

bayesstats summary — Bayesian summary statistics

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Description

`bayesstats summary` calculates and reports posterior summary statistics for model parameters and functions of model parameters using current Bayesian estimation results. Posterior summary statistics include posterior means, posterior standard deviations, MCMC standard errors (MCSE), posterior medians, and equal-tailed credible intervals or highest posterior density (HPD) credible intervals.

Quick start

Posterior summaries for all model parameters after a Bayesian regression model

```
bayesstats summary
```

As above, but only for parameters `{y:x1}` and `{y:x2}`

```
bayesstats summary {y:x1} {y:x2}
```

Same as above

```
bayesstats summary {y:x1 x2}
```

Posterior summaries for elements 1,1 and 2,1 of matrix parameter `{S}`

```
bayesstats summary {S_1_1 S_2_1}
```

Posterior summaries for all elements of matrix parameter `{S}`

```
bayesstats summary {S}
```

Posterior summaries with HPD instead of equal-tailed credible intervals and with credible level of 90%

```
bayesstats summary, hpd clevel(90)
```

Posterior summaries with MCSE calculated using batch means

```
bayesstats summary, batch(100)
```

Posterior summaries for functions of scalar model parameters

```
bayesstats summary ({y:x1}--{y:_cons}) (sd:sqrt({var}))
```

Posterior summaries for the log-likelihood and log-posterior functions

```
bayesstats summary _loglikelihood _logposterior
```

Posterior summaries for selected model parameters and functions of model parameters and for log-likelihood and log-posterior functions using abbreviated syntax

```
bayesstats summary {var} ({y:x1}--{y:_cons}) _ll _lp
```

Menu

Statistics > Bayesian analysis > Summary statistics

Syntax

Summary statistics for all model parameters

```
bayesstats summary [, options showeffects[(reref)]]
```

```
bayesstats summary _all [, options showeffects[(reref)]]
```

Summary statistics for selected model parameters

```
bayesstats summary paramspec [, options]
```

Summary statistics for functions of model parameters

```
bayesstats summary exprspec [, options]
```

Summary statistics of log-likelihood or log-posterior functions

```
bayesstats summary _loglikelihood|_logposterior [, options]
```

Full syntax

```
bayesstats summary spec [spec ...] [, options]
```

paramspec can be one of the following:

$\{eqname:param\}$ refers to a parameter *param* with equation name *eqname*;

$\{eqname:\}$ refers to all model parameters with equation name *eqname*;

$\{eqname:paramlist\}$ refers to parameters with names in *paramlist* and with equation name *eqname*;
or

$\{param\}$ refers to all parameters named *param* from all equations.

In the above, *param* can refer to a matrix name, in which case it will imply all elements of this matrix. See [Different ways of specifying model parameters](#) in [\[BAYES\] bayesian postestimation](#) for examples.

exprspec is an optionally labeled expression of model parameters specified in parentheses:

```
( [exprlabel: ]expr )
```

exprlabel is a valid Stata name, and *expr* is a scalar expression that may not contain matrix model parameters. See [Specifying functions of model parameters](#) in [\[BAYES\] bayesian postestimation](#) for examples.

_loglikelihood and _logposterior also have respective synonyms _ll and _lp.

spec is one of [paramspec](#), [exprspec](#), _loglikelihood (or _ll), or _logposterior (or _lp).

<i>options</i>	Description
Main	
<code>clevel(#)</code>	set credible interval level; default is <code>clevel(95)</code>
<code>hpd</code>	display HPD credible intervals instead of the default equal-tailed credible intervals
<code>batch(#)</code>	specify length of block for batch-means calculations; default is <code>batch(0)</code>
<code>skip(#)</code>	skip every # observations from the MCMC sample; default is <code>skip(0)</code>
<code>nolegend</code>	suppress table legend
<code>display_options</code>	control spacing, line width, and base and empty cells
Advanced	
<code>corrlag(#)</code>	specify maximum autocorrelation lag; default varies
<code>corrtol(#)</code>	specify autocorrelation tolerance; default is <code>corrtol(0.01)</code>

Options

Main

`clevel(#)` specifies the credible level, as a percentage, for equal-tailed and HPD credible intervals. The default is `clevel(95)` or as set by [\[BAYES\] set clevel](#).

`hpd` specifies the display of HPD credible intervals instead of the default equal-tailed credible intervals.

