**Description**

`bayesstats summary` calculates and reports posterior summary statistics for model parameters and functions of model parameters using current Bayesian estimation results. Posterior summary statistics include posterior means, posterior standard deviations, MCMC standard errors (MCSE), posterior medians, and equal-tailed credible intervals or highest posterior density (HPD) credible intervals.

**Quick start**

Posterior summaries for all model parameters after a Bayesian regression model

```
bayesstats summary
```

Same as above, but only for parameters \{y:x1\} and \{y:x2\}

```
bayesstats summary \{y:x1\} \{y:x2\}
```

Same as above

```
bayesstats summary \{y:x1 x2\}
```

Posterior summaries for elements 1,1 and 2,1 of matrix parameter \{S\}

```
bayesstats summary \{S_1_1 S_2_1\}
```

Posterior summaries for all elements of matrix parameter \{S\}

```
bayesstats summary \{S\}
```

Posterior summaries with HPD instead of equal-tailed credible intervals and with credible level of 90%

```
bayesstats summary, hpd clevel(90)
```

Posterior summaries with MCSE calculated using batch means

```
bayesstats summary, batch(100)
```

Posterior summaries for functions of scalar model parameters

```
bayesstats summary \{y:x1\}-(y:_cons) (sd:sqrt\{var\})
```

Posterior summaries for the log-likelihood and log-posterior functions

```
bayesstats summary _loglikelihood _logposterior
```

Posterior summaries for selected model parameters and functions of model parameters and for log-likelihood and log-posterior functions using abbreviated syntax

```
bayesstats summary \{var\} \{y:x1\}-(y:_cons) _ll _lp
```

Posterior summaries of the simulated outcome

```
bayespredict {_ysim}, saving(predres)
bayesstats summary {_ysim} using predres
```
Posterior summaries of the mean across observations of the simulated outcome labeled as mymean
bayesstats summary (mymean: @mean(_ysim)) using predres

Menu
Statistics > Bayesian analysis > Summary statistics

Syntax

Syntax is presented under the following headings:
Summary statistics for model parameters
Summary statistics for predictions

Summary statistics for model parameters

Summary statistics for all model parameters
bayesstats summary [ , options showreffects[ (reref) ] ]
bayesstats summary _all [ , options showreffects[ (reref) ] ]

Summary statistics for selected model parameters
bayesstats summary paramspec [ , options ]

Summary statistics for expressions of model parameters
bayesstats summary exprspec [ , options ]

Summary statistics of log-likelihood or log-posterior functions
bayesstats summary _loglikelihood|_logposterior [ , options ]

Full syntax
bayesstats summary spec [ spec ... ] [ , options ]

paramspec can be one of the following:
{eqname: param} refers to a parameter param with equation name eqname;
{eqname: } refers to all model parameters with equation name eqname;
{eqname: paramlist} refers to parameters with names in paramlist and with equation name eqname;
or
{param} refers to all parameters named param from all equations.
In the above, param can refer to a matrix name, in which case it will imply all elements of this matrix. See Different ways of specifying model parameters in [BAYES] Bayesian postestimation for examples.
exprspec is an optionally labeled expression of model parameters specified in parentheses:

\([\text{exprlabel}:\text{expr}]\)

exprlabel is a valid Stata name, and expr is a scalar expression that may not contain matrix model parameters. See Specifying functions of model parameters in [BAYES] Bayesian postestimation for examples.

_loglikelihood and _logposterior also have respective synonyms _ll and _lp.

spec is one of paramspec, exprspec, _loglikelihood (or _ll), or _logposterior (or _lp).

### Summary statistics for predictions

Summary statistics for simulated outcomes, residuals, and more

\[\text{bayesstats summary yspec [yspec ...] using predfile [, options]}\]

Summary statistics for expressions of simulated outcomes, residuals, and more

\[\text{bayesstats summary (yexprspec) [(yexprspec) ...] using predfile [, options]}\]

Summary statistics for Mata functions of simulated outcomes, residuals, and more

\[\text{bayesstats summary (funcspec) [(funcspec) ...] using predfile [, options]}\]

Full syntax

\[\text{bayesstats summary predspec [predspec ...] using predfile [, options]}\]

predfile is the name of the dataset created by bayespredict that contains prediction results.

yspec is \{ysimspec | residspec | muspec | label\}.

ysimspec is \{_ysim#\} or \{_ysim#[numlist]\}, where \{_ysim#\} refers to all observations of the #th simulated outcome and \{_ysim#[numlist]\} refers to the selected observations, numlist, of the #th simulated outcome. \{_ysim\} is a synonym for \{_ysim1\}.

residspec is \{_resid#\} or \{_resid#[numlist]\}, where \{_resid#\} refers to all residuals of the #th simulated outcome and \{_resid#[numlist]\} refers to the selected residuals, numlist, of the #th simulated outcome. \{_resid\} is a synonym for \{_resid1\}.

muspec is \{_mu#\} or \{_mu#[numlist]\}, where \{_mu#\} refers to all expected values of the #th outcome and \{_mu#[numlist]\} refers to the selected expected values, numlist, of the #th outcome. \{_mu\} is a synonym for \{_mu1\}.

label is the name of the function simulated using bayespredict.

