s_{\text{nls,om},1,i}\{x_i, w_i(t), \hat{\beta}_t\}', s_{\text{nls,om},2,i}\{x_i, w_i(t), \hat{\beta}_t\}'] \), and \( q \left( x_i, \hat{\beta}_t \right) = \left( \hat{x}_i\hat{\beta}_t'/\exp(\hat{x}_i\hat{\beta}_t) \right) \).

poisson

In the poisson case,

\[ s_{\text{nls,om},i}\{x_i, w_i(t), \hat{\beta}_t\} = w_i(t)t_i(t)\{y_i - \exp(x_i\hat{\beta}_t)\} \exp(x_i\hat{\beta}_t)x'_i \]

Effect estimating functions

We now describe the sample estimating functions for the effect parameters, which vary over estimator and effect parameter.
RA estimators

RA estimators estimate the effect parameters using means of the observation-level predictions of the conditional means of the outcomes. There is no model for the conditional probability of treatment.

The RA estimators use unweighted QML estimators to estimate the parameters of the conditional mean model. In other words, the RA estimators use the sample estimating functions $s_{\text{ml},om,i}(x_i, 1, \hat{\beta})$, given above.

For the RA estimators, the vector of sample estimating functions is the concatenation of the sample estimating functions for the effect parameters with the sample estimating functions for the OM parameters. Algebraically,

$$s_{\text{ra},i}(x_i, \hat{\theta})' = s_{\text{ra},e,i}(x_i, \hat{\theta}, \hat{\beta})', s_{\text{ml},om,i}(x_i, 1, \hat{\beta})'$$

The estimating functions $s_{\text{ra},e,i}(x_i, \hat{\theta}, \hat{\beta})'$ vary over the effect parameter.

RA for POM

The RA estimators for the POM parameters estimate $\theta' = (\alpha', \beta')$ using two types of estimating equations: 1) those for the POM parameters $\alpha$, and 2) those for the conditional-mean model parameters $\beta_t$ in $\mu(x, t, \beta_t)$.

The sample estimating functions for the $\hat{\beta}_t$ are given in $OM$ estimating functions above.

The elements of $s_{\text{ra},e,i}(x_i, \hat{\alpha}, \hat{\beta})$ for the POM parameters $\alpha$ are given by

$$\mu(x_i, t, \hat{\beta}_t) - \hat{\alpha}_t$$

(RAPOM)

RA for ATE

The RA estimators for the ATE parameters estimate $\theta' = (\tau', \beta')$ using two types of estimating equations: 1) those for the ATE parameters $\tau$, and 2) those for the OM parameters $\beta_t$ in $\mu(x, t, \beta_t)$.

The sample estimating functions that determine the $\hat{\beta}_t$ are given in $OM$ estimating functions with $w_i(t) = 1$.

The elements of $s_{\text{ra},e,i}(x_i, \hat{\tau}, \hat{\beta})$ for the ATE parameters $\tau$ are given by

$$\mu(x_i, t, \hat{\beta}_t) - \mu(x_i, 0, \hat{\beta}_t) - \hat{\tau}_t$$

(RAATE)

RA for ATET

The RA estimators for the ATET parameters estimate $\theta' = (\delta', \beta')$ using two types of estimating equations: 1) those for the ATET parameters $\delta$, and 2) those for the OM parameters $\beta_t$ in $\mu(x, t, \beta_t)$.

The sample estimating functions that determine the $\hat{\beta}_t$ are given in $OM$ estimating functions above with $w_i(t) = 1$.

The elements of $s_{\text{ra},e,i}(x_i, \hat{\delta}, \hat{\beta})$ for the ATET parameters $\delta$ are given by

$$\frac{N_{t_i}(\tilde{t})}{N_{\tilde{t}}} \left\{ \mu(x_i, t, \hat{\beta}_t) - \mu(x_i, 0, \hat{\beta}_t) - \delta_t \right\}$$

(RAATET)
IPW estimators

IPW estimators estimate the effect parameters using means of the observed outcomes weighted by the inverse probability of treatment. There is no outcome model. The IPW estimators use QML estimators to estimate the parameters of the conditional probability model.

The vector of estimating functions is the concatenation of the estimating functions for the effect parameters with the estimating functions for the conditional-probability parameters. The sample estimating functions used by the IPW estimators are

\[ s_{ipw,i}(x_i, \hat{\theta})' = s_{ipw,e,i}(x_i, \hat{\theta}, \hat{\gamma})', s_{tm,i}(z_i, 1, \hat{\gamma})' \]

The estimating functions \( s_{ipw,e,i}(z_i, \hat{\theta}, \hat{\gamma})' \) vary over the effect parameter.

All the IPW estimators use normalized inverse-probability weights. These weights are not related to the weights \( w_i(t) \) that were used in the OM equations. The functional form for the normalized inverse-probability weights varies over the effect parameters POM, ATE, and ATET.

The POM and ATE estimators use normalized inverse-probability weights. The unnormalized weights for individual \( i \) and treatment level \( t \) are \( d_i(t) = t_i(t)/p(z_i, t, \hat{\gamma}) \), and the normalized weights are \( \bar{d}_i(t) = N_t d_i(t)/\sum_i^N d_i(t) \).

The ATET estimator uses normalized treatment-adjusted inverse-probability weights. The treatment-adjusted inverse-probability weights adjust the inverse-probability weights for the probability of getting the conditional treatment \( \bar{t} \). The unnormalized weights are \( f_i = p(z_i, \bar{t}, \hat{\gamma})/p(z_i, t_i, \hat{\gamma}) \), and the normalized weights are \( \bar{f}_i = N f_i/\sum_i^N f_i \).

The IPW effect estimators are weighted cell averages. The sample estimating functions used in POM estimators are the sample estimating functions from weighted OLS regression on treatment-cell indicators. The POM estimators use a full set of \( q + 1 \) of treatment indicator variables \( a_i \). (The \( i \)th observation on the \( k \)th variable in \( a_i \) is 1 if \( i \) received treatment \( k - 1 \) and 0 otherwise, for \( k \in \{1, 2, \ldots, q + 1\} \).

The sample estimating functions used in the ATE and the ATET estimators are the sample estimating functions from weighted OLS regression on treatment-cell indicators, excluding the indicator for the control level and including a constant term. The variables \( \tilde{a}_i \) used in the ATE and ATET sample estimating functions include \( q \) of treatment indicator variables and a variable that is always 1. (The first \( q \) variables in \( \tilde{a}_i \) are treatment indicators: the \( i \)th observation on the \( k \)th variable in \( \tilde{a}_i \) is 1 if \( i \) received treatment \( k \) and 0 otherwise, for \( k \in \{1, 2, \ldots, q\} \). The \( (q + 1) \)th variable is always 1.) This definition of \( \tilde{a}_i \) sets the last treatment level to be the control; renaming the treatments handles the more general case allowed for by tefects.

Having defined \( a_i \) and \( \tilde{a}_i \), we can give the sample estimating functions that the IPW estimators use for the effects parameters.

IPW for POM

We begin with the IPW estimators for the potential-outcome means. In this case, \( \theta' = (\alpha', \gamma') \).

The sample estimating functions for the \( \hat{\gamma} \) are given in TM estimating functions above.

The sample estimating functions for \( \hat{\alpha} \) are given by

\[ s_{ipw,e,i}(z_i, \hat{\alpha}, \hat{\gamma})' = \bar{d}_i(t) (y_i - a_i \hat{\alpha}) a_i' \] (IPWPOM)
### IPW for ATE

The full parameter vector for the IPW estimators for the ATE is $\theta' = (\tau', \gamma')$.

The sample estimating functions for the $\hat{\gamma}$ are given in TM estimating functions above.

The sample estimating functions for $\hat{\tau}$ are given by

$$s_{ipw,e,i,t}(z_i, \hat{\tau}, \hat{\gamma})' = \bar{d}_i(t)(y_i - \bar{a}_i \hat{\tau})\bar{a}_i'$$

### IPW for ATET

The full parameter vector for the IPW estimators for the ATET is $\theta' = (\delta', \gamma')$.

The sample estimating functions for the $\hat{\gamma}$ are given in TM estimating functions above.

The sample estimating functions for $\hat{\delta}$ are given by

$$s_{ipw,e,i,t}(z_i, \hat{\delta}, \hat{\gamma})' = \bar{f}_i(t)(y_i - \bar{a}_i \hat{\delta})\bar{a}_i'$$

### AIPW estimators

This section documents the sample estimating functions used by the augmented inverse-probability-weighted (AIPW) estimators implemented in `teffects`. These AIPW estimators are efficient-influence-function estimators as discussed in [CAUSAL] teffects intro and [CAUSAL] teffects intro advanced. The teffects implementation was primarily inspired by Cattaneo, Drukker, and Holland (2013), which was based on Cattaneo (2010). Tan (2010) was influential by identifying the implemented weighted nonlinear least-squares estimator as having relatively good properties when both the conditional mean function and the conditional probability function are misspecified.

The AIPW estimating functions for the treatment parameters include terms from a conditional probability model and from a conditional mean model for the outcome.

The sample-estimation-equations vector has three parts for the AIPW estimators:

$$s_{aipw,i}(x_i, z_i, \hat{\theta})' = [s_{aipw,e,i}(x_i, z_i, \hat{\theta})', s_{aipw,tm,i}(z_i, \hat{\gamma})', s_{aipw,om,i}(x_i, w_i(t), \hat{\beta})']$$

The sample estimating functions for the parameters of the TM are the $s_{tm,i}(z_i, \hat{\gamma})$ given in TM estimating functions above.

`teffects aipw` implements three AIPW estimators: the standard AIPW estimator, the NLS AIPW estimator, and the WNLS AIPW estimator.

The three AIPW estimators differ in how they estimate the parameters of the OM.

By default, `teffects aipw` uses the standard AIPW estimator that estimates the parameters of the OM by the unweighted ML estimator, whose sample estimating functions, $s_{ml,om,i}(x_i, 1, \hat{\beta})$, are given in OM estimating functions above.

When the nls option is specified, `teffects aipw` uses the NLS AIPW estimator that estimates the parameters of the OM by the unweighted NLS estimator, whose sample estimating functions, $s_{nls,om,i}(x_i, 1, \hat{\beta})$, are given in OM estimating functions above.
When the \texttt{wnls} option is specified, \texttt{teffects aipw} uses the WNLS AIPW estimator that estimates the parameters of the OM by the WNLS estimator, whose sample estimating functions, \( s_{\text{nlso,om},i} \{ x_i, w_i(t), \theta \} \), are given in \textit{OM estimating functions} above. The weights for person \( i \) in treatment level \( t \) are

\[
\tilde{w}_i(t) = \frac{t_i(t)}{p(z_i, t, \gamma)} \left\{ \frac{t_i(t)}{p(z_i, t, \gamma)} - 1 \right\} \quad \text{(WNLSW)}
\]

Now we discuss the sample estimating functions for the effect parameters, the \( s_{e,i} \{ x_i, z_i, \hat{\theta} \} \).

**AIPW for POM**

We begin with the AIPW estimators for the potential-outcome means. In this case, \( \theta' = (\alpha', \gamma', \beta') \), and the elements of \( s_{\text{aipw},e,i} \{ x_i, z_i, \hat{\theta} \} \) are given by

\[
\frac{t_i(t)}{p(z_i, t, \gamma)} y_i - \mu(x_i, \hat{\beta}_i) \left\{ \frac{t_i(t)}{p(z_i, t, \gamma)} - 1 \right\} - \alpha_t
\]

**AIPW for ATE**

The AIPW estimators for the ATE estimate \( \theta' = (\tau', \gamma', \beta') \), and the elements of \( s_{\text{aipw},e,i} \{ x_i, z_i, \hat{\theta} \} \) are given by

\[
\frac{t_i(t)}{p(z_i, t, \gamma)} y_i - \mu(x_i, \hat{\beta}_i) \left\{ \frac{t_i(t)}{p(z_i, t, \gamma)} - 1 \right\} - \tau_0 \text{ if } t = 0
\]

\[
\frac{t_i(t)}{p(z_i, t, \gamma)} y_i - \mu(x_i, \hat{\beta}_i) \left\{ \frac{t_i(t)}{p(z_i, t, \gamma)} - 1 \right\} - \tau_t - \tau_0 \text{ if } t > 0
\]

**IPWRA estimators**

The IPWRA estimators combine inverse-probability weighting (IPW) with regression-adjustment (RA) methods. The sample estimating functions for IPWRA include sample estimating functions from both RA and IPW. The vector of sample estimating functions is

\[
s_{\text{ipwra},i} \{ x_i, \hat{\theta}' \} = s_{\text{ra},e,i} \{ x_i, \hat{\theta}', \hat{\beta}' \}, s_{\text{ml,om},i} \{ x_i, w_i(j), \hat{\beta}' \}, s_{\text{tm},i} \{ z_i, \hat{\gamma}' \}
\]

where \( \hat{\theta}' = (\hat{\theta}', \hat{\beta}', \hat{\gamma}') \), \( \hat{\theta} = \hat{\alpha} \) for POM, \( \hat{\theta} = \hat{\tau}_t \) for ATE, and \( \hat{\theta} = \hat{\delta}_t \) for ATET. The sample estimating functions, \( s_{\text{ra},e,i} \{ x_i, \hat{\theta}', \hat{\beta}' \} \), for POM, ATE, and ATET are given in equations (RAPOM), (RAATE), and (RAATET). For the OM sample estimating functions, \( s_{\text{ml,om},i}(\cdot) \), we replace the RA unity weights, \( w_i(t) = 1 \), with \( d_i(j) \) for POM or ATE and \( f_i \) for ATET, the normalized inverse-probability weights described in \textit{IPW estimators} above. The specific form of the OM sample estimating functions are given in \textit{OM estimating functions} above. The TM sample estimating functions are given in \textit{TM estimating functions} above.
References


Also see

[C] tteffects — Treatment-effects estimation for observational data
[C] tteffects postestimation — Postestimation tools for tteffects
[C] tteffects ipwra — Inverse-probability-weighted regression adjustment
[U] 20 Estimation and postestimation commands
**Description**

tffects ipw estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational data by inverse-probability weighting (IPW). IPW estimators use estimated probability weights to correct for missing data on the potential outcomes. tffects ipw accepts a continuous, binary, count, fractional, or nonnegative outcome and allows a multivalued treatment.

See [CAUSAL] tffects intro or [CAUSAL] tffects intro advanced for more information about estimating treatment effects from observational data.

**Quick start**

ATE of binary `treat2` on `y` by IPW using a logistic model of `treat2` on `x` and `w`

```
tffects ipw (y) (treat2 x w)
```

Same as above, but estimate ATET

```
tffects ipw (y) (treat2 x w), atet
```

Same as above, but estimate potential-outcome means

```
tffects ipw (y) (treat2 x w), pomeans
```

ATE of `treat2` on `y` using heteroskedastic probit for `treat2` as a function of `x` and `w`

```
tffects ipw (y) (treat2 x w, hetprobit(x w))
```

ATE for treatment levels 2 and 3 of three-valued treatment `treat3`

```
tffects ipw (y) (treat3 x w)
```

Same as above, and specify that `treat3 = 3` is the control level

```
tffects ipw (y) (treat3 x w), control(3)
```

Same as above, specified using the label "MyControl" corresponding to `treat3 = 3`

```
tffects ipw (y) (treat3 x w), control(MyControl)
```

**Menu**

Statistics > Causal inference/treatment effects > Continuous outcomes > Inverse-probability weighting (IPW)

Statistics > Causal inference/treatment effects > Binary outcomes > Inverse-probability weighting (IPW)

Statistics > Causal inference/treatment effects > Count outcomes > Inverse-probability weighting (IPW)

Statistics > Causal inference/treatment effects > Fractional outcomes > Inverse-probability weighting (IPW)

Statistics > Causal inference/treatment effects > Nonnegative outcomes > Inverse-probability weighting (IPW)
### Syntax

```
tffects ipw (ovar) (tvar tmvarlist [, tmodel noconstant]) [if] [in] [weight]

[ , stat options ]
```

*ovar* is a binary, count, continuous, fractional, or nonnegative outcome of interest.

*tvar* must contain integer values representing the treatment levels.

*tmvarlist* specifies the variables that predict treatment assignment in the treatment model.

#### tmodel

<table>
<thead>
<tr>
<th>Description</th>
<th>tmodel</th>
</tr>
</thead>
<tbody>
<tr>
<td>logistic treatment model; the default</td>
<td>logit</td>
</tr>
<tr>
<td>probit treatment model</td>
<td>probit</td>
</tr>
<tr>
<td>heteroskedastic probit treatment model</td>
<td>hetprobit(varlist)</td>
</tr>
</tbody>
</table>

*tmodel* specifies the model for the treatment variable.

For multivalued treatments, only *logit* is available and multinomial logit is used.

#### stat

<table>
<thead>
<tr>
<th>Description</th>
<th>stat</th>
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</thead>
<tbody>
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<td>ate</td>
</tr>
<tr>
<td>estimate average treatment effect on the treated</td>
<td>atet</td>
</tr>
<tr>
<td>estimate potential-outcome means</td>
<td>pomeans</td>
</tr>
</tbody>
</table>

#### options

<table>
<thead>
<tr>
<th>Description</th>
<th>options</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>vcetype</em> may be <em>robust</em>, <em>cluster clustvar</em>, <em>bootstrap</em>, or <em>jackknife</em></td>
<td>vce(vcetype)</td>
</tr>
<tr>
<td>set confidence level; default is level(95)</td>
<td>level(#)</td>
</tr>
<tr>
<td>display auxiliary-equation results</td>
<td>aequations</td>
</tr>
<tr>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
<td>display_options</td>
</tr>
<tr>
<td>control the maximization process; seldom used</td>
<td>maximize_options</td>
</tr>
<tr>
<td>set tolerance for overlap assumption</td>
<td>pstolerance(#)</td>
</tr>
<tr>
<td>newvar identifies observations that violate the overlap assumption</td>
<td>osample(newvar)</td>
</tr>
<tr>
<td>specify the level of <em>tvar</em> that is the control</td>
<td>control(#</td>
</tr>
<tr>
<td>specify the level of <em>tvar</em> that is the treatment</td>
<td>tlevel(#</td>
</tr>
<tr>
<td>display legend instead of statistics</td>
<td>coeflegend</td>
</tr>
</tbody>
</table>
teffects ipw — Inverse-probability weighting

`tvarlist` may contain factor variables; see [U] 11.4.3 Factor variables.

Bootstrap, by, collect, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

display does not appear in the dialog box.
See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

**Options**

---

**Model**

`noconstant`; see [R] Estimation options.

---

**Stat**

`stat` is one of three statistics: `ate`, `atet`, or `pomeans`. `ate` is the default.

- `ate` specifies that the average treatment effect be estimated.
- `atet` specifies that the average treatment effect on the treated be estimated.
- `pomeans` specifies that the potential-outcome means for each treatment level be estimated.

---

**SE/Robust**

`vce(vcetype)` specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (`.robust`), that allow for intragroup correlation (`.cluster clustvar`), and that use bootstrap or jackknife methods (`.bootstrap`, `.jackknife`); see [R] vce_option.

---

**Reporting**

`level(#)`; see [R] Estimation options.

`aequations` specifies that the results for the outcome-model or the treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

`display_option`:`s` are `.noci`, `.nopvalues`, `.noomitted`, `.vsquish`, `.noemptycells`, `.baselevels`, `.allbaselevels`, `.noflabel`, `.fwwrap(#), .fwwrap(style), .cformat(%fmt), .pformat(%fmt), .sformat(%fmt), and .nolstretch`; see [R] Estimation options.

---

**Maximization**

`maximize_options`: `.iterate(#), [no]log, and from(init_specs)`; see [R] Maximize. These options are seldom used.

- `init_specs` is one of
  - `matname [, skip copy]`
  - `# [, # ...], copy`

---

**Advanced**

`pstolerance(#)` specifies the tolerance used to check the overlap assumption. The default value is `pstolerance(1e-5)`. `teffects` will exit with an error if an observation has an estimated propensity score smaller than that specified by `pstolerance()`.

`osample(newvar)` specifies that indicator variable `newvar` be created to identify observations that violate the overlap assumption.
control(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with statistic pomeans. control() and tlevel() may not specify the same treatment level.

tlevel(# | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

The following option is available with teffects ipw but is not shown in the dialog box:

coefflegend; see [R] Estimation options.

Remarks and examples

Remarks are presented under the following headings:

Overview
Video example

Overview

IPW estimators use estimated probability weights to correct for the missing-data problem arising from the fact that each subject is observed in only one of the potential outcomes. IPW estimators use a two-step approach to estimating treatment effects:

1. They estimate the parameters of the treatment model and compute the estimated inverse-probability weights.
2. They use the estimated inverse-probability weights to compute weighted averages of the outcomes for each treatment level. The contrasts of these weighted averages provide the estimates of the ATEs. Using this weighting scheme corrects for the missing potential outcomes.

These steps produce consistent estimates of the effect parameters because the treatment is assumed to be independent of the potential outcomes after conditioning on the covariates. The overlap assumption ensures that predicted inverse-probability weights do not get too large. In fact, teffects ipw uses an estimation technique that implements both steps at once so that we do not need to correct the standard errors in the second step to reflect the uncertainty associated with the predicted treatment probabilities.

We will illustrate the use of teffects ipw by using data from a study of the effect of a mother’s smoking status during pregnancy (mbsmoke) on infant birthweight (bweight) as reported by Cattaneo (2010). This dataset also contains information about each mother’s age (mage), education level (medu), marital status (mmarried), whether the first prenatal exam occurred in the first trimester (prenatal1), and whether this baby was the mother’s first birth (fbaby).

Example 1: Estimating the ATE

We begin by using teffects ipw to estimate the average treatment effect of smoking on birthweight. We will use a probit model to predict treatment status, using prenatal1, mmarried, mage, the square of mage, and fbaby as explanatory variables:
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu, probit)
Iteration 0:  EE criterion = 4.621e-21
Iteration 1:  EE criterion = 7.358e-26
Treatment-effects estimation Number of obs = 4,642
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: probit

| bweight              | Robust Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|----------------------|--------------------|-----------|------|-------|---------------------|
| ATE                  |                    |           |      |       |                     |
| mbsmoke (Smoker vs Nonsmoker) | -230.6886 | 25.81524 | -8.94 | 0.000 | -281.2856 -180.0917 |
| POmean               |                    |           |      |       |                     |
| mbsmoke Nonsmoker    | 3403.463 | 9.571369  | 355.59 | 0.000 | 3384.703 3422.222  |

The average birthweight if all mothers were to smoke would be 231 grams less than the average of 3,403 grams that would occur if none of the mothers had smoked.

Sometimes, we are mainly concerned about those subjects that did in fact receive treatment, and we want to know how much the outcome changes as a result of treatment for that subpopulation. The ATET provides us with the answer. Moreover, the ATET can be estimated using weaker assumptions than are required to estimate the ATE; see [CAUSAL] teffects intro advanced.

Example 2: Estimating the ATET

. teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu, probit),
> atet
Iteration 0:  EE criterion = 4.636e-21
Iteration 1:  EE criterion = 6.467e-27
Treatment-effects estimation Number of obs = 4,642
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: probit

| bweight              | Robust Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|----------------------|--------------------|-----------|------|-------|---------------------|
| ATET                 |                    |           |      |       |                     |
| mbsmoke (Smoker vs Nonsmoker) | -225.1773 | 23.66458 | -9.52 | 0.000 | -271.559 -178.7955  |
| POmean               |                    |           |      |       |                     |
| mbsmoke Nonsmoker    | 3362.837 | 14.20149  | 236.79 | 0.000 | 3335.003 3390.671  |
The average birthweight is 225 grams less when all the mothers who smoke do so than the average of 3,363 grams that would have occurred if none of these mothers had smoked.

We often express statistics as percentages to alleviate scaling issues and aid interpretation. In the present context, we may wish to express an ATE as a percentage of the untreated POM to gain a more intuitive measure of the effect of treatment.

Example 3: Reporting the ATE as a percentage

Here we use the same model as in example 1, but we report the ATE as a percentage of the mean birthweight that would occur if no mothers smoke. First, we use `teffects ipw` to fit the model. We use the `coeflegend` option so that `teffects ipw` reports the names of the parameters. Then we use `nlcom` to obtain the statistic we want along with its delta-method-based standard error. We type

```stata
teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu, probit),
> coeflegend
```

Iteration 0: EE criterion = 4.621e-21
Iteration 1: EE criterion = 7.358e-26

```
Treatment-effects estimation Number of obs = 4,642
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: probit

<table>
<thead>
<tr>
<th>bweight</th>
<th>Coefficient Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATE</td>
<td></td>
</tr>
<tr>
<td>mbsmoke</td>
<td></td>
</tr>
<tr>
<td>(Smoker vs</td>
<td></td>
</tr>
<tr>
<td>Nonsmoker)</td>
<td></td>
</tr>
<tr>
<td>-230.6886</td>
<td>_b[ATE:r1vs0.mbsmoke]</td>
</tr>
<tr>
<td>POmean</td>
<td></td>
</tr>
<tr>
<td>mbsmoke</td>
<td></td>
</tr>
<tr>
<td>Nonsmoker</td>
<td></td>
</tr>
<tr>
<td>3403.463</td>
<td>_b[POmean:0.mbsmoke]</td>
</tr>
</tbody>
</table>

. nlcom _b[ATE:r1vs0.mbsmoke] / _b[POmean:0.mbsmoke]
:_nl_1: _b[ATE:r1vs0.mbsmoke] / _b[POmean:0.mbsmoke]

| bweight | Coefficient Std. err. z P>|z| [95% conf. interval] |
|---------|-----------------|----------------|--------------------|-----------------|
| _nl_1   | -.0677806   .0075169 -9.02 0.000 -.0825133 -.0530478 |
```

The average birthweight falls by an estimated 6.8% when every mother smokes relative to the case when no mothers smoke. We also obtain a 95% confidence interval of a 5.3% to 8.3% reduction.

Video example

Treatment effects: Inverse-probability weighting
Stored results

t/effects ipw stores the following in e():

Scalars
- e(N) number of observations
- e(n_j) number of observations for treatment level j
- e(N_clust) number of clusters
- e(k_eq) number of equations in e(b)
- e(k_levels) number of levels in treatment variable
- e(treated) level of treatment variable defined as treated
- e(control) level of treatment variable defined as control
- e(converged) 1 if converged, 0 otherwise

Macros
- e(cmd) teffects
- e(cmdline) command as typed
- e(depvar) name of outcome variable
- e(tvar) name of treatment variable
- e(subcmd) ipw
- e(tmodel) logit, probit, or hetprobit
- e(stat) statistic estimated, ate, atet, or pomeans
- e(wtype) weight type
- e(wexp) weight expression
- e(title) title in estimation output
- e(clustvar) name of cluster variable
- e(tlevels) levels of treatment variable
- e(vce) vcetype specified in vce()
- e(vcetype) title used to label Std. err.
- e(properties) b V
- e(estat_cmd) program used to implement estat
- e(predict) program used to implement predict
- e(marginsnotok) predictions disallowed by margins
- e(asbalanced) factor variables fvset as asbalanced
- e(asobserved) factor variables fvset as asobserved

Matrices
- e(b) coefficient vector
- e(V) variance–covariance matrix of the estimators

Functions
- e(sample) marks estimation sample

In addition to the above, the following is stored in r():

Matrices
- r(table) matrix containing the coefficients with their standard errors, test statistics, \(p\)-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

t/effects ipw implements a smooth treatment-effects estimator. All smooth treatment-effects estimators are documented in Methods and formulas of [CAUSAL] teffects aipw.
References


Also see

- [CAUSAL] `teffects postestimation` — Postestimation tools for `teffects`
- [CAUSAL] `teffects` — Treatment-effects estimation for observational data
- [U] 20 Estimation and postestimation commands
**teffects ipwra** estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational data by inverse-probability-weighted regression adjustment (IPWRA). IPWRA estimators use weighted regression coefficients to compute averages of treatment-level predicted outcomes, where the weights are the estimated inverse probabilities of treatment. The contrasts of these averages estimate the treatment effects. IPWRA estimators have the double-robust property. `teffects ipwra` accepts a continuous, binary, count, fractional, or nonnegative outcome and allows a multivalued treatment.

See [CAUSAL] `teffects intro` or [CAUSAL] `teffects intro advanced` for more information about estimating treatment effects from observational data.

**Quick start**

ATE of binary treatment `treat2` estimated by IPWRA using a linear model for outcome `y1` on `x1` and `x2` and a logistic model for `treat2` on `x1` and `w`

`teffects ipwra (y1 x1 x2) (treat2 x1 w)`

Same as above, but estimate the ATET

`teffects ipwra (y1 x1 x2) (treat2 x1 w), atet`

Probit model for binary outcome `y3`

`teffects ipwra (y3 x1 x2, probit) (treat2 x1 w)`

Same as above, but use a heteroskedastic probit model for `y3` and a probit model for `treat2`

`teffects ipwra (y3 x1 x2, hetprobit(x1 x2)) (treat2 x1 w, probit)`

Same as above, but use a fractional heteroskedastic probit model for `y4` and a probit model for `treat2`

`teffects ipwra (y4 x1 x2, fhetprobit(x1 x2)) (treat2 x1 w, probit)`

ATE for each level of a three-valued treatment `treat3`

`teffects ipwra (y1 x1 x2) (treat3 x1 w)`

Same as above, and specify that `treat3 = 3` is the control level using the value label “MyControl” for 3

`teffects ipwra (y1 x1 x2) (treat3 x1 w), control(MyControl)`
Menu

Statistics > Causal inference/treatment effects > Continuous outcomes > Regression adjustment with IPW
Statistics > Causal inference/treatment effects > Binary outcomes > Regression adjustment with IPW
Statistics > Causal inference/treatment effects > Count outcomes > Regression adjustment with IPW
Statistics > Causal inference/treatment effects > Fractional outcomes > Regression adjustment with IPW
Statistics > Causal inference/treatment effects > Nonnegative outcomes > Regression adjustment with IPW

Syntax

teffects ipwra (ovar omvarlist [, omodel noconstant])
    (tvar tmvarlist [, tmodel noconstant]) [if] [in] [weight]
    [, stat options]

ovar is a binary, count, continuous, fractional, or nonnegative outcome of interest.

omvarlist specifies the covariates in the outcome model.

tvar must contain integer values representing the treatment levels.

tmvarlist specifies the covariates in the treatment-assignment model.

omodel | Description
---|---
model | linear | linear outcome model; the default
     | logit | logistic outcome model
     | probit | probit outcome model
     | hetprobit(varlist) | heteroskedastic probit outcome model
     | poisson | exponential outcome model
     | flogit | fractional logistic outcome model
     | fprobit | fractional probit outcome model
     | fhetprobit(varlist) | fractional heteroskedastic probit outcome model

omodel specifies the model for the outcome variable.


tmodel | Description
---|---
modell | logit | logistic treatment model; the default
     | probit | probit treatment model
     | hetprobit(varlist) | heteroskedastic probit treatment model

For multi-valued treatments, only logit is available and multinomial logit is used.
### stteffects ipwra — Inverse-probability-weighted regression adjustment

**stat**  
<table>
<thead>
<tr>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ate</strong> estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td><strong>atet</strong> estimate average treatment effect on the treated</td>
</tr>
<tr>
<td><strong>pomeans</strong> estimate potential-outcome means</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>options</strong></th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>SE/Robust</strong></td>
<td><strong>vcetype</strong> may be robust, cluster clustvar, bootstrap, or jackknife</td>
</tr>
<tr>
<td><strong>Reporting</strong></td>
<td><strong>level(#)</strong> set confidence level; default is level(95)</td>
</tr>
<tr>
<td></td>
<td><strong>aequtions</strong> display auxiliary-equation results</td>
</tr>
<tr>
<td></td>
<td><strong>display_options</strong> control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td><strong>Maximization</strong></td>
<td><strong>maximize_options</strong> control the maximization process; seldom used</td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td><strong>pстolerance(#)</strong> set tolerance for overlap assumption</td>
</tr>
<tr>
<td></td>
<td><strong>osample(newvar)</strong> newvar identifies observations that violate the overlap assumption</td>
</tr>
<tr>
<td></td>
<td>**control(#)</td>
</tr>
<tr>
<td></td>
<td>**tlevel(#)</td>
</tr>
<tr>
<td></td>
<td><strong>coeflegend</strong> display legend instead of statistics</td>
</tr>
</tbody>
</table>

omvarlist and tmvarlist may contain factor variables; see [U] 11.4.3 Factor variables.

bootstrap, by, collect, jackknife, and statsby are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

### Options

#### Model

- **noconstant**: see [R] Estimation options.

#### Stat

**stat** is one of three statistics: **ate**, **atet**, or **pomeans**. **ate** is the default.

- **ate** specifies that the average treatment effect be estimated.
- **atet** specifies that the average treatment effect on the treated be estimated.
- **pomeans** specifies that the potential-outcome means for each treatment level be estimated.
vce(vcetype) specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce_option.

Reporting

level(#)'; see [R] Estimation options.

aequations specifies that the results for the outcome-model or the treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nolabels, fvwrap(#), fvwrapon(style), cformat(%,fmt), pformat(%,fmt), sformat(%,fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize_options: iterate(#), [no]log, and from(init_specs); see [R] Maximize. These options are seldom used.

init_specs is one of

matname [, skip copy]

# [, # ...], copy

Advanced

pstolerance(#) specifies the tolerance used to check the overlap assumption. The default value is pstolerance(1e-5). teffects will exit with an error if an observation has an estimated propensity score smaller than that specified by pstolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that violate the overlap assumption.

control(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with statistic pomeans. control() and tlevel() may not specify the same treatment level.

tlevel(# | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

The following option is available with teffects ipwra but is not shown in the dialog box:

coefflegend; see [R] Estimation options.

Remarks and examples

Remarks are presented under the following headings:

Overview

Video example
Overview

IPWRA estimators use probability weights to obtain outcome-regression parameters that account for the missing-data problem arising from the fact that each subject is observed in only one of the potential outcomes. The adjusted outcome-regression parameters are used to compute averages of treatment-level predicted outcomes. The contrasts of these averages provide estimates of the treatment effects.