`batch(#)` specifies the length of the block for calculating batch means, batch standard deviation, and MCSE using batch means. The default is `batch(0)`, which means no batch calculations. When `batch()` is not specified, MCSE is computed using effective sample sizes instead of batch means. Option `batch()` may not be combined with `corrlag()` or `corrtol()`.

`skip(#)` specifies that every # observations from the MCMC sample not be used for computation. The default is `skip(0)` or to use all observations in the MCMC sample. Option `skip()` can be used to subsample or thin the chain. `skip(#)` is equivalent to a thinning interval of #+1. For example, if you specify `skip(1)`, corresponding to the thinning interval of 2, the command will skip every other observation in the sample and will use only observations 1, 3, 5, and so on in the computation. If you specify `skip(2)`, corresponding to the thinning interval of 3, the command will skip every 2 observations in the sample and will use only observations 1, 4, 7, and so on in the computation. `skip()` does not thin the chain in the sense of physically removing observations from the sample, as is done by, for example, `bayesmh`'s `thinning()` option. It only discards selected observations from the computation and leaves the original sample unmodified.

`nolegend` suppresses the display of the table legend. The table legend identifies the rows of the table with the expressions they represent.

`showeffects` and `showeffects(reref)` are for use after multilevel models, and they specify that the results for all or a list *reref* of random-effects parameters be provided in addition to other model parameters. By default, all random-effects parameters are excluded from the results to conserve computation time.

`display_options`: `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, and `no!stretch`; see [\[R\] estimation options](#).

Advanced

`corrlag(#)` specifies the maximum autocorrelation lag used for calculating effective sample sizes. The default is $\min\{500, \text{mcmcsize}() / 2\}$. The total autocorrelation is computed as the sum of all lag-*k*

autocorrelation values for k from 0 to either `corrlag()` or the index at which the autocorrelation becomes less than `corrtol()` if the latter is less than `corrlag()`. Options `corrlag()` and `batch()` may not be combined.

`corrtol(#)` specifies the autocorrelation tolerance used for calculating effective sample sizes. The default is `corrtol(0.01)`. For a given model parameter, if the absolute value of the lag- k autocorrelation is less than `corrtol()`, then all autocorrelation lags beyond the k th lag are discarded. Options `corrtol()` and `batch()` may not be combined.

Remarks and examples

[stata.com](https://www.stata.com)

Remarks are presented under the following headings:

Introduction

Bayesian summaries for an auto data example

Introduction

`bayesstats summary` reports posterior summary statistics for model parameters and their functions using the current [Bayesian estimation](#) results. When typed without arguments, the command displays results for all model parameters. Alternatively, you can specify a subset of model parameters following the command name; see [Different ways of specifying model parameters](#) in [\[BAYES\] bayesian postestimation](#). You can also obtain results for scalar functions of model parameters; see [Specifying functions of model parameters](#) in [\[BAYES\] bayesian postestimation](#).

Sometimes, it may be useful to obtain posterior summaries of log-likelihood and log-posterior functions. This can be done by specifying `_loglikelihood` and `_logposterior` (or the respective synonyms `_ll` and `_lp`) following the command name.

`bayesstats summary` reports the following posterior summary statistics: posterior mean, posterior standard deviation, MCMC standard error, posterior median, and equal-tailed credible intervals or, if the `hpd` option is specified, HPD credible intervals. The default credible level is set to 95%, but you can change this by specifying the `clevel()` option. Equal-tailed and HPD intervals may produce very different results for asymmetric or highly skewed marginal posterior distributions. The HPD intervals are preferable in this situation.

You should not confuse the term “HPD interval” with the term “HPD region”. A $\{100 \times (1 - \alpha)\}\%$ HPD interval is defined such that it contains $\{100 \times (1 - \alpha)\}\%$ of the posterior density. A $\{100 \times (1 - \alpha)\}\%$ HPD region also satisfies the condition that the density inside the region is never lower than that outside the region. For multimodal univariate marginal posterior distributions, the HPD regions may include unions of nonintersecting HPD intervals. For unimodal univariate marginal posterior distributions, HPD regions are indeed simply HPD intervals. The `bayesstats summary` command thus calculates HPD intervals assuming unimodal marginal posterior distributions ([Chen and Shao 1999](#)).

Some authors use the term “posterior intervals” instead of “credible intervals” and the term “central posterior intervals” instead of “equal-tailed credible intervals” (for example, [Gelman et al. \[2014\]](#)).