With large datasets, specifications \{_ysim#\}, \{_resid#\}, and \{_mu#\} may use a lot of time and memory and should be avoided. See Generating and saving simulated outcomes in [BAYES] bayespredict.

yexprspec is \[\text{[exprlabel: yexpr]}\], where exprlabel is a valid Stata name and yexpr is a scalar expression that may contain individual observations of simulated outcomes, \{_ysim#[#]\}; individual expected outcome values, \{_mu#[#]\}; individual simulated residuals, \{_resid#[#]\}; and other scalar predictions, \{label\}.  

**Funcspec** is \[ \text{funcspec} @ \text{func}(\text{arg1}, \text{arg2}) \], where \text{label} is a valid Stata name; \text{func} is an official or user-defined Mata function that operates on column vectors and returns a real scalar; and \text{arg1} and \text{arg2} are one of \{\_ysim\}, \{\_resid\}, or \{\_mu\}. \text{arg2} is primarily for use with user-defined Mata functions; see *Defining test statistics using Mata functions* in [BAYES] bayespredict.

**Predspec** is one of \text{yspec}, \text{yexprspec}, or \text{funcspec}. See *Different ways of specifying predictions and their functions* in [BAYES] Bayesian postestimation.

**Options**

<table>
<thead>
<tr>
<th>Options</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Main</strong></td>
<td></td>
</tr>
<tr>
<td>\text{clevel(#)}</td>
<td>set credible interval level; default is \text{clevel}(95)</td>
</tr>
<tr>
<td>\text{hpd}</td>
<td>display HPD credible intervals instead of the default equal-tailed credible intervals</td>
</tr>
<tr>
<td>\text{batch(#)}</td>
<td>specify length of block for batch-means calculations; default is \text{batch}(0)</td>
</tr>
<tr>
<td>*\text{chains(_all</td>
<td>numlist)}</td>
</tr>
<tr>
<td>*\text{sepchains}</td>
<td>compute results separately for each chain</td>
</tr>
<tr>
<td>\text{skip(#)}</td>
<td>skip every # observations from the MCMC sample; default is \text{skip}(0)</td>
</tr>
<tr>
<td>\text{nolegend}</td>
<td>suppress table legend</td>
</tr>
<tr>
<td>\text{display_options}</td>
<td>control spacing, line width, and base and empty cells</td>
</tr>
<tr>
<td><strong>Advanced</strong></td>
<td></td>
</tr>
<tr>
<td>\text{corrlag(#)}</td>
<td>specify maximum autocorrelation lag; default varies</td>
</tr>
<tr>
<td>\text{corrtol(#)}</td>
<td>specify autocorrelation tolerance; default is \text{corrtol}(0.01)</td>
</tr>
</tbody>
</table>

*Options \text{chains()} and \text{sepchains} are relevant only when option \text{nchains()} is used with \text{bayesmh} or the \text{bayes} prefix.*

\text{collect} is allowed; see [U] 11.1.10 Prefix commands.