IPWRA estimators use a model to predict treatment status, and they use another model to predict outcomes. Because IPWRA estimators have the double-robust property, only one of the two models must be correctly specified for the IPWRA estimator to be consistent.

IPWRA estimators use a three-step approach to estimating treatment effects:

1. They estimate the parameters of the treatment model and compute inverse-probability weights.
2. Using the estimated inverse-probability weights, they fit weighted regression models of the outcome for each treatment level and obtain the treatment-specific predicted outcomes for each subject.
3. They compute the means of the treatment-specific predicted outcomes. The contrasts of these averages provide the estimates of the ATEs. By restricting the computations of the means to the subset of treated subjects, we can obtain the ATETs.

These steps produce consistent estimates of the effect parameters because the treatment is assumed to be independent of the potential outcomes after conditioning on the covariates. The overlap assumption ensures that predicted inverse-probability weights do not get too large. The standard errors reported by teffects ipwra correct for the three-step process. See [CAUSAL] teffects intro or [CAUSAL] teffects intro advanced for more information about this estimator.

We will illustrate the use of teffects ipwra by using data from a study of the effect of a mother’s smoking status during pregnancy (mbsmoke) on infant birthweight (bweight) as reported by Cattaneo (2010). This dataset also contains information about each mother’s age (mage), education level (medu), marital status (mmarried), whether the first prenatal exam occurred in the first trimester (prenatal1), and whether this baby was the mother’s first birth (fbaby).

Example 1: Estimating the ATE

We begin by using teffects ipwra to estimate the average treatment effect of smoking on birthweight. We will use a probit model to predict treatment status as a function of mmarried, mage, and fbaby; to maximize the predictive power of this model, we use factor-variable notation to incorporate quadratic effects of the mother’s age, the only continuous covariate in our model. We will use linear regression (the default) to model birthweight, using prenatal1, mmarried, mage, and fbaby as explanatory variables. We type
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ipwra (bweight prenatal1 mmarried mage fbaby)
   > (mbsmoke mmarried c.mage##c.mage fbaby medu, probit)
Iteration 0: EE criterion = 9.885e-21
Iteration 1: EE criterion = 7.847e-26
Treatment-effects estimation Number of obs = 4,642
Estimator : IPW regression adjustment
Outcome model : linear
Treatment model: probit

| bweight | Robust Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|---------|--------------------|-----------|------|-----|---------------------|
| ATE     | mbm smoke          |           |      |     |                     |
| vs non smoker |                | -229.9671 | 26.62668 | -8.64 | 0.000 | -282.1544 -177.7798 |
| PO mean | mbm smoke          |           |      |     |                     |
| vs non smoker |                | 3403.336  | 9.57126  | 355.58 | 0.000 | 3384.576 3422.095 |

The average birthweight if all mothers were to smoke would be 230 grams less than the average of 3,403 grams that would occur if none of the mothers had smoked.

By default, `teffects ipwra` displays the ATE and untreated POM. We can specify the `pomeans` option to display both the treated and untreated POMs, and we can use the `aequations` option to display the regression model coefficients used to predict the POMs as well as the coefficients from the model used to predict treatment.
Example 2: Displaying the POMs and equations

. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ipwra (bweight prenatal1 mmarried mage fbaby)
> (mbsmoke mmarried c.mage##c.mage fbaby, probit), pomeans aequations

Iteration 0: EE criterion = 9.885e-21
Iteration 1: EE criterion = 6.922e-26

Treatment-effects estimation

Number of obs = 4,642
Estimator : IPW regression adjustment
Outcome model : linear
Treatment model: probit

| bweight    | Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|------------|-------------|-----------|------|------|----------------------|
| POmeans    |             |           |      |      |                      |
| mbsmoke    |             |           |      |      |                      |
| Nonsmoker  | 3403.336    | 9.57126   | 355.58 | 0.000 | 3384.576 3422.095    |
| Smoker     | 3173.369    | 24.86997  | 127.60 | 0.000 | 3124.624 3222.113    |
| OME0       |             |           |      |      |                      |
| prenatal1  | 67.98549    | 28.78428  | 2.36 | 0.018 | 11.56933 124.4017    |
| mmarried   | 155.5893    | 26.46903  | 5.88 | 0.000 | 103.711 207.4677     |
| mage       | 2.893051    | 2.134788  | 1.36 | 0.175 | -1.291056 7.077158    |
| fbaby      | -71.9215    | 20.39317  | -3.53 | 0.000 | -111.8914 -31.95162  |
| _cons      | 3194.808    | 55.04911  | 58.04 | 0.000 | 3086.913 3302.702    |
| OME1       |             |           |      |      |                      |
| prenatal1  | 34.76923    | 43.18534  | 0.81 | 0.421 | -49.87248 119.4109    |
| mmarried   | 124.0941    | 40.29775  | 3.08 | 0.002 | 45.11193 203.0762    |
| mage       | -5.068833   | 5.954425  | -0.85 | 0.395 | -16.73929 6.601626   |
| fbaby      | 39.89692    | 56.82072  | 0.70 | 0.483 | -71.46966 151.2635   |
| _cons      | 3175.551    | 153.8312  | 20.64 | 0.000 | 2874.047 3477.054    |
| TME1       |             |           |      |      |                      |
| mmarried   | -.6484821   | .0554173  | -11.70 | 0.000 | -.757098 -.5398663   |
| mage       | .1744327    | .0363718  | 4.80 | 0.000 | .1031452 .2457202    |
| c.mage#    | -.0032559   | .0006678  | -4.88 | 0.000 | -.0045647 -.0019471  |
| c.mage     | -.2175962   | .0495604  | -4.39 | 0.000 | -.3147328 -.1204595  |
| fbaby      | -.0863631   | .0100148  | -8.62 | 0.000 | -.1059917 -.0667345  |
| _cons      | -1.558285   | .4639691  | -3.36 | 0.001 | -2.467618 -.6488926  |

As is well known, the standard probit model assumes that the error terms in the latent-utility framework are homoskedastic; the model is not robust to departures from this assumption. An alternative is to use the heteroskedastic probit model, which explicitly models the error variance as a function of a set of variables.
Example 3: Heteroskedastic probit treatment model

Here we use the variables as before, but we use a heteroskedastic probit model to predict treatment status, modeling the heteroskedasticity as a quadratic function of the mother’s age:

```
. teffects ipwra (bweight prenatal1 mmarried fbaby c.mage)
  > (mbsmoke mmarried c.mage##c.mage fbaby medu, hetprobit(c.mage##c.mage)),
  > equations
Iteration 0:  EE criterion = 3.904e-10
Iteration 1:  EE criterion = 4.647e-11
```

```
Number of obs = 4,642
Treatment-effects estimation
Estimator : IPW regression adjustment
Outcome model : linear
Treatment model: heteroskedastic probit
```

```

|                | Robust Coefficient | std. err. | z     | P>|z|   | [95% conf. interval] |
|----------------|--------------------|-----------|-------|-------|---------------------|
| ATE            |                    |           |       |       |                     |
| mbsmoke (Smoker vs Nonsmoker) | -229.6322 | 26.33451  | -8.72 | 0.000 | -281.2468 -178.0175 |
| P outcome mean |                    |           |       |       |                     |
| mbsmoke Nonsmoker| 3403.74 | 9.545798  | 356.57 | 0.000 | 3385.03 3422.449   |
| OME0           |                    |           |       |       |                     |
| prenatal1      | 64.95123           | 28.6216   | 2.27  | 0.023 | 8.853921 121.0485  |
| mmarried       | 154.2297           | 26.45867  | 5.83  | 0.000 | 102.3717 206.0878  |
| fbaby          | -71.61131          | 20.33774  | -3.52 | 0.000 | -111.4725 -31.75007|
| mage           | 3.010149           | 2.133812  | 1.41  | 0.158 | -1.172046 7.192344 |
| _cons          | 3195.355           | 55.05451  | 58.04 | 0.000 | 3087.45 3303.26   |
| OME1           |                    |           |       |       |                     |
| prenatal1      | 38.55275           | 43.57023  | 0.88  | 0.376 | -46.84334 123.9488 |
| mmarried       | 126.3378           | 40.7398   | 3.10  | 0.002 | 46.48921 206.1863 |
| fbaby          | 45.43551           | 56.44827  | 0.80  | 0.421 | -65.20106 156.0721 |
| mage           | -6.069917          | 5.952507  | -1.02 | 0.308 | -17.73662 5.596783 |
| _cons          | 3195.795           | 152.3978  | 20.97 | 0.000 | 2897.101 3494.489 |
| TME1           |                    |           |       |       |                     |
| mmarried       | -.0295517          | .0238767  | -1.24 | 0.216 | -.0763492 .0172459 |
| mage           | .015789            | .0105411  | 1.50  | 0.134 | -.0048711 .0364492 |
| c.mage         | -.0002837          | .0001899  | -1.49 | 0.135 | -.000656 .0000886 |
| c.mage#        | .0037613           | .001243   | 3.03  | 0.002 | .0013251 .0061976 |
| fbaby          | -.0093304          | .0079946  | -1.17 | 0.243 | -.0249996 .0063387 |
| medu           | -.0036772          | .0030294  | -1.21 | 0.225 | -.0096147 .0022602 |
| _cons          | -.1822172          | .117975   | -1.54 | 0.122 | -.413444 .0490095 |
| TME1_lnsigma   |                    |           |       |       |                     |
| mbage          | -.2211507          | .0631072  | -3.50 | 0.000 | -.3448385 -.097463 |
| c.mage#        | .0037613           | .001243   | 3.03  | 0.002 | .0013251 .0061976 |
The estimated ATE and base-level POM are essentially the same as those produced by the model that used a homoskedastic probit.

Video example

Treatment effects: Inverse-probability-weighted regression adjustment

Stored results

teffects ipwra stores the following in e():

Scalars
- e(N): number of observations
- e(nj): number of observations for treatment level j
- e(N_clust): number of clusters
- e(k_eq): number of equations in e(b)
- e(k_levels): number of levels in treatment variable
- e(treated): level of treatment variable defined as treated
- e(control): level of treatment variable defined as control
- e(converged): 1 if converged, 0 otherwise

Macros
- e(cmd): teffects
- e(cmdline): command as typed
- e(depvar): name of outcome variable
- e(tvar): name of treatment variable
- e(subcmd): ipwra
- e(tmodel): logit, probit, or hetprobit
- e(omodel): linear, logit, probit, hetprobit, poisson, flogit, fprobit, or fhetprobit
- e(stat): statistic estimated, ate, atet, or pomeans
- e(wtype): weight type
- e(wexp): weight expression
- e(titles): title in estimation output
- e(clustvar): name of cluster variable
- e(tlevels): levels of treatment variable
- e(vce): vcetype specified in vce()
- e(vcetype): title used to label Std. err.
- e(properties): b V
- e(estat_cmd): program used to implement estat
- e(predict): program used to implement predict
- e(marginsnotok): predictions disallowed by margins
- e(asbalanced): factor variables fvset as asbalanced
- e(asobserved): factor variables fvset as asobserved

Matrices
- e(b): coefficient vector
- e(V): variance–covariance matrix of the estimators
- e(sample): marks estimation sample

In addition to the above, the following is stored in r():

Matrices
- r(table): matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.
Methods and formulas

teffects ipwra implements a smooth treatment-effects estimator. All smooth treatment-effects estimators are documented in Methods and formulas of [CAUSAL] teffects aipw.

References


Also see

[CAUSAL] teffects postestimation — Postestimation tools for teffects
[CAUSAL] teffects — Treatment-effects estimation for observational data
[CAUSAL] teffects aipw — Augmented inverse-probability weighting
[U] 20 Estimation and postestimation commands
Description

This entry discusses the use of *teffects* when the treatment is multivalued. This entry presumes you are already familiar with the potential-outcome framework and the use of the *teffects* commands with binary treatments. See [CAUSAL] *teffects intro* or [CAUSAL] *teffects intro advanced* for more information.

Remarks and examples

Remarks are presented under the following headings:

- Introduction
- Parameters and notation
- Illustrating multivalued treatments
- Examples

Introduction

When the treatment is binary, each subject could either receive the treatment or not receive the treatment. In contrast, multivalued treatments refer to cases in which each subject could receive one of several different treatments or else not receive treatment at all. For example, in testing the efficacy of a drug, a patient could receive a 10 milligram (mg) dose, a 20 mg dose, a 30 mg dose, or no dose at all. We first want to be able to compare a patient receiving the 10 mg dose with a patient receiving no dose, a patient receiving the 20 mg dose with a patient receiving no dose, and a patient receiving the 30 mg dose with a patient receiving no dose. Once we can make those comparisons, we can then, for example, compare the efficacy of a 30 mg dose with that of a 20 mg dose or a 10 mg dose.

To highlight an example in economics, we consider an unemployed person who could participate in a comprehensive skills training program, attend a one-day workshop that helps job seekers write their résumés, or choose not to participate in either. We want to know how effective each of those programs is relative to not participating; once we know that, we can then compare the effectiveness of the comprehensive program with that of the one-day program.

Multivalued treatments increase the number of parameters that must be estimated and complicate the notation. Fortunately, however, using the *teffects* commands is not much more difficult with multivalued treatments than with binary treatments.

You can use *teffects ra*, *teffects ipw*, *teffects ipwra*, and *teffects aipw* to estimate multivalued treatment effects. However, the theory developed in Abadie and Imbens (2006, 2012) has not been extended to handle multivalued treatments, so you cannot use *teffects nnmatch* or *teffects psmatch* in these cases.

Cattaneo (2010), Imbens (2000), and Wooldridge (2010, sec. 21.6.3) discuss aspects of treatment-effect estimation with multivalued treatments.
Parameters and notation

We denote the potential outcome that subject $i$ would obtain if given treatment-level $t$ as $y_{ti}$, where $y_{ti}$ is the realization of the random variable $y_t$. Throughout this entry, $i$ subscripts denote realizations of the corresponding unsubscripted random variables. We again let $y_0$ denote the potential outcome of a subject who did not receive any treatment. To handle the case of multivalued treatments, we extend the definition of the unobservable, individual-level treatment effects to be $y_t - y_0$ for $t \in \{1, \ldots, q\}$.

As in the binary-valued case, we again focus on three parameters of interest: the average treatment effect (ATE), the potential-outcome mean (POM), and the average treatment effect on the treated (ATET).

ATE The ATE is the average effect of giving each individual treatment $t$ instead of treatment 0:

$$\text{ATE}_t = E(y_t - y_0)$$

POM The POM for each treatment level is an average of each potential outcome:

$$\text{POM}_t = E(y_t)$$

ATET The ATET is the average effect among those subjects that receive treatment level $\tilde{t}$ of giving each subject treatment $\tilde{t}$ instead of treatment 0:

$$\text{ATET}_{\tilde{t}, \tilde{t}} = E\{ (y_{\tilde{t}} - y_0) | t = \tilde{t} \}$$

The extra notation required to define the ATET in this case indicates the difficulties surrounding this parameter.

Defining the ATET in the multivalued treatment case requires three different treatment levels: $\tilde{t}$ defines the treatment level of the treated potential outcome; 0 is the treatment level of the control potential outcome; and $t = \tilde{t}$ restricts the expectation to include only those individuals who actually receive treatment level $\tilde{t}$.

Illustrating multivalued treatments

To illustrate the concept of a potential outcome and the parameters we would like to estimate, we consider the following table:

<table>
<thead>
<tr>
<th>$y$</th>
<th>$t$</th>
<th>$y_0$</th>
<th>$y_1$</th>
<th>$y_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>−0.50</td>
<td>0</td>
<td>−0.50</td>
<td>1.06</td>
<td>1.93</td>
</tr>
<tr>
<td>2.42</td>
<td>1</td>
<td>2.13</td>
<td>2.42</td>
<td>2.43</td>
</tr>
<tr>
<td>3.15</td>
<td>2</td>
<td>1.26</td>
<td>2.57</td>
<td>3.15</td>
</tr>
<tr>
<td>−0.39</td>
<td>0</td>
<td>−0.39</td>
<td>−0.18</td>
<td>0.52</td>
</tr>
<tr>
<td>2.22</td>
<td>2</td>
<td>−0.24</td>
<td>−0.01</td>
<td>2.22</td>
</tr>
</tbody>
</table>

We observe the outcome $y$ as well as the treatment indicator $t$. There are three levels of treatment: 0, 1, or 2. Ideally, we would observe $y_0$, $y_1$, and $y_2$, but in fact all we have is $y$. In the first row, the subject received treatment level 0, so $y = y_0$ for that subject. In the last row, the subject received treatment 2, so $y = y_2$. We reiterate that we do not actually observe $y_0$, $y_1$, or $y_2$. 
If we did have data on $y_0$, $y_1$, and $y_2$, then we could define subject-level treatment variables $te_1 = y_1 - y_0$ and $te_2 = y_2 - y_0$. Here we would be following the convention of taking treatment level 0 to be the control level. The following table adds these two variables:

<table>
<thead>
<tr>
<th>Y</th>
<th>T</th>
<th>Y0</th>
<th>Y1</th>
<th>Y2</th>
<th>Te1</th>
<th>Te2</th>
</tr>
</thead>
<tbody>
<tr>
<td>-0.50</td>
<td>0</td>
<td>-0.50</td>
<td>1.06</td>
<td>1.93</td>
<td>1.56</td>
<td>2.43</td>
</tr>
<tr>
<td>2.42</td>
<td>1</td>
<td>2.13</td>
<td>2.42</td>
<td>2.43</td>
<td>0.29</td>
<td>0.30</td>
</tr>
<tr>
<td>3.15</td>
<td>2</td>
<td>1.26</td>
<td>2.57</td>
<td>3.15</td>
<td>1.31</td>
<td>1.89</td>
</tr>
<tr>
<td>-0.39</td>
<td>0</td>
<td>-0.39</td>
<td>-0.18</td>
<td>0.52</td>
<td>0.21</td>
<td>0.91</td>
</tr>
<tr>
<td>2.22</td>
<td>2</td>
<td>-0.24</td>
<td>-0.01</td>
<td>2.22</td>
<td>0.23</td>
<td>2.46</td>
</tr>
</tbody>
</table>

Once we have $te_1$ and $te_2$, obtaining the ATEs is straightforward. The ATE of going from treatment 0 to treatment 1 is simply the mean of the five entries in the column labeled $te_1$, which here works out to 0.72. Going from treatment level 0 to treatment level 1 causes the outcome to increase an average of 0.72. Similarly, the ATE of going from treatment 0 to treatment 2 is the mean of the entries in the column labeled $te_2$, which is 1.60. Exposing all subjects to treatment level 2 would cause the outcome to rise by an average of 1.60 relative to the outcome obtained by exposing them to treatment level 0.

The ATET is the average difference in the potential outcomes among those that get a particular treatment level. To compute this, we must specify two treatment levels: the actual treatment level the subjects we are interested in received as well as the treatment level we want to compare them with. For example, suppose we are interested in the ATET of going from treatment 0 to treatment 1 for those who received treatment 0. This ATET is the average of $te_1$ for those subjects for which $t = 0$. Here that ATET is just $(1.56 + 0.21)/2 \approx 0.89$. If we exposed the subjects who received treatment 0 to treatment 1 instead, the outcome would increase an average of 0.89.

The ATET of going from treatment 0 to treatment 2 for those subjects who received treatment 2 is the mean of $te_2$ for those subjects for which $t = 2$, which is $(1.89 + 2.46)/2 \approx 2.18$. Receiving treatment 2 increased the outcome of those who received treatment 2 by an average of 2.18 relative to receiving the control.

### Examples

In the remainder of this entry, we provide several examples demonstrating how to estimate multivalued treatments using teffects.

#### Example 1: Potential outcomes with four treatment levels

bdsianesi5.dta contains an extract of data from Blundell, Dearden, and Sianesi (2005). In this dataset on individuals in the United Kingdom, wages records hourly wages in pounds; ed records the highest educational degree obtained; paed records the highest educational level obtained by each individual’s father; math7 records a score obtained on a standardized math test when the individual was seven; read7 records a score obtained on a standardized reading test when the individual was seven; and London and eastern are indicators for whether an individual lives in the expensive area of London or the east. We want to know how the level of education obtained affects a person’s wage.

We begin by using mean to report the estimated means of wages over the four education levels. The value labels on mean are coded as none for no degree, O for an O-level degree, A for an A-level degree, or H for a higher-education degree.
The output reveals that the estimated mean wage increases as the education level goes from no degree to an O-level degree, to an A-level degree, and to a higher-education degree, as we would expect. Once we control for other characteristics of each individual, do we still observe a positive effect of education on wage?

We use `teffects ra` (see [CAUSAL] `teffects ra`) to estimate the ATEs of the different education levels by regression adjustment (RA), controlling for each person’s location, math score, and father’s education level:

```
. teffects ra (wage london eastern paed math7, poisson) (ed)
Iteration 0:  EE criterion = 1.865e-18
Iteration 1:  EE criterion = 4.077e-30
```

```
Treatment-effects estimation
   Number of obs = 1,693

            Estimator : regression adjustment
   Outcome model : Poisson
   Treatment model: none

                      robust
                      Coefficient std. err.     z    P>|z|    [95% conf. interval]
ATE
     ed
     (O vs none)  1.181543  .3520371   3.36   0.001    .4915626   1.871523
     (A vs none)  1.743079  .3282152   5.31   0.000    1.099789   2.386369
     (H vs none)  3.972829  .3840024  10.35   0.000    3.220199   4.725463

POmean
     ed
     none  6.525873  .2931933  22.26   0.000    5.951224   7.100521
```

Because wages are necessarily positive, we used the `poisson` option inside the outcome-model specification. The estimated POM of the control level of no degree is 6.53 pounds per hour. The estimated ATE of going from no degree to an O-level degree is 1.18 pounds per hour; the estimated ATE of going from no degree to an A-level degree is 1.74 pounds per hour; and the estimated ATE of going from no degree to a higher-education degree is 3.97 pounds per hour. All of these effects are highly significant.

For comparison purposes, we also use `teffects aipw` (see [CAUSAL] `teffects aipw`). We use the same outcome model as before. We use a multinomial logit model to predict education level, using math and reading scores and both the father’s and the mother’s educational attainment levels as predictors:
. teffects aipw (wage london eastern paed math7, poisson)  
> (ed math7 read7 maed paed)  

Iteration 0: EE criterion = 1.877e-18  
Iteration 1: EE criterion = 1.029e-30  

Treatment-effects estimation  
Number of obs = 1,693  
Estimator : augmented IPW  
Outcome model : Poisson by ML  
Treatment model: (multinomial) logit  

| wage | Robust Coefficient | std. err. | z | P>|z| | [95% conf. interval] |
|------|--------------------|-----------|---|-----|-----------------|
| ATE  |                    |           |   |     |                 |
| ed   |                    |           |   |     |                 |
| (0 vs none) | 1.748197 | .3911167 | 4.47 | 0.000 | .9816221 | 2.514771 |
| (A vs none) | 2.363228 | .3741584 | 6.32 | 0.000 | 1.629891 | 3.096565 |
| (H vs none) | 4.359777 | .4133059 | 10.55 | 0.000 | 3.549712 | 5.169842 |
| POmean|                    |           |   |     |                 |
| ed   |                    |           |   |     |                 |
| none | 5.946184 | .3391531 | 17.53 | 0.000 | 5.281456 | 6.610912 |

The results indicate slightly higher treatment effects relative to those indicated by teffects ra. That is largely because the AIPW estimator predicts a lower no-higher-education POM than the RA estimator.

>> Example 2: Expressing ATEs as percentages

As in the binary-treatment case, expressing the ATEs as percentages of the POM for the control level often aids interpretation. Here we first use the replay facility of teffects aipw along with the coeflegend option to see how the parameters are named.

. teffects, coeflegend  

Treatment-effects estimation  
Number of obs = 1,693  
Estimator : augmented IPW  
Outcome model : Poisson by ML  
Treatment model: (multinomial) logit  

<table>
<thead>
<tr>
<th>wage</th>
<th>Coefficient</th>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(0 vs none)</td>
<td>1.748197</td>
<td>_b[ATE:r1vs0.ed]</td>
</tr>
<tr>
<td>(A vs none)</td>
<td>2.363228</td>
<td>_b[ATE:r2vs0.ed]</td>
</tr>
<tr>
<td>(H vs none)</td>
<td>4.359777</td>
<td>_b[ATE:r3vs0.ed]</td>
</tr>
<tr>
<td>POmean</td>
<td></td>
<td></td>
</tr>
<tr>
<td>ed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>none</td>
<td>5.946184</td>
<td>_b[POmean:0.ed]</td>
</tr>
</tbody>
</table>
Now that we know the names, we can use \texttt{nlcom} to obtain the ATEs relative to the base-level \texttt{POM}:

\begin{verbatim}
. nlcom (_b[ATE:r1vs0.ed] / _b[POmean:0.ed])
> (_b[ATE:r2vs0.ed] / _b[POmean:0.ed])
> (_b[ATE:r3vs0.ed] / _b[POmean:0.ed])
\end{verbatim}

\begin{verbatim}
  _nl_1: _b[ATE:r1vs0.ed] / _b[POmean:0.ed]  
  _nl_2: _b[ATE:r2vs0.ed] / _b[POmean:0.ed]  
  _nl_3: _b[ATE:r3vs0.ed] / _b[POmean:0.ed]  
\end{verbatim}

\begin{verbatim}

| wage | Coefficient | Std. err. | z     | P>|z|  | [95% conf. interval] |
|------|-------------|-----------|-------|------|---------------------|
| _nl_1 | .2940031   | .0808156  | 3.64  | 0.000 | .1356075 .4523988  |
| _nl_2 | .3974361   | .0840545  | 4.73  | 0.000 | .2326923 .5621799  |
| _nl_3 | .7332059   | .1068848  | 6.86  | 0.000 | .5237156 .9426962  |
\end{verbatim}

Wages are 29% higher when everyone receives an O-level degree than if no one receives a degree.
Wages are 40% higher when everyone receives an A-level degree than if no one receives a degree.
Wages are 73% higher when everyone receives an H-level degree than if no one receives a degree.

Although impressive, these changes are not presented in the way that is most commonly discussed.(There is a large amount of literature on the treatment effect of getting a higher-education degree.) In particular, we might rather want to know the percentage changes in wages relative to a person with an A-level degree. Next we estimate the ATEs treating an A-level degree as the control level; to do that, we use the control() option. We also specify coeflegend again because we are more interested in how the parameters are named rather than in their standard errors at this point:

\begin{verbatim}
. teffects aipw (wage london eastern paed math7, poisson)
> (ed math7 read7 maed paed), control(A) coeflegend
\end{verbatim}

\begin{verbatim}
Iteration 0: EE criterion = 1.870e-18
Iteration 1: EE criterion = 2.882e-30
\end{verbatim}

\texttt{Treatment-effects estimation}  \texttt{Number of obs} = 1,693
\texttt{Estimator} : augmented IPW
\texttt{Outcome model} : Poisson by ML
\texttt{Treatment model}: (multinomial) logit

\begin{verbatim}
<table>
<thead>
<tr>
<th>wage</th>
<th>Coefficient</th>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>\texttt{ATE}</td>
<td>\texttt{ed}</td>
<td></td>
</tr>
<tr>
<td>(none vs A)</td>
<td>-2.363228</td>
<td>_b[ATE:r0vs2.ed]</td>
</tr>
<tr>
<td>(O vs A)</td>
<td>-.6150312</td>
<td>_b[ATE:r1vs2.ed]</td>
</tr>
<tr>
<td>(H vs A)</td>
<td>1.996549</td>
<td>_b[ATE:r3vs2.ed]</td>
</tr>
</tbody>
</table>

| \texttt{POmean} | \texttt{ed} |         |
| A | 8.309412  | _b[POmean:2.ed] |
\end{verbatim}

Now we use \texttt{nlcom} to obtain the ATE of obtaining a higher-education degree as a percentage of the expected A-level wage:

\begin{verbatim}
. nlcom _b[ATE:r3vs2.ed] / _b[POmean:2.ed], noheader
\end{verbatim}

\begin{verbatim}
| wage | Coefficient | Std. err. | z     | P>|z|  | [95% conf. interval] |
|------|-------------|-----------|-------|------|---------------------|
| _nl_1 | .2402756   | .0355404  | 6.76  | 0.000 | .1706177 .3099335  |
\end{verbatim}
The average wage increases by 24% when everyone receives an H-level degree relative to when everyone receives an A-level degree.

### Example 3: Obtaining ATETs

In the previous example, we showed that on average, a higher-education degree increases a person’s wage by 24% relative to someone with only an A-level degree. Sometimes, though, we would rather know how much the higher-education degree increases wages among the people who actually have a higher-education degree. To answer that question, we want to examine the ATET rather than the ATE.

Here we use the IPWRA estimator to obtain our answer. We specify the `control(A)` option so that an A-level education is treated as the basis for comparisons. We specify the `atet` option to obtain ATETs rather than ATEs, and we specify the `tlevel(H)` option to indicate that we want the ATETs to be calculated for the subset of people who actually receive higher-education degrees.

```stata
. teffects ipwra (wage london eastern paed math7, poisson)>
   > (ed math7 read7 maed paed), atet control(A) tlevel(H)
```

The point estimates are similar to the ATEs we obtained above, suggesting that the means of the covariates among those with a higher-education degree are similar to the means for the entire population.

In output not shown to save space, we replayed the previous results with the `coeflegend` option to determine how the parameters are named. Armed with that information, we call `nlcom`:

```stata
. nlcom _b[ATET:r3vs2.ed] / _b[POmean:2.ed], noheader
```

Our estimate of the percentage increase is now noticeably smaller once we restrict ourselves to only those people who actually received a higher-education degree. However, because of the width of the confidence intervals, there is no evidence to suggest that the difference between the estimates is statistically significant.
Example 4: ATEs comparing adjacent treatments

In the first example, we obtained the three ATEs, and they were all expressed relative to the base level of no degree. Now we show how we can express the gains to an O-level degree relative to no degree, the gains to an A-level degree relative to an O-level degree, and the gains to a higher-education degree relative to an A-level degree.

First, we use an AIPW estimator to obtain all the POMs for our example dataset:

```plaintext
. teffects aipw (wage london eastern paed math7, poisson)
    > (ed math7 read7 maed paed), pom
```

Iteration 0:  EE criterion = 1.877e-18
Iteration 1:  EE criterion = 1.542e-30

Treatment-effects estimation  Number of obs = 1,693
Estimator : augmented IPW
Outcome model : Poisson by ML
Treatment model: (multinomial) logit

|              | Robust Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|--------------|--------------------|-----------|------|------|---------------------|
| P0means      |                    |           |      |      |                     |
|              | ed                 |           |      |      |                     |
| none         | 5.946184           | .3391531  | 17.53| 0.000| 5.281456 6.610912   |
| O            | 7.694381           | .1915192  | 40.18| 0.000| 7.31901 8.069752    |
| A            | 8.309412           | .1563348  | 53.15| 0.000| 8.003001 8.615823   |
| H            | 10.30596           | .2285837  | 45.09| 0.000| 9.857945 10.75398   |

ATEs are contrasts of POMs, and here we show how to use contrast to obtain the estimated ATEs:

```plaintext
. contrast r.ed, nowald
```

warning: cannot perform check for estimable functions.

Contrasts of marginal linear predictions
Margins: asbalanced

<table>
<thead>
<tr>
<th></th>
<th>Contrast</th>
<th>Std. err.</th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>P0means</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>ed</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(O vs none)</td>
<td>1.748197</td>
<td>.3911167</td>
<td>.9816221 2.514771</td>
</tr>
<tr>
<td>(A vs none)</td>
<td>2.363228</td>
<td>.3741584</td>
<td>1.629891 3.096665</td>
</tr>
<tr>
<td>(H vs none)</td>
<td>4.359777</td>
<td>.4133059</td>
<td>3.549712 5.169842</td>
</tr>
</tbody>
</table>

These estimated ATEs match those we obtained in example 2.

Now that we know how to use contrast to obtain the ATEs based on the POMs, we can take advantage of contrast's ability to obtain "reverse adjacent" contrasts, which compare each level with the previous level. We use the ar. operator with contrast to accomplish this:
. contrast ar.ed, nowald
warning: cannot perform check for estimable functions.

Contrasts of marginal linear predictions
Margins: asbalanced

<table>
<thead>
<tr>
<th></th>
<th>Contrast</th>
<th>Std. err.</th>
<th>[95% conf. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>POmeans</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ed IS (O vs none)</td>
<td>1.748197</td>
<td>.3911167</td>
<td>.9816221 2.514771</td>
</tr>
<tr>
<td>(A vs O)</td>
<td>.6150312</td>
<td>.2432806</td>
<td>.13821 1.091852</td>
</tr>
<tr>
<td>(H vs A)</td>
<td>1.996549</td>
<td>.2730712</td>
<td>1.461339 2.531759</td>
</tr>
</tbody>
</table>

These ATEs are for incremental increases. In contrast, the ATEs considered above had a common base.

Technical note

The multivalued treatment AIPW estimators implemented in teffects aipw are EIF estimators based on the results of Cattaneo (2010). The results in Cattaneo (2010) are for semiparametric estimators, and we implement parametric versions. Of more practical importance, Cattaneo (2010) contains results for quantile treatment effects that are not implemented in teffects but implemented in the community-contributed poparms command discussed in Cattaneo, Drukker, and Holland (2013). See Emsley et al. (2008) for another implementation of the AIPW estimator, and see Frölich and Melly (2010) for other commands that estimate quantile treatment effects.

References


Also see

[CAUSAL] teffects — Treatment-effects estimation for observational data
[CAUSAL] teffects intro — Introduction to treatment effects for observational data
[CAUSAL] teffects intro advanced — Advanced introduction to treatment effects for observational data
**teffects nnmatch —** Nearest-neighbor matching

### Description

`teffects nnmatch` estimates the average treatment effect (ATE) and average treatment effect on the treated (ATET) from observational data by nearest-neighbor matching (NNM). NNM estimators impute the missing potential outcome for each subject by using an average of the outcomes of similar subjects that receive the other treatment level. Similarity between subjects is based on a weighted function of the covariates for each observation. The treatment effect is computed by taking the average of the difference between the observed and imputed potential outcomes for each subject. `teffects nnmatch` accepts a continuous, binary, count, fractional, or nonnegative outcome.

See [CAUSAL] `teffects intro` or [CAUSAL] `teffects intro advanced` for more information about estimating treatment effects from observational data.

### Quick start

ATE of `treat` on `y` estimated by NNM on `x1` and indicators for levels of categorical variable `a`