Bayesian summaries for an auto data example

Recall our analysis of `auto.dta` from [example 4](#) in [\[BAYES\] bayesmh](#) using the mean-only normal model for `mpg` with a noninformative prior.

```
. use http://www.stata-press.com/data/r15/auto
(1978 Automobile Data)
. set seed 14
. bayesmh mpg, likelihood(normal({var}))
> prior({mpg:_cons}, flat) prior({var}, jeffreys)
Burn-in ...
Simulation ...
Model summary
```

```
Likelihood:
  mpg ~ normal({mpg:_cons},{var})
Priors:
  {mpg:_cons} ~ 1 (flat)
  {var} ~ jeffreys
```

```
Bayesian normal regression          MCMC iterations =    12,500
Random-walk Metropolis-Hastings sampling  Burn-in          =     2,500
                                          MCMC sample size =   10,000
                                          Number of obs    =     74
                                          Acceptance rate  =    .2668
                                          Efficiency: min  =    .09718
                                          avg              =    .1021
                                          max              =    .1071
```

Log marginal likelihood = -234.645

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]	
mpg						
_cons	21.29222	.6828864	.021906	21.27898	19.99152	22.61904
var	34.76572	5.91534	.180754	34.18391	24.9129	47.61286

► Example 1: Summaries for all parameters

If we type `bayesstats summary` without arguments after the `bayesmh` command, we will obtain the same summary table as reported by `bayesmh`.

```
. bayesstats summary
Posterior summary statistics          MCMC sample size =   10,000
```

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]	
mpg						
_cons	21.29222	.6828864	.021906	21.27898	19.99152	22.61904
var	34.76572	5.91534	.180754	34.18391	24.9129	47.61286

The posterior mean of `{mpg:_cons}` is 21.29 and of `{var}` is 34.8. They are close to their respective frequentist analogs (the sample mean of `mpg` is 21.297, and the sample variance is 33.47), because we used a noninformative prior. Posterior standard deviations are 0.68 for `{mpg:_cons}` and 5.92 for `{var}`, and they are comparable to frequentist standard errors under this noninformative prior. The standard error estimates of the posterior means, MCSEs, are low. For example, MCSE is 0.022 for `{mpg:_cons}`. This means that the precision of our estimate is, up to one decimal point, 21.3 provided that MCMC converged. The posterior means and medians of `{mpg:_cons}` are close, which suggests that the posterior distribution for `{mpg:_cons}` may be symmetric. According to the credible

intervals, we are 95% certain that the posterior mean of `{mpg:_cons}` is roughly between 20 and 23 and that the posterior mean of `{var}` is roughly between 25 and 48. We can infer from this that `{mpg:_cons}` is greater than, say, 15, and that `{var}` is greater than, say, 20, with a very high probability. (We can use [\[BAYES\] bayestest interval](#) to compute the actual probabilities.)

The above is also equivalent to typing

```
. bayesstats summary {mpg:_cons} {var}
(output omitted)
```

◀

▷ Example 2: Credible intervals

By default, `bayesstats summary` reports 95% equal-tailed credible intervals. We can change the default credible level by specifying the `clevel()` option.

```
. bayesstats summary, clevel(90)
Posterior summary statistics                                MCMC sample size = 10,000
```

		Mean	Std. Dev.	MCSE	Median	Equal-tailed [90% Cred. Interval]	
mpg	_cons	21.29222	.6828864	.021906	21.27898	20.18807	22.44172
	var	34.76572	5.91534	.180754	34.18391	26.28517	44.81732

As expected, 90% credible intervals are more narrow.

To calculate and report HPD intervals, we specify the `hpd` option.

```
. bayesstats summary, hpd
Posterior summary statistics                                MCMC sample size = 10,000
```

		Mean	Std. Dev.	MCSE	Median	HPD [95% Cred. Interval]	
mpg	_cons	21.29222	.6828864	.021906	21.27898	19.94985	22.54917
	var	34.76572	5.91534	.180754	34.18391	24.34876	46.12339

The posterior distribution of `{mpg:_cons}` is symmetric about the posterior mean; thus there is little difference between the 95% equal-tailed credible interval from [example 1](#) and this 95% HPD credible interval for `{mpg:_cons}`. The 95% HPD interval for `{var}` has a smaller width than the corresponding equal-tailed interval in [example 1](#).