**Options**

- \text{clevel(#)} specifies the credible level, as a percentage, for equal-tailed and HPD credible intervals. The default is \text{clevel}(95) or as set by [BAYES] set \text{clevel}.
- \text{hpd} displays the HPD credible intervals instead of the default equal-tailed credible intervals.
- \text{batch(#)} specifies the length of the block for calculating batch means and an MCSE using batch means. The default is \text{batch}(0), which means no batch calculations. When \text{batch()} is not specified, the MCSE is computed using effective sample sizes instead of batch means. \text{batch()} may not be combined with \text{corrlag()} or \text{corrtol()}.
- \text{chains(_all | numlist)} specifies which chains from the MCMC sample to use for computation. The default is \text{chains(_all)} or to use all simulated chains. Using multiple chains, provided the chains have converged, generally improves MCMC summary statistics. Option \text{chains()} is relevant only when option \text{nchains()} is specified with \text{bayesmh} or the \text{bayes} prefix.
- \text{sepchains} specifies that the results be computed separately for each chain. The default is to compute results using all chains as determined by option \text{chains()}. Option \text{sepchains} is relevant only when option \text{nchains()} is specified with \text{bayesmh} or the \text{bayes} prefix.
- \text{showreffects} and \text{showreffects(reref)} are for use after multilevel models, and they specify that the results for all or a list \text{reref} of random-effects parameters be provided in addition to other model parameters. By default, all random-effects parameters are excluded from the results to conserve computation time.
skip(#) specifies that every # observations from the MCMC sample not be used for computation. The default is skip(0) or to use all observations in the MCMC sample. Option skip() can be used to subsample or thin the chain. skip(#) is equivalent to a thinning interval of #+1. For example, if you specify skip(1), corresponding to the thinning interval of 2, the command will skip every other observation in the sample and will use only observations 1, 3, 5, and so on in the computation. If you specify skip(2), corresponding to the thinning interval of 3, the command will skip every 2 observations in the sample and will use only observations 1, 4, 7, and so on in the computation. skip() does not thin the chain in the sense of physically removing observations from the sample, as is done by, for example, bayesmh’s thinning() option. It only discards selected observations from the computation and leaves the original sample unmodified.

nolegend suppresses the display of the table legend, which identifies the rows of the table with the expressions they represent.

display_options: vsquish, noemptycells, baselevels, allbaselevels,nofvlabel, fvwrap(#), fvwrapon(style), and nolstretch; see [R] Estimation options.

Advanced

corrlag(#) specifies the maximum autocorrelation lag used for calculating effective sample sizes. The default is min{500,mcmcsizer}/2}. The total autocorrelation is computed as the sum of all lag-k autocorrelation values for k from 0 to either corrlag() or the index at which the autocorrelation becomes less than corrtol() if the latter is less than corrlag(). Options corrlag() and batch() may not be combined.

corrtol(#) specifies the autocorrelation tolerance used for calculating effective sample sizes. The default is corrtol(0.01). For a given model parameter, if the absolute value of the lag-k autocorrelation is less than corrtol(), then all autocorrelation lags beyond the kth lag are discarded. Options corrtol() and batch() may not be combined.

Remarks and examples

stata.com

Remarks are presented under the following headings:

Introduction
Bayesian summaries for an auto data example

Introduction

bayesstats summary reports posterior summary statistics for model parameters and their functions using the current Bayesian estimation results. When typed without arguments, the command displays results for all model parameters. Alternatively, you can specify a subset of model parameters following the command name; see Different ways of specifying model parameters in [BAYES] Bayesian postestimation. You can also obtain results for scalar functions of model parameters; see Specifying functions of model parameters in [BAYES] Bayesian postestimation.

Sometimes, it may be useful to obtain posterior summaries of log-likelihood and log-posterior functions. This can be done by specifying _loglikelihood and _logposterior (or the respective synonyms _ll and _lp) following the command name.

You can also obtain the posterior summaries for prediction quantities when you specify the prediction dataset in the using specification; see Different ways of specifying predictions and their functions in [BAYES] Bayesian postestimation for how to specify prediction quantities with bayesstats summary.
bayesstats summary reports the following posterior summary statistics: posterior mean, posterior standard deviation, MCMC standard error, posterior median, and equal-tailed credible intervals or, if the hpd option is specified, HPD credible intervals. The default credible level is set to 95%, but you can change this by specifying the clevel() option. Equal-tailed and HPD intervals may produce very different results for asymmetric or highly skewed marginal posterior distributions. The HPD intervals are preferable in this situation.

You should not confuse the term “HPD interval” with the term “HPD region”. A \( \{100 \times (1 - \alpha)\}% \) HPD interval is defined such that it contains \( \{100 \times (1 - \alpha)\}% \) of the posterior density. A \( \{100 \times (1 - \alpha)\}% \) HPD region also satisfies the condition that the density inside the region is never lower than that outside the region. For multimodal univariate marginal posterior distributions, the HPD regions may include unions of nonintersecting HPD intervals. For unimodal univariate marginal posterior distributions, HPD regions are indeed simply HPD intervals. The bayesstats summary command thus calculates HPD intervals assuming unimodal marginal posterior distributions (Chen and Shao 1999).

Some authors use the term “posterior intervals” instead of “credible intervals” and the term “central posterior intervals” instead of “equal-tailed credible intervals” (for example, Gelman et al. [2014]).