```
teffects nnmatch (y x1 i.a) (treat)
```

Same as above, but estimate the ATET

```
teffects nnmatch (y x1 i.a) (treat), atet
```

Add continuous covariate `x2` and perform bias correction

```
teffects nnmatch (y x1 x2 i.a) (treat), biasadj(x1 x2)
```

Same as above, and match exactly on values of `a`

```
teffects nnmatch (y x1 x2 i.a) (treat), biasadj(x1 x2) ematch(i.a)
```

With robust standard errors

```
teffects nnmatch (y x1 x2 i.a) (treat), vce(robust)
```

With four matches per observation

```
teffects nnmatch (y x1 x2 i.a) (treat), nneighbor(4)
```

### Menu

Statistics > Causal inference/treatment effects > Continuous outcomes > Nearest-neighbor matching

Statistics > Causal inference/treatment effects > Binary outcomes > Nearest-neighbor matching

Statistics > Causal inference/treatment effects > Count outcomes > Nearest-neighbor matching

Statistics > Causal inference/treatment effects > Fractional outcomes > Nearest-neighbor matching

Statistics > Causal inference/treatment effects > Nonnegative outcomes > Nearest-neighbor matching
Syntax

teffects nnmatch (ovar omvarlist) (tvar) [ if ] [ in ] [ weight ]

[ , stat options ]

ovar is a binary, count, continuous, fractional, or nonnegative outcome of interest.

omvarlist specifies the covariates in the outcome model.

tvar must contain integer values representing the treatment levels. Only two treatment levels are allowed.

stat Description

<table>
<thead>
<tr>
<th>Stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
</tr>
</tbody>
</table>

options Description

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>nneighbor( )</td>
<td>specify number of matches per observation; default is nneighbor(1)</td>
</tr>
<tr>
<td>biasadj(varlist)</td>
<td>correct for large-sample bias using specified variables</td>
</tr>
<tr>
<td>ematch(varlist)</td>
<td>match exactly on specified variables</td>
</tr>
</tbody>
</table>

SE/Robust

vce(vcetype) vcetype may be

vce(robust [ , nn(#) ]); use robust Abadie–Imbens standard errors with # matches
vce(iid); use independently and identically distributed Abadie–Imbens standard errors

Reporting

level(#) set confidence level; default is level(95)
dmvariables display names of matching variables
display_options control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Advanced

caliper(#) specify the maximum distance for which two observations are potential neighbors
dtolerance(#) set maximum distance between individuals considered equal
osample(newvar) newvar identifies observations that violate the overlap assumption
control(# | label) specify the level of tvar that is the control
tlevel(# | label) specify the level of tvar that is the treatment
generate(stub) generate variables containing the observation numbers of the nearest neighbors
metric(metric) select distance metric for covariates
coeflegend display legend instead of statistics
### Options

**Model**

- `nneighbor(#)` specifies the number of matches per observation. The default is `nneighbor(1)`. Each observation is matched with at least the specified number of observations from the other treatment level. `nneighbor()` must specify an integer greater than or equal to 1 but no larger than the number of observations in the smallest treatment group.

- `biasadj(varlist)` specifies that a linear function of the specified covariates be used to correct for a large-sample bias that exists when matching on more than one continuous covariate. By default, no correction is performed.

  Abadie and Imbens (2006, 2011) show that nearest-neighbor matching estimators are not consistent when matching on two or more continuous covariates and propose a bias-corrected estimator that is consistent. The correction term uses a linear function of variables specified in `biasadj()`; see example 3.

- `ematch(varlist)` specifies that the variables in `varlist` match exactly. All variables in `varlist` must be numeric and may be specified as factors. `teffects nnmatch` exits with an error if any observations do not have the requested exact match.

**Stat**

- `stat` is one of two statistics: `ate` or `atet`. `ate` is the default.

  - `ate` specifies that the average treatment effect be estimated.
  - `atet` specifies that the average treatment effect on the treated be estimated.

**SE/Robust**

- `vce(vcetype)` specifies the standard errors that are reported. By default, `teffects nnmatch` uses robust standard errors estimated using two matches.

  - `vce(robust [ , nn(#)] )` specifies that robust standard errors be reported and that the requested number of matches be used optionally.

  - `vce(iid)` specifies that standard errors for independently and identically distributed data be reported.

The standard derivative-based standard-error estimators cannot be used by `teffects nnmatch`, because these matching estimators are not differentiable. The implemented methods were derived by Abadie and Imbens (2006, 2011, 2012); see *Methods and formulas*. 

---

**metric**

<table>
<thead>
<tr>
<th>metric</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>maha</td>
<td>inverse sample covariate covariance; the default</td>
</tr>
<tr>
<td>lanobis</td>
<td>inverse diagonal sample covariate covariance</td>
</tr>
<tr>
<td>ivariance</td>
<td>identity</td>
</tr>
<tr>
<td>euclidean</td>
<td>identity</td>
</tr>
<tr>
<td>matrix matname</td>
<td>user-supplied scaling matrix</td>
</tr>
</tbody>
</table>

`omvarlist` may contain factor variables; see [U] 11.4.3 Factor variables.

by, collect, and statsby are allowed; see [U] 11.1.10 Prefix commands.

fweights are allowed; see [U] 11.1.10 weight.

coefflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.
As discussed in Abadie and Imbens (2008), bootstrap estimators do not provide reliable standard errors for the estimator implemented by `teffects nnmatch`.

`Reporting` level(#); see [R] `Estimation options`.

`dmvariables` specifies that the matching variables be displayed.

`display_options`: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlavel, fwrap(#), fwrapon(style), cformat(%,fmt), pformat(%,fmt), sformat(%,fmt), and nolstretch; see [R] `Estimation options`.

`Advanced` caliper(#) specifies the maximum distance at which two observations are a potential match. By default, all observations are potential matches regardless of how dissimilar they are.

The distance is based on `omvarlist`. If an observation does not have at least `nneighbor(#)` matches, `teffects nnmatch` exits with an error message. Use option `osample(newvar)` to identify all observations that are deficient in matches.

`dtolerance(#)` specifies the tolerance used to determine exact matches. The default value is `dtolerance(sqrt(c(epsdouble)))`.

Integer-valued variables are usually used for exact matching. The `dtolerance()` option is useful when continuous variables are used for exact matching.

`osample(newvar)` specifies that indicator variable `newvar` be created to identify observations that violate the overlap assumption. This variable will identify all observations that do not have at least `nneighbor(#)` matches in the opposite treatment group within `caliper(#)` (for `metric()` distance matching) or `dtolerance(#)` (for `ematch(varlist)` exact matches).

The `vce(robust, nn(#))` option also requires at least # matches in the same treatment group within the distance specified by `caliper(#)` or within the exact matches specified by `dtolerance(#)`.

The average treatment effect on the treated, option `atet`, using `vce(iid)` requires only `nneighbor(#)` control group matches for the treated group.

`control(# | label)` specifies the level of `tvar` that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. `control()` and `tlevel()` may not specify the same treatment level.

`tlevel(# | label)` specifies the level of `tvar` that is the treatment for the statistic `atet`. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. `tlevel()` may only be specified with statistic `atet`. `tlevel()` and `control()` may not specify the same treatment level.

`generate(stub)` specifies that the observation numbers of the nearest neighbors be stored in the new variables `stub1`, `stub2`, .... This option is required if you wish to perform postestimation based on the matching results. The number of variables generated may be more than `nneighbor(#)` because of tied distances. These variables may not already exist.

`metric(metric)` specifies the distance matrix used as the weight matrix in a quadratic form that transforms the multiple distances into a single distance measure; see `Nearest-neighbor matching estimator` in `Methods and formulas` for details.

The following option is available with `teffects nnmatch` but is not shown in the dialog box: `coeflegend`; see [R] `Estimation options`. 
The NNM method of treatment-effect estimation imputes the missing potential outcome for each individual by using an average of the outcomes of similar subjects that receive the other treatment level. Similarity between subjects is based on a weighted function of the covariates for each observation. The average treatment effect (ATE) is computed by taking the average of the difference between the observed and potential outcomes for each subject.

`tffects nnmatch` determines the “nearest” by using a weighted function of the covariates for each observation. By default, the Mahalanobis distance is used, in which the weights are based on the inverse of the covariates’ variance–covariance matrix. `teffects nnmatch` also allows you to request exact matching for categorical covariates. For example, you may want to force all matches to be of the same gender or race.

NNM is nonparametric in that no explicit functional form for either the outcome model or the treatment model is specified. This flexibility comes at a price; the estimator needs more data to get to the true value than an estimator that imposes a functional form. More formally, the NNM estimator converges to the true value at a rate slower than the parametric rate, which is the square root of the sample size, when matching on more than one continuous covariate. `teffects nnmatch` uses bias correction to fix this problem. `teffects psmatch` implements an alternative to bias correction; the method matches on a single continuous covariate, the estimated treatment probabilities. See [CAUSAL] `teffects intro` or [CAUSAL] `teffects intro advanced` for more information about this estimator.

We will illustrate the use of `teffects nnmatch` by using data from a study of the effect of a mother’s smoking status during pregnancy (`mbsmoke`) on infant birthweight (`bweight`) as reported by Cattaneo (2010). This dataset also contains information about each mother’s age (`mage`), education level (`medu`), marital status (`mmarried`), whether the first prenatal exam occurred in the first trimester (`prenatal1`), whether this baby was the mother’s first birth (`fbaby`), and the father’s age (`fage`).

### Example 1: Estimating the ATE

We begin by using `teffects nnmatch` to estimate the average treatment effect of `mbsmoke` on `bweight`. Subjects are matched using the Mahalanobis distance defined by covariates `mage`, `prenatal1`, `mmarried`, and `fbaby`.

```
. use https://www.stata-press.com/data/r18/cattaneo3
. teffects nnmatch (bweight mage prenatal1 mmarried fbaby) (mbsmoke)
```

<table>
<thead>
<tr>
<th>Treatment-effects estimation</th>
<th>Number of obs = 4,642</th>
</tr>
</thead>
<tbody>
<tr>
<td>Estimator</td>
<td>nearest-neighbor matching</td>
</tr>
<tr>
<td>Outcome model</td>
<td>matching</td>
</tr>
<tr>
<td>Distance metric</td>
<td>Mahalanobis</td>
</tr>
</tbody>
</table>

| bweight | Coefficient | std. err. | z     | P>|z| | [95% conf. interval] |
|---------|-------------|-----------|-------|------|---------------------|
| ATE     |             |           |       |      |                     |
| mbsmoke (smoker vs nonsmoker) | -240.3306 | 28.43006 | -8.45 | 0.000 | -296.0525 | -184.6087 |

The average birthweight if all mothers were to smoke would be 240 grams less than the average that would occur if none of the mothers had smoked.
When the model includes indicator and categorical variables, you may want to restrict matches to only those subjects who are in the same category. The `ematch()` option of `teffects nnmatch` allows you to specify such variables that must match exactly.

### Example 2: Exact matching

Here we use the `ematch()` option to require exact matches on the binary variables `prenatal1`, `mmarried`, and `fbaby`. We also use Euclidean distance, rather than the default Mahalanobis distance, to match on the continuous variable `mage`.

```
. teffects nnmatch (bweight mage) (mbsmoke),
> ematch(prenatal1 mmarried fbaby) metric(euclidean)
```

Treatment-effects estimation

<table>
<thead>
<tr>
<th>Outcome model : matching</th>
</tr>
</thead>
<tbody>
<tr>
<td>Distance metric: Euclidean</td>
</tr>
<tr>
<td>max = 139</td>
</tr>
</tbody>
</table>

```
| Coefficient std. err. | z | P>|z| | 95% conf. interval |
|------------------------|---|-------|-------------------|
| ATE                    |
| mbsmoke (smoker vs nonsmoker) | -240.3306 28.43006 | -8.45 | 0.000 | -296.0525 -184.6087 |
```

Abadie and Imbens (2006, 2011) have shown that nearest-neighbor matching estimators are not consistent when matching on two or more continuous covariates. A bias-corrected estimator that uses a linear function of variables can be specified with `biasadj()`.

### Example 3: Bias adjustment

Here we match on two continuous variables, `mage` and `fage`, and we use the bias-adjusted estimator:

```
. teffects nnmatch (bweight mage fage) (mbsmoke),
> ematch(prenatal1 mmarried fbaby) biasadj(mage fage)
```

Treatment-effects estimation

| Outcome model : matching |
| Distance metric: Mahalanobis |
| max = 25 |

```
| Coefficient std. err. | z | P>|z| | 95% conf. interval |
|------------------------|---|-------|-------------------|
| ATE                    |
| mbsmoke (smoker vs nonsmoker) | -223.8389 26.19973 | -8.54 | 0.000 | -275.1894 -172.4883 |
```

These results are similar to those reported in example 1.
Example 4: NNM can reduce to RA

NNM reduces to RA when matching exactly and all the covariates are discrete. We begin our illustration of this point by estimating the ATE by NNM using exact matching on `mmarried` and the mother’s age-categories `magecat`.

```
teffects nnmatch (bweight) (mbsmoke), ematch(i.mmarried i.magecat)
```

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Outcome model</th>
<th>Treatment model</th>
</tr>
</thead>
<tbody>
<tr>
<td>nearest-neighbor matching</td>
<td>matching</td>
<td>none</td>
</tr>
</tbody>
</table>

The RA estimator that includes the interactions among the discrete covariates produces the same point estimate.

```
teffects ra (bweight i.mmarried##i.magecat) (mbsmoke)
```

The two estimates of the ATE are the same. The standard errors differ in finite samples because the RA and NNM estimators use different robust estimators of the variance of the estimator.

With exact matching on discrete covariates, the NNM estimator reduces to an average of differences in cell means. With fully interacted discrete covariates, the RA estimator reduces to the same average of difference in cell means.
Video example

Treatment effects in Stata: Nearest-neighbor matching

Stored results

teffects nnmatch stores the following in e():

Scalars
- e(N) number of observations
- e(nj) number of observations for treatment level j
- e(k_levels) number of levels in treatment variable
- e(treated) level of treatment variable defined as treated
- e(control) level of treatment variable defined as control
- e(k_nneighbor) requested number of matches
- e(k_nnmin) minimum number of matches
- e(k_nnmax) maximum number of matches
- e(k_robust) matches for robust VCE

Macros
- e(cmd) teffects
- e(cmdline) command as typed
- e(depvar) name of outcome variable
- e(tvar) name of treatment variable
- e(emvarlist) exact match variables
- e(bavarlist) variables used in bias adjustment
- e(mvarlist) match variables
- e(subcmd) nnmatch
- e(metric) mahalanobis, ivariance, euclidean, or matrix matname
- e(stat) statistic estimated, ate or atet
- e(wtype) weight type
- e(wexp) weight expression
- e(tlevels) levels of treatment variable
- e(vce) vcetype specified in vce()
- e(vcetype) title used to label Std. err.
- e(datasignature) the checksum
- e(datasignaturevars) variables used in calculation of checksum
- e(properties) b V
- e(estat_cmd) program used to implement estat
- e(predict) program used to implement predict
- e(marginsnotok) predictions disallowed by margins
- e(asbalanced) factor variables fvset as asbalanced
- e(asobserved) factor variables fvset as asobserved

Matrices
- e(b) coefficient vector
- e(V) variance–covariance matrix of the estimators

Functions
- e(sample) marks estimation sample

In addition to the above, the following is stored in r():

Matrices
- r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.
Methods and formulas

The methods and formulas presented here provide the technical details underlying the estimators implemented in `teffects nnmatch` and `teffects psmatch`. See Methods and formulas of [CAUSAL] `teffects aipw` for the methods and formulas used by `teffects aipw`, `teffects ipw`, `teffects ipwra`, and `teffects ra`.

Methods and formulas are presented under the following headings:

- Nearest-neighbor matching estimator
- Bias-corrected matching estimator
- Propensity-score matching estimator
- PSM, ATE, and ATET variance adjustment

Nearest-neighbor matching estimator

teffects nnmatch implements the nearest-neighbor matching (NNM) estimator for the average treatment effect (ATE) and the average treatment effect on the treated (ATET). This estimator was derived by Abadie and Imbens (2006, 2011) and was previously implemented in Stata as discussed in Abadie et al. (2004).

teffects psmatch implements nearest-neighbor matching on an estimated propensity score. A propensity score is a conditional probability of treatment. The standard errors implemented in `teffects psmatch` were derived by Abadie and Imbens (2012).

teffects nnmatch and `teffects psmatch` permit two treatment levels: the treatment group with $t = 1$ and a control group with $t = 0$.

Matching estimators are based on the potential-outcome model, in which each individual has a well-defined outcome for each treatment level; see [CAUSAL] `teffects intro`. In the binary-treatment potential-outcome model, $y_1$ is the potential outcome obtained by an individual if given treatment-level 1 and $y_0$ is the potential outcome obtained by each individual $i$ if given treatment-level 0. The problem posed by the potential-outcome model is that only $y_{1i}$ or $y_{0i}$ is observed, never both. $y_{0i}$ and $y_{1i}$ are realizations of the random variables $y_0$ and $y_1$. Throughout this document, $i$ subscripts denote realizations of the corresponding, unsubscripted random variables.

Formally, the ATE is

$$\tau_1 = E(y_1 - y_0)$$

and the ATET is

$$\delta_1 = E(y_1 - y_0 | t = 1)$$

These expressions imply that we must have some solution to the missing-data problem that arises because we only observe either $y_{1i}$ or $y_{0i}$, not both.

For each individual, NNM uses an average of the individuals that are most similar, but get the other treatment level, to predict the unobserved potential outcome. NNM uses the covariates $\{x_1, x_2, \ldots, x_p\}$ to find the most similar individuals that get the other treatment level.

More formally, consider the vector of covariates $x_i = \{x_{i,1}, x_{i,2}, \ldots, x_{i,p}\}$ and frequency weight $w_i$ for observation $i$. The distance between $x_i$ and $x_j$ is parameterized by the vector norm

$$\|x_i - x_j\|_S = \{(x_i - x_j)^\prime S^{-1} (x_i - x_j)\}^{1/2}$$

where $S$ is a given symmetric, positive-definite matrix.

Using this distance definition, we find that the set of nearest-neighbor indices for observation $i$ is

$$\Omega_x^m(i) = \{j_1, j_2, \ldots, j_m | t_{j_k} = 1 - t_i, \|x_i - x_{j_k}\|_S < \|x_i - x_l\|_S, \ t_l = 1 - t_i, \ l \neq j_k\}$$
Here \( m_i \) is the smallest number such that the number of elements in each set, \( m_i = |\Omega^x_m(i)| = \sum_{j \in \Omega^x_m(i)} w_j \), is at least \( m \), the desired number of matches. You set the size of \( m \) using the \texttt{nnneighbor(#)} option. The number of matches for the \( i \)th observation may not equal \( m \) because of ties or if there are not enough observations with a distance from observation \( i \) within the caliper limit, \( c, \|x_i - x_j\|_S \leq c \). You may set the caliper limit by using the \texttt{caliper(#)} option. For ease of notation, we will use the abbreviation \( \Omega(i) = \Omega^x_m(i) \).

With the \texttt{metric(string)} option, you have three choices for the scaling matrix \( S \): Mahalanobis, inverse variance, or Euclidean.

\[
S = \begin{cases} 
\frac{(X - \bar{x}'1_n)'W(X - \bar{x}'1_n)}{\sum_i w_i - 1} & \text{if metric = mahalanobis} \\
\text{diag} \left\{ \frac{(X - \bar{x}'1_n)'W(X - \bar{x}'1_n)}{\sum_i w_i - 1} \right\} & \text{if metric = ivariance} \\
I_p & \text{if metric = euclidean}
\end{cases}
\]

where \( 1_n \) is an \( n \times 1 \) vector of ones, \( I_p \) is the identity matrix of order \( p \), \( \bar{x} = (\sum_i^n w_i x_i)/(\sum_i^n w_i) \), and \( W \) is an \( n \times n \) diagonal matrix containing frequency weights.

The NNM method predicting the potential outcome for the \( i \)th observation as a function of the observed \( y_i \) is

\[
\hat{y}_{ti} = \begin{cases} 
y_i & \text{if } t_i = t \\
\sum_{j \in \Omega(i)} w_j y_j / \sum_{j \in \Omega(i)} w_j & \text{otherwise}
\end{cases}
\]

for \( t \in \{0, 1\} \).

We are now set to provide formulas for estimates \( \hat{\tau}_1 \), the ATE, and \( \hat{\delta}_1 \), the ATET,

\[
\hat{\tau}_1 = \frac{\sum_{i=1}^n w_i (\hat{y}_{1i} - \hat{y}_{0i})}{\sum_{i=1}^n w_i} = \frac{\sum_{i=1}^n w_i (2t_i - 1)\{1 + K_m(i)\} y_i}{\sum_{i=1}^n w_i}
\]

\[
\hat{\delta}_1 = \frac{\sum_{i=1}^n t_i w_i (\hat{y}_{1i} - \hat{y}_{0i})}{\sum_{i=1}^n t_i w_i} = \frac{\sum_{i=1}^n t_i (1 - t_i) K_m(i) y_i}{\sum_{i=1}^n t_i w_i}
\]

where

\[
K_m(i) = \sum_{j=1}^n I\{i \in \Omega(j)\} \frac{w_j}{\sum_{k \in \Omega(j)} w_k}
\]

The estimated variance of \( \hat{\tau}_1 \) and \( \hat{\delta}_1 \) are computed as

\[
\hat{\sigma}^2_\tau = \frac{\sum_{i=1}^n w_i \left[ (\hat{y}_{1i} - \hat{y}_{0i} - \hat{\tau}_1)^2 + \tilde{\xi}^2_i \{K_m^2(i) + 2K_m(i) - K'_m(i)\} \right]}{(\sum_{i=1}^n w_i)^2}
\]

\[
\hat{\sigma}^2_\delta = \frac{\sum_{i=1}^n t_i w_i \left[ (\hat{y}_{1i} - \hat{y}_{0i} - \hat{\delta}_1)^2 + \tilde{\xi}^2_i \{K_m^2(i) - K'_m(i)\} \right]}{(\sum_{i=1}^n t_i w_i)^2}
\]
where

\[ K'_m(i) = \sum_{j=1}^{n} I\{i \in \Omega(j)\} \left( \sum_{k \in \Omega(j)} w_k \right)^2 \]

and \( \xi^2_i = \text{var}(y_{it} | x_i) \) is the conditional outcome variance. If we can assume that \( \xi^2_i \) does not vary with the covariates or treatment (homoskedastic), then we can compute an ATE estimate of \( \xi^2_T \) as

\[
\hat{\xi}^2_T = \frac{1}{2 \sum_i n_i} \sum_{i=1}^{n} \sum_{j=\in(i)} \frac{\sum w_j \{ y_i - y_j (1 - t_i) - \hat{\tau}_1 \}^2}{\sum_{j \in \in(i)} w_j}
\]

and an ATET estimate of \( \xi^2_\delta \) as

\[
\hat{\xi}^2_\delta = \frac{1}{2 \sum_i t_i n_i} \sum_{i=1}^{n} t_i \frac{\sum j w_j \{ y_i - y_j (1 - t_i) - \hat{\delta}_1 \}^2}{\sum_{j \in \in(i)} t_j w_j}
\]

If the conditional outcome variance is dependent on the covariates or treatment, we require an estimate for \( \xi^2_i \) at each observation. In this case, we require a second matching procedure, where we match on observations within the same treatment group.

Define the within-treatment matching set

\[ \Psi^k_h(i) = \{ j_1, j_2, \ldots, j_{h_i} | t_{j_k} = t_i, \| x_i - x_{j_k} \| < \| x_i - x_l \|, t_l = t_i, l \neq j_k \} \]

where \( h \) is the desired set size. As before, the number of elements in each set, \( h_i = |\Psi^k_h(i)| \), may vary depending on ties and the value of the caliper. You set \( h \) using the \texttt{vce(robust, nn(#))} option. As before, we will use the abbreviation \( \Psi(i) = \Psi^k_h(i) \) where convenient.

We estimate \( \xi^2_i \) by

\[
\hat{\xi}^2_{t_i}(x_i) = \frac{\sum_{j \in \Psi(i)} w_j (y_j - \bar{y}_{\Psi})^2}{\sum_{j \in \Psi(i)} w_j - 1} \quad \text{where} \quad \bar{y}_{\Psi} = \frac{\sum_{j \in \Psi(i)} w_j y_j}{\sum_{j \in \Psi(i)} w_j - 1}
\]

### Bias-corrected matching estimator

When matching on more than one continuous covariate, the matching estimator described above is biased, even in infinitely large samples; in other words, it is not \( \sqrt{n} \)-consistent; see Abadie and Imbens (2006, 2011). Following Abadie and Imbens (2011) and Abadie et al. (2004), \texttt{teffects nnmatch} makes an adjustment based on the regression functions \( \mu_t(x_i) = E(y_{it} | \tilde{x} = \tilde{x}_i) \), for \( t = 0, 1 \) and the set of covariates \( \tilde{x}_i = (\tilde{x}_{i,1}, \ldots, \tilde{x}_{i,q}) \). The bias-correction covariates may be the same as the NNM covariates \( x_i \). We denote the least-squares estimates as \( \tilde{\mu}_t(\tilde{x}_i) = \tilde{\nu}_t + \tilde{\beta}_t \tilde{x}_i \), where we regress \( \{ y_{it} | t_i = t \} \) onto \( \{ \tilde{x}_i | t_i = t \} \) with weights \( w_i K_m(i) \), for \( t = 0, 1 \).
Given the estimated regression functions, the bias-corrected predictions for the potential outcomes are computed as

\[
\hat{y}_{ti} = \begin{cases} 
    y_i & \text{if } t_i = t \\
    \frac{\sum_{j \in \Omega_{m}^p(i)} w_j \{ y_j + \hat{\mu}_t(\tilde{x}_i) - \hat{\mu}_t(\tilde{x}_j) \}}{\sum_{j \in \Omega_{m}^p(i)} w_j} & \text{otherwise}
\end{cases}
\]

The \texttt{biasadj(varlist)} option specifies the bias-adjustment covariates \(\tilde{x}_i\).

### Propensity-score matching estimator

The propensity-score matching (PSM) estimator uses a treatment model (TM), \(p(z_i, t, \gamma)\), to model the conditional probability that observation \(i\) receives treatment \(t\) given covariates \(z_i\). The literature calls \(p(z_i, t, \gamma)\) a propensity score, and PSM matches on the estimated propensity scores.

When matching on the estimated propensity score, the set of nearest-neighbor indices for observation \(i\), \(i = 1, \ldots, n\), is

\[
\Omega_{m}^p(i) = \{j_1, j_2, \ldots, j_{m_i} \mid t_{jk} = 1 - t_i, |\hat{p}_i(t) - \hat{p}_{jk}(t)| < |\hat{p}_i(t) - \hat{p}_l(t)|, t_l = 1 - t_i, l \neq j_k\}
\]

where \(\hat{p}_i(t) = p(z_i, t, \hat{\gamma})\). As was the case with the NNM estimator, \(m_i\) is the smallest number such that the number of elements in each set, \(m_i = |\Omega_{m}^p(i)| = \sum_{j \in \Omega_{m}^p(i)} w_j\), is at least \(m\), the desired number of matches, set by the \texttt{nnneighbor(#)} option.

We define the within-treatment matching set analogously,

\[
\Psi_{h}^p(i) = \{j_1, j_2, \ldots, j_{h_i} \mid t_{jk} = t_i, |\hat{p}_i(t) - \hat{p}_{jk}(t)| < |\hat{p}_i(t) - \hat{p}_l(t)|, t_l = t_i, l \neq j_k\}
\]

where \(h\) is the desired number of within-treatment matches, and \(h_i = |\Psi_{h}^p(i)|\), for \(i = 1, \ldots, n\), may vary depending on ties and the value of the caliper. The sets \(\Psi_{h}^p(i)\) are required to compute standard errors for \(\hat{\tau}_1\) and \(\hat{\delta}_1\).

Once a matching set is computed for each observation, the potential-outcome mean, ATE, and ATET computations are identical to those of NNM. The ATE and ATET standard errors, however, must be adjusted because the TM parameters were estimated; see Abadie and Imbens (2012).

### PSM, ATE, and ATET variance adjustment

The variances for \(\hat{\tau}_1\) and \(\hat{\delta}_1\) must be adjusted because we use \(\hat{\gamma}\) instead of \(\gamma\). The adjusted variances for \(\hat{\tau}_1\) and \(\hat{\delta}_1\) have the following forms, respectively:

\[
\hat{\sigma}_{\tau, \text{adj}}^2 = \hat{\sigma}_{\tau}^2 + \hat{\mathbf{c}}_{\tau}' \hat{\mathbf{V}}_\gamma \hat{\mathbf{c}}_{\tau}
\]

\[
\hat{\sigma}_{\delta, \text{adj}}^2 = \hat{\sigma}_{\delta}^2 - \hat{\mathbf{c}}_{\delta}' \hat{\mathbf{V}}_\gamma \hat{\mathbf{c}}_{\delta} + \frac{\partial \hat{\delta}_1}{\partial \gamma} \hat{\mathbf{V}}_\gamma \frac{\partial \hat{\delta}_1}{\partial \gamma}
\]

In both equations, the matrix \(\hat{\mathbf{V}}_\gamma\) is the TM coefficient variance–covariance matrix.
The adjustment term for ATE can be expressed as

$$\hat{c}_\tau = \frac{1}{\sum_{i=1}^n w_i} \sum_{i=1}^n w_i f(z'_i \hat{\gamma}) \left( \frac{\text{cov} (z_i, \hat{y}_{i1})}{\hat{p}_i(1)} + \frac{\text{cov} (z_i, \hat{y}_{i0})}{\hat{p}_i(0)} \right)$$

where

$$f(z'_i \hat{\gamma}) = \frac{dp(z_i, 1, \hat{\gamma})}{d(z'_i \hat{\gamma})}$$

is the derivative of $p(z_i, 1, \hat{\gamma})$ with respect to $z'_i \hat{\gamma}$, and

$$\text{cov} (z_i, \hat{y}_{ti}) = \begin{cases} \sum_{j \in \Psi_h(i)} w_j (z_j - z_{\Psi_i}) (y_j - \bar{y}_{\Psi_i}) & \text{if } t_i = t \\ \sum_{j \in \Omega_h(i)} w_j (z_j - z_{\Omega_i}) (y_j - \bar{y}_{\Omega_i}) & \text{otherwise} \end{cases}$$

is a $p \times 1$ vector with

$$z_{\Psi_i} = \frac{\sum_{j \in \Psi_h(i)} w_j z_j}{\sum_{j \in \Psi_h(i)} w_j}, \quad z_{\Omega_i} = \frac{\sum_{j \in \Omega_h(i)} w_j z_j}{\sum_{j \in \Omega_h(i)} w_j} \quad \text{and} \quad \bar{y}_{\Omega_i} = \frac{\sum_{j \in \Omega_h(i)} w_j y_j}{\sum_{j \in \Omega_h(i)} w_j}$$

Here we have used the notation $\Psi_h(i) = \Psi^p_h(i)$ and $\Omega_h(i) = \Omega^p_h(i)$ to stress that the within-treatment and opposite-treatment clusters used in computing $\tilde{\sigma}_{\tau,\text{adj}}^2$ and $\delta_{\tau,\text{adj}}^2$ are based on $h$ instead of the cluster $\Omega^m_h(i)$ based on $m$ used to compute $\hat{\tau}_1$ and $\hat{\delta}_1$, although you may desire to have $h = m$.

The adjustment term $c_\delta$ for the ATET estimate has two components, $c_\delta = c_{\delta,1} + c_{\delta,2}$, defined as

$$c_{\delta,1} = \frac{1}{\sum_{i=1}^n t_i w_i} \sum_{i=1}^n w_i z_i f(z'_i \hat{\gamma}) \left( \hat{y}_{i1} - \hat{y}_{i0} - \hat{\delta}_1 \right)$$

$$c_{\delta,2} = \frac{1}{\sum_{i=1}^n t_i w_i} \sum_{i=1}^n w_i f(z'_i \hat{\gamma}) \left\{ \text{cov} (z_i, \hat{y}_{i1}) + \frac{\hat{p}_i(1)}{\hat{p}_i(0)} \text{cov} (z_i, \hat{y}_{i0}) \right\}$$

where

$$\tilde{y}_{ti} = \begin{cases} \sum_{j \in \Psi_h(-i)} w_j y_j & \text{if } t_i = t \\ \sum_{j \in \Omega_h(-i)} w_j y_j & \text{otherwise} \end{cases}$$

and the within-treatment matching sets $\Psi_h(-i) = \Psi^p_h(-i)$ are similar to $\Psi^p_h(i)$ but exclude observation $i$:

$$\Psi^p_h(-i) = \{j_1, j_2, \ldots, j_{n_i} \mid j_k \neq i, \ t_{j_k} = t_i, \ |\hat{p}_i - \hat{p}_{j_k}| < |\hat{p}_i - \hat{p}_l|, \ t_l = t_i, \ l \notin \{i, j_k\} \}$$
Finally, we cover the computation of $\widehat{\frac{\partial \delta_1}{\partial \gamma'}}$ in the third term on the right-hand side of $\widehat{\sigma_{\delta, \text{adj}}^2}$. Here we require yet another clustering, but we match on the opposite treatment by using the covariates $z_i = (z_{i,1}, \ldots, z_{i,p})'$. We will denote these cluster sets as $\Omega_m^z(i)$, for $i = 1, \ldots, n$.

The estimator of the $p \times 1$ vector $(\frac{\partial \delta_1}{\partial \gamma'})$ is computed as

$$\widehat{\frac{\partial \delta_1}{\partial \gamma'}} = \frac{1}{\sum_i t_i w_i} \sum_{i=1}^{n} z_i f(z' \hat{\gamma}) \left\{ \left(2t_i - 1\right)(y_i - \bar{y}_{\Omega_m^z(i)} - \hat{\delta}_1 \right\}$$

where

$$\bar{y}_{\Omega_m^z(i)} = \frac{\sum_{j \in \Omega_m^z(i)} w_j y_j}{\sum_{j \in \Omega_m^z(i)} w_j}$$

References


Also see

[CAUSAL] teffects postestimation — Postestimation tools for teffects
[CAUSAL] teffects — Treatment-effects estimation for observational data
[CAUSAL] teffects psmatch — Propensity-score matching

[U] 20 Estimation and postestimation commands
### Postestimation commands

The following postestimation command is of special interest after `teffects`:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>teoverlap</code></td>
<td>overlap plots</td>
</tr>
<tr>
<td><code>tebalance</code></td>
<td>check balance of covariates</td>
</tr>
</tbody>
</table>

The following standard postestimation commands are also available:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>estat summarize</code></td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td><code>estat vce</code></td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td><code>estimates</code></td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td><code>etable</code></td>
<td>table of estimation results</td>
</tr>
<tr>
<td><code>hausman</code></td>
<td>Hausman’s specification test</td>
</tr>
<tr>
<td><code>lincom</code></td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td><code>nlcom</code></td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td><code>predict</code></td>
<td>treatment effects, conditional means at treatment, propensity scores, etc.</td>
</tr>
<tr>
<td><code>predictnl</code></td>
<td>point estimates, standard errors, testing, and inference for generalized predictions</td>
</tr>
<tr>
<td><code>test</code></td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td><code>testnl</code></td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>
predict

Description for predict

predict creates a new variable containing predictions such as treatment effects, potential outcomes, conditional means, propensity scores, linear predictions, nearest-neighbor distances, and log square root of latent variances.

Menu for predict

Statistics > Postestimation

Syntaxes for predict

Syntaxes are presented under the following headings:

- Syntax for predict after aipw and ipwra
- Syntax for predict after ipw
- Syntax for predict after nnmatch and psmatch
- Syntax for predict after ra

Syntax for predict after aipw and ipwra

```
predict [type] { stub* | newvar | newvarlist } [if] [in]
               [ , statistic tlevel(treat_level) ]
```

```
predict [type] stub* [if] [in], scores
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>te</strong></td>
<td>treatment effect; the default</td>
</tr>
<tr>
<td><strong>cmean</strong></td>
<td>conditional mean at treatment level</td>
</tr>
<tr>
<td><strong>ps</strong></td>
<td>propensity score</td>
</tr>
<tr>
<td><strong>xb</strong></td>
<td>linear prediction</td>
</tr>
<tr>
<td><strong>psxb</strong></td>
<td>linear prediction for propensity score</td>
</tr>
<tr>
<td><strong>lnsigma</strong></td>
<td>log square root of conditional latent variance (for outcome model</td>
</tr>
<tr>
<td></td>
<td>hetprobit()) at treatment level</td>
</tr>
<tr>
<td><strong>pslnsigma</strong></td>
<td>log square root of latent variance (for treatment model hetprobit())</td>
</tr>
</tbody>
</table>

If you do not specify tlevel() and only specify one new variable, te and psxb assume tlevel() specifies the first noncontrol treatment level.