◀

▷ Example 3: Batch-means estimator

`bayesstats summary` provides two estimators for MCSE: effective-sample-size and batch-means. Estimation using effective sample sizes is the default. You can use the `batch(#)` option to request the batch-means estimator, where `#` is the batch size. The optimal batch size depends on the autocorrelation in the MCMC sample. For example, if we observe that the autocorrelation for the parameters of interest is negligible after lag 100, we can specify `batch(100)` to estimate MCSE.

In our example, autocorrelation dies out after about lag 10 (see, for example, [Autocorrelation plots](#) in [BAYES] [bayesgraph](#) and [example 1](#) in [BAYES] [bayesstats ess](#)), so we use 10 as our batch size:

```
. bayesstats summary, batch(10)
Posterior summary statistics          MCMC sample size = 10,000
                                     Batch size       = 10
```

		Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]
mpg	_cons	21.29222	.4842889	.015315	21.27898	19.99152 22.61904
	var	34.76572	4.278417	.135295	34.18391	24.9129 47.61286

Note: Mean, Std. Dev., and MCSE are estimated using batch means.

The batch-means MCSE estimates are somewhat smaller than those obtained by default using effective sample sizes.

Use caution when choosing the batch size for the batch-means method. For example, if you use the batch size of 1, you will obtain MCSE estimates under the assumption that the draws in the MCMC sample are independent, which is not true.



▷ Example 4: Subsampling or thinning the chain

You can reduce correlation between MCMC draws by thinning or subsampling the MCMC chain. You can use the `skip(#)` option to skip every # observations from the MCMC sample, which is equivalent to a thinning interval of # + 1. For example, if you specify `skip(1)`, corresponding to the thinning interval of 2, `bayesstats summary` will skip every other observation in the sample and will use only observations 1, 3, 5, and so on in the computation. If you specify `skip(2)`, corresponding to the thinning interval of 3, `bayesstats summary` will skip every two observations in the sample and will use only observations 1, 4, 7, and so on in the computation. By default, no observations are skipped—`skip(0)`. Note that `skip()` does not thin the chain in the sense of physically removing observations from the sample, as is done by `bayesmh`'s `thinning()` option. It discards only selected observations from the computation and leaves the original sample unmodified.

```
. bayesstats summary, skip(9)
note: skipping every 9 sample observations; using observations 1,11,21,...
Posterior summary statistics          MCMC sample size = 1,000
```

		Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]
mpg	_cons	21.29554	.6813796	.029517	21.27907	19.98813 22.58582
	var	34.7396	5.897313	.206269	33.91782	24.9554 48.11452

We selected to skip every 9 observations, which led to a significant reduction of the MCMC sample size and thus increased our standard deviations. In some cases, with larger MCMC sample sizes, subsampling may decrease standard deviations because of the decreased autocorrelation in the reduced MCMC sample.



▷ Example 5: Summaries for functions of model parameters

`bayesstats summary` accepts expressions to provide summaries of functions of model parameters. For example, we can use expression `(sd:sqrt({var}))` with a label, `sd`, to summarize the standard deviation of `mpg` in addition to the variance.

```
. bayesstats summary (sd:sqrt({var})) {var}
Posterior summary statistics                MCMC sample size =    10,000
      sd : sqrt({var})
```

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]	
sd	5.87542	.4951654	.014972	5.846701	4.991282	6.900207
var	34.76572	5.91534	.180754	34.18391	24.9129	47.61286

Expressions can also be used for calculating posterior probabilities, although this can be more easily done using `bayestest interval` (see [BAYES] [bayestest interval](#)). For illustration, let's verify the probability that `{var}` is within the endpoints of the reported credible interval, indeed 0.95.

```
. bayesstats summary (prob:{var}>24.913 & {var}<47.613)
Posterior summary statistics                MCMC sample size =    10,000
      prob : {var}>24.913 & {var}<47.613
```

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]	
prob	.9502	.2175424	.005301	1	0	1

◀

▷ Example 6: Summaries for log likelihood and log posterior

We can use reserved names `_loglikelihood` (or the synonym `_ll`) and `_logposterior` (or the synonym `_lp`) to obtain summaries of the log likelihood and log posterior for the simulated MCMC sample.

```
. bayesstats summary _ll _lp
Posterior summary statistics                MCMC sample size =    10,000
      _ll : _loglikelihood
      _lp : _logposterior
```