Bayesian summaries for an auto data example

Recall our analysis of auto.dta from example 4 in [BAYES] bayesmh using the mean-only normal model for mpg with a noninformative prior.

\[ . \text{use https://www.stata-press.com/data/r18/auto} \]
\[ . \text{(1978 automobile data)} \]
\[ . \text{set seed 14} \]
\[ . \text{bayesmh mpg, likelihood(normal({var}))} \]
\[ > \text{prior({mpg:_cons}, flat) prior({var}, jeffreys)} \]
\[ \text{Burn-in ...} \]
\[ \text{Simulation ...} \]
\[ \text{Model summary} \]

| Likelihood: | mpg ~ normal({mpg:_cons},{var}) |
| Priors: | {mpg:_cons} ~ 1 (flat) {var} ~ jeffreys |

Bayesian normal regression
Random-walk Metropolis-Hastings sampling

| MCMC iterations = 12,500 |
| Burn-in = 2,500 |
| MCMC sample size = 10,000 |
| Number of obs = 74 |
| Acceptance rate = .2668 |
| Efficiency: min = .09718 |
| avg = .1021 |
| max = .1071 |

Log marginal-likelihood = -234.645

<table>
<thead>
<tr>
<th>mpg</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>Equal-tailed [95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>var</td>
<td>34.76572</td>
<td>5.91534</td>
<td>.180754</td>
<td>34.18391</td>
<td>24.9129 47.61286</td>
</tr>
</tbody>
</table>
Example 1: Summaries for all parameters

If we type `bayesstats summary` without arguments after the `bayesmh` command, we will obtain the same summary table as reported by `bayesmh`.

```
. bayesstats summary
Posterior summary statistics
 Equal-tailed
<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mpg</td>
<td>cons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>var</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34.76572</td>
<td>5.91534</td>
<td>.180754</td>
<td>34.18391</td>
<td>24.9129 47.61286</td>
</tr>
</tbody>
</table>
```

The posterior mean of `{mpg:_cons}` is 21.29 and of `{var}` is 34.8. They are close to their respective frequentist analogs (the sample mean of `mpg` is 21.297, and the sample variance is 33.47), because we used a noninformative prior. Posterior standard deviations are 0.68 for `{mpg:_cons}` and 5.92 for `{var}`, and they are comparable to frequentist standard errors under this noninformative prior. The standard error estimates of the posterior means, MCSEs, are low. For example, MCSE is 0.022 for `{mpg:_cons}`. This means that the precision of our estimate is, up to one decimal point, 21.3 provided that MCMC converged. The posterior means and medians of `{mpg:_cons}` are close, which suggests that the posterior distribution for `{mpg:_cons}` may be symmetric. According to the credible intervals, we are 95% certain that the posterior mean of `{mpg:_cons}` is roughly between 20 and 23 and that the posterior mean of `{var}` is roughly between 25 and 48. We can infer from this that `{mpg:_cons}` is greater than, say, 15, and that `{var}` is greater than, say, 20, with a very high probability. (We can use [BAYES] bayestest interval to compute the actual probabilities.)

The above is also equivalent to typing

```
. bayesstats summary {mpg:_cons} {var}
(output omitted)
```

Example 2: Credible intervals

By default, `bayesstats summary` reports 95% equal-tailed credible intervals. We can change the default credible level by specifying the `clevel()` option.

```
. bayesstats summary, clevel(90)
Posterior summary statistics
 Equal-tailed
<table>
<thead>
<tr>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[90% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>mpg</td>
<td>cons</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>var</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>34.76572</td>
<td>5.91534</td>
<td>.180754</td>
<td>34.18391</td>
<td>26.28517 44.81732</td>
</tr>
</tbody>
</table>
```

As expected, 90% credible intervals are more narrow.
To calculate and report HPD intervals, we specify the `hpd` option.

```
.bayesstats summary, hpd
```

**Posterior summary statistics**  
MCMC sample size = 10,000

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>_cons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>var</strong></td>
<td>34.7657</td>
<td>5.91534</td>
<td>.180754</td>
<td>34.18391</td>
<td>24.34876  46.12339</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The posterior distribution of `{mpg:_cons}` is symmetric about the posterior mean; thus there is little difference between the 95% equal-tailed credible interval from example 1 and this 95% HPD credible interval for `{mpg:_cons}`. The 95% HPD interval for `{var}` has a smaller width than the corresponding equal-tailed interval in example 1.

---

**Example 3: Batch-means estimator**

`bayesstats summary` provides two estimators for MCSE: effective-sample-size and batch-means. Estimation using effective sample sizes is the default. You can use the `batch(#)` option to request the batch-means estimator, where # is the batch size. The optimal batch size depends on the autocorrelation in the MCMC sample. For example, if we observe that the autocorrelation for the parameters of interest is negligible after lag 100, we can specify `batch(100)` to estimate MCSE.