If you do not specify tlevel() and only specify one new variable, cmean, ps, xb, and lnsigma assume tlevel() specifies the first treatment level.

You specify one or t new variables with cmean, ps, xb, and lnsigma, where t is the number of treatment levels.

You specify one or t−1 new variables with te, psxb, and pslnsigma.
Syntax for predict after `ipw`

```
predict [type] { stub* | newvar | newvarlist } [if] [in]  
[ , statistic tlevel(treat_level) ]
```

```
predict [type] stub* [if] [in], scores
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ps</td>
<td>propensity score; the default</td>
</tr>
<tr>
<td>xb</td>
<td>linear prediction for the propensity score</td>
</tr>
<tr>
<td>lnsigma</td>
<td>log square root of latent variance (for treatment model <code>hetprobit()</code>)</td>
</tr>
</tbody>
</table>

If you do not specify `tlevel()` and only specify one new variable, `ps` assumes `tlevel()` specifies the first treatment level.

If you do not specify `tlevel()` and only specify one new variable, `xb` assumes `tlevel()` specifies the first noncontrol treatment level.

You specify one or `t` new variables with `ps`, where `t` is the number of treatment levels.

You specify one or `t-1` new variables with `xb` and `lnsigma`.

Syntax for predict after `nnmatch` and `psmatch`

```
predict [type] { stub* | newvarlist } [ , statistic tlevel(treat_level) ]
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>te</td>
<td>treatment effect; the default</td>
</tr>
<tr>
<td>po</td>
<td>potential outcome</td>
</tr>
<tr>
<td>distance</td>
<td>nearest-neighbor distance</td>
</tr>
<tr>
<td>ps</td>
<td>propensity score (psmatch only)</td>
</tr>
<tr>
<td>lnsigma</td>
<td>log square root of latent variance (for treatment model <code>hetprobit()</code>)</td>
</tr>
</tbody>
</table>

These statistics are available for the estimation sample only and require the estimation option `generate(stub)`. This is because of the nonparametric nature of the matching estimator.

If you do not specify `tlevel()` and only specify one new variable, `po` and `ps` assume `tlevel()` specifies the first treatment level.

You specify one new variable with `te` and `lnsigma`.

You specify one or two new variables with `po` and `ps`. 
Syntax for predict after ra

\begin{verbatim}
predict [type] { stub* | newvar | newvarlist } [if] [in] [, statistic tlevel(treat_level) ]
predict [type] stub* [if] [in], scores
\end{verbatim}

\textbf{statistic} \hspace{2cm} \textbf{Description}

\begin{tabular}{ll}
\hline
Main & \\
te & treatment effect; the default \\
\texttt{cmean} & conditional mean at treatment level \\
xb & linear prediction \\
\texttt{lnsigma} & log square root of conditional latent variance (for outcome model \texttt{hetprobit()}) at treatment level \\
\hline
\end{tabular}

If you do not specify \texttt{tlevel()} and only specify one new variable, \texttt{te} assumes \texttt{tlevel()} specifies the first noncontrol treatment level.

If you do not specify \texttt{tlevel()} and only specify one new variable, \texttt{cmean}, \texttt{xb}, and \texttt{lnsigma} assume \texttt{tlevel()} specifies the first treatment level.

You specify one or \texttt{t} new variables with \texttt{cmean}, \texttt{xb}, and \texttt{lnsigma}, where \texttt{t} is the number of treatment levels.

You specify one or \texttt{t}−\texttt{1} new variables with \texttt{te}.

\section*{Options for predict}

Options are presented under the following headings:

Options for predict after \texttt{aipw} and \texttt{ipwra}
Options for predict after \texttt{ipw}
Options for predict after \texttt{nnmatch} and \texttt{psmatch}
Options for predict after \texttt{ra}

\section*{Options for predict after \texttt{aipw} and \texttt{ipwra}}

\begin{verbatim}
Main
\end{verbatim}

te, the default, calculates the treatment effect for each noncontrol treatment level or the treatment level specified in \texttt{tlevel()}. If you specify the \texttt{tlevel()} option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

\texttt{cmean} calculates the conditional mean for each treatment level or the treatment level specified in \texttt{tlevel()}. If you specify the \texttt{tlevel()} option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

\texttt{ps} calculates the propensity score of each treatment level or the treatment level specified in \texttt{tlevel()}. If you specify the \texttt{tlevel()} option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

\texttt{xb} calculates the linear prediction at each treatment level or the treatment level specified in \texttt{tlevel()}. If you specify the \texttt{tlevel()} option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.
psxb calculates the linear prediction for the propensity score at each noncontrol level of the treatment or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

lnsigma calculates the log square root of the conditional latent variance for each treatment level or the treatment level specified in tlevel(). This option is valid when outcome model hetprobit() was used. If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

pslnsigma calculates the log square root of the latent variance for the propensity score. This option is only valid when treatment model hetprobit() was used. Specify only one new variable.

tlevel(treat_level) specifies the treatment level for prediction.

scores calculates the score variables. Parameter-level scores are computed for the treatment mean and average treatment-effect equations. Equation-level scores are computed for the propensity-score equations.

The jth new variable will contain the scores for the jth parameter in the coefficient table if $j \leq t$, where t is the number of treatment levels. Otherwise, it will contain the scores for fitted equation $j - t$ following the first t parameters in the coefficient table.

Options for predict after ipw

ps, the default, calculates the propensity score of each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

xb calculates the linear prediction for the propensity score at each noncontrol level of the treatment or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

lnsigma calculates the log square root of the latent variance. This option is only valid when treatment model hetprobit() was used. Specify only one new variable.

tlevel(treat_level) specifies the treatment level for prediction.

scores calculates the score variables. Parameter-level scores are computed for the treatment mean and average treatment-effect equations. Equation-level scores are computed for the propensity-score equations.

The jth new variable will contain the scores for the jth parameter in the coefficient table if $j \leq t$, where t is the number of treatment levels. Otherwise, it will contain the scores for fitted equation $j - t$ following the first t parameters in the coefficient table.

Options for predict after nnmatch and psmatch

te, the default, calculates the treatment effect.

po calculates the predicted potential outcomes for each observation and treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify new variables for the control and treated groups.
**distance** calculates the distances of the nearest neighbors for each observation. The number of variables generated is equal to the maximum number of nearest-neighbor matches. This is equal to the number of index variables generated by the estimation option `generate(stub)`. You may use the `stub*` syntax to set the distance variable prefix: `stub1, stub2, ...`.

**ps** calculates the propensity score of each treatment level or the propensity score of the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify new variables for the control and treated groups.

**lnsigma** calculates the log square root of the latent variance. This option is only valid when treatment model `hetprobit()` was used. Specify only one new variable.

**tlevel(treat_level)** restricts potential-outcome estimation to either the treated group or the control group. This option may only be specified with options `po` and `ps`.

### Options for predict after ra

#### Main

**te**, the default, calculates the treatment effect for each noncontrol treatment level or the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level (except the control level).

**cmean** calculates the conditional mean for each treatment level or the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

**xb** calculates the linear prediction at each treatment level or the treatment level specified in `tlevel()`. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

**lnsigma** calculates the log square root of the conditional latent variance for each treatment level or the treatment level specified in `tlevel()`. This option is valid when outcome model `hetprobit()` was used. If you specify the `tlevel()` option, you need to specify only one new variable; otherwise, you must specify a new variable for each treatment level.

**tlevel(treat_level)** specifies the treatment level for prediction.

**scores** calculates the score variables. Parameter-level scores are computed for the treatment mean and average treatment-effect equations. Equation-level scores are computed for the regression equations.

The $j$th new variable will contain the scores for the $j$th parameter in the coefficient table if $j \leq t$, where $t$ is the number of treatment levels. Otherwise, it will contain the scores for fitted equation $j - t$ following the first $t$ parameters in the coefficient table.

### Remarks and examples

Checking model specification is the most frequent reason for postestimation computation after `teffects`. `teoverlap` provides a graphical method for checking the overlap assumption; see [CAUSAL] `teoverlap`. Summarizing the estimated probabilities provides another check. Recall that the reciprocals of these estimated probabilities are used as weights by some of the estimators. If the estimated probabilities are too small, the weights blow up.
We estimate the ATE of maternal smoking on infant birthweight by inverse-probability weighting; see example 1 of [CAUSAL] `teffects ipw` for background.

```
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ipw (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu, probit)
Iteration 0:  EE criterion =  4.621e-21
Iteration 1:  EE criterion =  7.358e-26

Treatment-effects estimation
                              Number of obs   =   4,642
Estimator                   : inverse-probability weights
Outcome model : weighted mean
Treatment model: probit

                      bweight          Robust
                      Coefficient std. err.    z    P>|z|    [95% conf. interval]
ATE
   mbsmoke
      (Smoker vs Nonsmoker)  -230.6886    25.81524   -8.94    0.000   -281.2856   -180.0917

POmean
   mbsmoke
      Nonsmoker              3403.463      9.571369  355.59    0.000   3384.703    3422.222
```

Below we compute and summarize the estimated treatment probabilities.

```
. predict pr1
(option ps assumed; propensity score)
. summarize pr1 if mbsmoke==1, detail

propensity score, mbsmoke=Non
Percentiles Smallest
  1%    .2991634    .2196947
  5%    .544155    .2258079
 10%    .5973879    .2258079    Obs    864
 25%    .637770    .2409025    Sum of wgt. 864
 50%    .7601717         Mean    .7456264
 75%    .8453946         Std. dev.   .1276102
 90%    .8943686    .9533503    Variance   .0162844
 95%    .9096801    .9610222    Skewness  -.7701643
 99%    .9367017    .9665684    Kurtosis   3.858214

The smallest values do not imply very large weights.
```
Below we compute and summarize the estimated probabilities of not getting the treatment.

```
. generate pr0 = 1 -pr1
. summarize pr0 if mbsmoke==0, detail
```

<table>
<thead>
<tr>
<th>Percentiles</th>
<th>Smallest</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>.0351884</td>
</tr>
<tr>
<td>5%</td>
<td>.0578012</td>
</tr>
<tr>
<td>10%</td>
<td>.0674359</td>
</tr>
<tr>
<td>25%</td>
<td>.0950869</td>
</tr>
<tr>
<td>50%</td>
<td>.1372589</td>
</tr>
<tr>
<td>75%</td>
<td>.2211142</td>
</tr>
<tr>
<td>90%</td>
<td>.3242757</td>
</tr>
<tr>
<td>95%</td>
<td>.3883457</td>
</tr>
<tr>
<td>99%</td>
<td>.501537</td>
</tr>
</tbody>
</table>

```

Although there are two small probabilities, overall the small values do not imply large weights.

**Also see**

- [CAUSAL] `teoverlap` — Overlap plots
- [CAUSAL] `teffects aipw` — Augmented inverse-probability weighting
- [CAUSAL] `teffects ipw` — Inverse-probability weighting
- [CAUSAL] `teffects ipwra` — Inverse-probability-weighted regression adjustment
- [CAUSAL] `teffects nnmatch` — Nearest-neighbor matching
- [CAUSAL] `teffects psmatch` — Propensity-score matching
- [CAUSAL] `teffects ra` — Regression adjustment
- [U] 20 Estimation and postestimation commands
teffects psmatch — Propensity-score matching

Description

`teffects psmatch` estimates the average treatment effect (ATE) and average treatment effect on the treated (ATET) from observational data by propensity-score matching (PSM). PSM estimators impute the missing potential outcome for each subject by using an average of the outcomes of similar subjects that receive the other treatment level. Similarity between subjects is based on estimated treatment probabilities, known as propensity scores. The treatment effect is computed by taking the average of the difference between the observed and potential outcomes for each subject. `teffects psmatch` accepts a continuous, binary, count, fractional, or nonnegative outcome.

See [CAUSAL] `teffects intro` or [CAUSAL] `teffects intro advanced` for more information about estimating treatment effects from observational data.

Quick start

ATE of `treat` on `y` estimated by PSM using a logistic model for `treat` on `x` and indicators for levels of categorical variable `a`

```
teffects psmatch (y) (treat x i.a)
```

Same as above, but estimate the ATET

```
teffects psmatch (y) (treat x i.a), atet
```

ATE of `treat` using a heteroskedastic probit model for treatment

```
teffects psmatch (y) (treat x i.a, hetprobit(x i.a))
```

With 4 matches per observation

```
teffects psmatch (y) (treat x i.a), nneighbor(4)
```

Menu

Statistics > Causal inference/treatment effects > Continuous outcomes > Propensity-score matching
Statistics > Causal inference/treatment effects > Binary outcomes > Propensity-score matching
Statistics > Causal inference/treatment effects > Count outcomes > Propensity-score matching
Statistics > Causal inference/treatment effects > Fractional outcomes > Propensity-score matching
Statistics > Causal inference/treatment effects > Nonnegative outcomes > Propensity-score matching
Syntax

teffects psmatch (ovar) (tvar tmvarlist [, tmodel]) [if] [in] [weight]
[ , stat options ]

*ovar* is a binary, count, continuous, fractional, or nonnegative outcome of interest.
*tvar* must contain integer values representing the treatment levels.
*tmvarlist* specifies the variables that predict treatment assignment in the treatment model. Only two treatment levels are allowed.

<table>
<thead>
<tr>
<th>tmodel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>logit</td>
<td>logistic treatment model; the default</td>
</tr>
<tr>
<td>probit</td>
<td>probit treatment model</td>
</tr>
<tr>
<td>hetprobit(varlist)</td>
<td>heteroskedastic probit treatment model</td>
</tr>
</tbody>
</table>

*tmodel* specifies the model for the treatment variable.

<table>
<thead>
<tr>
<th>stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
</tr>
</tbody>
</table>
Model

\texttt{nneighbor(#)} specify number of matches per observation; default is \texttt{nneighbor(1)}

SE/Robust

\texttt{vce(vcetype)} \texttt{vcetype} may be

\texttt{vce(robust [, nn(#) ])}; use robust Abadie–Imbens standard errors with \texttt{#} matches

\texttt{vce(iid)}; use independent and identically distributed Abadie–Imbens standard errors

Reporting

\texttt{level(#)} set confidence level; default is \texttt{level(95)}

\texttt{display_options} control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

Advanced

\texttt{caliper(#)} specify the maximum distance for which two observations are potential neighbors

\texttt{pstorlenace(#)} set tolerance for overlap assumption

\texttt{osample(newvar)} \texttt{newvar} identifies observations that violate the overlap assumption

\texttt{control(# | label)} specify the level of \texttt{tvar} that is the control

\texttt{tlevel(# | label)} specify the level of \texttt{tvar} that is the treatment

\texttt{generate(stub)} generate variables containing the observation numbers of the nearest neighbors

\texttt{coeflegend} display legend instead of statistics

\texttt{tmvarlist} may contain factor variables; see \texttt{[U] 11.4.3 Factor variables.}

\texttt{by}, \texttt{collect}, and \texttt{statsby} are allowed; see \texttt{[U] 11.1.10 Prefix commands.}

\texttt{fweight} are allowed; see \texttt{[U] 11.1.6 weight.}

\texttt{coeflegend} does not appear in the dialog box.

See \texttt{[U] 20 Estimation and postestimation commands} for more capabilities of estimation commands.

Options

\texttt{nneighbor(#)} specifies the number of matches per observation. The default is \texttt{nneighbor(1)}. Each individual is matched with at least the specified number of individuals from the other treatment level. \texttt{nneighbor()} must specify an integer greater than or equal to 1 but no larger than the number of observations in the smallest group.

\texttt{stat} is one of two statistics: \texttt{ate} or \texttt{atet}. \texttt{ate} is the default.

\texttt{ate} specifies that the average treatment effect be estimated.

\texttt{atet} specifies that the average treatment effect on the treated be estimated.
teffects psmatch — Propensity-score matching

vce(vcetype) specifies the standard errors that are reported. By default, teffects psmatch uses two matches in estimating the robust standard errors.

vce(robust [, nn(#)]) specifies that robust standard errors be reported and that the requested number of matches be used optionally.

vce(iid) specifies that standard errors for independent and identically distributed data be reported.

The standard derivative-based standard-error estimators cannot be used by teffects psmatch, because these matching estimators are not differentiable. The implemented methods were derived by Abadie and Imbens (2006, 2011, 2012); see Methods and formulas.

As discussed in Abadie and Imbens (2008), bootstrap estimators do not provide reliable standard errors for the estimator implemented by teffects psmatch.

level(#); see [R] Estimation options.

display options: noc, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nol, fvwrap(#), fvwrap(style), cformat(%,fmt), pformat(%,fmt), sformat(%,fmt), and nolstretch; see [R] Estimation options.

Advanced

caliper(#) specifies the maximum distance at which two observations are a potential match. By default, all observations are potential matches regardless of how dissimilar they are.

In teffects psmatch, the distance is measured by the estimated propensity score. If an observation has no matches, teffects psmatch exits with an error.

pstolerance(#) specifies the tolerance used to check the overlap assumption. The default value is pstolerance(1e-5). teffects will exit with an error if an observation has an estimated propensity score smaller than that specified by pstolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that violate the overlap assumption. Two checks are made to verify the assumption. The first ensures that the propensity scores are greater than pstolerance(#) and less than 1 − pstolerance(#). The second ensures that each observation has at least nneighbor(#) matches in the opposite treatment group within the distance specified by caliper(#).

The vce(robust, nn(#)) option also requires at least # matches in the same treatment group within the distance specified by caliper(#).

The average treatment effect on the treated, option atet, using vce(iid) requires only nneighbor(#) control group matches for the treated group.

control(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() and tlevel() may not specify the same treatment level.

tlevel(# | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

generate(stub) specifies that the observation numbers of the nearest neighbors be stored in the new variables stub1, stub2, .... This option is required if you wish to perform postestimation based
on the matching results. The number of variables generated may be more than \texttt{nneighbor(#)} because of tied distances. These variables may not already exist.

The following option is available with \texttt{teffects psmatch} but is not shown in the dialog box: \texttt{coeflegend}; see \texttt{[R] Estimation options}.

\section*{Remarks and examples}

Propensity-score matching uses an average of the outcomes of similar subjects who get the other treatment level to impute the missing potential outcome for each subject. The \textit{ATE} is computed by taking the average of the difference between the observed and potential outcomes for each subject. \texttt{teffects psmatch} determines how near subjects are to each other by using estimated treatment probabilities, known as propensity scores. This type of matching is known as propensity-score matching (PSM).

PSM does not need bias correction, because PSM matches on a single continuous covariate. In contrast, the nearest-neighbor matching estimator implemented in \texttt{teffects nnmatch} uses a bias-correction term when matching on more than one continuous covariate. In effect, the PSM estimator parameterizes the bias-correction term in the treatment probability model. See \texttt{[CAUSAL] teffects intro} or \texttt{[CAUSAL] teffects intro advanced} for more information about this estimator.

We will illustrate the use of \texttt{teffects psmatch} by using data from a study of the effect of a mother’s smoking status during pregnancy (\texttt{mbsmoke}) on infant birthweight (\texttt{bweight}) as reported by Cattaneo (2010). This dataset also contains information about each mother’s age (\texttt{mage}), education level (\texttt{medu}), marital status (\texttt{mmarried}), whether the first prenatal exam occurred in the first trimester (\texttt{prenatal1}), whether this baby was the mother’s first birth (\texttt{fbaby}), and the father’s age (\texttt{fage}).

\subsection*{Example 1: Estimating the ATE}

We begin by using \texttt{teffects psmatch} to estimate the ATE of \texttt{mbsmoke} on \texttt{bweight}. We use a logistic model (the default) to predict each subject’s propensity score, using covariates \texttt{mage}, \texttt{medu}, \texttt{mmarried}, and \texttt{fbaby}. Because the performance of PSM hinges upon how well we can predict the propensity scores, we will use factor-variable notation to include both linear and quadratic terms for \texttt{mage}, the only continuous variable in our model:

\begin{verbatim}
  . use https://www.stata-press.com/data/r18/cattaneo2
  . teffects psmatch (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu)
Treatment-effects estimation
  Number of obs = 4,642
  Estimator : propensity-score matching
  Matches: requested = 1
  Outcome model : matching
  Treatment model: logit
  min =  1
  max =  74

|                | Coefficient | std. err. |    z  |    P>|z|   | [95% conf. interval] |
|----------------|-------------|-----------|-------|--------|---------------------|
| \textbf{ATE}   |             |           |       |        |                     |
| mbsmoke (Smoker vs Nonsmoker) | -210.9683   |  32.021   | -6.59 | 0.000  | -273.7284  -148.2083 |

The average birthweight if all mothers were to smoke would be 211 grams less than the average that would occur if none of the mothers had smoked.
By default, \texttt{teffects psmatch} estimates the ATE by matching each subject to a single subject with the opposite treatment whose propensity score is closest. Sometimes, however, we may want to ensure that matching occurs only when the propensity scores of a subject and a match differ by less than a specified amount. To do that, we use the \texttt{caliper()} option. If a match within the distance specified in the \texttt{caliper()} option cannot be found, \texttt{teffects psmatch} exits.

\section*{Example 2: Specifying the caliper}

Here we reconsider the previous example, first specifying that we only want to consider a pair of observations a match if the absolute difference in the propensity scores is less than 0.03:

\begin{verbatim}
. teffects psmatch (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu),
> caliper(0.03)
\end{verbatim}

The error arose because there is not a smoking mother whose propensity score is within 0.03 of the propensity score of the nonsmoking mother in observation 4504. If we instead raise the caliper to 0.10, we have matches for all subjects and therefore obtain the same results as in example 1:

\begin{verbatim}
. teffects psmatch (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu),
> caliper(0.1)
\end{verbatim}

\begin{verbatim}
Treatment-effects estimation Number of obs = 4,642
Estimator : propensity-score matching Matches: requested = 1
Outcome model : matching min = 1
Treatment model: logit max = 74

| bweight | AI robust Coefficient | std. err. | z | P>|z| | [95% conf. interval] |
|---|---|---|---|---|---|
| ATE | mbsmoke (Smoker vs Nonsmoker) | \( -210.9683 \) | 32.021 | -6.59 | 0.000 | \( -273.7284 \) \( -148.2083 \) |
\end{verbatim}

\section*{Technical note}

Example 2 highlights that estimating the ATE requires finding matches for both the treated and control subjects. In contrast, estimating the ATET only requires finding matches for the treated subjects. Because subject 4504 is a control subject, we can estimate the ATET using \texttt{caliper(0.03)}. We must also specify \texttt{vce(iid)} because the default robust standard errors for the estimated ATET require viable matches for both treated subjects and control subjects. (This requirement comes from the nonparametric method derived by Abadie and Imbens [2012].)
. teffects psmatch (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu),
   > atet vce(iid) caliper(0.03)

Treatment-effects estimation
   Number of obs = 4,642
Estimator : propensity-score matching
   Matches: requested = 1
Outcome model : matching
   min = 1
Treatment model: logit
   max = 74

| bweight | Coefficient | Std. err. | z   | P>|z|   | [95% conf. interval] |
|---------|-------------|-----------|-----|--------|---------------------|
| ATET    |             |           |     |        |                     |
| mbsmoke (Smoker vs Nonsmoker) | -236.7848  | 26.11698 | -9.07| 0.000  | -287.9731 -185.5964 |

In the previous examples, each subject was matched to at least one other subject, which is the default behavior for `teffects psmatch`. However, we can request that `teffects psmatch` match each subject to multiple subjects with the opposite treatment level by specifying the `nneighbor()` option. Matching on more distant neighbors can reduce the variance of the estimator at a cost of an increase in bias.

**Example 3**

Now we request that `teffects psmatch` match a mother to four mothers in the opposite treatment group:

. teffects psmatch (bweight) (mbsmoke mmarried c.mage##c.mage fbaby medu),
   > nneighbor(4)

Treatment-effects estimation
   Number of obs = 4,642
Estimator : propensity-score matching
   Matches: requested = 4
Outcome model : matching
   min = 4
Treatment model: logit
   max = 74

| bweight | Coefficient | std. err. | z   | P>|z|   | [95% conf. interval] |
|---------|-------------|-----------|-----|--------|---------------------|
| ATE     |             |           |     |        |                     |
| mbsmoke (Smoker vs Nonsmoker) | -224.006  | 29.88627 | -7.50| 0.000  | -282.582 -165.43   |

These results are similar to those reported in example 1.

**Video example**

Treatment effects in Stata: Propensity-score matching
## Stored results

`teffects psmatch` stores the following in `e()`:

### Scalars
- `e(N)` number of observations
- `e(nj)` number of observations for treatment level $j$
- `e(k_levels)` number of levels in treatment variable
- `e(caliper)` maximum distance between matches
- `e(treated)` level of treatment variable defined as treated
- `e(control)` level of treatment variable defined as control
- `e(k_neighbor)` requested number of matches
- `e(k_min)` minimum number of matches
- `e(k_max)` maximum number of matches
- `e(k_robust)` matches for robust VCE

### Macros
- `e(cmd)` `teffects`
- `e(cmdline)` command as typed
- `e(depvar)` name of outcome variable
- `e(tvar)` name of treatment variable
- `e(subcmd)` `psmatch`
- `e(tmodel)` `logit`, `probit`, or `hetprobit`
- `e(stat)` statistic estimated, `ate` or `atet`
- `e(wtype)` weight type
- `e(wexp)` weight expression
- `e(title)` title in estimation output
- `e(tlevels)` levels of treatment variable
- `e(psvrlist)` variables in propensity-score model
- `e(hvrlist)` variables for variance, only if `hetprobit`
- `e(vce)` `vctype` specified in `vce()`
- `e(vcetype)` title used to label Std. err.
- `e(datasignature)` the checksum
- `e(datasignaturevars)` variables used in calculation of checksum
- `e(properties)` `b V` (with or without `psmatch`)
- `e(estat_cmd)` program used to implement `estat`
- `e(predict)` program used to implement `predict`
- `e(marginsnotok)` predictions disallowed by `margins`
- `e(asbalanced)` factor variables `fvset` as `asbalanced`
- `e(asobserved)` factor variables `fvset` as `asobserved`

### Matrices
- `e(b)` coefficient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(bps)` coefficient vector from propensity-score model
- `e(Vps)` variance–covariance matrix of the estimators from propensity-score model

### Functions
- `e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

### Matrices
- `r(table)` matrix containing the coefficients with their standard errors, test statistics, $p$-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

## Methods and formulas

The methods and formulas used by `teffects psmatch` are documented in the Methods and formulas of [CAUSAL] `teffects nnmatch`.
References


Also see

[CAUSAL] teffects psmatch — Propensity-score matching

[CAUSAL] teeffects postestimation — Postestimation tools for teffects

[CAUSAL] teffects — Treatment-effects estimation for observational data

[CAUSAL] teffects nnmatch — Nearest-neighbor matching

[U] 20 Estimation and postestimation commands
Description

teffects ra estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational data by regression adjustment (RA). RA estimators use contrasts of averages of treatment-specific predicted outcomes to estimate treatment effects. teffects ra accepts a continuous, binary, count, fractional, or nonnegative outcome and allows a multivalued treatment.

See [CAUSAL] teffects intro or [CAUSAL] teffects intro advanced for more information about estimating treatment effects from observational data.

Quick start

ATE from a linear model of y1 on x1 and x2 with binary treatment treat2

teffects ra (y1 x1 x2) (treat2)

Same as above, but estimate the ATET

teffects ra (y1 x1 x2) (treat2), atet

Same as above, but estimate the potential-outcome means

teffects ra (y1 x1 x2) (treat2), pomeans

ATE of treat2 using a heteroskedastic probit model for binary outcome y2

teffects ra (y2 x1 x2, hetprobit(x1 x2)) (treat2)

ATE of treat2 using a Poisson model for count outcome y3

teffects ra (y3 x1 x2, poisson) (treat2)

ATE for each level of three-valued treatment treat3

teffects ra (y1 x1 x2) (treat3)

Same as above, and specify that treat3 = 3 is the control level

teffects ra (y1 x1 x2) (treat3), control(3)

Same as above, specified using the label “MyControl” corresponding to treat3 = 3

teffects ra (y1 x1 x2) (treat3), control("MyControl")
teffects ra (ovar omvarlist [, omodel noconstant]) (tvar) [if] [in] [weight] [, stat options]

ovar is a binary, count, continuous, fractional, or nonnegative outcome of interest.

omvarlist specifies the covariates in the outcome model.

tvar must contain integer values representing the treatment levels.

<table>
<thead>
<tr>
<th>omodel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>linear outcome model; the default</td>
</tr>
<tr>
<td>logit</td>
<td>logistic outcome model</td>
</tr>
<tr>
<td>probit</td>
<td>probit outcome model</td>
</tr>
<tr>
<td>hetprobit(varlist)</td>
<td>heteroskedastic probit outcome model</td>
</tr>
<tr>
<td>poisson</td>
<td>exponential outcome model</td>
</tr>
<tr>
<td>flogit</td>
<td>fractional logistic outcome model</td>
</tr>
<tr>
<td>fprobit</td>
<td>fractional probit outcome model</td>
</tr>
<tr>
<td>fhetprobit(varlist)</td>
<td>fractional heteroskedastic probit outcome model</td>
</tr>
</tbody>
</table>

omodel specifies the model for the outcome variable.

<table>
<thead>
<tr>
<th>stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
</tr>
<tr>
<td>pomeans</td>
<td>estimate potential-outcome means</td>
</tr>
</tbody>
</table>
### Options

#### Model

- **noconstant**: see [R] Estimation options.

#### Stat

- **stat**: is one of three statistics: *ate*, *atet*, or *pomeans*. *ate* is the default.
  - *ate* specifies that the average treatment effect be estimated.
  - *atet* specifies that the average treatment effect on the treated be estimated.
  - *pomeans* specifies that the potential-outcome means for each treatment level be estimated.

#### SE/Robust

- **vce(vcetype)**: specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (*robust*), that allow for intragroup correlation (*cluster clustvar*), and that use bootstrap or jackknife methods (*bootstrap, jackknife*); see [R] vce_option.

#### Reporting

- **level(#)**: see [R] Estimation options.
- **aequations**: specifies that the results for the outcome-model or the treatment-model parameters be displayed. By default, the results for these auxiliary parameters are not displayed.
display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

maximize_options: iterate(#), [no] log, and from(init_specs); see [R] Maximize. These options are seldom used.

init_specs is one of

matname [, skip copy]
# [, # ...], copy

control(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with statistic pomeans. control() and tlevel() may not specify the same treatment level.

tlevel(# | label) specifies the level of tvar that is the treatment for the statistic atet. The default is the second treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. tlevel() may only be specified with statistic atet. tlevel() and control() may not specify the same treatment level.

The following option is available with teffects ra but is not shown in the dialog box:
coeflegend; see [R] Estimation options.

### Remarks and examples

Remarks are presented under the following headings:

**Overview**

**Video example**

### Overview

Regression adjustment (RA) estimators use the contrasts of the averages of treatment-specific predicted outcomes to estimate treatment effects. RA estimators use a two-step approach to estimating treatment effects:

1. They fit separate regression models of the outcome on a set of covariates for each treatment level.
2. They compute the averages of the predicted outcomes for each subject and treatment level. These averages reflect the POMs. The contrasts of these averages provide estimates of the ATEs. By restricting the computations of the means to the subset of treated subjects, we obtain the ATETs.

RA estimators are consistent as long as the treatment is independent of the potential outcomes after conditioning on the covariates. In fact, teffects ra uses an estimation technique that implements both steps at once so that we do not need to correct the standard errors in the second step to reflect the uncertainty surrounding the predicted outcomes.
We will illustrate the use of `teffects ra` by using data from a study of the effect of a mother’s smoking status during pregnancy (`mbsmoke`) on infant birthweight (`bweight`) as reported by Cattaneo (2010). This dataset also contains information about each mother’s age (`mage`), education level (`medu`), marital status (`mmarried`), whether the first prenatal exam occurred in the first trimester (`prenatal1`), and whether this baby was the mother’s first birth (`fbaby`).

**Example 1: Estimating the ATE**

We begin by using `teffects ra` to estimate the average treatment effect of smoking, controlling for first-trimester exam status, marital status, mother’s age, and first-birth status. In Stata, we type

```
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ra (bweight prenatal1 mmarried mage fbaby) (mbsmoke)
Iteration 0: EE criterion = 7.734e-24
Iteration 1: EE criterion = 1.196e-25
Treatment-effects estimation Number of obs = 4,642
Estimator : regression adjustment
Outcome model : linear
Treatment model: none

| bweight | Coefficient | std. err. | z | P>|z| | [95% conf. interval] |
|---------|-------------|-----------|---|--------|---------------------|
| ATE     |             |           |   |        |                     |
| mbsmoke (Smoker vs Nonsmoker) | -239.6392  | 23.82402  | -10.06 | 0.000 | -286.3334  | -192.945  |
| POmean mbsmoke Nonsmoker     | 3403.242   | 9.525207  | 357.29 | 0.000 | 3384.573   | 3421.911  |
```

The average birthweight if all mothers were to smoke would be 240 grams less than the average of 3,403 grams that would occur if none of the mothers had smoked.

The previous results showed us the average amount by which infants’ weights are affected by their mothers’ decision to smoke. We may instead be interested in knowing the average amount by which the weight of babies born to smoking mothers was decreased as a result of smoking. The ATET provides us with the answer.
Example 2: Estimating the ATET

To obtain the ATET rather than the ATE, we use the atet option:

```
. teffects ra (bweight prenatal1 mmarried mage fbaby) (mbsmoke), atet
```

```
Iteration 0:  EE criterion =  7.629e-24
Iteration 1:  EE criterion =  2.697e-26
```

```
Treatment-effects estimation         Number of obs  =   4,642
Estimator :  regression adjustment
Outcome model :  linear
Treatment model:  none
```

| bweight  | Robust | Coefficient | std. err. | z    | P>|z| | [95% conf. interval] |
|----------|--------|-------------|-----------|------|-----|----------------------|
| ATET     |        | -223.3017   | 22.7422   | -9.82| 0.000| -267.8755 -178.7278  |
|          | mbsmoke| (Smoker     | vs        | Nonsmoker) |
|          |        | -223.3017   | 22.7422   | -9.82| 0.000| -267.8755 -178.7278  |
| POmean   |        | 3360.961    | 12.75749  | 263.45| 0.000| 3335.957 3385.966   |
|          | mbsmoke| Nonsmoker   |           |      |     |                      |
|          |        | 3360.961    | 12.75749  | 263.45| 0.000| 3335.957 3385.966   |

The average birthweight is 223 grams less when all the mothers who smoke do so than the average of 3,361 grams that would have occurred if none of these mothers had smoked.

The ATET differs from the ATE because the distribution of the covariates among mothers who smoke differs from the distribution for nonsmoking mothers. For example, in [CAUSAL] teffects intro, we remarked that in our sample, mothers who smoked tended to be older than those who did not. The differing distributions of covariates also affect the estimated POMs.

By default, teffects ra reports the ATE, which is the difference between the two POMs in the case of a binary treatment variable. Sometimes, we want to know the estimated POMs themselves. We might also want to see the actual regression equations used to estimate the POMs. Obtaining this information is easy, as the next example illustrates.
Example 3: Estimating the POMs

Here we use the `pomeans` option to display the POMs and the `aequations` option to display the estimated regression coefficients for the treated and untreated subjects.

