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**bayestest model** — Hypothesis testing using model posterior probabilities

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# **Description**

bayestest model computes posterior probabilities of Bayesian models fit using the bayesmh command or the bayes prefix. These posterior probabilities can be used to test hypotheses about model parameters. The command reports marginal likelihoods, prior probabilities, and posterior probabilities for all tested models.

### **Quick start**

Compute posterior probabilities of models corresponding to previously saved estimation results M1 and M2

bayestest model M1 M2

Same as above, but specify prior probabilities for models

bayestest model M1 M2, prior(0.3 0.7)

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# **Syntax**

bayestest model [namelist] [, options]

where *namelist* is a name, a list of names, \_all, or \*. A name may be ., meaning the current (active) estimates. \_all and \* mean the same thing.

options	Description
Main	
<pre>prior(numlist)</pre>	specify prior probabilities for tested models; default is all models are equally likely
* chains(_all   numlist) * sepchains	specify which chains to use for computation; default is chains(_all) compute results separately for each chain
Advanced	
<pre>marglmethod(method)</pre>	specify marginal-likelihood approximation method; default is to use Laplace-Metropolis approximation, lmetropolis; rarely used

<sup>\*</sup>Options chains() and sepchains are relevant only when option nchains() is used with bayesmh or the bayes prefix.

collect is allowed; see [U] 11.1.10 Prefix commands.

method	Description	
<pre>lmetropolis hmean</pre>	Laplace-Metropolis approximation; default harmonic-mean approximation	

# **Options**

Main

prior(numlist) specifies prior probabilities for models. By default, all models are assumed to be equally likely. You may specify probabilities for all tested models, in which case the probabilities must sum to one. Alternatively, you may specify probabilities for all but the last model, in which case the sum of the specified probabilities must be less than one, and the probability for the last model is computed as one minus this sum.

chains(\_all|numlist) specifies which chains from the MCMC sample to use for computation. The default is chains(\_all) or to use all simulated chains. Using multiple chains, provided the chains have converged, generally improves MCMC summary statistics. Option chains() is relevant only when option nchains() is specified with bayesmh or the bayes prefix.

sepchains specifies that the results be computed separately for each chain. The default is to compute results using all chains as determined by option chains(). Option sepchains is relevant only when option nchains() is specified with bayesmh or the bayes prefix.

Advanced

marglmethod(*method*) specifies a method for approximating the marginal likelihood. *method* is either lmetropolis, the default, for Laplace-Metropolis approximation or hmean for harmonic-mean approximation. This option is rarely used.

# Remarks and examples

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Remarks are presented under the following headings:

Introduction Testing nested hypotheses Comparing models with different priors

#### Introduction

In this entry, we describe hypothesis testing by computing model posterior probabilities, probabilities of Bayesian models given observed data. For interval hypothesis testing, see [BAYES] bayestest interval.

The bayestest model command computes posterior probabilities for specified models. The computed probabilities can be used to compare which model is more likely among considered models given observed data. You can compare models that differ only in several covariates or models with completely different regression functions, such as linear and nonlinear models. You can compare models with different outcome distributions or with different prior distributions or both. The only requirements are that the considered models have proper posterior distributions and that the same data are used to fit the models. If MCMC is used to approximate posterior distributions, convergence of MCMC should also be verified before model comparison.

The results reported by bayestest model are related to Bayes factors; see [BAYES] bayestats ic to compute Bayes factors.

To use bayestest model, you must store estimation results after each Bayesian model of interest. You can use estimates store (see [R] estimates store) to store estimation results after bayesmh or the bayes prefix, as you can with other estimation commands, provided you also saved simulation results from bayesmh or the bayes prefix using the saving() option. See Storing estimation results after Bayesian estimation in [BAYES] **Bayesian postestimation** for details.

## Testing nested hypotheses

Consider the following Bayesian regression model for auto.dta,

$$\mathtt{mpg} = \beta_0 + \beta_1 \mathtt{weight1} + \beta_2 \mathtt{length1} + \epsilon$$

where weight1 and length1 are the original weight and length variables rescaled to have similar scale as mpg.

We assume that errors are normally distributed:  $\epsilon \sim \text{normal}(0, \sigma^2)$ . We also assume a noninformative Jeffreys prior for the parameters:  $(\beta, \sigma^2) \sim 1/\sigma^2$ . Suppose that we are interested in testing whether there is a relationship between mileage and weight and length of cars. We will consider four models: the mean-only model, the model with weight only, the model with length only, and the full model with both covariates.