In our example, autocorrelation dies out after about lag 10 (see, for example, Autocorrelation plots in [BAYES] bayesgraph and example 1 in [BAYES] bayesstats ess), so we use 10 as our batch size:

```
.bayesstats summary, batch(10)
```

**Posterior summary statistics**  
MCMC sample size = 10,000  
Batch size = 10

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>_cons</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>var</strong></td>
<td>34.7657</td>
<td>5.91534</td>
<td>.135295</td>
<td>34.18391</td>
<td>24.9129   47.61286</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: Mean and MCSE are estimated using batch means.

The batch-means MCSE estimates are somewhat smaller than those obtained by default using effective sample sizes.

Use caution when choosing the batch size for the batch-means method. For example, if you use the batch size of 1, you will obtain MCSE estimates under the assumption that the draws in the MCMC sample are independent, which is not true.
Example 4: Subsampling or thinning the chain

You can reduce correlation between MCMC draws by thinning or subsampling the MCMC chain. You can use the `skip(#)` option to skip every # observations from the MCMC sample, which is equivalent to a thinning interval of # + 1. For example, if you specify `skip(1)`, corresponding to the thinning interval of 2, `bayesstats summary` will skip every other observation in the sample and will use only observations 1, 3, 5, and so on in the computation. If you specify `skip(2)`, corresponding to the thinning interval of 3, `bayesstats summary` will skip every two observations in the sample and will use only observations 1, 4, 7, and so on in the computation. By default, no observations are skipped—`skip(0)`. Note that `skip()` does not thin the chain in the sense of physically removing observations from the sample, as is done by `bayesmh`’s `thinning()` option. It discards only selected observations from the computation and leaves the original sample unmodified.

```
. bayesstats summary, skip(9)
 note: skipping every 9 sample observations; using observations 1,11,21,...
```

```
Posterior summary statistics  MCMC sample size =  1,000

      | Mean  | Std. dev. | MCSE  | Median  | [95% cred. interval]
---|------|---------|------|--------|---------------------
mpg | 21.29554 | .6813796 | .029517 | 21.27907 | 19.98813 to 22.58582 |
    | .cons | 5.87542 | .4951654 | .014972 | 5.846701 | 5.991282 to 6.900207 |
```

We selected to skip every 9 observations, which led to a significant reduction of the MCMC sample size and thus increased our standard deviations. In some cases, with larger MCMC sample sizes, subsampling may decrease standard deviations because of the decreased autocorrelation in the reduced MCMC sample.

Example 5: Summaries for expressions of model parameters

`bayesstats summary` accepts expressions to provide summaries of functions of model parameters. For example, we can use expression `(sd:sqrt({var}))` with a label, `sd`, to summarize the standard deviation of `mpg` in addition to the variance.

```
. bayesstats summary (sd:sqrt({var})) {var}
```

```
Posterior summary statistics  MCMC sample size =  10,000

sd : sqrt({var})

      | Mean   | Std. dev. | MCSE   | Median   | [95% cred. interval]
---|-------|---------|-------|---------|---------------------
    | 5.87542 | .4951654 | .014972 | 5.846701 | 4.991282 to 6.900207 |
var | 34.76572 | 5.91534 | .180754 | 34.18391 | 24.9129 to 47.61286 |
```
Expressions can also be used for calculating posterior probabilities, although this can be more easily done using `bayestest interval` (see [BAYES bayestest interval]). For illustration, let’s verify the probability that `var` is within the endpoints of the reported credible interval, indeed 0.95.

```stata
.bayesstats summary (prob:{var}>24.913 & {var}<47.613)
Posterior summary statistics MCMC sample size = 10,000
prob : {var}>24.913 & {var}<47.613

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>prob</td>
<td>.9502</td>
<td>.217542</td>
<td>.005301</td>
<td>1</td>
<td>0</td>
</tr>
</tbody>
</table>
```

**Example 6: Summaries for log likelihood and log posterior**

We can use reserved names `_loglikelihood` (or the synonym `_ll`) and `_logposterior` (or the synonym `_lp`) to obtain summaries of the log likelihood and log posterior for the simulated MCMC sample.

```stata
.bayesstats summary _ll _lp
Posterior summary statistics MCMC sample size = 10,000
_ll : _loglikelihood
_lp : _logposterior

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>_ll</td>
<td>-235.4162</td>
<td>.990654</td>
<td>.032232</td>
<td>-235.1379</td>
<td>-238.1236 -234.4345</td>
</tr>
<tr>
<td>_lp</td>
<td>-238.9507</td>
<td>1.037785</td>
<td>.034535</td>
<td>-238.6508</td>
<td>-241.7889 -237.9187</td>
</tr>
</tbody>
</table>
```

**Example 7: Summaries for predicted outcomes**

We continue our series of examples by computing summaries for Bayesian predictions. Let’s generate Bayesian predictions of `mpg` and summarize them.