```
. teffects ra (bweight prenatal mmarried mage fbaby) (mbsmoke),
  > pomeans aequations
Iteration 0: EE criterion = 7.734e-24
Iteration 1: EE criterion = 2.850e-26
```

```
Treatment-effects estimation
Number of obs = 4,642
Estimator : regression adjustment
Outcome model : linear
Treatment model: none
```

|                | Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|----------------|-------------|-----------|------|------|----------------------|
| **POmeans**    |             |           |      |      |                      |
| Nonsmoker      | 3403.242    | 9.525207  | 357.29 | 0.000 | 3384.573  3421.911  |
| Smoker         | 3163.603    | 21.86351  | 144.70 | 0.000 | 3120.751  3206.455  |
| **OME0**       |             |           |      |      |                      |
| prenatal1      | 64.40859    | 27.52699  | 2.34 | 0.019 | 10.45669  118.3605  |
| mmarried       | 160.9513    | 26.6162   | 6.05 | 0.000 | 108.7845  213.1181  |
| mage           | 2.546828    | 2.084324  | 1.22 | 0.222 | -1.538373 6.632028  |
| fbaby          | -71.3286    | 19.64701  | -3.63 | 0.000 | -109.836 -32.82117 |
| _cons          | 3202.746    | 54.01082  | 59.30 | 0.000 | 3096.886  3308.605  |
| **OME1**       |             |           |      |      |                      |
| prenatal1      | 25.11133    | 40.37541  | 0.62 | 0.534 | -54.02302 104.2457  |
| mmarried       | 133.6617    | 40.86443  | 3.27 | 0.001 | 53.5689  213.7545  |
| mage           | -7.370881   | 4.21817   | -1.75 | 0.081 | -15.63834 .8965804 |
| fbaby          | 41.43991    | 39.70712  | 1.04 | 0.297 | -36.38461 119.2644  |
| _cons          | 3227.169    | 104.4059  | 30.91 | 0.000 | 3022.537  3431.801  |

The nonsmoker POM for infant birthweight is 3,403 grams; that means that if none of the women in our sample smoked during pregnancy, the expected average birthweight would be 3,403 grams. The POM if all mothers did smoke during pregnancy is 3,164 grams, a difference of 240 grams, as we established in example 1. The coefficients for the equation labeled OME0 represent the linear equation used to estimate the nontreated POM, and the coefficients for the equation labeled OME1 represent the linear equation used to estimate the treated POM. The coefficients are identical to those we would obtain using `regress`, but the standard errors differ slightly because `teffects ra` does not make the small-sample adjustment that `regress` does.

We often express statistics as percentages to alleviate scaling issues and aid interpretation. In the present context, we may wish to express an ATE as a percentage of the untreated POM to gain a more intuitive measure of efficacy.
Example 4: Reporting the ATE as a percentage

Sometimes, we are interested in reporting the estimated treatment effect as a percentage of the untreated POM. We continue to use the same model as in the previous examples, but we specify the coeflegend option so that `teffects ra` reports the names of the parameters. Knowing the correct names to use, we can then use `nlcom` to obtain the percentage change along with its delta-method-based standard error. We type

```
use https://www.stata-press.com/data/r18/cattaneo2
.teffects ra (bweight prenatal1 mmarried mage fbaby) (mbsmoke), coeflegend
```

```
Iteration 0: EE criterion = 7.73e-24
Iteration 1: EE criterion = 1.196e-25
```
```
Treatment-effects estimation Number of obs = 4,642
Estimator : regression adjustment
Outcome model : linear
Treatment model: none

<table>
<thead>
<tr>
<th>bweight</th>
<th>Coefficient</th>
<th>Legend</th>
</tr>
</thead>
<tbody>
<tr>
<td>ATE mbsmoke</td>
<td>-239.6392</td>
<td>_b[ATE:r1vs0.mbsmoke]</td>
</tr>
<tr>
<td>(Smoker vs</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonsmoker)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>POmean</td>
<td>3403.242</td>
<td>_b[POmean:0.mbsmoke]</td>
</tr>
<tr>
<td>mbsmoke</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nonsmoker</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

```
.nlcom _b[ATE:r1vs0.mbsmoke] / _b[POmean:0.mbsmoke]

```
| bweight | Coefficient | Std. err. | z     | P>|z|   | 95% conf. interval |
|---------|-------------|-----------|-------|-------|-------------------|
| _nl_1   | -.070415    | .0069245  | -10.17| 0.000 | -.0839867 to -.0568433 |
```

The average birthweight falls by an estimated 7.0% when every mother smokes relative to the case when no mothers smoke. We also obtain a 95% confidence interval of a 5.7% to 8.4% reduction.

Birthweights cannot be negative, though it is possible for a linear regression model to make negative predictions. A common way to enforce nonnegative predictions is to use an exponential conditional-mean model, which is commonly fitted using the Poisson quasimaximum likelihood estimator, as discussed in Cameron and Trivedi (2005, sec. 5.7), Wooldridge (2010, sec. 18.2), and Pawitan (2001, chap. 14). `teffects ra` provides an option to use this model rather than linear regression for the outcomes.
Example 5: Modeling nonnegative outcomes

Now we refit our model of smoking behavior on birthweight, but we specify the poisson option in the outcome-model equation so that `teffects ra` uses the Poisson exponential model rather than linear regression:

```
. teffects ra (bweight prenatal1 mmarried mage fbaby, poisson) (mbsmoke)
Iteration 0:   EE criterion = 3.950e-17
Iteration 1:   EE criterion = 1.244e-23
```

```
Treatment-effects estimation Number of obs   =  4,642
Estimator      : regression adjustment
Outcome model  : Poisson
Treatment model: none

|            | Coefficient | std. err. | z     | P>|z| | [95% conf. interval] |
|------------|-------------|-----------|-------|-----|----------------------|
| ATE        |             |           |       |     |                      |
| mbsmoke    |             |           |       |     |                      |
| (Smoker vs Nonsmoker) | -239.6669  | 23.83757  | -10.05 | 0.000 | -286.3877 -192.9462  |
| POmean     |             |           |       |     |                      |
| mbsmoke    |             |           |       |     |                      |
| Nonsmoker  | 3403.178    | 9.526006  | 357.25 | 0.000 | 3384.508 3421.849    |
```

In this case, using a model that forces outcomes to be nonnegative did not make any substantive difference. In this dataset, nearly 90% of babies weigh at least 2,700 grams, and even the smallest baby weighs 340 grams. When the dependent variable is so large, the predictions from Poisson and linear regression models are remarkably similar.

We now consider models for fractional outcomes. Fractional responses concern outcomes between 0 and 1. These responses may be averaged 0/1 outcomes such as participation rates, or they may be variables that are naturally on a 0 to 1 scale such as pollution levels, patient oxygen saturation, and Gini coefficients (income inequality measures).

Example 6: Modeling fractional outcomes

We will illustrate the use of `teffects ra` with the outcome-model option `fprobit` by using simulated data. The observations are 543 cities at least 200 miles apart. The data contain information about each city’s level of industrialization (`industrial`), average annual rainfall in millimeters (`rainfall`), whether or not the city has a metro or train (`train`), and traffic congestion measured by an index (`traffic`).

Our outcome is the level of pollution (`pollution`) measured on a 0 to 1 scale. Values of pollution between 0 and 0.3 have no public health implications, but values greater than 0.7 imply that people with breathing or health problems should remain indoors. We study the effect of a tax on gas-guzzler cars (`guzzler`) on air pollution. A tax that is effective in reducing pollution improves public health.

We estimate the ATE of a gas-guzzler tax on pollution, controlling for average yearly rainfall, traffic congestion, the level of industrialization, and whether the city has a train or a metro by using a fractional probit model.
. use https://www.stata-press.com/data/r18/pollution
(Simulated Urban Pollution Data)
. teffects ra (pollution rainfall i.traffic industrial i.train, fprobit) (guzzler)
Iteration 0:  EE criterion = 3.023e-16
Iteration 1:  EE criterion = 9.917e-32
Treatment-effects estimation               Number of obs  =  534
Estimator      :  regression adjustment
Outcome model  :  fractional probit
Treatment model:  none

<table>
<thead>
<tr>
<th></th>
<th>Robust</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Coefficient</td>
</tr>
<tr>
<td>ATE</td>
<td></td>
</tr>
<tr>
<td>guzzler</td>
<td>-.0960214</td>
</tr>
<tr>
<td></td>
<td>(tax vs no tax)</td>
</tr>
<tr>
<td>POmean</td>
<td></td>
</tr>
<tr>
<td>guzzler</td>
<td>.3879346</td>
</tr>
<tr>
<td></td>
<td>(no tax)</td>
</tr>
</tbody>
</table>

The POM if no city were to implement a gas-guzzler tax is an air pollution index of 0.39. If all cities implement a gas-guzzler tax, the air pollution index would decrease by 0.096 relative to a scenario where no city implements the tax.

Video example

Treatment effects: Regression adjustment
Stored results

t/effects ra stores the following in e():

Scalars
- e(N) number of observations
- e(n_j) number of observations for treatment level j
- e(N_clust) number of clusters
- e(k_eq) number of equations in e(b)
- e(k_levels) number of levels in treatment variable
- e(treated) level of treatment variable defined as treated
- e(control) level of treatment variable defined as control
- e(converged) 1 if converged, 0 otherwise

Macros
- e(cmd) t/effects
- e(cmdline) command as typed
- e(depvar) name of outcome variable
- e(tvar) name of treatment variable
- e(subcmd) ra
- e(omodel) linear, logit, probit, hetprobit, poisson, flogit, fprobit, or fhetprobit
- e(stat) statistic estimated, ate, atet, or pomeans
- e(wtype) weight type
- e(wexp) weight expression
- e(title) title in estimation output
- e(clustvar) name of cluster variable
- e(tlevels) levels of treatment variable
- e(vce) vcetype specified in vce()
- e(vcetype) title used to label Std. err.
- e(properties) b V
- e(estat_cmd) program used to implement estat
- e(predict) program used to implement predict
- e(marginsnotok) predictions disallowed by margins
- e(asbalanced) factor variables fvset as asbalanced
- e(asobserved) factor variables fvset as asobserved

Matrices
- e(b) coefficient vector
- e(V) variance–covariance matrix of the estimators

Functions
- e(sample) marks estimation sample

In addition to the above, the following is stored in r():

Matrices
- r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

t/effects ra implements a smooth treatment-effects estimator. All smooth treatment-effects estimators are documented in [CAUSAL] t/effects aipw.
References


Also see

[CAUSAL] tefects postestimation — Postestimation tools for tefects

[CAUSAL] tefects — Treatment-effects estimation for observational data

[ERM] eregress — Extended linear regression

[U] 20 Estimation and postestimation commands
**Description**

telasso estimates the average treatment effect (ATE), the average treatment effect on the treated (ATET), and the potential-outcome means (POMs) from observational data by augmented inverse-probability weighting (AIPW) while using lasso methods to select from potential control variables to be included in the model.

AIPW estimators combine aspects of regression-adjustment and inverse-probability-weighted methods. AIPW estimators have the double-robust property.

telasso accepts a continuous, binary, count, or nonnegative outcome.

See [CAUSAL] teffects intro or [CAUSAL] teffects intro advanced for more information about estimating treatment effects from observational data. See [LASSO] Lasso inference intro for more information on estimating coefficients and standard errors for a subset of variables while using lasso methods to select from a set of control variables.

**Quick start**

ATE of binary treatment treat using a linear model for outcome y1 on x1–x100 and a logistic model for treat on w1–w100; use lassos to select variables from x1–x100 for the outcome model and from w1–w100 for the treatment model

telasso (y1 x1-x100) (treat w1-w100)

Same as above, but estimate the ATET
telasso (y1 x1-x100) (treat w1-w100), atet

Use a Poisson model for count outcome y2

telasso (y2 x1-x100, poisson) (treat w1-w100)

Same as above, but use a probit model for treat
telasso (y2 x1-x100, poisson) (treat w1-w100, probit)

Use BIC instead of a plugin iterative formula to select the optimal $\lambda^*$ in each lasso
telasso (y1 x1-x100) (treat w1-w100), selection(bic)

Perform cross-fitting with five folds
telasso (y1 x1-x100) (treat w1-w100), xfolds(5)

Same as above, but repeat the cross-fitting procedure 15 times and average the results
telasso (y1 x1-x100) (treat w1-w100), xfolds(5) resample(15)

Use BIC to select covariates in the lasso for y1 for treatment level 1 only
telasso (y1 x1-x100, lasso(1, selection(bic))) (treat w1-w100)
Use cross-validation (CV) for the lasso for treat only

telasso (y1 x1-x100) (treat w1-w100, lasso(selection(cv)))

Menu

Statistics > Causal inference/treatment effects > Continuous outcomes > AIPW with lasso covariate selection
Statistics > Causal inference/treatment effects > Binary outcomes > AIPW with lasso covariate selection
Statistics > Causal inference/treatment effects > Count outcomes > AIPW with lasso covariate selection
Statistics > Causal inference/treatment effects > Nonnegative outcomes > AIPW with lasso covariate selection

Syntax

    telasso (ovar omvarlist [, omodel om_options])
             (tvar tmvarlist [, tmodel tm_options]) [if] [in] [weight] [, stat options]

ovar is a binary, count, continuous, or nonnegative outcome of interest.

omvarlist specifies the covariates in the outcome model.

tvar must contain a binary value representing the treatment levels.

tmvarlist specifies the covariates in the treatment model.

<table>
<thead>
<tr>
<th>omodel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>linear outcome model; the default</td>
</tr>
<tr>
<td>logit</td>
<td>logistic outcome model</td>
</tr>
<tr>
<td>probit</td>
<td>probit outcome model</td>
</tr>
<tr>
<td>poisson</td>
<td>exponential-mean outcome model</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>om_options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>lasso([#,] lasso_options)</td>
<td>specify options for the lassos for ovar at treatment level #; may be repeated</td>
</tr>
<tr>
<td>*sqrtlasso</td>
<td>use square-root lassos instead of lassos for ovar</td>
</tr>
<tr>
<td>*sqrtlasso([#,] lasso_options)</td>
<td>specify options for the square-root lassos for ovar at treatment level #; may be repeated</td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant term</td>
</tr>
<tr>
<td>ainclude(varlist)</td>
<td>specify variables that should always be included in the outcome model</td>
</tr>
<tr>
<td>noselection</td>
<td>suppress model selection for the outcome model</td>
</tr>
</tbody>
</table>

*sqrtlasso() may be specified only when omodel is linear.
**tmmodel**

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>logit</td>
<td>logistic treatment model; the default</td>
</tr>
<tr>
<td>probit</td>
<td>probit treatment model</td>
</tr>
</tbody>
</table>

**tm_options**

<table>
<thead>
<tr>
<th>Model</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>lasso(lasso_options)</td>
<td>specify options for the lasso for <em>tvar</em></td>
</tr>
<tr>
<td>noconstant</td>
<td>suppress constant term</td>
</tr>
<tr>
<td>ainclude(varlist)</td>
<td>specify variables that should always be included in the treatment model</td>
</tr>
<tr>
<td>noselation</td>
<td>suppress model selection for the treatment model</td>
</tr>
</tbody>
</table>

**stat**

<table>
<thead>
<tr>
<th>Stat</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ate</td>
<td>estimate average treatment effect in population; the default</td>
</tr>
<tr>
<td>atet</td>
<td>estimate average treatment effect on the treated</td>
</tr>
<tr>
<td>pomeans</td>
<td>estimate potential-outcome means</td>
</tr>
</tbody>
</table>

**options**

<table>
<thead>
<tr>
<th>Selection</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>selection(plugin)</td>
<td>use a plugin iterative formula to select an optimal value of the lasso penalty parameter $\lambda^*$ for each lasso; the default</td>
</tr>
<tr>
<td>selection(cv)</td>
<td>use CV to select an optimal value of the lasso penalty parameter $\lambda^*$ for each lasso</td>
</tr>
<tr>
<td>selection(adaptive)</td>
<td>use adaptive lasso to select an optimal value of the lasso penalty parameter $\lambda^*$ for each lasso</td>
</tr>
<tr>
<td>selection(bic)</td>
<td>use BIC to select an optimal value of the lasso penalty parameter $\lambda^*$ for each lasso</td>
</tr>
<tr>
<td>xfolds[#]</td>
<td>use # folds for cross-fitting</td>
</tr>
<tr>
<td>resample[#]</td>
<td>repeat sample splitting # times and average results</td>
</tr>
</tbody>
</table>

**SE/Robust**

| vce(vcetype)                | *vcetype* may be _robust_ or _cluster_ clustvar                            |

**Reporting**

| level(#)                    | set confidence level; default is _level(95)_                               |
| display_options             | control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling |

**Optimization**

| rseed(#)                    | set random-number seed                                                     |
| verbose                    | display a verbose iteration log                                           |
| [no] log                   | display or suppress an iteration log                                       |
### Advanced

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>ptolerance(#)</code></td>
<td>set tolerance for overlap assumption</td>
</tr>
<tr>
<td><code>osample(newvar)</code></td>
<td><code>newvar</code> identifies observations that violate the overlap assumption</td>
</tr>
<tr>
<td>`control(#</td>
<td>label)`</td>
</tr>
<tr>
<td><code>reestimate</code></td>
<td>refit the model after using <code>lassoselect</code> to select a different $\lambda^*$</td>
</tr>
<tr>
<td><code>noheader</code></td>
<td>do not display the header on the coefficient table</td>
</tr>
<tr>
<td><code>coeflegend</code></td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>

- `omvarlist` and `tmvarlist` may contain factor variables; see [U] 11.4.3 Factor variables.
- `collect` is allowed; see [U] 11.1.10 Prefix commands.
- Default weights are not allowed. `iweights` are allowed when `selection(plugin)`, `selection(adaptive)`, `selection(cv)`, or `selection(bic)` is specified. `fweights` are allowed when `selection(plugin)` or `selection(bic)` is specified. See [U] 11.1.6 weight.

- `reestimate`, `noheader`, and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## Options

Options specific to the outcome model (`om_options`) are the following:

- `lasso([#, ] lasso_options)` lets you set different options for different lassos or set advanced options for all lassos for the outcome model. Specify a `#` followed by the options to apply settings to the lasso of `ovar` at treatment level `#` only. This option is repeatable as long as a different treatment level is given in each specification. If `#` is not specified, the lasso options apply to lassos of `ovar` at all treatment levels. `lasso_options` are `selection()`, `grid()`, `stop()`, `cvtolerance()`, `bictolerance()`, `tolerance()`, and `dtolerance()`; see [LASSO] lasso options. When `lasso([#, ] selection())` is specified, it overrides any global `selection()` option for the outcome variable at the specified treatment level.

- `sqrtlasso` specifies that square-root lassos rather than lassos should be used in selecting variables in models for the outcome at both treatment levels. This option is not allowed if `omodel` is logit, probit, or poisson.

- `sqrtlasso([#, ] lasso_options)` works like the option `lasso()`, except square-root lassos for the outcome variable are done rather than regular lassos. This option is not allowed if `omodel` is logit, probit, or poisson. Specify a `#` followed by the options to apply settings to the lasso of `ovar` at treatment level `#` only. This option is repeatable as long as a different treatment level is given in each specification. If `#` is not specified, the lasso options apply to lassos of `ovar` at all treatment levels. `lasso_options` are `selection()`, `grid()`, `stop()`, `cvtolerance()`, `bictolerance()`, `tolerance()`, and `dtolerance()`; see [LASSO] lasso options. When `sqrtlasso([#, ] selection())` is specified, it overrides any global `selection()` option for the outcome variable at the specified treatment level.

- `noconstant` suppresses the constant term in the outcome model.

- `ainclude(varlist)` specifies variables that are always included in the outcome model. Lassos will choose to include or exclude other variables in `omvarlist`.

- `noselection` specifies not to perform model selection in the outcome model.
Options specific to the treatment model (tm_options) are the following:

- **lasso(lasso_options)** lets you set advanced options for the lasso for the treatment model. *lasso_options are selection(), grid(), stop(), cvtolerance(), bictolerance(), tolerance(), and dtolerance*; see [LASSO] lasso options. When lasso(selection()) is specified, it overrides any global selection() option for the treatment variable.

- **noconstant** suppresses the constant term in the treatment model.

- **ainclude(varlist)** specifies variables that are always included in the treatment model. Lassos will choose to include or exclude other variables in tmvarlist.

- **noselection** specifies not to perform model selection in the treatment model.

- **selection(plugin|cv|adaptive|bic)** specifies the selection method for choosing an optimal value of the lasso penalty parameter $\lambda^*$ for each lasso or square-root lasso estimation. Separate lassos are estimated for the outcome variable at each treatment level and for the treatment variable. Specifying selection() changes the selection method for all these lassos. You can specify different selection methods for different lassos by using lasso() or sqrtlasso() in the outcome model specification or the option lasso() in the treatment model specification. When lasso() or sqrtlasso() is used to specify a different selection method for the lassos of outcome or treatment variables, they override the global setting made using selection() for the specified model.

  - **selection(plugin)** is the default. It selects $\lambda^*$ based on a “plugin” iterative formula dependent on the data. See [LASSO] lasso options.
  - **selection(cv)** selects the $\lambda^*$ that gives the minimum of the CV function. See [LASSO] lasso options.
  - **selection(adaptive)** selects $\lambda^*$ using the adaptive lasso selection method. It cannot be specified when sqrtlasso() is specified for the outcome model. See [LASSO] lasso options.
  - **selection(bic)** selects the $\lambda^*$ that gives the minimum of the BIC function. See [LASSO] lasso options.

- **xfolds[#]** specifies the number of folds for cross-fitting. Not specifying xfolds or xfolds(#) is equivalent to specifying xfolds(1). In other words, by default no cross-fitting is done. Specifying xfolds alone is equivalent to specifying xfolds(10); that is, cross-fitting is done by randomly dividing the original data into 10 folds.

- **resample[#]** specifies that sample splitting be repeated and results averaged. This reduces the effects of the randomness of sample splitting on the estimated coefficients. Not specifying resample or resample(#) is equivalent to specifying resample(1). In other words, by default no resampling is done. Specifying resample alone is equivalent to specifying resample(10); that is, sample splitting is repeated 10 times. For each sample split, lassos are computed. So when this option is not specified, lassos are repeated xfolds(#) times. But when resample(#) is specified, lassos are repeated xfolds(#) $\times$ resample(#) times. Thus, while we recommend using resample to get final results, note that it can be an extremely time-consuming procedure.

- **vce(vcetype)** specifies the type of standard error reported, which includes types that are robust to some kinds of misspecification (robust) and that allow for intragroup correlation (cluster clustvar); see [R] vce_option.
Reporting

level(#) ; see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, noflabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Optimization

rseed(#) sets the random-number seed. This option can be used to reproduce results. rseed(#) is equivalent to typing set seed # prior to running telasso. Random numbers are used to produce split samples for cross-fitting. So for all selection() options, if you want to reproduce your results, you must either use this option or use set seed. See [R] set seed.

verbose displays a verbose log showing the iterations of each lasso estimation. This option is useful when doing selection(cv) or selection(adaptive). It allows monitoring the progress of the lasso estimations for these selection methods, which can be time consuming when there are many variables specified in omvarlist or tmvarlist.

[no]log displays or suppresses a log showing the progress of the estimation. By default, one-line messages indicating when each lasso estimation begins are shown. Specify verbose to see a more detailed log.

Advanced

pstolerance(#) specifies the tolerance used to check the overlap assumption. The default value is pstolerance(1e-5). telasso will exit with an error if an observation has an estimated propensity score smaller than that specified by pstolerance().

osample(newvar) specifies that indicator variable newvar be created to identify observations that violate the overlap assumption.

ccontrol(# | label) specifies the level of tvar that is the control. The default is the first treatment level. You may specify the numeric level # (a nonnegative integer) or the label associated with the numeric level. control() may not be specified with statistic pomeans.

The following options are available with telasso but are not shown in the dialog box:

reestimate is an advanced option that refits the telasso model based on changes made to the underlying lassos using lassoselect. After running telasso, you can select a different λ* for one or more of the lassos estimated by telasso. After selecting λ*, you type telasso, reestimate to refit the telasso model based on the newly selected λ's.

reestimate may be combined only with reporting options.

noheader prevents the coefficient table header from being displayed.

coeflegend; see [R] Estimation options.

Remarks and examples

Remarks are presented under the following headings:

Overview

Estimating the ATE with lassos for covariate selection

Choosing the tuning parameter

Estimating the ATET

High-dimensional semiparametric models
**Overview**

telasso estimates treatment effects while using lasso techniques to select the variables in the model. Like the lasso for inference commands, inferences drawn about the ATEs, ATETs, and POMs reported by telasso are robust to model selection mistakes made by lasso. In addition, like the teffects aipw command, the estimated effects are consistent if the functional form of only one of the outcome or treatment model is correctly specified. Thus, telasso is a version of the AIPW estimator that also allows for covariate selection via lasso and is robust to functional form misspecification in one of the models as well as errors in selection of covariates by lasso. It is, however, required that the true covariates are a subset of the specified list of potential covariates. See [CAUSAL] teffects intro or [CAUSAL] teffects intro advanced for an introduction to treatment effects estimation. See [CAUSAL] teffects aipw for more information on the AIPW estimator. See [LASSO] Lasso intro for an introduction to lasso. See [LASSO] Lasso inference intro for an introduction to making inference after using lasso for model selection.

Researchers who have datasets with many variables, sometimes more than observations, face difficult decisions. They cannot use all the variables in their dataset as covariates in a model, and therefore, they may want to use a model selection method such as lasso to select covariates. In addition, rather than simply allowing covariates to enter a model linearly, researchers may want to fit a more flexible and realistic model to their data by including higher-order terms, interactions, spline basis functions, and the like, but this can also produce more terms than can be included in a model and require a method such as lasso for selection. The lasso methods used in telasso allow for making valid inference after selecting from among the potential covariates.

In the context of treatment-effect estimation, why do we need model selection? It is because of the intrinsic conflicts between two crucial assumptions used to identify treatment effects: conditional independence (CI) and overlap assumptions. CI means that, dependent on a set of control variables, the potential outcome is independent of the treatment assignment. The more variables there are in the model, the more plausible it is that the CI assumption is satisfied. On the other hand, the overlap assumption implies that there is always a positive probability that any given unit is treated or untreated. The fewer variables there are in the model, the more comfortable we can be that the overlap assumption is satisfied. To summarize, the conflict is that the CI assumption expects many variables in the model, but the overlap assumption expects few variables. By including many potential variables in the model and allowing lasso to select from among them, we can reconcile this conflict. For a more detailed discussion, see Farrell (2015) and Chernozhukov et al. (2018).

Model selection, however, does not come for free. If researchers use model selection but conduct inference ignoring the fact that they did model selection, the inference results are possibly wrong. This is because model selection techniques make mistakes. Making inferences without considering the variability in model selection is a dangerous practice (see Leeb and Pötscher [2005] and Leeb and Pötscher [2006]). Instead, the inference should be robust to model selection mistakes.

Recently, Farrell (2015) and Chernozhukov et al. (2018) proposed that modern model selection techniques, such as lasso methods for inference, be combined with the doubly robust AIPW estimator in Rosenbaum and Rubin (1983) to estimate the ATEs. The intuition behind this method is twofold. On one hand, lasso techniques resolve the conflicts between the CI and overlap assumptions. Although CI assumption expects many variables, we only need the covariates that matter for the outcome. If lasso selects a sparse model, the overlap assumption is more plausible to be satisfied. On the other hand, the proposed estimator should guard against model selection mistakes. The doubly robust AIPW estimator happens to satisfy this property. Double robustness also means that the estimates of treatment effects are consistent if either the outcome model or the treatment model is correctly specified.

Telasso implements estimators in Chernozhukov et al. (2018) for the ATEs, ATETs, and POMs from observational data.
The `telasso` estimator uses a five-step approach to estimating ATE:

1. It uses lasso techniques to select variables in the outcome model for each treatment level.
2. Based on the selected variables in step 1, it fits separate regression models of the outcome for each treatment level and obtains the treatment-specific predicted outcomes for each subject.
3. It uses lasso techniques to select variables in the treatment model.
4. Based on the selected variables in step 3, it estimates the parameters of the treatment model and computes inverse-probability weights.
5. It computes the weighted means of the treatment-specific predicted outcomes, where the weights are the inverse-probability weights computed in step 4. The contrasts of these weighted averages provide the estimates of the ATE.

Steps 1 and 3 perform the model selection for the outcome and treatment models, respectively. Using the selected variables, steps 2, 4, and 5 construct moment conditions to estimating ATEs. The resulting estimator is consistent under CI, overlap, and independent and identically distributed assumptions. The inference is robust to the mild model selection mistakes that could happen in steps 1 and 3. This estimator is also robust to model misspecification in either the outcome or the treatment model because of the double robust moment condition used in step 5.

`telasso` also implements the double machine learning estimator in Chernozhukov et al. (2018) for ATEs, ATETs, and POMs. Double machine learning relaxes the sparsity assumption needed for lasso methods. The sparsity assumption implies that lasso can only have good properties if it selects a few variables from a potentially large number of candidates. Double machine learning allows `telasso` to select more variables in the models and still be valid.

`telasso` allows the outcome variable to be modeled using a linear, logistic, probit, or Poisson model. For a linear outcome model, square-root lasso can be used instead of lasso in step 1.

### Estimating the ATE with lassos for covariate selection

In some cases, we want to estimate a treatment effect when we have several variables in our dataset that, when interacted, create a large set of candidate covariates in our model.

#### Example 1: ATE of bilateral lung transplant

We first illustrate `telasso` with an example that compares two types of lung transplants. Bilateral lung transplant (BLT) is usually associated with a higher death rate in the short term after the operation but with a more significant improvement in the quality of life compared with the single lung transplant (SLT). As a result, for patients who need to decide between these two treatment options, knowing the effect of BLT (versus SLT) on quality of life is essential. We can measure quality of life based on an individual’s forced expiratory volume in one second (FEV1).

We have a fictional dataset (`lung.dta`) inspired by Koch, Vock, and Wolfson (2018). The outcome (`fev1p`) is FEV1% measured one year after the operation. FEV1% is the percentage of FEV1 that the patient has relative to a healthy person with similar characteristics. The treatment variable (`transtype`) indicates whether the treatment is BLT or SLT.
To open the dataset and describe it, we type