In a frequentist setting, the four models correspond to the following hypotheses:  $H_0$ :  $\beta_1 = 0$ ,  $\beta_2 = 0$ ,  $H_0$ :  $\beta_1 = 0$ , and  $H_0$ :  $\beta_2 = 0$ . In a Bayesian setting, we cannot formulate point hypotheses for parameters with continuous distributions; see [BAYES] bayestest interval for examples. However, we can compute probabilities of how likely each of the four models is given the observed data.

Let's load auto.dta and generate rescaled versions of weight and length.

```
. use https://www.stata-press.com/data/r18/auto
(1978 automobile data)
. generate weight1 = weight/100
. generate length1 = length/10
```

Next, we fit the four models using bayesmh. We use the saving() option to save the simulation datasets so that we can store estimation results of each model for later use with bayestest model.

The first model we fit is the mean-only model. We store its estimation results as meanonly.

```
. set seed 14
. bayesmh mpg, likelihood(normal({var}))
> prior({mpg:}, flat) prior({var}, jeffreys)
> saving(meanonly_simdata) burnin(3500)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  mpg ~ normal({mpg:_cons},{var})
  {mpg:_cons} ~ 1 (flat)
        {var} ~ jeffreys
Bayesian normal regression
                                                   MCMC iterations =
                                                                           13,500
Random-walk Metropolis-Hastings sampling
                                                   Burn-in
                                                                            3,500
                                                   MCMC sample size =
                                                                           10,000
                                                   Number of obs
                                                                               74
                                                                            .2627
                                                   Acceptance rate =
                                                   Efficiency:
                                                                min =
                                                                             .105
                                                                 avg =
                                                                            .1064
Log marginal-likelihood = -234.64617
                                                                 max =
                                                                            .1078
                                                                 Equal-tailed
                     Mean
                            Std. dev.
                                          MCSE
                                                    Median
                                                             [95% cred. interval]
mpg
                 21.29355
                            .6768607
                                        .020887
                                                  21.28059
                                                              20.00132
                                                                         22.61904
       _cons
                 34.80707
                            5.963995
                                        .181615
                                                  34.23247
                                                              24.9129
                                                                          47.6883
         var
```

file meanonly\_simdata.dta saved.

. estimates store meanonly

To accommodate the Jeffreys prior for the parameters, we specify suboption flat within the prior() option for coefficients to request the flat prior with the density of 1 and suboption jeffreys within prior() for the variance parameter to request a Jeffreys prior. We also specify a longer burn-in period to improve convergence of MCMC samples for all examples. (Remember to use bayesgraph to check convergence of MCMC.)

We fit the second model containing only covariate length1 and store its results as length:

```
. set seed 14
. bayesmh mpg length1, likelihood(normal({var}))
> prior({mpg:}, flat) prior({var}, jeffreys)
> saving(length_simdata) burnin(3500)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  mpg ~ normal(xb_mpg,{var})
Priors:
                                                                              (1)
  {mpg:length1 _cons} ~ 1 (flat)
                {var} ~ jeffreys
(1) Parameters are elements of the linear form xb_mpg.
Bayesian normal regression
                                                  MCMC iterations =
                                                                          13,500
                                                                           3,500
Random-walk Metropolis-Hastings sampling
                                                  Burn-in
                                                  MCMC sample size =
                                                                          10,000
                                                  Number of obs
                                                                              74
                                                  Acceptance rate =
                                                                           .2865
                                                  Efficiency:
                                                               min =
                                                                           .0771
                                                                avg =
                                                                          .07938
Log marginal-likelihood = -198.7678
                                                                max =
                                                                          .08286
                                                                Equal-tailed
                    Mean
                            Std. dev.
                                          MCSE
                                                   Median
                                                            [95% cred. interval]
mpg
               -2.069861
                            .1882345
                                       .006539
                                                -2.068094
                                                             -2.44718
                                                                       -1.706264
     length1
                60.20346
                            3.562119
                                       .127411
                                                 60.20927
                                                             53.34306
                                                                        67.22423
       _cons
         var
                12.88852
                            2.273808
                                       .081887
                                                  12.62042
                                                             9.169482
                                                                        18.16685
```