We use `bayespredict` to simulate outcome values for `mpg` for the first 10 observations from the fitted `bayesmh` model. To use `bayespredict`, we must first save the simulation results from `bayesmh` in a Stata dataset, `autosim.dta`. We then use `bayespredict` to save the prediction results in the dataset `mpgreps.dta`.

```stata
.bayesmh, saving(autosim)
  note: file autosim.dta saved.
.bayespredict {_ysim[1/10]}, saving(mpgreps) rseed(16)
  Computing predictions ...  
  file mpgreps.dta saved.
  file mpgreps.ster saved.
```

We can now summarize the prediction results by using `bayesstats summary`. We specify the prediction quantity we wish to summarize, the simulated outcome `{_ysim}` in our example, and the prediction dataset, `mpgreps.dta`, which contains the prediction quantity, in the `using` specification.
. bayesstats summary {_ysim} using mpgreps

Posterior summary statistics  MCMC sample size = 10,000

<table>
<thead>
<tr>
<th></th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>_ysim1_1</td>
<td>21.24878</td>
<td>6.018783</td>
<td>.062648</td>
<td>21.23939</td>
<td>9.444973 33.07051</td>
</tr>
<tr>
<td>_ysim1_2</td>
<td>21.2539</td>
<td>5.944206</td>
<td>.060415</td>
<td>21.21638</td>
<td>9.421932 32.90605</td>
</tr>
<tr>
<td>_ysim1_3</td>
<td>21.3256</td>
<td>5.910363</td>
<td>.061595</td>
<td>21.31499</td>
<td>9.801655 33.02746</td>
</tr>
<tr>
<td>_ysim1_4</td>
<td>21.40651</td>
<td>5.963456</td>
<td>.059479</td>
<td>21.45933</td>
<td>9.794156 33.39388</td>
</tr>
<tr>
<td>_ysim1_5</td>
<td>21.19781</td>
<td>5.926335</td>
<td>.061197</td>
<td>21.26437</td>
<td>9.759916 32.80291</td>
</tr>
<tr>
<td>_ysim1_6</td>
<td>21.34776</td>
<td>5.94413</td>
<td>.059441</td>
<td>21.32314</td>
<td>9.771529 33.30251</td>
</tr>
<tr>
<td>_ysim1_7</td>
<td>21.34043</td>
<td>5.898474</td>
<td>.058985</td>
<td>21.34119</td>
<td>9.821613 32.97709</td>
</tr>
<tr>
<td>_ysim1_8</td>
<td>21.25329</td>
<td>5.957051</td>
<td>.05886</td>
<td>21.26176</td>
<td>9.476474 32.96236</td>
</tr>
<tr>
<td>_ysim1_9</td>
<td>21.25284</td>
<td>5.866096</td>
<td>.05962</td>
<td>21.3052</td>
<td>9.714165 32.82636</td>
</tr>
<tr>
<td>_ysim1_10</td>
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<td>5.931401</td>
<td>.060853</td>
<td>21.30528</td>
<td>9.670334 33.10769</td>
</tr>
</tbody>
</table>

bayesstats summary reports posterior summaries for all simulated outcomes in the prediction dataset, mpgreps.dta. Estimated posterior means and standard deviations are similar to the corresponding observed values for mpg, 21.30 and 5.79, respectively.

We can specifically examine the first observation of the replicated sample, {_ysim_1}, and compare it with the observed value, mpg[1], of 22.

. bayesstats summary ({_ysim_1}>='=mpg[1]') using mpgreps

Posterior summary statistics  MCMC sample size = 10,000

<table>
<thead>
<tr>
<th>expr1</th>
<th>Mean</th>
<th>Std. dev.</th>
<th>MCSE</th>
<th>Median</th>
<th>[95% cred. interval]</th>
</tr>
</thead>
<tbody>
<tr>
<td>expr1</td>
<td>.4479</td>
<td>.497303</td>
<td>.004973</td>
<td>0</td>
<td>0 1</td>
</tr>
</tbody>
</table>

We find that 45% of the replicates of mpg[1] are greater than 22. The reported probability of 0.45 is known as the posterior predictive $p$-value and is used for goodness-of-fit checking; see [BAYES] bayesstats ppvalues.