```
. use https://www.stata-press.com/data/r18/lung
(Fictional data on lung transplant)
. describe *, short
```

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Storage type</th>
<th>Display format</th>
<th>Value label</th>
<th>Variable label</th>
</tr>
</thead>
<tbody>
<tr>
<td>agep</td>
<td>byte</td>
<td>%10.0g</td>
<td></td>
<td>Patient age (years)</td>
</tr>
<tr>
<td>bmi</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Patient body mass index</td>
</tr>
<tr>
<td>diabetesp</td>
<td>byte</td>
<td>%12.0g</td>
<td>lbdiab</td>
<td>Patient diabetes status</td>
</tr>
<tr>
<td>heightp</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Patient height (cm)</td>
</tr>
<tr>
<td>o2amt</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Oxygen delivered</td>
</tr>
<tr>
<td>karn</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Karnofsky score &gt; 60</td>
</tr>
<tr>
<td>lungals</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Lung allocation score</td>
</tr>
<tr>
<td>racep</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbrace</td>
<td>Patient race</td>
</tr>
<tr>
<td>sexp</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbsex</td>
<td>Patient gender</td>
</tr>
<tr>
<td>lifesvent</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Life support ventilator needed</td>
</tr>
<tr>
<td>assisvent</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Assisted ventilation needed</td>
</tr>
<tr>
<td>centervol</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Center volume</td>
</tr>
<tr>
<td>walkdist</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Walking distance in 6 minutes</td>
</tr>
<tr>
<td>o2rest</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Oxygen needed at rest</td>
</tr>
<tr>
<td>aged</td>
<td>byte</td>
<td>%10.0g</td>
<td></td>
<td>Donor age (years)</td>
</tr>
<tr>
<td>raced</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbrace</td>
<td>Donor race</td>
</tr>
<tr>
<td>bmid</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Donor body mass index</td>
</tr>
<tr>
<td>smoked</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Donor if has history of smoking</td>
</tr>
<tr>
<td>cmv</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Positive cytomegalovirus test</td>
</tr>
<tr>
<td>deathcause</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Cause of death - traumatic brain injury</td>
</tr>
<tr>
<td>diabetesd</td>
<td>byte</td>
<td>%12.0g</td>
<td>lbdiab</td>
<td>Donor diabetes status</td>
</tr>
<tr>
<td>expandd</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbyes</td>
<td>Expanded donor needed</td>
</tr>
<tr>
<td>heightd</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Donor height (cm)</td>
</tr>
<tr>
<td>sexd</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbsex</td>
<td>Donor gender</td>
</tr>
<tr>
<td>distd</td>
<td>int</td>
<td>%10.0g</td>
<td></td>
<td>Donor to treatment center distance</td>
</tr>
<tr>
<td>lungpo2</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Lung PO2</td>
</tr>
<tr>
<td>lungalloc</td>
<td>byte</td>
<td>%8.0g</td>
<td>lballo</td>
<td>Lung allocation status</td>
</tr>
<tr>
<td>hratio</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Height ratio</td>
</tr>
<tr>
<td>ischemict</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Ischemic time</td>
</tr>
<tr>
<td>genderm</td>
<td>byte</td>
<td>%19.0g</td>
<td>lbgm</td>
<td>Matching gender status</td>
</tr>
<tr>
<td>racem</td>
<td>byte</td>
<td>%17.0g</td>
<td>lbm</td>
<td>Matching race status</td>
</tr>
<tr>
<td>transtype</td>
<td>byte</td>
<td>%8.0g</td>
<td>lbtau</td>
<td>Lung transplant type</td>
</tr>
<tr>
<td>fev1p</td>
<td>double</td>
<td>%10.0g</td>
<td></td>
<td>Percentage of predicted value of FEV1</td>
</tr>
</tbody>
</table>

Thirty-one variables measure some characteristics of the patients and donors. To construct control variables, we want to use these variables and the interactions among them. It would be tedious to type these variable names one by one to distinguish between continuous and categorical variables. vl is a suite of commands that simplifies this process.

First, we use vl set to automatically partition the variables into continuous and categorical variables. The global macro $vlcategorical contains all the categorical variable names, and $vlcontinuous contains all the continuous variable names.
. vl set

<table>
<thead>
<tr>
<th>Macro</th>
<th># Vars</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$vlcategorical</td>
<td>18</td>
<td>categorical variables</td>
</tr>
<tr>
<td>$vlcontinuous</td>
<td>13</td>
<td>continuous variables</td>
</tr>
<tr>
<td>$vluncertain</td>
<td>2</td>
<td>perhaps continuous, perhaps categorical variables</td>
</tr>
<tr>
<td>$vlother</td>
<td>0</td>
<td>all missing or constant variables</td>
</tr>
</tbody>
</table>

Notes

1. Review contents of $vlcategorical and $vlcontinuous to ensure they are correct. Type vl list $vlcategorical and type vl list $vlcontinuous.

2. If there are any variables in $vluncertain, you can reallocate them to $vlcategorical, $vlcontinuous, or $vlother. Type vl list $vluncertain.

3. Use vl move to move variables among classifications. For example, type vl move (x50 x80) $vlcontinuous to move variables x50 and x80 to the continuous classification.

4. vlnames are global macros. Type the vlname without the leading dollar sign ($) when using vl commands. Example: vlcreate not $vlcategorical. Type the dollar sign with other Stata commands to get a varlist.

   . display "$vlcontinuous"
   bmip heightp o2amt lungals centervol walkdist bmid heightd distd lungpo2
   > hratio ischemict fev1p
   . display "$vlcategorical"
   diabetesp karn racep sexp lifesevent assisvent o2rest raced smoked cmv
   > deathcause diabetesd expandd sexd lungalloc genderm racem transtype

Second, we use vl create to create customized variable lists. Specifically, $cvars contains all the continuous variables except the outcome (fev1p), and $fvars consists of all the categorical variables except the treatment (transtype). Finally, vl sub substitutes the global macro $allvars with the full second-order interaction between the continuous variables in $cvars and categorical variables in $fvars. We will use $allvars as the control variables for both outcome model and treatment model.

   . vl create cvars = vlcontinuous - (fev1p)
   note: $cvars initialized with 12 variables.

   . vl create fvars = vlcategorical - (transtype)
   note: $fvars initialized with 17 variables.

   . vl sub allvars = c.cvars i.fvars c.cvars#i.fvars
Now we are ready to use `telasso` to estimate the ATEs. We assume a linear outcome model and a logit treatment model, the defaults for `telasso`. We type

```plaintext
.telasso (fev1p $allvars) (transtype $allvars)
```

Estimating lasso for outcome `fev1p` if `transtype = 0` using plugin method ...
Estimating lasso for outcome `fev1p` if `transtype = 1` using plugin method ...
Estimating lasso for treatment `transtype` using plugin method ...
Estimating ATE ...

<table>
<thead>
<tr>
<th>Treatment-effects lasso estimation</th>
<th>Number of observations = 937</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome model: linear</td>
<td>Number of controls = 454</td>
</tr>
<tr>
<td>Treatment model: logit</td>
<td>Number of selected controls = 8</td>
</tr>
</tbody>
</table>

| fev1p | Robust Coefficient | std. err. | z | P>|z| | [95% conf. interval] |
|-------|-------------------|-----------|---|-----|------------------|
| ATE transtype (BLT vs SLT) | 37.51841 | .1606703 | 233.51 | 0.000 | 37.20351 37.83332 |
| POmean transtype SLT | 46.4938 | .2021582 | 229.99 | 0.000 | 46.09757 46.89002 |

The FEV1% if all the patients were to choose BLT is expected to be 38 percentage points higher than the 46% average expected if all patients were to choose a SLT. Among the 454 control variables, `telasso` selects only 8 of them. To summarize the model selection for both the outcome and the treatment models, we can use `lassoinfo`.

```plaintext
. lassoinfo
    Estimate: active
    Command: telasso
```

<table>
<thead>
<tr>
<th>Variable</th>
<th>Selection Model</th>
<th>selection method</th>
<th>lambda</th>
<th>No. of selected variables</th>
</tr>
</thead>
<tbody>
<tr>
<td>fev1p</td>
<td>linear</td>
<td>plugin</td>
<td>.2239121</td>
<td>5</td>
</tr>
<tr>
<td>transtype-0</td>
<td>linear</td>
<td>plugin</td>
<td>.1986153</td>
<td>6</td>
</tr>
<tr>
<td>transtype</td>
<td>logit</td>
<td>plugin</td>
<td>.0748279</td>
<td>3</td>
</tr>
</tbody>
</table>
If we want to see which variables are selected by each lasso, we can use `lassocoef`. Notice that there are two lassos associated with the outcome of `fev1p`. One is for outcome `fev1p` when the treatment `transtype` is 0, and the other is for `fev1p` when `transtype` is 1. So, in `lassocoef`, we need to use the options `tlevel()` and `for()` to refer to the lasso for the outcome variable at a specified treatment level. In contrast, for the treatment of `transtype`, there is only one lasso; therefore, we only need to use option `for()` to specify the lasso for the treatment variable.

```
. lassocoef (., for(fev1p) tlevel(0)) (., for(fev1p) tlevel(1))
> (., for(transtype))
```

<table>
<thead>
<tr>
<th></th>
<th>fev1p(0)</th>
<th>fev1p(1)</th>
<th>transtype</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>heightp</code></td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td><code>centervol</code></td>
<td>x</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td><code>walkdist</code></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td><code>lungpo2</code></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
<tr>
<td><code>diabetesd#c.lungpo2</code></td>
<td></td>
<td>x</td>
<td></td>
</tr>
<tr>
<td><code>diabetesp#c.walkdist</code></td>
<td>0</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td><code>assisvent#c.walkdist</code></td>
<td>0</td>
<td>x</td>
<td></td>
</tr>
<tr>
<td><code>ischemict</code></td>
<td></td>
<td></td>
<td>x</td>
</tr>
<tr>
<td><code>_cons</code></td>
<td>x</td>
<td>x</td>
<td>x</td>
</tr>
</tbody>
</table>

Legend:
- b - base level
- e - empty cell
- o - omitted
- x - estimated

The lassos selected `walkdist` and `lungpo2` in all three models, while `heightp`, `centervol`, `0.diabetesd#c.lungpo2`, `0.diabetesp#c.walkdist`, `0.assisvent#c.walkdist`, and `ischemict` were each selected as covariates in one or two models.
Choosing the tuning parameter

By default, `telasso` uses a plugin method to choose the tuning parameter $\lambda$ in the lasso steps. We can also use BIC, cross-validation, or adaptive lasso to select the optimal $\lambda$.

Example 2: Choosing $\lambda$ via BIC

Here we use the `selection(bic)` option to select $\lambda$ by minimizing BIC.

```
. telasso (fev1p $allvars) (transtype $allvars), selection(bic)
```

Estimating lasso for outcome `fev1p` if `transtype = 0` using BIC ...
Estimating lasso for outcome `fev1p` if `transtype = 1` using BIC ...
Estimating lasso for treatment `transtype` using BIC ...
Estimating ATE ...

<table>
<thead>
<tr>
<th>Treatment-effects lasso estimation</th>
<th>Number of observations</th>
<th>937</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome model: linear</td>
<td>Number of controls</td>
<td>454</td>
</tr>
<tr>
<td>Treatment model: logit</td>
<td>Number of selected controls</td>
<td>18</td>
</tr>
</tbody>
</table>

| fev1p | Coefficient | std. err. | z      | P>|z| | [95% conf. interval] |
|-------|-------------|-----------|--------|-----|---------------------|
| ATE transtype (BLT vs SLT)        |             |           |       |                 |                     |
| 37.54872 | .2222001    | 168.99    | 0.000  | 37.11322     | 37.98423            |
| POmean transtype SLT              |             |           |       |                 |                     |
| 46.44739 | .2282797    | 203.47    | 0.000  | 45.99997     | 46.89481            |

We can interpret the estimation results in a similar way as in example 1. The FEV1% if all the patients were to choose BLT is expected to be 38 percentage points higher than the average of 46% that would be expected if all patients were to choose a SLT. This result is similar to example 1, where we used the default plugin method to select the tuning parameter. However, among 454 controls, `telasso` with BIC selects 18 of them, which is more than the plugin method selected.
We can use bicplot for a global view about how the BIC function changes as the \( \lambda \)'s change. Here we show the BIC plot for the outcome fev1p when treatment transtype is 1. In bicplot, we use options for() and tlevel() to refer to the lasso for the outcome variable at a specified treatment level.

```
. bicplot, for(fev1p) tlevel(1)
```

From this plot, we can see that the optimal \( \lambda = 0.11 \) is chosen by the minimum BIC. If we want to investigate this further, we can use lassoknots to see which variables are selected or dropped for each \( \lambda \).

```
. lassoknots, for(fev1p) tlevel(1) display(nonzero var bic)
```

We see that the 43rd \( \lambda \), with value 0.1064, minimizes the BIC function, and there are seven selected variables at this \( \lambda \).
Estimating the ATET

Sometimes, we want to estimate the ATETs to determine the effect on those who actually received the treatment.

Example 3: ATET for BLT

We use the `atet` option to estimate the ATETs. We type

```
. telasso (fev1p $allvars) (transtype $allvars), atet
```

```
Estimating lasso for outcome fev1p if transtype = 0 using plugin method ...
Estimating lasso for outcome fev1p if transtype = 1 using plugin method ...
Estimating lasso for treatment transtype using plugin method ...
Estimating ATET ...
```

<table>
<thead>
<tr>
<th>Treatment-effects lasso estimation</th>
<th>Number of observations = 937</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome model: linear</td>
<td>Number of controls = 454</td>
</tr>
<tr>
<td>Treatment model: logit</td>
<td>Number of selected controls  = 8</td>
</tr>
</tbody>
</table>

| fev1p | Robust Coefficient | std. err. | z   | P>|z|  | [95% conf. interval] |
|-------|--------------------|-----------|-----|------|------------------|
| ATET  |                    |           |     |      |                  |
| transtype | (BLT vs SLT) | 35.78157  | 0.1831478 | 195.37 | 0.000  | 35.42261  | 36.14053 |
| POmean | transtype | 43.35214  | 1.268976  | 34.16  | 0.000  | 40.86499  | 45.83929 |

For the patients who have a BLT, we expect the average FEV1% to be 36 percentage points higher than if all of them choose an SLT.

High-dimensional semiparametric models

Sometimes, the theory suggests that some variables are essential to control the confounders, but it is silent on the functional form of the model. In the next example, we will illustrate how to use `telasso` to estimate the ATE in a high-dimensional semiparametric model.

Example 4: ATE of 401(k) eligibility

We want to estimate the effect of 401(k) eligibility (e401) on net financial assets (asset) using data reported by Chernozhukov and Hansen (2004). These data are from a sample of households in the 1990 Survey of Income and Program Participation (SIPP). The data contain information on the head of the household: income (income), age (age), years of education (educ), whether to receive pension benefit (pension), marital status (married), and whether to participate in the IRA (ira).

One concern when determining the effect of 401(k) eligibility on financial assets is that choosing to work for a company that offers a 401(k) plan is not randomly assigned. To overcome this issue, Poterba, Venti, and Wise (1994, 1995) propose that after conditioning on income, we can take working for a company that offers a 401(k) as exogenous.
We want to include income and age as covariates in the model, but we do not want to assume that they enter the model linearly. Because we want a more flexible model, we use makespline to create B-splines of order 3 with 3 knots at percentiles of variables income and age. The generated variables will be used to form a semiparametric model. These variables have the common stub _bs for easy reference later.

```
use https://www.stata-press.com/data/r18/assets, clear
(Excerpt from Chernozhukov and Hansen (2004))
makespline bspline income age, basis(_bs) knots(3)
```

We describe the generated B-spline variables. Seven terms are generated for each of the original variables.

```
describe _bs*
```

<table>
<thead>
<tr>
<th>Variable name</th>
<th>Storage type</th>
<th>Display format</th>
<th>Value label</th>
</tr>
</thead>
<tbody>
<tr>
<td>_bs_1_1</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 1 for income</td>
</tr>
<tr>
<td>_bs_1_2</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 2 for income</td>
</tr>
<tr>
<td>_bs_1_3</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 3 for income</td>
</tr>
<tr>
<td>_bs_1_4</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 4 for income</td>
</tr>
<tr>
<td>_bs_1_5</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 5 for income</td>
</tr>
<tr>
<td>_bs_1_6</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 6 for income</td>
</tr>
<tr>
<td>_bs_1_7</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 7 for income</td>
</tr>
<tr>
<td>_bs_2_1</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 1 for age</td>
</tr>
<tr>
<td>_bs_2_2</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 2 for age</td>
</tr>
<tr>
<td>_bs_2_3</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 3 for age</td>
</tr>
<tr>
<td>_bs_2_4</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 4 for age</td>
</tr>
<tr>
<td>_bs_2_5</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 5 for age</td>
</tr>
<tr>
<td>_bs_2_6</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 6 for age</td>
</tr>
<tr>
<td>_bs_2_7</td>
<td>double</td>
<td>%10.0g</td>
<td>B-spline basis term 7 for age</td>
</tr>
</tbody>
</table>

Next, we define the control variables as the categorical variables (pension, married, and ira), the generated spline variables (_bs*), and all interactions among these variables. The global macro $controls contains the defined control variables, and we will use it in both the outcome and the treatment model.

```
local vars c._(bs*) i.(pension married ira)
local controls $vars ($vars)#($vars)
```

Now, we are ready to use telasso to estimate the ATEs of 401(k) eligibility on net financial assets. We use a linear nonparametric series to approximate the outcome model. Moreover, we use a logit nonparametric series to approximate the treatment model. The global macro $controls defines terms in the nonparametric series.
. telasso (assets $controls) (e401 $controls), rseed(111)

Estimating lasso for outcome **assets** if **e401k = 0** using plugin method ...
Estimating lasso for outcome **assets** if **e401k = 1** using plugin method ...
Estimating lasso for treatment **e401k** using plugin method ...
Estimating ATE ...

<table>
<thead>
<tr>
<th>Treatment-effects lasso estimation</th>
<th>Number of observations</th>
<th>9,913</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outcome model:</td>
<td>linear</td>
<td>221</td>
</tr>
<tr>
<td>Treatment model: logit</td>
<td></td>
<td>48</td>
</tr>
</tbody>
</table>

| assets      | Coefficient | std. err. | z   | P>|z|  | [95% conf. interval] |
|------------|-------------|-----------|-----|------|----------------------|
| ATE        |             |           |     |      |                      |
| e401k      | 8391.237    | 1136.779  | 7.38| 0.000| 6163.191 10619.28    |
| (Eligible vs Not elig..)          |             |           |     |      |                      |
| POmean     |             |           |     |      |                      |
| e401k      | 13853.67    | 796.7734  | 17.39| 0.000| 12292.03 15415.32    |
| Not eligi..|             |           |     |      |                      |

The net financial assets if all the workers work for the companies with a 401(k) plan is expected to be $8,391 more than the average of $13,854 that is expected if all the workers work for the companies without a 401(k) plan.

### Stored results

telasso stores the following in **e()**:

** Scalars**
- `e(N)` number of observations
- `e(N0)` number of observations for treatment level 0
- `e(N1)` number of observations for treatment level 1
- `e(N_clust)` number of clusters
- `e(k_omvars)` number of potential control variables in the outcome model
- `e(k_omvars_sel)` number of selected control variables in the outcome model
- `e(k_tmvars)` number of potential control variables in the treatment model
- `e(k_tmvars_sel)` number of selected control variables in the treatment model
- `e(k_controls)` number of potential control variables in the outcome and treatment models
- `e(k_controls_sel)` number of selected control variables in the outcome and treatment models
- `e(k_levels)` number of levels in treatment variable
- `e(n_xfolds)` number of folds for cross-fitting
- `e(n_resample)` number of resamples
- `e(treated)` level of treatment variable defined as treated
- `e(control)` level of treatment variable defined as control
- `e(converged)` 1 if converged, 0 otherwise

** Macros**
- `e(cmd)` telasso
- `e(cmdline)` command as typed
- `e(depvar)` name of outcome variable
- `e(tvar)` name of treatment variable
- `e(tmodel)` logit or probit
- `e(omodel)` linear, logit, probit, or poisson
- `e(omvars)` potential control variables in the outcome model
- `e(omvars_sel)` selected control variables in the outcome model
The model

We consider estimating the ATEs, ATETs, and POMs when the treatment effects are heterogeneous and the treatment is binary. For notational simplicity, we drop the subscript \(i\) indicating the \(i\)th observation.

The outcome model is

\[
y = g_0(\tau, x) + u \quad E(u|\tau, x) = 0
\]

where \(y\) is the outcome variable, \(\tau\) is the binary treatment variable, \(x\) are potentially high-dimensional control variables in the outcome model, and \(g_0(\tau, x)\) is the expected potential outcome given a level of treatment and covariates \(x\). Because each unit can only be treated or not treated, the observed outcome \(y\) can only be one of \(g_0(1, x) + u\) or \(g_0(0, x) + u\).

The treatment model is

\[
\tau = m_0(z) + v \quad E(v|z) = 0
\]
where \( z \) are potentially high-dimensional covariates in the treatment model. \( m_0(z) \) is the expected value of \( \tau \) given \( z \). In other words, \( m_0(z) \) is the probability of a unit getting treated given \( z \).

The parameter of interest \( \theta_0 \) is ATE, ATET, or POMs.

ATE is

\[
\theta_0 = E \{ g_0(1, x) - g_0(0, x) \}
\]

ATET is

\[
\theta_0 = E \{ g_0(1, x) - g_0(0, x) | \tau = 1 \}
\]

POMs when \( \tau = 1 \) is

\[
\theta_0 = E \{ g_0(1, x) \}
\]

POMs when \( \tau = 0 \) is

\[
\theta_0 = E \{ g_0(0, x) \}
\]

We have several remarks about the above model.

1. The outcome model can be one of linear, logit, probit, or poisson. Let \( \beta_t \) be the outcome model parameters when treatment \( \tau = t \). The table below provides details about the available functional form of \( g_0(\tau, x) \).

<table>
<thead>
<tr>
<th>Outcome model</th>
<th>Functional form for ( g_0(\tau = t, x) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>linear</td>
<td>( x'\beta_t )</td>
</tr>
<tr>
<td>logit</td>
<td>( \exp(x'\beta_t)/(1 + \exp(x'\beta_t)) )</td>
</tr>
<tr>
<td>probit</td>
<td>( \Phi(x'\beta_t) )</td>
</tr>
<tr>
<td>poisson</td>
<td>( \exp(x'\beta_t) )</td>
</tr>
</tbody>
</table>

2. The treatment model can be either logit or probit. Let \( \gamma \) be the parameters in the treatment model. The table below provides details about the available functional form of \( m_0(z) \).

<table>
<thead>
<tr>
<th>Treatment model</th>
<th>Functional form for ( m_0(z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>logit</td>
<td>( \exp(z'\gamma)/(1 + \exp(z'\gamma)) )</td>
</tr>
<tr>
<td>probit</td>
<td>( \Phi(z'\gamma) )</td>
</tr>
</tbody>
</table>

3. Both \( x \) and \( z \) can be high dimensional, and there may be more variables than the number of observations. However, the outcome model parameter \( \beta_t \) and the treatment model parameter \( \gamma \) are assumed to be sparse. This means that there are only a few nonzero elements in both \( \beta_t \) and \( \gamma \).

4. Although we assume that the functional forms of \( g_0(\cdot) \) and \( m_0(\cdot) \) are known, we do not know which variables should enter the model. We use lasso techniques to select variables from the potential high-dimensional controls \( x \) and \( z \). The resulting estimators should guard against possible model selection errors made bylassos.

Next we discuss the methods that telasso uses to estimate ATEs, ATETs, and POMs.
Neyman orthogonal moments

Denote $\theta_0$ as the parameter of interest, which can be ATE, ATET, or POM. Following Chernozhukov et al. (2018), we use the following moment conditions to estimate $\theta_0$.

The moment condition for estimating ATE is

$$
\psi(w; \theta, \eta) = \{g(1, x) - g(0, x)\} + \frac{\tau \{y - g(1, x)\}}{m(z)} - \frac{(1 - \tau) \{y - g(0, x)\}}{1 - m(z)} - \theta
$$

where $w = (y, x', z', \tau)'$ and $\eta$ is the nuisance parameter consisting of $g(\tau, x)$ and $m(z)$. Using the variables selected by lassos, $g(1, x)$, $g(0, x)$, and $m(z)$ are approximations to the true functions $g_0(1, x)$, $g_0(0, x)$, and $m_0(z)$, respectively.

The moment condition for estimating ATET is

$$
\psi(w; \theta; \eta) = \tau \{y - g(0, x)\} \frac{p}{p(1 - m(z))} - \frac{\tau \theta}{p}
$$

where $p = E(\tau)$.

The moment condition for estimating POM for $\tau = 1$ is

$$
\psi(w; \theta, \eta) = g(1, x) + \frac{\tau \{y - g(1, x)\}}{m(z)} - \theta
$$

The moment condition for estimating POM for $\tau = 0$ is

$$
\psi(w; \theta, \eta) = g(0, x) + \frac{(1 - \tau) \{y - g(0, x)\}}{1 - m(z)} - \theta
$$

To estimate $\theta_0$, we first use lasso techniques to select variables in the outcome and treatment models. Based on the selected variables, we can estimate the post-lasso predictions for $g(\tau, x)$ and $m(z)$. Denote $\tilde{\eta}$ as the estimates for $g(\tau, x)$ and $m(z)$.

The estimator $\hat{\theta}$ for $\theta_0$ is the solution to

$$
\frac{1}{n} \sum_{i=1}^{n} [\psi_i(w; \theta, \tilde{\eta})] = 0
$$

The variance estimator for $\hat{\theta}$ is

$$
\frac{1}{n^2} \sum_{i=1}^{n} [\psi_i(w; \hat{\theta}, \tilde{\eta})^2] = 0
$$

To identify $\theta_0$, we need to assume the conditional mean independence, overlap, and independent treatment assignment. For a detailed discussion of these assumptions, see [CAUSAL] teffects intro or [CAUSAL] teffects intro advanced.

The moment conditions defined in (1) to (4) are Neyman orthogonal. Intuitively, Neyman orthogonal moments mean that the estimator or parameter of interest is still consistent even if model selection makes some mild mistakes. For a formal introduction on Neyman orthogonality, see section 2.1 in Chernozhukov et al. (2018).

The moment conditions in (1) to (4) also imply the AIPW estimator for treatment effects. The AIPW estimator is doubly robust in the sense that only one of the outcome model or the treatment model is required to be correctly specified.
Double machine learning

telasso also implements the double machine learning estimator in Chernozhukov et al. (2018) for ATEs, ATETs, and POMs. One advantage of double machine learning is that it allows telasso to select more variables in the models. The sparsity assumption is crucial for the validity of the estimators implemented in telasso. In other words, the sparsity assumption allows only a few variables to be selected. Double machine learning techniques relax this sparsity requirement to some extent such that more variables can be selected and the estimation results are still valid.

telasso with the xfold() option implements cross-fit estimation for ATEs, ATETs, and POMs. Cross-fit estimation fits the nuisance parameters and parameter of interest in different samples. It has the following structure.

Let \( K \) be the specified number of cross-fitting folds.

1. Randomly partition the sample into \( K \) subsamples called folds.
2. Define \( I_k \) to be the observations in fold \( k \), and define \( IC_k \) to be the sample observations not in fold \( k \).
3. For each \( k = 1, \ldots, K \), fill in the observations of \( i \in I_k \) for the moment condition \( \psi(\cdot) \) in (1), (2), (3), or (4) depending on the specified statistic.
   a. Using observations \( i \in IC_k \), perform model selection, and estimate the parameters in the outcome and treatment models.
   b. Based on the estimates in step 3a, predict the nuisance parameter \( \tilde{\eta}_k \) for observations \( i \in I_k \).
   c. Based on \( \tilde{\eta}_k \), fill in the moment conditions \( \psi(\cdot) \) for observations \( i \in I_k \).
4. The estimator \( \hat{\theta} \) is the solution in (5).

Resampling the partitions

The \( K \) folds are chosen once by default. Specify the resample(#) option to have the \( K \) folds randomly selected # times. This resampling removes the dependence of the estimator on any specifically selected folds, at the cost of more computer time.

Let \( S \) be the specified number of resamples.

1. For each random partition \( s = 1, \ldots, S \), use a cross-fit estimator to obtain point estimate \( \hat{\theta}_s \) and the estimated VCE \( \hat{\text{Var}}(\hat{\theta}_s) \).
2. The mean resampling-corrected point estimates are

\[
\tilde{\theta} = \frac{1}{S} \sum_{s=1}^{S} \hat{\theta}_s
\]

3. The mean resampling-corrected estimate of the VCE is

\[
\tilde{\text{Var}}(\tilde{\theta}) = \frac{1}{S} \sum_{s=1}^{S} \left\{ \text{Var}(\hat{\theta}_s) + (\hat{\theta}_s - \tilde{\theta})(\hat{\theta}_s - \tilde{\theta})' \right\}
\]
References


Also see

[CAUSAL] **telasso postestimation** — Postestimation tools for telasso

[CAUSAL] **teffects aipw** — Augmented inverse-probability weighting

[LASSO] **Lasso inference intro** — Introduction to inferential lasso models

[U] **20 Estimation and postestimation commands**
telasso postestimation — Postestimation tools for telasso

**Postestimation commands**

The following postestimation commands are of special interest after telasso:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>teoverlap</td>
<td>overlap plots</td>
</tr>
<tr>
<td>tebalance</td>
<td>check balance of covariates</td>
</tr>
<tr>
<td>bicplot</td>
<td>plot Bayesian information criterion function</td>
</tr>
<tr>
<td>*coefpath</td>
<td>plot path of coefficients</td>
</tr>
<tr>
<td>*cvplot</td>
<td>plot cross-validation function</td>
</tr>
<tr>
<td>lassocoef</td>
<td>display selected coefficients</td>
</tr>
<tr>
<td>lassoinfo</td>
<td>information about lasso estimation results</td>
</tr>
<tr>
<td>lassoknots</td>
<td>knot table of coefficient selection and measures of fit</td>
</tr>
<tr>
<td>*lassoselect</td>
<td>select alternative $\lambda$*</td>
</tr>
</tbody>
</table>

*coefpath, cvplot, and lassoselect require the selection method of the lasso to be selection(cv), selection(adaptive), or selection(bic). See [LASSO] lasso options.*

The following standard postestimation commands are also available:

<table>
<thead>
<tr>
<th>Command</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>estat summarize</td>
<td>summary statistics for the estimation sample</td>
</tr>
<tr>
<td>estat vce</td>
<td>variance–covariance matrix of the estimators (VCE)</td>
</tr>
<tr>
<td>estimates</td>
<td>cataloging estimation results</td>
</tr>
<tr>
<td>etable</td>
<td>table of estimation results</td>
</tr>
<tr>
<td>lincom</td>
<td>point estimates, standard errors, testing, and inference for linear combinations of coefficients</td>
</tr>
<tr>
<td>nlcom</td>
<td>point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients</td>
</tr>
<tr>
<td>predict</td>
<td>treatment effects, conditional means at treatment, propensity scores, etc.</td>
</tr>
<tr>
<td>predictnl</td>
<td>point estimates for generalized predictions</td>
</tr>
<tr>
<td>test</td>
<td>Wald tests of simple and composite linear hypotheses</td>
</tr>
<tr>
<td>testnl</td>
<td>Wald tests of nonlinear hypotheses</td>
</tr>
</tbody>
</table>
predict

Description for predict

predict creates a new variable containing predictions such as treatment effects, potential outcomes, conditional means, propensity scores, and linear predictions.

Menu for predict

Statistics > Postestimation

Syntax for predict

```
predict [type] { stub* | newvar | newvarlist } [if] [in] 
[ , statistic tlevel(treat_level) ]
```

<table>
<thead>
<tr>
<th>statistic</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main</strong></td>
<td></td>
</tr>
<tr>
<td>te</td>
<td>treatment effect; the default</td>
</tr>
<tr>
<td>cm</td>
<td>mean conditional mean at treatment level</td>
</tr>
<tr>
<td>ps</td>
<td>propensity score</td>
</tr>
<tr>
<td>xb</td>
<td>linear prediction</td>
</tr>
<tr>
<td>psxb</td>
<td>linear prediction for propensity score</td>
</tr>
</tbody>
</table>

Option tlevel() may not be combined with te or psxb.

If you do not specify tlevel() and only specify one new variable, then cm, ps, and xb assume tlevel() specifies the control.

You specify one or two new variables with cm, ps, and xb.
You specify one new variable with te and psxb.

Options for predict

<table>
<thead>
<tr>
<th>Main</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>te, the default, calculates the treatment effect for the noncontrol treatment level. You need to specify only one new variable.</td>
<td></td>
</tr>
<tr>
<td>cm calculates the conditional mean for each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify two new variables corresponding to the control and noncontrol treatment levels.</td>
<td></td>
</tr>
<tr>
<td>ps calculates the propensity score of each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify two new variables corresponding to the control and noncontrol treatment levels.</td>
<td></td>
</tr>
<tr>
<td>xb calculates the linear prediction at each treatment level or the treatment level specified in tlevel(). If you specify the tlevel() option, you need to specify only one new variable; otherwise, you must specify two new variables corresponding to the control and noncontrol treatment levels.</td>
<td></td>
</tr>
</tbody>
</table>
psxb calculates the linear prediction for the propensity score at the noncontrol level of the treatment. You need to specify only one new variable.

tlevel(treat_level) specifies the treatment level for prediction.

Remarks and examples

Some of the telasso postestimation commands explore the lasso results computed within telasso. Here is a list of such commands: coefpath, cvplot, bicplot, lassoknots, lassoselect, and lassocoef.

When referring to a lasso result computed by telasso, there is a distinction between the outcome model and the treatment model. To refer to the lasso result for the treatment model, we need to specify the treatment variable with the for(tvar) option. In contrast, to refer to the lasso result for the outcome model, we need to specify the outcome variable at a specific treatment level with the for(ovar) and tlevel(#) options. In summary, for the treatment model, the for(tvar) option is required; for the outcome model, both the for(ovar) and the tlevel(#) options are required.

Examples that demonstrate how to use the telasso command and explore the lasso results using the postestimation tools can be found in Remarks and examples in [CAUSAL] telasso.

Also see

[CAUSAL] telasso — Treatment-effects estimation using lasso

[U] 20 Estimation and postestimation commands
Description

One of the assumptions required to use the `teffects` and `stteffects` estimators is the overlap assumption, which states that each individual has a positive probability of receiving each treatment level. `teoverlap`, a postestimation command, plots the estimated densities of the probability of getting each treatment level. These plots can be used to check whether the overlap assumption is violated.

Quick start

Visually check whether the overlap assumption is violated