#### file length\_simdata.dta saved.

<sup>.</sup> estimates store length

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We fit the third model containing only covariate weight1 and store its results as weight:

```
. set seed 14
. bayesmh mpg weight1, likelihood(normal({var}))
> prior({mpg:}, flat) prior({var}, jeffreys)
> saving(weight_simdata) burnin(3500)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  mpg ~ normal(xb_mpg,{var})
Priors:
                                                                              (1)
  {mpg:weight1 _cons} ~ 1 (flat)
                {var} ~ jeffreys
(1) Parameters are elements of the linear form xb_mpg.
Bayesian normal regression
                                                  MCMC iterations =
                                                                          13,500
                                                                           3,500
Random-walk Metropolis-Hastings sampling
                                                  Burn-in
                                                   MCMC sample size =
                                                                          10,000
                                                   Number of obs
                                                                              74
                                                   Acceptance rate =
                                                                           .1735
                                                   Efficiency:
                                                                min =
                                                                            .0463
                                                                avg =
                                                                           .06694
Log marginal-likelihood = -198.20751
                                                                max =
                                                                           .07989
                                                                Equal-tailed
                    Mean
                            Std. dev.
                                          MCSE
                                                   Median
                                                            [95% cred. interval]
mpg
               -.6014409
                            .0506121
                                       .001791
                                                 -.6013071
                                                            -.6996976
                                                                         -.50121
     weight1
                39.45934
                            1.574673
                                       .057646
                                                 39.49735
                                                             36.31386
                                                                        42.33547
       _cons
         var
                12.13997
                            2.141741
                                       .099534
                                                  11.87332
                                                             8.883221
                                                                        17.14041
```

#### file weight\_simdata.dta saved.

<sup>.</sup> estimates store weight

Finally, we fit the last model containing both covariates and store its results as full:

```
. set seed 14
. bayesmh mpg weight1 length1, likelihood(normal({var}))
> prior({mpg:}, flat) prior({var}, jeffreys)
> saving(full_simdata) burnin(3500)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  mpg ~ normal(xb_mpg,{var})
Priors:
  {mpg:weight1 length1 _cons} ~ 1 (flat)
                                                                               (1)
                         {var} ~ jeffreys
(1) Parameters are elements of the linear form xb_mpg.
Bayesian normal regression
                                                   MCMC iterations =
                                                                           13,500
Random-walk Metropolis-Hastings sampling
                                                   Burn-in
                                                                            3,500
                                                   MCMC sample size =
                                                                           10,000
                                                   Number of obs
                                                                               74
                                                   Acceptance rate
                                                                             .2323
                                                   Efficiency:
                                                                 min =
                                                                            .05455
                                                                 avg =
                                                                            .06647
Log marginal-likelihood = -196.86195
                                                                            .08085
                                                                 max =
                                                                 Equal-tailed
                     Mean
                            Std. dev.
                                           MCSE
                                                    Median
                                                             [95% cred. interval]
mpg
     weight1
               -.3977027
                             .1580411
                                        .005558
                                                  -.401646
                                                             -.6965175
                                                                        -.0721332
               -.7599159
                             .5546754
                                        .021944
                                                 -.7502182
                                                             -1.907818
                                                                          .3106868
     length1
                                        .262563
       _cons
                  47.5913
                            6.132597
                                                   47.5656
                                                              35.89593
                                                                         60.18002
                                         .07608
         var
                 11.81753
                             1.96315
                                                  11.59273
                                                              8.729182
                                                                         16.14065
```

file full\_simdata.dta saved.

. estimates store full

### Example 1: Computing posterior probabilities of models

We now use bayestest model to compute posterior probabilities of the four models.

. bayestest model meanonly length weight full Bayesian model tests

	log(ML)	P(M)	P(M y)
meanonly	-234.6462	0.2500	0.0000
length	-198.7678	0.2500	0.1055
weight	-198.2075	0.2500	0.1848
full	-196.8619	0.2500	0.7097

Note: Marginal likelihood (ML) is computed using Laplace-Metropolis approximation.

The mean-only model is very unlikely compared with other models. The length and weight models are somewhat likely with the respective posterior probabilities of 0.11 and 0.18, and the full model has the highest posterior probability of 0.71.

## Example 2: Specifying prior probabilities of models

If we have some prior knowledge about each of the models, we can use the prior() option to specify prior probabilities for each model. For example, suppose that we have prior knowledge that the weight model is much more likely than the full model so that the prior probabilities are 0.1 for the mean-only model and the length model, 0.6 for the weight model, and only 0.2 for the full model.

. bayestest model meanonly length weight full, prior(0.1 0.1 0.6 0.2) Bayesian model tests

	log(ML)	P(M)	P(M y)
meanonly	-234.6462	0.1000	0.0000
length	-198.7678	0.1000	0.0401
weight	-198.2075	0.6000	0.4210
full	-196.8619	0.2000	0.5389

Note: Marginal likelihood (ML) is computed using Laplace-Metropolis approximation.