	Mean	Std. Dev.	MCSE	Median	Equal-tailed [95% Cred. Interval]	
_ll	-235.4162	.990654	.032232	-235.1379	-238.1236	-234.4345
_lp	-238.9507	1.037785	.034535	-238.6508	-241.7889	-237.9187

◀

Stored results

`bayesstats summary` stores the following in `r()`:

Scalars

<code>r(clevel)</code>	credible interval level
<code>r(hpd)</code>	1 if <code>hpd</code> is specified, 0 otherwise
<code>r(batch)</code>	batch length for batch-means calculations
<code>r(skip)</code>	number of MCMC observations to skip in the computation; every <code>r(skip)</code> observations are skipped
<code>r(corrflag)</code>	maximum autocorrelation lag
<code>r(corrtol)</code>	autocorrelation tolerance

Macros

<code>r(expr_#)</code>	#th expression
<code>r(names)</code>	names of model parameters and expressions
<code>r(exprnames)</code>	expression labels

Matrices

<code>r(summary)</code>	matrix with posterior summaries statistics for parameters in <code>r(names)</code>
-------------------------	--

Methods and formulas

Methods and formulas are presented under the following headings:

Point estimates
Credible intervals

Most of the summary statistics employed in Bayesian analysis are based on the marginal posterior distributions of individual model parameters or functions of model parameters.

Let θ be a scalar model parameter and $\{\theta_t\}_{t=1}^T$ be an MCMC chain of size T drawn from the marginal posterior distribution of θ . For a function $g(\theta)$, substitute $\{\theta_t\}_{t=1}^T$ with $\{g(\theta_t)\}_{t=1}^T$ in the formulas below. If θ is a covariance matrix model parameter, the formulas below are applied to each element of the lower-diagonal portion of θ .

Point estimates

Marginal posterior moments are approximated using the Monte Carlo integration applied to the simulated samples $\{\theta_t\}_{t=1}^T$.

Sample posterior mean and sample standard deviation are defined as follows,

$$\hat{\theta} = \frac{1}{T} \sum_{t=1}^T \theta_t, \quad \hat{s}^2 = \frac{1}{T-1} \sum_{t=1}^T (\theta_t - \hat{\theta})^2$$

where $\hat{\theta}$ and \hat{s}^2 are sample estimators of the population posterior mean $E(\theta_t)$ and posterior variance $\text{Var}(\theta_t)$.

The precision of the sample posterior mean is evaluated by its standard error, also known as the Monte Carlo standard error (MCSE). Note that MCSE cannot be estimated using the classical formula for the standard error, \hat{s}/\sqrt{T} , because of the dependence between θ_t 's.

Let

$$\sigma^2 = \text{Var}(\theta_t) + 2 \sum_{k=1}^{\infty} \text{Cov}(\theta_t, \theta_{t+k})$$

Then, $\sqrt{T} \times \text{MCSE}$ approaches σ^2 asymptotically in T .

`bayesstats summary` provides two different approaches for estimating MCSE. Both approaches try to adjust for the existing autocorrelation in the MCMC sample. The first one uses the so-called effective sample size (ESS), and the second one uses batch means (Roberts 1996; Jones et al. 2006).

The ESS-based estimator for MCSE, the default in `bayesstats summary`, is given by

$$\text{MCSE}(\hat{\theta}) = \hat{s} / \sqrt{\text{ESS}}$$

ESS is defined as

$$\text{ESS} = T / (1 + 2 \sum_{k=1}^{\text{max_lags}} \rho_k)$$

where ρ_k is the lag- k autocorrelation, and `max_lags` is the maximum number less than or equal to ρ_{lag} such that for all $k = 1, \dots, \text{max_lags}$, $|\rho_k| > \rho_{\text{tol}}$, where ρ_{lag} and ρ_{tol} are specified in options `corrlag()` and `corrtol()` with the respective default values of 500 and 0.01. ρ_k is estimated as γ_k / γ_0 , where

$$\gamma_k = \frac{1}{T} \sum_{t=1}^{T-k} (\theta_t - \hat{\theta})(\theta_{t+k} - \hat{\theta})$$

is the lag- k empirical autocovariance.