```
teoverlap
```

Same as above, but use the Epanechnikov kernel function

```
teoverlap, kernel(epanechnikov)
```

Plot probability of getting treatment 3 for subjects receiving treatments 2 or 3 of a multivalued treatment

```
teoverlap, ptlevel(3) tlevels(2 3)
```

Same as above, and change legend labels to “Treated 2” and “Treated 3”

```
teoverlap, ptlevel(3) tlevels(2 3) ///
    legend(label(1 "Treated 2") label(2 "Treated 3"))
```

Menu

Statistics > Causal inference/treatment effects > Overlap plots
Syntax

teoverlap [, treat_options kden_options]

<table>
<thead>
<tr>
<th>treat_options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main</td>
<td></td>
</tr>
<tr>
<td><code>ptlevel(treat_level)</code></td>
<td>calculate predicted probabilities for treatment level <code>treat_level</code>; by default, <code>ptlevel()</code> corresponds to the first treatment level</td>
</tr>
<tr>
<td><code>tlevels(treatments)</code></td>
<td>specify conditioning treatment levels; default is all treatment levels</td>
</tr>
<tr>
<td><code>nolabel</code></td>
<td>use treatment level values and not value labels in legend and axis titles</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>kden_options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>Main</td>
<td></td>
</tr>
<tr>
<td><code>kernel(kernel)</code></td>
<td>specify kernel function; default is <code>kernel(triangle)</code></td>
</tr>
<tr>
<td><code>n(#)</code></td>
<td>estimate densities using # points; default is <code>e(N)</code>, the number of</td>
</tr>
<tr>
<td></td>
<td>observations in the estimation sample</td>
</tr>
<tr>
<td><code>bwwidth(#)</code></td>
<td>half-width of kernel</td>
</tr>
<tr>
<td><code>at(var_x)</code></td>
<td>estimate densities using the values specified by <code>var_x</code></td>
</tr>
<tr>
<td>Kernel plots</td>
<td></td>
</tr>
<tr>
<td><code>line#opts(cline_options)</code></td>
<td>affect rendition of density for conditioning treatment #</td>
</tr>
<tr>
<td>Add plots</td>
<td></td>
</tr>
<tr>
<td><code>addplot(plot)</code></td>
<td>add other plots to the generated graph</td>
</tr>
<tr>
<td>Y axis, X axis, Titles, Legend, Overall</td>
<td></td>
</tr>
<tr>
<td><code>twoway_options</code></td>
<td>any options other than by() documented in [G-3] <code>twoway_options</code></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>kernel</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>triangle</code></td>
<td>triangle kernel function; the default</td>
</tr>
<tr>
<td><code>epanechnikov</code></td>
<td>Epanechnikov kernel function</td>
</tr>
<tr>
<td><code>epan2</code></td>
<td>alternative Epanechnikov kernel function</td>
</tr>
<tr>
<td><code>biweight</code></td>
<td>biweight kernel function</td>
</tr>
<tr>
<td><code>cosine</code></td>
<td>cosine trace kernel function</td>
</tr>
<tr>
<td><code>gaussian</code></td>
<td>Gaussian kernel function</td>
</tr>
<tr>
<td><code>parzen</code></td>
<td>Parzen kernel function</td>
</tr>
<tr>
<td><code>rectangle</code></td>
<td>rectangle kernel function</td>
</tr>
</tbody>
</table>

collect is allowed; see [U] 11.1.10 Prefix commands.

Options

`ptlevel(treat_level)` specifies that predicted probabilities be calculated for treatment level `treat_level`. The default is `ptlevel(first)`, where `first` is the first treatment level.
tlevels(treatments) specifies the observations for which to obtain predicted probabilities. By default, all treatment levels are used. Specify treatments as a space-delimited list.

For instance,

```
  . teoverlap, ptlevel(1) tlevels(1 2)
```

says to predict the probability of getting treatment level 1 for those subjects who actually obtained treatment levels 1 or 2.

nlabel specifies that treatment level values and not value labels be used in legend and axis titles.

kernel(kernel) specifies the kernel function for use in calculating the kernel density estimates. The default kernel is the triangle kernel (triangle).

n(#) specifies the number of points at which the density estimate is to be evaluated. The default is e(N), the estimation sample size.

bwidth(#) specifies the half-width of the kernel, the width of the density window around each point. If bwidth() is not specified, the “optimal” width is calculated and used; see [R] kdensity. The optimal width is the width that would minimize the mean integrated squared error if the data were Gaussian and a Gaussian kernel were used, so it is not optimal in any global sense. In fact, for multimodal and highly skewed densities, this width is usually too wide and oversmooths the density (Silverman 1986).

at(var_x) specifies a variable that contains the values at which the density should be estimated. This option allows you to more easily obtain density estimates for different variables or different subsamples of a variable and then overlay the estimated densities for comparison.

Kernel plots

line#opts(cline_options) affect the rendition of the plotted kernel density estimates. See [G-3] cline_options.

Add plots

addplot(plot) provides a way to add other plots to the generated graph. See [G-3] addplot_option.

Y axis, X axis, Titles, Legend, Overall

twoway_options are any of the options documented in [G-3] twoway_options, excluding by(). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

Remarks and examples

tteoverlap plots the estimated densities of the probability of getting each treatment level after tteffects.

These plots can be used to check whether the overlap assumption is violated. The overlap assumption is satisfied when there is a chance of seeing observations in both the control and the treatment groups at each combination of covariate values; see [CAUSAL] tteffects intro or [CAUSAL] tteffects intro advanced.

The overlap assumption is required by the estimators implemented in tteffects. Intuitively, when the overlap assumption is violated, we cannot predict, or otherwise account for, the unobserved outcomes for some individuals.
There is evidence that the overlap assumption is violated when an estimated density has too much mass around 0 or 1; see Busso, DiNardo, and McCrary (2014). An implication of this point is that when the overlap assumption is violated, the estimated densities will have relatively little mass in the regions in which they overlap.

Example 1: Assumption not violated

Continuing with example 1 of [CAUSAL] teffects ipw, we estimate the average treatment effect of smoking on birthweight and then draw the overlap plot:

```
. use https://www.stata-press.com/data/r18/cattaneo2
. teffects ipw (bweight) (mbsmoke mmarrried c.mage##c.mage fbaby medu, probit)
Iteration 0:   EE criterion = 4.621e-21
Iteration 1:   EE criterion = 7.358e-26
Treatment-effects estimation Number of obs   =    4,642
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: probit

|               | Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|---------------|-------------|-----------|------|------|---------------------|
| ATE mbsmoke   | -230.6886   | 25.81524  | -8.94| 0.000| -281.2856 -180.0917 |
| (Smoker vs Nonsmoker) |           |           |      |      |                     |
| P0mean mbsmoke Nonsmoker | 3403.463   | 9.571369  | 355.59| 0.000| 3384.703 3422.222   |
```

The graph displays the estimated density of the predicted probabilities that a nonsmoking mother is a nonsmoker and the estimated density of the predicted probabilities that a smoking mother is a nonsmoker.
Neither plot indicates too much probability mass near 0 or 1, and the two estimated densities have most of their respective masses in regions in which they overlap each other. Thus there is no evidence that the overlap assumption is violated.

Example 2: Assumption violated

This example produces an overlap plot that indicates a failure of the overlap assumption. We will use simulated data, so we know that the assumption is not true.

In our simulated dataset, some of the 1,000 adult males were given drug XY1 for high blood pressure and others were not. A scatterplot of systolic blood pressure (systolic) and weight (weight) reveals that heavier men were given the treatment. (The scatterplots corresponding to the treatment group are colored red, while the scatterplots corresponding to the control group are colored blue.)

```
. use https://www.stata-press.com/data/r18/systolic2
(Systolic blood pressure)
. twoway (scatter systolic weight if xy1==1, mcolor(stred))
>     (scatter systolic weight if xy1==0, mcolor(stblue)),
>     legend(label(1 "Treated") label(2 "Untreated"))
```

There are no observations in the treated group for small weights, and there are no observations in the control group for large weights. There is clear evidence that the overlap assumption is violated.

Drawing an overlaid scatterplot is a straightforward way to check the overlap assumption in this example because there is only one covariate. This method is not available when there is more than one covariate. The predicted probability is a one-dimensional measure that captures the relevant multivariate information.
Below we estimate the parameters needed to calculate the predicted probabilities. The \texttt{pstolerance(1e-8)} option is specified to ensure that estimation is performed as long as the predicted probabilities are at least as large as $1e^{-8}$.

\begin{verbatim}
. teffects ipw (systolic) (xy1 weight), pstolerance(1e-8)
Iteration 0:  EE criterion = 9.523e-18
Iteration 1:  EE criterion = 3.489e-28
Treatment-effects estimation Number of obs = 1,000
Estimator : inverse-probability weights
Outcome model : weighted mean
Treatment model: logit

|     systolic | Coefficient | std. err. | z    | P>|z|  | [95% conf. interval] |
|-------------|-------------|-----------|------|------|----------------------|
| ATE         |             |           |      |      |                      |
| xy1         |             |           |      |      |                      |
| (Treated vs |             |           |      |      |                      |
| Untreated)  | -16.23679   | 2.191703  | -7.41| 0.000| [-20.53245 -11.94114]|
| POmean xy1  |             |           |      |      |                      |
| Untreated   | 127.9094    | .7004533 | 182.61| 0.000| [126.5365 129.2822]  |
\end{verbatim}

Now we can obtain the overlap plot.

\begin{verbatim}
. teoverlap
\end{verbatim}

The estimated density of the predicted probabilities that a treated individual is not assigned to \textit{XY1} treatment has most of its mass near 0. The estimated density of the predicted probabilities that an untreated individual is not assigned to \textit{XY1} treatment has most of its mass near 1. Note that the two have very little mass in the region in which they overlap. There is clear evidence that the overlap assumption is violated.
Stored results

teoverlap stores the following in r():

Scalars
- $r(b\text{width}_j)$: kernel bandwidth for treatment level $j$
- $r(n_j)$: number of points at which the estimate was evaluated for treatment level $j$
- $r(scale_j)$: density bin width for treatment level $j$

Macros
- $r(kernel)$: name of kernel

References


Also see

[CAUSAL] stteffects — Treatment-effects estimation for observational survival-time data
[CAUSAL] stteffects ipw — Survival-time inverse-probability weighting
[CAUSAL] stteffects ipwra — Survival-time inverse-probability-weighted regression adjustment
[CAUSAL] teffects — Treatment-effects estimation for observational data
[CAUSAL] teffects aipw — Augmented inverse-probability weighting
[CAUSAL] teffects ipw — Inverse-probability weighting
[CAUSAL] teffects ipwra — Inverse-probability-weighted regression adjustment
[CAUSAL] teffects nnmatch — Nearest-neighbor matching
[CAUSAL] teffects psmatch — Propensity-score matching
[CAUSAL] teffects ra — Regression adjustment
[CAUSAL] telasso — Treatment-effects estimation using lasso
**Description**

`xthdidregress` estimates average treatment effects on the treated (ATETs) that may vary over time and over treatment cohorts. Treatment cohorts are groups subject to treatment at different points in time. `xthdidregress` provides four estimators: extended two-way fixed effects (TWFE), regression adjustment (RA), inverse-probability weighting (IPW), and augmented inverse-probability weighting (AIPW). See [CAUSAL] `teffects intro` for a discussion of RA, IPW, and AIPW estimators.

`xthdidregress` is for panel data. For repeated cross-sectional data, see [CAUSAL] `hdidregress`.

**Quick start**

Estimate ATETs of treatment `treat` on outcome `y` with group `grpvar`; use the RA estimator and model `y` with covariate `x` on `xtset` data

```
xthdidregress ra (y x) (treat), group(grpvar)
```

Same as above, but use the TWFE estimator

```
xthdidregress twfe (y x) (treat), group(grpvar)
```

Use the IPW estimator and model `treat` using covariate `z`

```
xthdidregress ipw (y) (treat z), group(grpvar)
```

Use the AIPW estimator, model `y` using covariate `x`, and model `treat` using covariate `z`

```
xthdidregress aipw (y x) (treat z), group(grpvar)
```

Same as above, but use the not-yet-treated group as the control group

```
xthdidregress aipw (y x) (treat z), group(grpvar) ///  
controlgroup(notyet)
```

Same as above, but cluster at the county level

```
xthdidregress aipw (y x) (treat z), group(grpvar) ///  
controlgroup(notyet) vce(cluster county)
```

**Menu**

Statistics > Causal inference/treatment effects > Continuous outcomes > Heterogeneous DID > Panel-data heterogeneous DID (TWFE)

Statistics > Causal inference/treatment effects > Continuous outcomes > Heterogeneous DID > Panel-data heterogeneous DID (RA)

Statistics > Causal inference/treatment effects > Continuous outcomes > Heterogeneous DID > Panel-data heterogeneous DID (IPW)

Statistics > Causal inference/treatment effects > Continuous outcomes > Heterogeneous DID > Panel-data heterogeneous DID (AIPW)
Syntax

Two-way fixed effects

\[
\texttt{xthdidregress\ twfe (ovar [omvarlist]) (tvar) [if] [in] [weight],}
\]
\[
\quad \text{\texttt{group(groupvar)}\ [options]}\]

Regression adjustment

\[
\texttt{xthdidregress\ ra (ovar [omvarlist]) (tvar) [if] [in] [weight],}
\]
\[
\quad \text{\texttt{group(groupvar)}\ [options]}\]

Inverse-probability weighting

\[
\texttt{xthdidregress\ ipw (ovar) (tvar [tmvarlist]) [if] [in] [weight],}
\]
\[
\quad \text{\texttt{group(groupvar)}\ [options]}\]

Augmented inverse-probability weighting

\[
\texttt{xthdidregress\ aipw (ovar [omvarlist]) (tvar [tmvarlist]) [if] [in] [weight],}
\]
\[
\quad \text{\texttt{group(groupvar)}\ [options]}\]

\textit{ovar} is a continuous outcome of interest.

\textit{omvarlist} specifies the covariates in the outcome model and may contain factor variables; see [\texttt{U}\ 11.4.3 Factor variables].

\textit{tvar} must be a binary variable indicating observations subject to treatment.

\textit{tmvarlist} specifies the covariates in the treatment model and may contain factor variables; see [\texttt{U}\ 11.4.3 Factor variables].

\textit{groupvar} is a categorical variable that indicates the group level at which the treatment occurs.
### Model

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>*group(groupvar)</code></td>
<td>specify group variable</td>
</tr>
<tr>
<td><code>controlgroup(cgtype)</code></td>
<td>specify the type of control group; default is <code>never</code></td>
</tr>
<tr>
<td><code>cohortvar(cvar [, replace])</code></td>
<td>specify the variable name for the generated cohort</td>
</tr>
<tr>
<td><code>hettype(hetspec)</code></td>
<td>type of heterogeneity; default is <code>timecohort</code></td>
</tr>
</tbody>
</table>

### SE/Robust

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>vce(vcetype)</code></td>
<td><code>vcetype</code> may be <code>cluster clustvar</code>, <code>robust</code>, <code>bootstrap</code>, or <code>jackknife</code></td>
</tr>
</tbody>
</table>

### Reporting

<table>
<thead>
<tr>
<th>Option</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><code>level(#)</code></td>
<td>set confidence level; default is <code>level(95)</code></td>
</tr>
<tr>
<td><code>[no] log</code></td>
<td>suppress iteration log</td>
</tr>
<tr>
<td><code>nodots</code></td>
<td>suppress replication dots</td>
</tr>
<tr>
<td><code>display_options</code></td>
<td>control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling</td>
</tr>
<tr>
<td><code>coeflegend</code></td>
<td>display legend instead of statistics</td>
</tr>
</tbody>
</table>

- `*group(groupvar)` is required.
- `‡`This option may be specified only when `twfe` is specified.

### cgtype

<table>
<thead>
<tr>
<th>Description</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><code>never</code></td>
<td>use the never-treated group as the control group; the default</td>
</tr>
<tr>
<td><code>notyet</code></td>
<td>use the not-yet-treated group as the control group</td>
</tr>
</tbody>
</table>

### hetspec

<table>
<thead>
<tr>
<th>Description</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><code>timecohort</code></td>
<td>heterogeneous treatment effects over time and cohort; the default</td>
</tr>
<tr>
<td><code>time</code></td>
<td>heterogeneous treatment effects over time</td>
</tr>
<tr>
<td><code>cohort</code></td>
<td>heterogeneous treatment effects over cohort</td>
</tr>
</tbody>
</table>

A panel variable and a time variable must be specified using `xtset`; see [XT] `xtset`. by, `collect`, and `statsby` are allowed; see [U] 11.1.10 `Prefix commands`. `fweights`, `aweights`, and `pweights` are allowed; see [U] 11.1.6 `weight`. Weights must be constant within panel. `coeflegend` does not appear in the dialog box.

See [U] 20 `Estimation and postestimation commands` for more capabilities of estimation commands.
**Options**

**Model**

- **group(groupvar)** specifies a group variable that indicates the group level at which the treatment occurs. groupvar may be, for example, states, counties, or hospitals. group() also defines the level clustering for the default cluster–robust standard errors. group() is required. You may specify only one group variable.

- **controlgroup(cgtype)** specifies the type of control group. A control group can be either a never-treated group or a not-yet-treated group. A never-treated group refers to the units that are untreated from the first to the last period. A not-yet-treated group refers to the units that are untreated up to a specific period. cgtype can be one of never, referring to the never-treated group, or notyet, referring to the not-yet-treated group. By default, cgtype is never.

- **cohortvar(cvar [, replace])** specifies the variable name cvar for the generated cohort variable. The cohort variable is a categorical variable indicating the period when the unit is first treated. By default, _did_coort is used as the name of the cohort variable. If _did_coort already exists in the dataset, it is replaced if option cohortvar() is not specified.

  If suboption replace is specified, cvar is replaced.

- **hettype(hetspec)** specifies time or cohort heterogeneity for the twfe estimator. By default, treatment is interacted with time and cohort. You may choose to keep one of time or cohort interactions using hetspec.

  hetspec may be one of timecohort for heterogeneous treatment effects over both time and cohort, time for heterogeneous treatment effects over time only, or cohort for heterogeneous treatment effects over cohort only. By default, hetspec is timecohort.

**SE/Robust**

- **vce(vcetype)** specifies the type of standard error reported, which includes types that allow for intragroup correlation (cluster clustvar), that are robust to intragroup correlation among group variable (robust), and that use bootstrap or jackknife sampling done at the panel level (bootstrap, jackknife); see [R] vce_option.

  vce(cluster clustvar), the default, uses the variable specified in group(groupvar).

  Specifying vce(robust) is equivalent to specifying vce(cluster clustvar), where clustvar is the variable specified in the group(groupvar) option.

**Reporting**

- **level(#)**; see [R] Estimation options.

  log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

  nodots suppresses display of the replication dots.

  display_options: noci, nopvalues, cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

The following option is available with xthdidregress but is not shown in the dialog box:

- **coeflegend**; see [R] Estimation options.
Remarks and examples

It is common to study the effects of a treatment, for example, a policy or intervention, on a group. `xthdidregress` is for data where the treated groups are subject to the treatment at different points in time and they remain exposed to the treatment. For example, a health policy such as an increase in the age to purchase cigarettes is implemented in a given region, and over time, other regions decide to imitate the initiative. Another example is change in work policies across industries. Perhaps airlines implement a minimum number of hours between shifts for safety reasons. The policy is subsequently adopted by other similar industries. Some similar industries may never adopt the policy, remaining untreated, or it might be that all similar industries eventually adopt the policy.

`xthdidregress` estimates ATET parameters that change over time and treatment cohorts (groups treated at different points in time). Each one of these ATETs has the same interpretation that the parameters of a two-time two-group difference-in-differences (DID) parameter would have. Because there are multiple DID parameters, we refer to them as heterogeneous treatment effects or as heterogeneous DID. This is in contrast to estimating only one ATET, which assumes there is no variation across time or cohort. If you assume no variation across time or cohort, you may use `xtdidregress`; see `[CAUSAL] didregress`.

`xthdidregress` provides four estimators: TWFE, outlined in Wooldridge (2021); RA, IPW, and AIPW, outlined in Callaway and Sant’Anna (2021). Each one of these estimators fits a model for the outcome of interest, a model for the treatment, or a model for both. For example, RA and TWFE model the outcome; IPW models the treatment; and AIPW models both. If the model for the outcome is correctly specified, RA and TWFE are best, with TWFE being more efficient. If the treatment model is correctly specified, IPW should be best. AIPW models both treatment and outcome. If at least one of the models is correctly specified, it provides consistent estimates. Thus, it allows us to misspecify one of the models and still get consistent estimates, a property called double robustness. See `[CAUSAL] teffects intro` for a discussion of RA, IPW, and AIPW estimators.

`xthdidregress` is for panel data. For repeated cross-sectional data, see `[CAUSAL] hdidregress`. Below, we illustrate how to use `xthdidregress`. For more information about the methods used below, see `[CAUSAL] DID intro`. For general discussions about the methods, see Roth et al. (2022) and de Chaisemartin and D’Haultfœuille (forthcoming) and the references therein.

Example 1: Fitting a heterogeneous DID model

We are interested in how the number of registrations of a dog breed with the American Kennel Club (AKC), registered, is affected by dogs being the protagonists in a movie, movie. We conjecture that the number of registrations increases if the dog breed appears as the protagonist in a movie. We also conjecture that registrations increase if the dog has won the Best in Show award from the Westminster Kennel Club, best, in the 10 years before 2034. We use simulated data, but there is some evidence of the effect of movies on dog breed registrations. See, for example, Ghirlanda, Acerbi, and Herzog (2014).

There are 141 dog breeds in our sample, which extends between the years 2031 and 2040. At the beginning of the sample, none of the breeds are featured in a movie. This changes in 2034, when four breeds are featured in movies. The next year in which we see an increase of breeds featured in movies is 2036, when 7 more breeds are featured. In 2037, there is a substantial increase, with 22 more breeds featured. There is no increase in breeds in movies thereafter. The table below illustrates this.
We have a panel dataset. To obtain the effect of movie on registered controlling for best, we first xtset our data:

```
. xtset breed year
  Panel variable: breed (strongly balanced)
  Time variable: year, 2031 to 2040
  Delta: 1 unit
```

We chose regression adjustment, ra, to fit our model (see [CAUSAL] teffects ra for details on this model). In the first set of parentheses, we define the outcome, registered, and any covariates that affect the outcome directly. In the second set of parentheses, we define the observation-level treatment variable, movie. Other models allow us to define covariates that affect the treatment, but this is not the case of the ra model. After the comma, we need to define the group variable in group(); this is a required option. The group variable defines at which level the treatment occurs and also identifies the clustering variable, which in this case is breed.

```
. xthdidregress ra (registered best) (movie), group(breed)
  note: variable _did_cohort, containing cohort indicators formed by treatment
        variable movie and group variable breed, was added to the dataset.
```

Computing ATET for each cohort and time:
```
Cohort 2034 (9): ........ done
Cohort 2036 (9): ........ done
Cohort 2037 (9): ........ done
```

Treatment and time information
```
Time variable: year
Time interval: 2031 to 2040
Control:   _did_cohort = 0
Treatment: _did_cohort > 0
```

<table>
<thead>
<tr>
<th>_did_cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cohorts</td>
</tr>
<tr>
<td>Number of obs</td>
</tr>
<tr>
<td>Never treated</td>
</tr>
<tr>
<td>2034</td>
</tr>
<tr>
<td>2036</td>
</tr>
<tr>
<td>2037</td>
</tr>
</tbody>
</table>
Heterogeneous-treatment-effects regression  
Number of obs = 1,410  
Number of panels = 141  

Estimator: Regression adjustment  
Panel variable: breed  
Treatment level: breed  
Control group: Never treated  

(Std. err. adjusted for 141 clusters in breed)

| Cohort | ATET | std. err. | z    | P>|z| | [95% conf. interval] |
|--------|------|-----------|------|--------|----------------------|
| 2034   |      |           |      |        |                      |
|        | 2032 | -254.8927| 266.1024 | -0.96 | 0.338 | -776.4439 | 266.6584 |
|        | 2033 | -257.5329| 217.9389 | -1.18 | 0.237 | -684.6852 | 169.6194 |
|        | 2034 | 701.1318 | 127.0935 | 5.52  | 0.000 | 452.0331 | 950.2304 |
|        | 2035 | 1099.044 | 282.0704 | 3.90  | 0.000 | 546.196 | 1651.892 |
|        | 2036 | 1367.632 | 225.8702 | 6.05  | 0.000 | 924.9343 | 1810.329 |
|        | 2037 | 2008.294 | 237.2396 | 8.47  | 0.000 | 1543.313 | 2473.275 |
|        | 2038 | 2472.624 | 278.2949 | 8.88  | 0.000 | 1927.176 | 3018.072 |
|        | 2039 | 2689.615 | 504.3324 | 5.33  | 0.000 | 1701.142 | 3678.088 |
|        | 2040 | 3110.97  | 568.916 | 5.47  | 0.000 | 1995.915 | 4226.025 |
| 2036   |      |           |      |        |                      |
|        | 2032 | 216.0259 | 122.9107 | 1.76  | 0.079 | -24.8742 | 456.9265 |
|        | 2033 | -172.5154| 372.0776 | -0.46 | 0.643 | -901.7741 | 556.7433 |
|        | 2034 | -218.0495| 504.5267 | -0.43 | 0.666 | -1206.904 | 770.8045 |
|        | 2035 | 621.033  | 156.1306 | 3.98  | 0.000 | 315.0227 | 927.0434 |
|        | 2036 | 999.0781 | 180.1055 | 5.55  | 0.000 | 646.0779 | 1352.078 |
|        | 2037 | 1003.333 | 250.5916 | 4.00  | 0.000 | 512.1829 | 1494.484 |
|        | 2038 | 1556.669 | 451.6914 | 3.45  | 0.001 | 671.3697 | 2441.967 |
|        | 2039 | 2590.674 | 662.6979 | 3.91  | 0.000 | 1291.81 | 3889.538 |
|        | 2040 | 2225.712 | 486.9917 | 4.57  | 0.000 | 1271.225 | 3180.198 |
| 2037   |      |           |      |        |                      |
|        | 2032 | -114.582 | 160.0972 | -0.72 | 0.474 | -428.3668 | 199.2028 |
|        | 2033 | -127.9856| 183.3941 | -0.70 | 0.485 | -487.4315 | 231.4603 |
|        | 2034 | 33.40901 | 168.0312 | 0.20  | 0.842 | -295.9262 | 362.7442 |
|        | 2035 | 130.3495 | 166.226 | 0.78  | 0.433 | -195.4477 | 456.1468 |
|        | 2036 | -10.48288| 167.5059 | -0.06 | 0.950 | -338.7884 | 317.8226 |
|        | 2037 | 1717.016 | 268.5592 | 6.39  | 0.000 | 1190.65 | 2243.383 |
|        | 2038 | 2086.798 | 278.0215 | 7.51  | 0.000 | 1541.886 | 2631.71 |
|        | 2039 | 2473.611 | 268.186 | 9.22  | 0.000 | 1947.976 | 2999.246 |
|        | 2040 | 2835.117 | 378.6699 | 7.49  | 0.000 | 2092.938 | 3577.296 |

Note: ATET computed using covariates.

Notice the note below the command. A variable with the name _did_cohort has been generated. Using the group variable and the observation-level treatment, xthdidregress generated treatment-time cohorts. The new variable creates treatment groups based on the time when a group was first treated. For instance, if a Boxer and a Rottweiler are featured in movies in 2034, they are grouped in the 2034 cohort. The variable also contains a category for a control group. In this case, the control group is formed by the breeds that are not featured in a movie. Cohorts are an important input for estimation and for postestimation commands. You do not need to adhere to the default name, _did_cohort, and may provide your own name using the cohortvar() option.

Next appears a table that gives you a sense of the treatment groups and time. You see the time variable, year, and its range, 2031 to 2040. Then we see what defines a treated or a control group.
The table after provides group-level information about the cohort-time groups. The first row tells you the number of cohorts. Following the number of cohorts is a tabulation showing how many observations are in each cohort. For instance, 1,190 observations are never treated in our data. The table gives you a sense of the amount of information available in each cohort and might hint at the variability of cohort-level estimates.

The next table presents the ATET estimates. The first panel shows the ATETs for the 2034 cohort. We first have the 2032 ATET of $-255$, with a confidence interval that includes 0. This is as expected; before treatment, the effect should be 0. We should interpret the ATET to mean that among the breeds about which a movie was made in 2034, the expected AKC registrations in 2032 are 255 lower than if the breed had never been in a movie. At treatment onset, in 2034, we observe a treatment effect for the number of registrations of 701, and on the last year of our data, for the 2034 cohort, we see a treatment effect of 3,111 registrations. We interpret the results for the remaining cohorts similarly.

Example 2: Visualizing estimation results

In the example above, in the output table, we had three cohorts and nine time periods. There is a lot of information to process, and it can get even more daunting if we had more cohorts and time periods. To better visualize the results, we can use `estat atetplot`:

```
. estat atetplot
```

![Figure 1. ATETs by cohort over time](image)

The graph shows the pretreatment and the posttreatment ATETs for each cohort and their pointwise confidence intervals. ATETs increase over time after treatment and are flat before treatment.
Example 3: Less heterogeneity; aggregating and summarizing treatment effects

So far, we have allowed treatment effects to change over cohort and over time. But we might want to obtain only one treatment effect for each cohort, abstracting away from time variation within cohorts. You would get this using the postestimation command `estat aggregation`.

```
estat aggregation, cohort
ATET over cohort
(Std. err. adjusted for 141 clusters in breed)

| Cohort | ATET  | std. err. | z    | P>|z| | [95% conf. interval] |
|--------|-------|-----------|------|------|---------------------|
| 2034   | 1921.33 | 187.2787  | 10.26 | 0.000 | 1554.271 - 2288.389 |
| 2036   | 1675.093 | 130.4929  | 12.84 | 0.000 | 1419.332 - 1930.855 |
| 2037   | 2278.136 | 166.5283  | 13.68 | 0.000 | 1951.746 - 2604.525 |
```

We now have 3 treatment effects to analyze instead of 27. For the 2034 cohort, we have a treatment effect of 1,921 registrations for the periods after treatment. For the 2036 cohort, the effect is 1,675 registrations, and for the 2037 cohort, it is 2,278. We cannot see how the treatment evolves over time for each cohort, but we have a sense of the average effect over time for each of them.

We could instead want to see the treatment effect at each point in time, abstracting from cohort-level variation.

```
estat aggregation, time
ATET over time
(Std. err. adjusted for 141 clusters in breed)

| Time   | ATET  | std. err. | z    | P>|z| | [95% conf. interval] |
|--------|-------|-----------|------|------|---------------------|
| 2034   | 701.1318 | 127.0935  | 5.52 | 0.000 | 452.0331 - 950.2304 |
| 2035   | 1099.044 | 282.0704  | 3.90 | 0.000 | 546.196 - 1651.892 |
| 2036   | 1209.68 | 170.2043  | 7.11 | 0.000 | 876.0858 - 1543.275 |
| 2037   | 1672.655 | 202.1854  | 8.27 | 0.000 | 1276.379 - 2068.932 |
| 2038   | 2084.658 | 214.5072  | 9.72 | 0.000 | 1664.232 - 2505.084 |
| 2039   | 2528.847 | 225.8763  | 11.20 | 0.000 | 2086.138 - 2971.557 |
| 2040   | 2802.171 | 291.8412  | 9.60 | 0.000 | 2230.173 - 3374.17  |
We see the treatment effects for each one of the posttreatment periods. As before, we have the option to look at the effects graphically. We just need to use the `graph` option.

```
estat aggregation, time graph
```

![Figure 2. ATETs over time](image)

### Example 4: Dynamic treatment effects

We could also ask what the evolution of the treatment effect is after treatment. For instance, we might want to know what happens one period after the onset of treatment, two periods after treatment, and so forth. It might be the case that treatment effects vanish over time or even change patterns. We might also want to see whether, before treatment, we observe a treatment effect or a pattern that might suggest that there is anticipation of treatment. `estat aggregation` allows us to answer these questions by using the `dynamic` option.

```
estat aggregation, dynamic graph
```

```
| Exposure | ATET | Robust std. err. | z     | P>|z|  | [95% conf. interval] |
|----------|------|------------------|-------|------|----------------------|
| -5       | -114.582 | 160.0972      | -0.72 | 0.474 | -428.3668 199.2028 |
| -4       | -70.65034 | 156.3185    | -0.45 | 0.651 | -377.029 235.7283 |
| -3       | -9117242 | 153.0999    | -0.01 | 0.995 | -300.982 299.1585 |
| -2       | 12.79653 | 144.8216   | 0.09  | 0.930 | -271.0486 296.6417 |
| -1       | 30.71473 | 132.8508  | 0.23  | 0.817 | -229.668 291.0975 |
| 0        | 1434.409 | 206.3277  | 6.95  | 0.000 | 1030.014 1838.804 |
| 1        | 1759.461 | 221.903   | 7.85  | 0.000 | 1320.385 2198.538 |
| 2        | 2147.486 | 221.903   | 9.68  | 0.000 | 1712.564 2582.408 |
| 3        | 2651.452 | 284.8928  | 9.31  | 0.000 | 2093.073 3209.832 |
| 4        | 2366.805 | 267.4253  | 8.85  | 0.000 | 1842.661 2890.949 |
| 5        | 2689.615 | 504.3324  | 5.33  | 0.000 | 1701.142 3678.088 |
| 6        | 3110.97 | 568.916   | 5.47  | 0.000 | 1995.915 4226.025 |
```

Note: Exposure is the number of periods since the first treatment time.
From the output table, we see that 5 periods before treatment onset, there is no significant treatment effect or, for that matter, no significant effect at any time of exposure before treatment. This suggests there is no anticipation of treatment. At treatment onset, 0, there is a positive effect, which seems to increase as exposure time to treatment augments.