Under the specified prior, posterior probabilities of the weight and full models are now more similar: 0.42 and 0.54, respectively, but the full model is still preferable.

The above is equivalent to the following prior specification:

. bayestest model meanonly length weight full, prior(0.1 0.1 0.6)
 (output omitted)

Using our results, we conclude that mpg is related to both weight and length and would proceed with the full model.

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After your analysis, remember to erase the saved simulation datasets you no longer need. For example, we erase all of them by typing

- . erase meanonly\_simdata.dta
- . erase weight\_simdata.dta
- . erase length\_simdata.dta
- . erase full\_simdata.dta

## Comparing models with different priors

In the previous section, we used bayestest model to compare nested hypotheses about which covariates to include in the regression function. We can use bayestest model to compare models with not only different covariates but also different outcome distributions and priors for parameters.

We continue our analysis of auto.dta, but for simplicity, we now consider the mean-only model for mpg. Let's compare models with two slightly different informative priors. We use an informative normal—inverse-gamma prior for both models,

$$(\beta_0|\sigma^2) \sim N(\mu_0, \sigma^2/n_0)$$
 
$$\sigma^2 \sim \text{InvGamma}(\nu_0/2, \nu_0 \sigma_0^2/2)$$

with  $\mu_0=25$ ,  $n_0=10$ , and  $\sigma_0^2=30$ , but we consider two different values for the degrees of freedom:  $\nu_0=5$  and  $\nu_0=1$ .

We use bayesmh to fit our models. Following the formulas, we specify a normal() prior for the constant {mpg:\_cons} (mean parameter) and an inverse-gamma prior igamma() for the variance parameter {var}. We specify an expression for the variance of the normal prior distribution in parentheses.

We fit the first model with  $\nu_0 = 5$  and store its estimation results as informative1.

```
. set seed 14
. bayesmh mpg, likelihood(normal({var}))
> prior({mpg:}, normal(25,{var}/10))
> prior({var}, igamma(2.5,75)) saving(inf1_simdata)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  mpg ~ normal({mpg:_cons},{var})
Priors:
  {mpg:_cons} ~ normal(25,{var}/10)
        \{var\} \sim igamma(2.5,75)
Bayesian normal regression
                                                   MCMC iterations =
                                                                           12,500
Random-walk Metropolis-Hastings sampling
                                                   Burn-in
                                                                            2,500
                                                   MCMC sample size =
                                                                           10,000
                                                   Number of obs
                                                                               74
                                                   Acceptance rate =
                                                                            .2548
                                                   Efficiency:
                                                                           .09065
                                                                min =
                                                                            .1049
                                                                avg =
                                                                            .1192
Log marginal-likelihood = -238.55856
                                                                max =
                                                                Equal-tailed
                     Mean
                            Std. dev.
                                          MCSE
                                                    Median
                                                            [95% cred. interval]
mpg
                21.71853
                            .6592655
                                        .019091
                                                  21.69554
                                                             20.44644
                                                                         23.04896
       _cons
```

.193417

34.72454

25.84419

48.228

#### file inf1\_simdata.dta saved.

var

35.47405

5.823372

<sup>.</sup> estimates store informative1

We fit the second model with  $\nu_0 = 1$  and store its estimation results as informative2.

```
. set seed 14
. bayesmh mpg, likelihood(normal({var}))
> prior({mpg:}, normal(25,{var}/10))
> prior({var}, igamma(0.5,15)) saving(inf2_simdata)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  mpg ~ normal({mpg:_cons},{var})
  {mpg:_cons} ~ normal(25,{var}/10)
        \{var\} \sim igamma(0.5,15)
Bayesian normal regression
                                                   MCMC iterations =
                                                                          12,500
Random-walk Metropolis-Hastings sampling
                                                   Burn-in
                                                                           2,500
                                                   MCMC sample size =
                                                                          10,000
                                                   Number of obs
                                                                              74
                                                   Acceptance rate =
                                                                            .2261
                                                   Efficiency:
                                                                min =
                                                                            .0941
                                                                avg =
                                                                            .109
Log marginal-likelihood =
                          -239.4049
                                                                max =
                                                                            .1239
                                                                Equal-tailed
                                          MCSE
                    Mean
                            Std. dev.
                                                   Median [95% cred. interval]
mpg
       cons
                 21.7175
                            .6539814
                                       .021319
                                                   21.7295
                                                             20.47311
                                                                        23.02638
                35.89504
                            6.288571
                                       .178665
                                                  35.17056
                                                             25.86084
                                                                        50.21624
         var
```

file inf2\_simdata.dta saved.