Stored results

bayesstats summary stores the following in r():

Scalars
- r(mcmcsize)  MCMC sample size used in the computation
- r(clevel)  credible interval level
- r(hpd)  1 if hpd is specified, 0 otherwise
- r(batch)  batch length for batch-means calculations
- r(skip)  number of MCMC observations to skip in the computation; every r(skip) observations are skipped
- r(corrlag)  maximum autocorrelation lag
- r(corrtol)  autocorrelation tolerance
- r(nchains)  number of chains used in the computation

Macros
- r(names)  names of model parameters and expressions
- r(expr_#)  #th expression
- r(exprnames)  expression labels
- r(chains)  chains used in the computation, if chains() is specified
Methods and formulas

Methods and formulas are presented under the following headings:

Point estimates
Credible intervals

Most of the summary statistics employed in Bayesian analysis are based on the marginal posterior distributions of individual model parameters or functions of model parameters.

Let $\theta$ be a scalar model parameter and $\{\theta_t\}_{t=1}^T$ be an MCMC chain of size $T$ drawn from the marginal posterior distribution of $\theta$. For a function $g(\theta)$, substitute $\{\theta_t\}_{t=1}^T$ with $\{g(\theta_t)\}_{t=1}^T$ in the formulas below. If $\theta$ is a covariance matrix model parameter, the formulas below are applied to each element of the lower-diagonal portion of $\theta$.

Point estimates

Marginal posterior moments are approximated using the Monte Carlo integration applied to the simulated samples $\{\theta_t\}_{t=1}^T$.

Sample posterior mean and sample posterior standard deviation are defined as follows,

$$\hat{\theta} = \frac{1}{T} \sum_{t=1}^T \theta_t, \quad \hat{s}^2 = \frac{1}{T-1} \sum_{t=1}^T (\theta_t - \hat{\theta})^2$$

where $\hat{\theta}$ and $\hat{s}^2$ are sample estimators of the population posterior mean $E(\theta_t)$ and posterior variance $\text{Var}(\theta_t)$.

With multiple chains, the posterior mean and standard deviation are estimated using the combined sample of all chains or of those that are requested in the `chains()` option as follows. Let $\{\theta_{jt}\}_{t=1}^T$ be the $j$th Markov chain, $j = 1, \ldots, M$, with sample mean $\hat{\theta}_j$ and variance $\hat{s}_j^2$. The overall sample posterior mean is

$$\hat{\theta} = \frac{1}{MT} \sum_{j=1}^M \sum_{t=1}^T \theta_{jt}$$

and equals the average of the sample means of individual chains. Let $B$ and $W$ be the respective between-chains and within-chain variances

$$B = \frac{T}{M-1} \sum_{j=1}^M (\hat{\theta}_j - \hat{\theta})^2, \quad W = \frac{1}{M} \sum_{j=1}^M \hat{s}_j^2$$

The estimator of the posterior variance is

$$\hat{s}^2 = \frac{T-1}{T} W + \frac{1}{T} B \quad (1)$$
When the chains are strongly stationary, $\hat{s}^2$ is an unbiased estimator of the marginal posterior variance of $\theta$ (Gelman et al. 2014, sec. 11.4).

The precision of the sample posterior mean is evaluated by its standard error, also known as the Monte Carlo standard error (MCSE). Note that MCSE cannot be estimated using the classical formula for the standard error, $\hat{s}/\sqrt{T}$, because of the dependence between $\theta_t$'s.

Let

$$\sigma^2 = \text{Var}(\theta_t) + 2 \sum_{k=1}^{\infty} \text{Cov}(\theta_t, \theta_{t+k})$$

Then, $\sqrt{T} \times \text{MCSE}$ approaches $\sigma$ asymptotically in $T$.

bayesstats summary provides two different approaches for estimating MCSE. Both approaches try to adjust for the existing autocorrelation in the MCMC sample. The first one uses the so-called effective sample size (ESS), and the second one uses batch means (Roberts 1996; Jones et al. 2006).

The ESS-based estimator for MCSE, the default in bayesstats summary, is given by

$$\text{MCSE}(\hat{\theta}) = \frac{\hat{s}}{\sqrt{\text{ESS}}}$$

ESS is defined as

$$\text{ESS} = T/(1 + 2 \sum_{k=1}^{\text{max}_\text{lags}} \rho_k)$$

where $\rho_k$ is the lag-$k$ autocorrelation, and $\text{max}_\text{lags}$ is the maximum number less than or equal to $\rho_{\text{lag}}$ such that for all $k = 1, \ldots, \text{max}_\text{lags}$, $|\rho_k| > \rho_{\text{tol}}$, where $\rho_{\text{lag}}$ and $\rho_{\text{tol}}$ are specified in options corrlag() and corrtol() with the respective default values of 500 and 0.01. $\rho_k$ is estimated as $\gamma_k/\gamma_0$, where

$$\gamma_k = \frac{1}{T} \sum_{t=1}^{T-k} (\theta_t - \hat{\theta})(\theta_{t+k} - \hat{\theta})$$

is the lag-$k$ empirical autocovariance.