Example 5: TWFE estimation

The literature on heterogeneous DID started by pointing out the problems that arise when one assumes erroneously that the treatment effects are homogeneous. It suggested that TWFE estimation was inadequate. Wooldridge (2021) suggests that fixed-effects estimation can be used if we extend it to include interactions between treatment-time cohorts and time.

Another important insight of Wooldridge (2021) is that you can use pooled ordinary least squares and add panel-level averages of covariates and obtain the same point estimates as one would get with fixed-effects estimation in the context of DID estimation. This is an extension of the intuition by Mundlak (1978). xthdidregress and hdidregress fit pooled ordinary least-squares models using these ideas. Below, we present the results we obtain using the twfe estimator.
. xthdidregress twfe (registered best) (movie), group(breed)

note: variable _did_cohort, containing cohort indicators formed by treatment
variable movie and group variable breed, was added to the dataset.

Computing ATETs using margins ...

Treatment and time information

Time variable: year
Time interval: 2031 to 2040
Control: _did_cohort = 0
Treatment: _did_cohort > 0

| _did_cohort | ATET | std. err. | t    | P>|t|   | [95% conf. interval] |
|-------------|------|-----------|------|-------|----------------------|
| 2034        |      |           |      |       |                      |
| 2034 year   | 469.2023 | 148.8998   | 3.15 | 0.002 | 174.8195 763.5852   |
| 2035        | 823.8532 | 211.7491   | 3.89 | 0.000 | 405.2138 1242.493   |
| 2036        | 1108.669 | 179.8404   | 6.16 | 0.000 | 753.1144 1464.223   |
| 2037        | 1752.287 | 283.487    | 6.18 | 0.000 | 1191.818 2312.756   |
| 2038        | 2216.617 | 173.4446   | 12.78| 0.000 | 1873.708 2559.526   |
| 2039        | 2433.608 | 521.4074   | 4.67 | 0.000 | 1402.757 3464.458   |
| 2040        | 2854.963 | 494.1892   | 5.78 | 0.000 | 1877.924 3832.001   |

2036 year

| 2036        |      |           |      |       |                      |
| 2036        | 1336.121 | 96.75296  | 13.81| 0.000 | 1144.835 1527.406   |
| 2037        | 1343.004 | 297.7487  | 4.51 | 0.000 | 754.3383 1931.669   |
| 2038        | 1896.339 | 399.2574  | 4.75 | 0.000 | 1106.985 2685.692   |
| 2039        | 2930.344 | 591.0712  | 4.96 | 0.000 | 1761.765 4098.924   |
| 2040        | 2565.382 | 578.9303  | 4.43 | 0.000 | 1420.806 3709.958   |

2037 year

| 2037        |      |           |      |       |                      |
| 2037        | 1750.126 | 216.4288  | 8.09 | 0.000 | 1322.234 2178.017   |
| 2038        | 2119.908 | 217.1685  | 9.76 | 0.000 | 1690.554 2549.262   |
| 2039        | 2506.72  | 303.9392  | 8.25 | 0.000 | 1905.816 3107.624   |
| 2040        | 2868.227 | 313.1679  | 9.16 | 0.000 | 2249.077 3487.377   |

Note: ATET computed using covariates.

The output is almost the same as the one for the ra estimator in example 1. There are a couple of noteworthy differences. First, the estimator fits an extended TWFE regression and then uses margins
to compute the treatment effects. The command tells us Computing ATETs using margins when it begins this more computationally intensive calculation.

Second, the ATET parameters are shown for each cohort only at the time of treatment exposure and for the periods thereafter but not for the pretreatment periods. As discussed in Wooldridge (2021), these are the parameters identified using the parallel-trends assumption he derives.

As we did before, we could use estat aggregation to explore different ways of looking at our treatment effects and estat atetplot to visualize the estimated ATETs.

Example 6: Reducing model complexity

When we fit the ra model, we had to estimate ATET parameters for each cohort over time. The complexity of the model grows with the number of cohorts and the number of time periods. As is described in Methods and formulas, the ra estimator uses a different subset of the data to obtain each parameter. To get a reliable estimator of each parameter, you need sufficient data for each subsample. Sometimes, there are few observations for a given cohort in a given set of time periods.

We can ameliorate this problem by reducing the amount of heterogeneity we assume. For the twfe estimator, the complexity of the model comes from the interactions between the observation-level treatment with cohort and time and the interactions between the observation-level treatment, cohort, time, and covariates. This allows us to decide which interactions to include in our model. We could, for instance, allow for heterogeneity at the cohort level instead of at the cohort and time level. We use the hettype() option with the argument cohort() to do this:
. xthdidregress twfe (registered best) (movie), group(breed) hettype(cohort)
  note: variable _did_cohort, containing cohort indicators formed by treatment
variable movie and group variable breed, was added to the dataset.

Computing ATETs using margins ...

Treatment and time information

Time variable: year
Time interval: 2031 to 2040
Control: _did_cohort = 0
Treatment: _did_cohort > 0

<table>
<thead>
<tr>
<th></th>
<th>_did_cohort</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cohorts</td>
<td>4</td>
</tr>
<tr>
<td>Number of obs</td>
<td></td>
</tr>
<tr>
<td>Never treated</td>
<td>1190</td>
</tr>
<tr>
<td>2034</td>
<td>40</td>
</tr>
<tr>
<td>2036</td>
<td>30</td>
</tr>
<tr>
<td>2037</td>
<td>150</td>
</tr>
</tbody>
</table>

Heterogeneous-treatment-effects regression

Number of obs = 1,410
Number of panels = 141

Estimator: Two-way fixed effects
Panel variable: breed
Treatment level: breed
Control group: Never treated
Heterogeneity: Cohort

(Std. err. adjusted for 141 clusters in breed)

| Cohort | ATET     | Robust std. err. | t       | P>|t| | 95% conf. interval |
|--------|----------|------------------|---------|-----|-------------------|
| 2034   | 1662.492 | 108.002          | 15.39   | 0.000 | 1448.966 1876.017 |
| 2036   | 1978.645 | 54.21043         | 36.50   | 0.000 | 1871.468 2085.822 |
| 2037   | 2276.223 | 70.63244         | 32.23   | 0.000 | 2136.579 2415.867 |

Note: ATET computed using covariates.

You fit a regression model with fewer terms and obtain treatment effects only at the cohort level. You could also have the treatment effect change over time but not over cohort by typing cohort(time).

For the estimators proposed by Callaway and Sant’Anna (2021), heterogeneity is built in, so we need to estimate all the ATET parameters.
Stored results

`xthdidregress` stores the following in `e()`:

Scalars
- `e(N)` number of observations
- `e(N_clust)` number of clusters
- `e(N_panels)` number of panels
- `e(tmin)` first time period
- `e(tmax)` last time period
- `e(rank)` rank of `e(V)`

Macros
- `e(cmd)` `xthdidregress`
- `e(cmdline)` command as typed
- `e(clustvar)` name of cluster variable
- `e(control_group)` control group
- `e(het_type)` heterogeneity type for `twfe` estimator
- `e(cohortvar)` name of cohort variable
- `e(ovar)` name of outcome variable
- `e(wtype)` weight type
- `e(wexp)` weight expression
- `e(marginsnotok)` predictions disallowed by `margins`
- `e(ivar)` panel identifier variable
- `e(timevar)` time variable
- `e(treatname)` name of treatment variable
- `e(estat_cmd)` program used to implement `estat`
- `e(vce)` `vcetype` specified in `vce()`
- `e(vcetype)` title used to label Std. err.
- `e(method)` estimator method
- `e(properties) b V`

Matrices
- `e(b)` coefficient vector
- `e(V)` variance–covariance matrix of the estimators
- `e(cohort_count)` matrix with cohort count information

Functions
- `e(sample)` marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices
- `r(table)` matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

Methods and formulas are presented under the following headings:

- **The model**
- **The RA, IPW, and AIPW estimators**
- **Panel data**
- **The TWFE estimator**
The model

\texttt{xthdidregress} estimates the treatment-effects parameters using the DID approach 1) with multiple periods, 2) with different timing when a unit is first treated, and 3) when the treatment effects can be heterogeneous across time and cohort. In particular, \texttt{xthdidregress} provides the RA, IPW, and AIPW estimators, outlined in Callaway and Sant’Anna (2021), and the TWFE estimator, outlined in Wooldridge (2021).

We observe data \{y_{it}, x_{it}, d_{it}, z_{it}\} for unit \(i\) at time \(t\) with unit \(i, i = 1, \ldots, N\), and time \(t, t = 1, \ldots, T\), where

1. \(y_{it}\) is the observed outcome;
2. \(x_{it}\) are covariates for the outcome model;
3. \(d_{it}\) is an indicator that equals one if an observation is treated or zero otherwise; and
4. \(z_{it}\) are the covariates for the treatment-assignment model.

Denote \(y_{it}(g)\) as the potential outcome for unit \(i\) at time \(t\) if it was first treated at time \(g\). Denote \(y_{it}(0)\) as the potential outcome for unit \(i\) and \(t\) if it is never treated. All the units can be grouped into cohorts, and each cohort starts the treatment simultaneously. For example, cohort \(g\) means all the units start treatment at time \(g\). Let \(G_{ig}\) be an indicator that equals one if unit \(i\) is first treated at time \(g\). Then the units in cohort \(g\) can be denoted by \(G_{ig} = 1\). When a unit \(i\) is never treated, we denote \(G_{i0} = 1\). Thus, cohort 0 indicates all the units that are never treated. We assume that once a unit is treated, it will remain treated.

To reveal how the heterogeneous treatment effects evolve across cohorts and time, we are interested in estimating the ATET for each combination of cohort and time. Let \(\theta(g, t)\) be the ATET for cohort \(g\) at time \(t\), which is defined as

\[ \theta(g, t) = E\{y_{t}(g) - y_{t}(0)|G_{g} = 1\} \]  

where \(G_{g}\) equals 1 if a unit belongs to cohort \(g\). All the four estimators provided in \texttt{xthdidregress} and \texttt{hdidregress} estimate \(\theta(g, t)\) in equation \(\text{(ATET)}\). We cannot directly estimate \(\theta(g, t)\) using equation \(\text{(ATET)}\) because the potential outcomes \(y_{t}(g)\) and \(y_{t}(0)\) are not observable.

Next, we will describe the RA, IPW, and AIPW estimators.

The RA, IPW, and AIPW estimators

To define treatment effect, we need a control group. There are two ways to define the control group. One way is to use the units that are never treated as the control group. Let \(C^{\text{NEV}}\) be an indicator that equals one if a unit belongs to the never-treated group. In particular, \(C^{\text{NEV}} = G_{0}\). Another way is to use the units not in cohort \(g\) and not yet treated at time \(t\) as the control group. Let \(C^{\text{NY}}_{g,t}\) be an indicator that equals one if a unit belongs to the not-yet-treated group by time \(t\). In particular, \(C^{\text{NY}}_{g,t} = (1 - G_{g})(1 - d_{t})\). To simplify, we indicate control, in both cases, as \(C^{*}_{g,t}\).

The definitions of the RA, IPW, and AIPW estimators depend on the definition of \(C^{*}_{g,t}\), which can either be \(C^{\text{NEV}}\) or \(C^{\text{NY}}_{g,t}\). However, regardless of the control group’s choice, the estimators’ definitions can always be written using the general notation \(C^{*}_{g,t}\).
Panel data

The estimands for RA, IPW, and AIPW are defined as follows.

The RA estimand is

$$\theta_{RA}(g, t) = \mathbb{E} \left[ \frac{G_g}{E(G_g)} \{y_t - y_{g-1} - m_{g,t}(x)\} \right]$$

(RA)

where $m_{g,t}(x) = \mathbb{E}(y_t - y_{g-1}|x, C_{g,t}^* = 1)$.

The IPW estimand is

$$\theta_{IPW}(g, t) = \mathbb{E} \left[ \frac{G_g}{E(G_g)} - \frac{p_{g,t}(z)C_{g,t}^*}{1 - p_{g,t}(z)} \right] \left( y_t - y_{g-1} \right)$$

(IPW)

where

$$p_{g,t}(z) = Pr(G_g = 1|z, G_g + C_{g,t}^* = 1)$$

(Pz)

The AIPW estimand is

$$\theta_{AIPW}(g, t) = \mathbb{E} \left[ \frac{G_g}{E(G_g)} - \frac{p_{g,t}(z)C_{g,t}^*}{1 - p_{g,t}(z)} \right] \left( y_t - y_{g-1} - m_{g,t}(x) \right)$$

(AIPW)

Callaway and Sant’Anna (2021) show that the estimand for RA, IPW, and AIPW is the same as $\theta(g, t)$ in equation (ATET). In other words,

$$\theta(g, t) = \theta_{RA}(g, t) = \theta_{IPW}(g, t) = \theta_{AIPW}(g, t)$$

However, unlike the nonestimable $\theta(g, t)$ in equation (ATET), the estimands in equation (RA), (IPW), and (AIPW) are estimable because only the observed random variables such as $x$, $y$, $z$, $G_g$, and $C_{g,t}^*$ are used.

The identification of the estimators sheds light on how to estimate $\theta(g, t)$. The estimator can be generally divided into three steps:

1. Restrict the sample to time $t$ and $t_0$, and keep only the units in cohort $g$ or in control group $C_{g,t}^*$, where $t_0 = g - 1$ if $t \geq g$ or $t_0 = t - 1$ if $t < g$. Only the paired observations are used. In other words, unit $i$ will be used if it is observed at both times $t$ and $t_0$.

2. Use a parametric model to estimate the nuisance function for $p_{g,t}(z)$ and $m_{g,t}(x)$. In practice, we use a linear regression model to fit $m_{g,t}(x)$ and a logit regression model to fit $p_{g,t}(z)$. In particular, do the following:

   a. Run a linear regression of $y_t - y_{t_0}$ on $x$ when $C_{g,t}^* = 1$.
   b. Denote the linear predictor as $\hat{m}_{g,t}(x)$.
   c. Run a logit regression of $G_g$ on $z$.
   d. Denote the prediction of the probability of a positive outcome as $\hat{p}_{g,t}(z)$.

3. Plug in the nuisance function estimates $\hat{m}_{g,t}(x)$, $\hat{p}_{g,t}(z)$, or both into the estimating equation in equation (RA), (IPW), or (AIPW). Notice that the expectation operator $\mathbb{E}(\cdot)$ is replaced by the sample average.
The variance–covariance matrix for $\hat{\theta}(g,t)$ for each cohort $g$ and time $t$ is computed using the influence function approach proposed in theorem 2 in Callaway and Sant’Anna (2021). The influence function approach is numerically equivalent to the generalized method-of-moments approach. However, it is much faster because it avoids computing the covariance matrix for the parameters in the nuisance functions $m_{g,t}(x)$ and $p_{g,t}(z)$. For more discussions on influence functions, see Hampel et al. (1986), Newey and McFadden (1994), and Jann (2020).

The TWFE estimator

A TWFE estimator fits
\[
y_{it} = \alpha_i + \gamma t + x_{it}\beta + d_{it}\tau + \varepsilon_{it}
\]

Wooldridge (2021) extends this model to incorporate interactions between the treatment, $d_{it}$, cohort, $G_{ig}$, posttreatment periods, and covariates. We define indicators for posttreatment period as $f_s$ with $s$ going from $q$ to $T$, where $q$ is the first time period we observe treatment. For instance, $f_q$ equals 1 if we are in time period $q$ and 0 otherwise. To simplify this notation, we show the model without covariates. The extended fixed-effects model is given by
\[
y_{it} = \eta + \sum_{g=q}^{T} G_{ig}\alpha_g + \sum_{s=q}^{T} f_s\gamma_s + \sum_{g=q}^{T} \sum_{s=g}^{T} d_{it}G_{ig}f_s\tau_{gs} + \varepsilon_{it} \tag{TWFE}
\]

We can fit equation (TWFE) using pooled ordinary least squares or a within estimator. We are going to use the estimator proposed by Mundlak (1978). This gives the same point estimates as using the within estimator of xtreg ..., fe for the parameters in equation (TWFE) but has different degrees of freedom because of the additional terms added by the Mundlak approach. Unlike within estimation, the Mundlak approach works for both repeated cross-sectional data and panel data. Also, it has good properties to obtain partial effect under various data-generating processes, as pointed out in Wooldridge (2019).

Above, the $\tau_{gs}$ are the cohort-time treatment effects. When we have covariates, we interact them with all the relevant variables in the model. To get the treatment effects in this case, we need to control for the variation in the covariates. We can obtain both effects using margins.

In particular, we are going to type
\[
.margins, dydx(d) at(year=q ... year=T) over(cohort) vce(unconditional)
\]
where $d$ is the treatment indicator, $year$ indicates treatment times at which treatment will be evaluated using at(), and cohort is the treatment-time cohorts. We use vce(unconditional) to account for the variation in the covariates.

With the hettype() option, we reduce the complexity of (TWFE). In particular, if we ask for hettype(time), we have
\[
y_{it} = \eta + \sum_{g=q}^{T} G_{ig}\alpha_g + \sum_{s=q}^{T} f_s\gamma_s + \sum_{s=q}^{T} d_{it}f_s\tau_s + \varepsilon_{it}
\]
Now treatment varies over time but not over cohort, that is, $\tau_s$. If we use the hettype(cohort) option, we have
\[ y_{it} = \eta + \sum_{g=q}^{T} G_{ig} \alpha_{g} + \sum_{s=q}^{T} f_{s} \gamma_{s} + \sum_{g=q}^{T} d_{it} G_{ig} \tau_{g} + \varepsilon_{it} \]

Now treatment varies over cohort but not over time, that is, \( \tau_{g} \).

When the controlgroup(notyet) option is specified, the \( G_{ig} \) indicator excludes the last treated cohort. As discussed in Wooldridge (2021), when every group is eventually treated, we cannot identify the treatment effect for this cohort. It is therefore sensible to use the last treated cohort as a control group. When some of the units in our sample are never treated, we can always identify all cohorts, and the \texttt{twfe} estimator will always revert to using controlgroup(never).

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**References**


Also see

[CAUSAL]  **hdidregress postestimation** — Postestimation tools for hdidregress and xthdidregress

[CAUSAL]  **hdidregress** — Heterogeneous difference in differences

[CAUSAL]  **DID intro** — Introduction to difference-in-differences estimation

[CAUSAL]  **didregress** — Difference-in-differences estimation

[CAUSAL]  **teffects intro** — Introduction to treatment effects for observational data

[CAUSAL]  **teffects intro advanced** — Advanced introduction to treatment effects for observational data

[U]  **20 Estimation and postestimation commands**
hديدregress and xثdidgregress allow the same postestimation commands; see [CAUSAL] hديدregress postestimation.
Glossary

ADTE. See average direct treatment effect.

ADTET. See average direct treatment effect with respect to the treated.

AIPW estimator. See augmented inverse-probability-weighted estimator.

AITE. See average indirect treatment effect.

AITEC. See average indirect treatment effect with respect to controls.

analysis time. Analysis time is like time, except that 0 has a special meaning: $t = 0$ is the time of onset of risk, the time when failure first became possible.

Analysis time is usually not what is recorded in a dataset. A dataset of patients might record calendar time. Calendar time must then be mapped to analysis time.

The letter $t$ is reserved for time in analysis-time units. The term time is used for time measured in other units.

The origin is the time corresponding to $t = 0$, which can vary subject to subject. Thus $t = \text{time} - \text{origin}$.

ATE. See average treatment effect.

ATET. See average treatment effect on the treated.

augmented inverse-probability-weighted estimator. An augmented inverse-probability-weighted (AIPW) estimator is an inverse-probability-weighted estimator that includes an augmentation term that corrects the estimator when the treatment model is misspecified. When the treatment is correctly specified, the augmentation term vanishes as the sample size becomes large. An AIPW estimator uses both an outcome model and a treatment model and is a doubly robust estimator.

average direct treatment effect. The average direct treatment effect (ADTE) is the average direct effect of the treatment on the outcome when the mediator is held at its value associated with being untreated.

For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The average direct treatment effect measures the average difference in potential outcomes for well-being when everyone in the population exercised but with endorphins set to levels that would be observed if no one exercised compared with when no one in the population exercised.

average direct treatment effect with respect to the treated. The average direct treatment effect with respect to the treated (ADTET) is the average direct treatment effect when the mediator is held at its value associated with being treated.

For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The average direct treatment effect with respect to the treated measures the average difference in the potential outcomes for well-being when everyone in the population exercised compared with when no one exercised but with endorphins set to levels that would be observed if everyone exercised.

average indirect treatment effect. The average indirect treatment effect (AITE) is the average indirect effect through a mediator.
For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The average indirect treatment effect measures the average difference in the potential outcomes for well-being when everyone in the population exercised compared with when everyone exercised but with endorphins set to levels that would be observed if no one exercised.

**average indirect treatment effect with respect to controls.** The average indirect treatment effect with respect to controls (AITEC) is the average indirect effect of a mediator under the control condition.

For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The average indirect treatment effect with respect to controls measures the average difference in the potential outcomes for well-being when no one in the population exercised but with endorphins set to levels that would be observed if everyone exercised compared with the case where no one in the population exercised.

**average treatment effect.** The average treatment effect is the average effect of the treatment among all individuals in a population.

**average treatment effect on the treated.** The average treatment effect on the treated is the average effect of the treatment among those individuals who actually get the treatment.

**causal mediation analysis.** Causal mediation analysis is a method that disentangles the causal mechanisms that operate between a cause and an effect by decomposing causal effects of a treatment, or exposure, on an outcome variable into direct and indirect effects. The indirect effects are due to the treatment affecting the outcome via intermediary variables, also known as mediators. Direct effects are causal effects that are due to mechanisms other than those captured by the mediators.

censored, left-censored, and right-censored. An observation is left-censored when the exact time of failure is not known; it is merely known that the failure occurred before \( t_l \). Suppose that the event of interest is becoming employed. If a subject is already employed when first interviewed, his outcome is left-censored.

An observation is right-censored when the time of failure is not known; it is merely known that the failure occurred after \( t_r \). If a patient survives until the end of a study, the patient’s time of death is right-censored.

In common usage, censored without a modifier means right-censored.

Also see truncation, left-truncation, and right-truncation.

**CI assumption.** See conditional-independence assumption.

**conditional mean.** The conditional mean expresses the average of one variable as a function of some other variables. More formally, the mean of \( y \) conditional on \( x \) is the mean of \( y \) for given values of \( x \); in other words, it is \( E(y|x) \).

A conditional mean is also known as a regression or as a conditional expectation.

**conditional-independence assumption.** The conditional-independence assumption requires that the common variables that affect treatment assignment and treatment-specific outcomes be observable. The dependence between treatment assignment and treatment-specific outcomes can be removed by conditioning on these observable variables.

This assumption is also known as a selection-on-observables assumption because its central tenet is the observability of the common variables that generate the dependence.

**counterfactual.** A counterfactual is an outcome a subject would have obtained had that subject received a different level of treatment. In the binary-treatment case, the counterfactual outcome
for a person who received treatment is the outcome that person would have obtained had the person instead not received treatment; similarly, the counterfactual outcome for a person who did not receive treatment is the outcome that person would have obtained had the person received treatment.

Also see potential outcome.

doubly robust estimator. A doubly robust estimator only needs one of two auxiliary models to be correctly specified to estimate a parameter of interest.

Doubly robust estimators for treatment effects are consistent when either the outcome model or the treatment model is correctly specified.

EE estimator. See estimating-equation estimator.

estimating-equation estimator. An estimating-equation (EE) estimator calculates parameters estimates by solving a system of equations. Each equation in this system is the sample average of a function that has mean zero.

These estimators are also known as M estimators or Z estimators in the statistics literature and as generalized method of moments (GMM) estimators in the econometrics literature.

failure event. Survival analysis is really time-to-failure analysis, and the failure event is the event under analysis. The failure event can be death, heart attack, myopia, or finding employment. Many authors—including Stata—write as if the failure event can occur only once per subject, but when we do, we are being sloppy. Survival analysis encompasses repeated failures, and all of Stata’s survival analysis features can be used with repeated-failure data.

hazard, cumulative hazard, and hazard ratio. The hazard or hazard rate at time $t$, $h(t)$, is the instantaneous rate of failure at time $t$ conditional on survival until time $t$. Hazard rates can exceed 1. Say that the hazard rate were 3. If an individual faced a constant hazard of 3 over a unit interval and if the failure event could be repeated, the individual would be expected to experience three failures during the time span.

The cumulative hazard, $H(t)$, is the integral of the hazard function $h(t)$, from 0 (the onset of risk) to $t$. It is the total number of failures that would be expected to occur up until time $t$, if the failure event could be repeated. The relationship between the cumulative hazard function, $H(t)$, and the survivor function, $S(t)$, is

$$S(t) = \exp\{-H(t)\}$$

$$H(t) = -\ln\{S(t)\}$$

The hazard ratio is the ratio of the hazard function evaluated at two different values of the covariates: $h(t \mid x)/h(t \mid x_0)$. The hazard ratio is often called the relative hazard, especially when $h(t \mid x_0)$ is the baseline hazard function.

i.i.d. sampling assumption. See independent and identically distributed sampling assumption.

independent and identically distributed sampling assumption. The independent and identically distributed (i.i.d.) sampling assumption specifies that each observation is unrelated to (independent of) all the other observations and that each observation is a draw from the same (identical) distribution.

individual-level treatment effect. An individual-level treatment effect is the difference in an individual’s outcome that would occur because this individual is given one treatment instead of another. In other words, an individual-level treatment effect is the difference between two potential outcomes for an individual.
For example, the blood pressure an individual would obtain after taking a pill minus the blood pressure an individual would obtain had that person not taken the pill is the individual-level treatment effect of the pill on blood pressure.

**inverse-probability-weighted estimators.** Inverse-probability-weighted (IPW) estimators use weighted averages of the observed outcome variable to estimate the potential-outcome means. The weights are the reciprocals of the treatment probabilities estimated by a treatment model.

**inverse-probability-weighted regression-adjustment estimators.** Inverse-probability-weighted regression-adjustment (IPWRA) estimators use the reciprocals of the estimated treatment probability as weights to estimate missing-data-corrected regression coefficients that are subsequently used to compute the potential-outcome means.

**IPW estimators.** See *inverse-probability-weighted estimators.*

**IPWRA estimators.** See *inverse-probability-weighted regression-adjustment estimators.*

**left-censored.** See *censored, left-censored, and right-censored.*

**left-truncation.** See *truncation, left-truncation, and right-truncation.*

**matching estimator.** An estimator that compares differences between the outcomes of similar—that is, matched—individuals. Each individual that receives a treatment is matched to a similar individual that does not get the treatment, and the difference in their outcomes is used to estimate the individual-level treatment effect. Likewise, each individual that does not receive a treatment is matched to a similar individual that does get the treatment, and the difference in their outcomes is used to estimate the individual-level treatment effect.

**mediator.** A mediator is an intermediary variable on the causal pathway between a treatment, or exposure, and an outcome variable. For example, endorphin production may be a mediator in the relationship between exercise (the treatment) and well-being (the outcome). In this case, exercise affects endorphin production, which, in turn, affects well-being.

**multiple-record st data.** See *st data.*

**multivalued treatment effect.** A multivalued treatment refers to a treatment that has more than two values. For example, a person could have taken a 20 mg dose of a drug, a 40 mg dose of the drug, or not taken the drug at all.

**natural direct effect.** The natural direct effect (NDE) is the average direct effect of the treatment on the outcome when the mediator is held at its value associated with being untreated.

For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The natural direct effect measures the average difference in the potential outcomes for well-being when everyone in the population exercised but with endorphins set to levels that would be observed if no one exercised compared with when no one in the population exercised. In the causal mediation literature, the natural direct effect is also referred to as the pure natural direct effect; it is synonymous with the *average direct treatment effect.*

**natural indirect effect.** The natural indirect effect (NIE) is the average indirect effect through a mediator.

For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The natural indirect effect measures the average difference in the potential outcomes for well-being when everyone in the population exercised compared with when everyone exercised but with endorphins set to levels that would be observed if no one exercised. In the causal mediation literature, the natural indirect effect is also referred to as the total natural indirect effect; it is synonymous with the *average indirect treatment effect.*

**NDE.** See *natural direct effect.*
nearest-neighbor matching. Nearest-neighbor matching uses the distance between observed variables to find similar individuals.

NIE. See natural indirect effect.

observational data. In observational data, treatment assignment is not controlled by those who collected the data; thus some common variables affect treatment assignment and treatment-specific outcomes.

outcome model. An outcome model is a model used to predict the outcome as a function of covariates and parameters.

overlap assumption. The overlap assumption requires that each individual have a positive probability of each possible treatment level.

PNIE. See pure natural indirect effect.

POMs. See potential-outcome means.

potential outcome. The potential outcome is the outcome an individual would obtain if given a specific treatment.

For example, an individual has one potential blood pressure after taking a pill and another potential blood pressure had that person not taken the pill.

potential-outcome means. The potential-outcome means refers to the means of the potential outcomes for a specific treatment level.

The mean blood pressure if everyone takes a pill and the mean blood pressure if no one takes a pill are two examples.

The average treatment effect is the difference between potential-outcome mean for the treated and the potential-outcome mean for the not treated.

propensity score. The propensity score is the probability that an individual receives a treatment.

propensity-score matching. Propensity-score matching uses the distance between estimated propensity scores to find similar individuals.

pure natural indirect effect. The pure natural indirect effect (PNIE) is the average indirect effect of a mediator under the control condition.

For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The pure natural indirect effect measures the average difference in the potential outcomes for well-being when no one in the population exercised but with endorphins set to levels that would be observed if everyone exercised compared with the case when no one in the population exercised. The pure natural indirect effect is synonymous with the average indirect treatment effect with respect to controls.

regression-adjustment estimators. Regression-adjustment estimators use means of predicted outcomes for each treatment level to estimate each potential-outcome mean.

right-censored. See censored, left-censored, and right-censored.

right-truncation. See truncation, left-truncation, and right-truncation.

selection-on-observables. See conditional-independence assumption.

shape parameter. A shape parameter governs the shape of a probability distribution. One example is the parameter $p$ of the Weibull model.

single-record st data. See st data.
smooth treatment-effects estimator. A smooth treatment-effects estimator is a smooth function of the data so that standard methods approximate the distribution of the estimator. The RA, IPW, AIPW, and IPWRA estimators are all smooth treatment-effects estimators while the nearest-neighbor matching estimator and the propensity-score matching estimator are not.

st data. st stands for survival time. In survival-time data, each observation represents a span of survival, recorded in variables $t_0$ and $t$. For instance, if in an observation $t_0$ were 3 and $t$ were 5, the span would be $(t_0, t]$, meaning from just after $t_0$ up to and including $t$.

Sometimes variable $t_0$ is not recorded; $t_0$ is then assumed to be 0. In such a dataset, an observation that had $t = 5$ would record the span $(0, 5]$.

Each observation also includes a variable $d$, called the failure variable, which contains 0 or nonzero (typically, 1). The failure variable records what happened at the end of the span: 0, the subject was still alive (had not yet failed) or 1, the subject died (failed).

Sometimes variable $d$ is not recorded; $d$ is then assumed to be 1. In such a dataset, all time-span observations would be assumed to end in failure.

Finally, each observation in an st dataset can record the entire history of a subject or each can record a part of the history. In the latter case, groups of observations record the full history. One observation might record the period $(0, 5]$ and the next, $(5, 8]$. In such cases, there is a variable ID that records the subject for which the observation records a time span. Such data are called multiple-record st data. When each observation records the entire history of a subject, the data are called single-record st data. In the single-record case, the ID variable is optional.

See [ST] stset.

survival-time data. See st data.

survivor function. Also known as the survivorship function and the survival function, the survivor function, $S(t)$, is 1) the probability of surviving beyond time $t$, or equivalently, 2) the probability that there is no failure event prior to $t$, 3) the proportion of the population surviving to time $t$, or equivalently, 4) the reverse cumulative distribution function of $T$, the time to the failure event: $S(t) = \Pr(T > t)$. Also see hazard, cumulative hazard, and hazard ratio.

TE. See total effect.

TNDE. See total natural direct effect.

total effect. The total effect (TE) is the average difference in outcomes that we expect when everyone receives the treatment versus when no one receives the treatment. In the causal mediation literature, the TE is also referred to as the total average treatment effect and the marginal total effect; it is synonymous with the average treatment effect.

total natural direct effect. The total natural direct effect (TNDE) is the average direct treatment effect when the mediator is held at its value associated with being treated.

For example, suppose that exercise has a direct effect on well-being as well as an indirect effect via the production of endorphins. The total natural direct effect measures the average difference in the potential outcomes for well-being when everyone in the population exercised compared with when no one exercised but with endorphins set to levels that would be observed if everyone exercised. The total natural direct effect is synonymous with the average direct treatment effect with respect to the treated.

treatment model. A treatment model is a model used to predict treatment-assignment probabilities as a function of covariates and parameters.

truncation, left-truncation, and right-truncation. In survival analysis, truncation occurs when subjects are observed only if their failure times fall within a certain observational period of a study. Censoring,
on the other hand, occurs when subjects are observed for the whole duration of a study, but the exact times of their failures are not known; it is known only that their failures occurred within a certain time span.

Left-truncation occurs when subjects come under observation only if their failure times exceed some time $t_l$. It is only because they did not fail before $t_l$ that we even knew about their existence. Left-truncation differs from left-censoring in that, in the censored case, we know that the subject failed before time $t_l$, but we just do not know exactly when.

Imagine a study of patient survival after surgery, where patients cannot enter the sample until they have had a post-surgical test. The patients’ survival times will be left-truncated. This is a “delayed entry” problem, one common type of left-truncation.

Right-truncation occurs when subjects come under observation only if their failure times do not exceed some time $t_r$. Right-truncated data typically occur in registries. For example, a cancer registry includes only subjects who developed a cancer by a certain time, and thus survival data from this registry will be right-truncated.

**unconfoundedness.** See *conditional-independence assumption*.

**weighted-regression-adjustment estimator.** Weighted-regression-adjustment estimators use means of predicted outcomes for each treatment level to estimate each potential-outcome mean. The weights are used to estimate censoring-adjusted regression coefficients.
Subject and author index

See the combined subject index and the combined author index in the Stata Index.