The batch-means estimator of MCSE is obtained as follows. For a given batch of length b , the initial MCMC chain is split into m batches of size b ,

$$\{\theta_{j'+1}, \dots, \theta_{j'+b}\} \{\theta_{j'+b+1}, \dots, \theta_{j'+2b}\} \dots \{\theta_{T-b+1}, \dots, \theta_T\}$$

where $j' = T - m \times b$ and m batch means $\hat{\mu}_1, \dots, \hat{\mu}_m$ are calculated as sample means of each batch. m is chosen as the maximum number such that $m \times b \leq T$. If m is not a divisor of T , the first $T - m \times b$ observations of the sample are not used in the batch-means computation. The batch-means estimator of the posterior variance, \hat{s}_{batch}^2 , is based on the assumption that $\hat{\mu}_j$ s are much less correlated than the original sample draws.

The batch-means estimators of the posterior mean and posterior variance are

$$\hat{\theta}_{\text{batch}} = \frac{1}{m} \sum_{j=1}^m \hat{\mu}_j, \quad \hat{s}_{\text{batch}}^2 = \frac{1}{m-1} \sum_{j=1}^m (\hat{\mu}_j - \hat{\theta}_{\text{batch}})^2$$

We have $\hat{\theta}_{\text{batch}} = \hat{\theta}$, whenever $m \times b = T$. Under the assumption that the batch means are uncorrelated, \hat{s}_{batch}^2 can be used as an estimator of σ^2/b . This fact justifies the batch-means estimator of MCSE given by

$$\text{MCSE}_{\text{batch}}(\hat{\theta}) = \frac{\hat{s}_{\text{batch}}}{\sqrt{m}}$$

The accuracy of the batch-means estimator depends on the choice of the batch length b . The higher the autocorrelation in the original MCMC sample, the larger the batch length b should be, provided that the number of batches m does not become too small; \sqrt{T} is typically used as the maximum value for b . The batch length is commonly determined by inspecting the autocorrelation plot for θ . Under certain assumptions, Flegal and Jones (2010) establish that an asymptotically optimal batch size is of order $T^{1/3}$.

Credible intervals

Let $\theta_{(1)}, \dots, \theta_{(T)}$ be an MCMC sample ordered from smallest to largest. Let $(1 - \alpha)$ be a credible level. Then, a $\{100 \times (1 - \alpha)\}\%$ equal-tailed credible interval is

$$(\theta_{(\lceil T\alpha/2 \rceil)}, \theta_{(\lceil T(1-\alpha/2) \rceil)})$$

where $\lceil \cdot \rceil$ in the above imply an integer number.

A $\{100 \times (1 - \alpha)\}\%$ HPD interval is defined as the shortest interval among the $\{100 \times (1 - \alpha)\}\%$ credible intervals $(\theta_{(j)}, \theta_{(j+\lceil T(1-\alpha) \rceil)})$, $j = 1, \dots, T - \lceil T(1 - \alpha) \rceil$.

References

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- Flegal, J. M., and G. L. Jones. 2010. Batch means and spectral variance estimators in Markov chain Monte Carlo. *Annals of Statistics* 38: 1034–1070.
- Gelman, A., J. B. Carlin, H. S. Stern, D. B. Dunson, A. Vehtari, and D. B. Rubin. 2014. *Bayesian Data Analysis*. 3rd ed. Boca Raton, FL: Chapman & Hall/CRC.
- Jones, G. L., M. Haran, B. S. Caffo, and R. Neath. 2006. Fixed-width output analysis for Markov chain Monte Carlo. *Journal of the American Statistical Association* 101: 1537–1547.
- Roberts, G. O. 1996. Markov chain concepts related to sampling algorithms. In *Markov Chain Monte Carlo in Practice*, ed. W. R. Gilks, S. Richardson, and D. J. Spiegelhalter, 45–57. Boca Raton, FL: Chapman and Hall.

Also see

- [BAYES] [bayes](#) — Bayesian regression models using the bayes prefix
- [BAYES] [bayesmh](#) — Bayesian models using Metropolis–Hastings algorithm
- [BAYES] [bayesian estimation](#) — Bayesian estimation commands
- [BAYES] [bayesian postestimation](#) — Postestimation tools for bayesmh and the bayes prefix
- [BAYES] [bayesgraph](#) — Graphical summaries and convergence diagnostics
- [BAYES] [bayesstats ess](#) — Effective sample sizes and related statistics
- [BAYES] [bayestest interval](#) — Interval hypothesis testing