With multiple chains, the overall ESS is given by the sum of the effective sample sizes of individual chains. The MCSE is then calculated using the formula

$$\text{MCSE}(\hat{\theta}) = \frac{\hat{s}}{\sqrt{\sum_{j=1}^{M} \text{ESS}_j}}$$

where $\hat{s}$ is computed using (1) and $\text{ESS}_j$ is the effective sample size of the $j$th chain.

The batch-means estimator of MCSE is obtained as follows. For a given batch of length $b$, the initial MCMC chain is split into $m$ batches of size $b$,

$$\{\theta_{j'+1}, \ldots, \theta_{j'+b}\} \{\theta_{j'+b+1}, \ldots, \theta_{j'+2b}\} \ldots \{\theta_{T-b+1}, \ldots, \theta_T\}$$

where $j' = T - m \times b$ and $m$ batch means $\hat{\mu}_1, \ldots, \hat{\mu}_m$ are calculated as sample means of each batch. $m$ is chosen as the maximum number such that $m \times b \leq T$. If $m$ is not a divisor of $T$, the first $T - m \times b$ observations of the sample are not used in the batch-means computation. The batch-means estimator of the posterior variance, $\hat{s}^2_{\text{batch}}$, is based on the assumption that $\hat{\mu}_j$s are much less correlated than the original sample draws.

The batch-means estimator of the posterior mean is

$$\hat{\theta}_{\text{batch}} = \frac{1}{m} \sum_{j=1}^{m} \hat{\mu}_j$$
We have \( \hat{\theta}_{\text{batch}} = \hat{\theta} \), whenever \( m \times b = T \). Under the assumption that the batch means are uncorrelated, \( \hat{s}_{\text{batch}}^2 = \left\{ 1/(m - 1) \right\} \sum_{j=1}^{m} (\hat{\mu}_j - \hat{\theta}_{\text{batch}})^2 \) can be used as an estimator of \( \sigma^2/b \). This fact justifies the batch-means estimator of MCSE given by

\[
\text{MCSE}_{\text{batch}}(\hat{\theta}) = \frac{\hat{s}_{\text{batch}}}{\sqrt{m}}
\]

The accuracy of the batch-means estimator depends on the choice of the batch length \( b \). The higher the autocorrelation in the original MCMC sample, the larger the batch length \( b \) should be, provided that the number of batches \( m \) does not become too small; \( \sqrt{T} \) is typically used as the maximum value for \( b \). The batch length is commonly determined by inspecting the autocorrelation plot for \( \theta \). Under certain assumptions, Flegal and Jones (2010) establish that an asymptotically optimal batch size is of order \( T^{1/3} \).

With multiple chains, the batch-means estimator is calculated using the combined sample of all chains or of those that are requested in the `chains()` option.

**Credible intervals**

Let \( \theta_{(1)}, \ldots, \theta_{(T)} \) be an MCMC sample ordered from smallest to largest. Let \( (1 - \alpha) \) be a credible level. Then, a \{100 \times (1 - \alpha)\}\% equal-tailed credible interval is

\[
(\theta_{([T\alpha/2])}, \theta_{([T(1-\alpha/2)])})
\]

where \([ \cdot ]\) in the above imply an integer number.

A \{100 \times (1 - \alpha)\}\% HPD interval is defined as the shortest interval among the \{100 \times (1 - \alpha)\}% credible intervals \( (\theta_{(j)}, \theta_{(j+[T(1-\alpha)])}), j = 1, \ldots, T - [T(1 - \alpha)] \).

With multiple chains, credible intervals are computed using the combined sample of all chains or of those requested with the `chains()` option; see Brooks and Gelman (1998, sec. 1.1).

**References**


Also see

[BAYES] bayes — Bayesian regression models using the bayes prefix
[BAYES] bayesmh — Bayesian models using Metropolis–Hastings algorithm
[BAYES] Bayesian estimation — Bayesian estimation commands
[BAYES] Bayesian postestimation — Postestimation tools for bayesmh and the bayes prefix
[BAYES] bayesgraph — Graphical summaries and convergence diagnostics
[BAYES] bayespredict — Bayesian predictions
[BAYES] bayesstats ess — Effective sample sizes and related statistics
[BAYES] bayesstats ppvalues — Bayesian predictive p-values and other predictive summaries
[BAYES] bayestest interval — Interval hypothesis testing