. estimates store informative2

# Example 3: Comparing models with informative priors

We now use bayestest model to compare our models with two different informative priors.

. bayestest model informative1 informative2 Bayesian model tests

	log(ML)	P(M)	P(M y)
informative1 informative2	-238.5586	0.5000	0.6998
	-239.4049	0.5000	0.3002

Note: Marginal likelihood (ML) is computed using Laplace-Metropolis approximation.

Assuming that both models are equally likely a priori, the posterior probability of the informative1 stored results, 0.70, is much higher than the probability of the informative2 stored results, 0.3.

### Example 4: Comparing a model with noninformative prior

A note of caution regarding comparing models with informative and noninformative priors—models with noninformative priors will often win because they are typically in most agreement with the observed data. For models with noninformative priors, most of the information about parameters is contained in a likelihood. As such, any model with an informative prior that is not in perfect agreement with the data will not fit data as well as a model with a noninformative prior.

For example, let's fit our constant-only model using a noninformative Jeffreys prior for the parameters.

```
. bayesmh mpg, likelihood(normal({var}))
> prior({mpg:}, flat) prior({var}, jeffreys)
> saving(jeffreys_simdata)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  mpg ~ normal({mpg:_cons},{var})
Priors:
  {mpg:_cons} ~ 1 (flat)
        {var} ~ jeffreys
Bayesian normal regression
                                                  MCMC iterations =
                                                                          12,500
Random-walk Metropolis-Hastings sampling
                                                                          2,500
                                                  Burn-in
                                                  MCMC sample size =
                                                                          10,000
                                                  Number of obs
                                                                              74
                                                                           .2668
                                                  Acceptance rate =
                                                                          .09718
                                                  Efficiency: min =
```

		Mean	Std. dev.	MCSE	Median	Equal- [95% cred.	
mpg	_cons	21.29222	.6828864	.021906	21.27898	19.99152	22.61904
	var	34.76572	5.91534	.180754	34.18391	24.9129	47.61286

-234.645

avg =

max =

.1021

.1071

file jeffreys\_simdata.dta saved.

. estimates store jeffreys

Log marginal-likelihood =

. set seed 14

Let's now compare this model with our two informative models.

. bayestest model informative1 informative2 jeffreys Bayesian model tests

	log(ML)	P(M)	P(M y)
informative1	-238.5586	0.3333	0.0194
informative2	-239.4049	0.3333	0.0083
jeffreys	-234.6450	0.3333	0.9723

Note: Marginal likelihood (ML) is computed using Laplace-Metropolis approximation.

The posterior probability of the Jeffreys model is 0.97.

Finally, at the end of our analysis, we erase all the simulation datasets we no longer need. We erase all of them by typing

- . erase inf1\_simdata.dta
- . erase inf2\_simdata.dta
- . erase jeffreys\_simdata.dta

### Stored results

bayestest model stores the following in r():

Macros

r(names) names of estimation results used

r(marglmethod) method for approximating marginal likelihood: lmetropolis or hmean

r(chains) chains used in the computation, if chains() is specified

Matrices

r(test) test results for models in r(names)

r(test\_chain#) matrix test for chain #, if sepchains is specified

#### Methods and formulas

Suppose we have r models  $M_j$  for  $j=1,\ldots,r$  with prior probabilities  $P(M_j)$  such that  $\sum_{j=1}^r p(M_j) = 1$ . Then, posterior probability for model J is

$$P(M_j|\mathbf{y}) = \frac{P(\mathbf{y}|M_j)P(M_j)}{P(\mathbf{y})}$$

where  $P(\mathbf{y}|M_j) = m_j(y)$  is the marginal likelihood of  $M_j$  with respect to  $\mathbf{y}$ , and  $P(\mathbf{y}) = \sum_{j=1}^r P(\mathbf{y}|M_j)P(M_j)$ . See *Methods and formulas* in [BAYES] **bayesmh** for details about computing marginal likelihood.

With multiple chains, the bayestest model command uses the averaged across chains log marginal-likelihood for calculations. If the sepchains option is specified, the results are calculated and reported separately for each chain.

### Also see

[BAYES] bayes — Bayesian regression models using the bayes prefix

[BAYES] bayesmh — Bayesian models using Metropolis-Hastings algorithm

[BAYES] **Bayesian estimation** — Bayesian estimation commands

[BAYES] **Bayesian postestimation** — Postestimation tools for bayesmh and the bayes prefix

[BAYES] bayesstats ic — Bayesian information criteria and Bayes factors

[BAYES] bayesstats summary — Bayesian summary statistics

[BAYES] bayestest interval — Interval hypothesis testing

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