bayes: qreg — Bayesian quantile regression

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Description

bayes: qreg fits a Bayesian quantile regression to a continuous outcome; see [BAYES] bayes and [R] qreg for details.

Quick start

Bayesian median regression of y on x1 and x2, using default normal priors for regression coefficients

bayes: qreg y x1 x2

Same as above, and fix the scale σ equal to 1

bayes, sigma(1): qreg y x1 x2

Use a standard deviation of 10 instead of 100 for the default normal priors

bayes, normalprior(10): qreg y x1 x2

Use uniform priors for the slopes and a normal prior for the intercept

bayes, prior({y_q50: x1 x2}, uniform(-10,10)) ///
prior({y_q50: _cons}, normal(0,10)): qreg y x1 x2

Bayesian quantile regression of the 75th percentile of y conditional on x1 and x2

bayes: qreg y x1 x2, quantile(0.75)

Same as above, but use uniform priors for the slopes and a normal prior for the intercept

bayes, prior({y_q75: x1 x2}, uniform(-10,10)) ///
prior({y_q75: _cons}, normal(0,10)): greg y x1 x2, quantile(0.75)

Save simulation results to simdata.dta, and use a random-number seed for reproducibility

bayes, saving(simdata) rseed(123): qreg y x1 x2

Specify 20,000 Markov chain Monte Carlo (MCMC) samples, set length of the burn-in period to 5,000, and request that a dot be displayed every 500 simulations

bayes, mcmcsize(20000) burnin(5000) dots(500): qreg y x1 x2

In the above, request that the 90% highest posterior density (HPD) credible interval be displayed instead of the default 95% equal-tailed credible interval

bayes, clevel(90) hpd

Also see Quick start in [BAYES] bayes and Quick start in [R] qreg.

Menu

 $Statistics > Linear \ models \ and \ related > Bayesian \ regression > Quantile \ regression$

Syntax

bayes [, bayesopts]: qreg depvar [indepvars] [if] [in] [weight] [, options]

options	Description
Model quantile(#)	estimate # quantile; default is quantile(.5)
<u>nocons</u> tant	suppress constant term
Reporting	control appring line width and base and apprty calls
display_options	control spacing, line width, and base and empty cells
<u>l</u> evel(#)	set credible level; default is level(95)
indepvars may contain fa	ctor variables; see [U] 11.4.3 Factor variables.
C	

fweights are allowed; see [U] 11.1.6 weight.

bayes: qreg, level() is equivalent to bayes, clevel(): qreg. For a detailed description of options, see *Options* in [R] qreg.

bayesopts	Description
Priors	
* sigma(#)	specify a fixed scale σ ; default is random σ parameter with inverse-gamma prior
* <u>normalpr</u> ior(#)	specify standard deviation of default normal priors for regression coefficients; default is normalprior(100)
* <pre>igammaprior(##)</pre>	specify shape and scale of default inverse-gamma prior for scaling factor σ ; default is igammaprior(0.010.01)
prior(<i>priorspec</i>)	prior for model parameters; this option may be repeated
dryrun	show model summary without estimation
Simulation	
nchains(#)	number of chains; default is to simulate one chain
<pre>mcmcsize(#)</pre>	MCMC sample size; default is mcmcsize(10000)
<u>burn</u> in(#)	burn-in period; default is burnin(2500)
<u>thin</u> ning(#)	thinning interval; default is thinning(1)
rseed(#)	random-number seed
<pre><u>excl</u>ude(paramref)</pre>	specify model parameters to be excluded from the simulation results
Blocking	
*blocksize(#)	maximum block size; default is blocksize(50)
<pre>block(paramref[, blockopts])</pre>	
<u>blocksumm</u> ary	display block summary
* <u>noblock</u> ing	do not block parameters by default

Initialization <u>init</u> ial(<i>initspec</i>)	specify initial values for model parameters with a single chain
<pre>init#(initspec)</pre>	specify initial values for #th chain; requires nchains()
<pre>initall(initspec)</pre>	specify initial values for all chains; requires nchains()
<u>nomleinit</u> ial	suppress the use of linear programming estimates as starting values
<u>initrand</u> om	specify random initial values
<u>initsumm</u> ary	display initial values used for simulation
* <u>noi</u> sily	display output from the estimation command during initialization
Reporting	
<u>clev</u> el(#)	set credible interval level; default is clevel(95)
hpd	display HPD credible intervals instead of the default equal-tailed credible intervals
batch(#)	specify length of block for batch-means calculations; default is batch(0)
<pre>saving(filename[, replace])</pre>	save simulation results to <i>filename</i> .dta
nomodelsummary	suppress model summary
chainsdetail	display detailed simulation summary for each chain
[no]dots	suppress dots or display dots every 100 iterations and iteration numbers every 1,000 iterations; default is nodots
dots(#[, every(#)])	display dots as simulation is performed
[no]show(paramref)	specify model parameters to be excluded from or included in the output
<u>notab</u> le	suppress estimation table
<u>nohead</u> er	suppress output header
<pre>title(string)</pre>	display string as title above the table of parameter estimates
display_options	control spacing, line width, and base and empty cells
Advanced	
<pre>search(search_options)</pre>	control the search for feasible initial values
corrlag(#)	specify maximum autocorrelation lag; default varies
corrtol(#)	specify autocorrelation tolerance; default is corrtol(0.01)

* Starred options are specific to the bayes prefix; other options are common between bayes and bayesmh. priorspec and paramref are defined in [BAYES] bayesmh.

paramref may contain factor variables; see [U] 11.4.3 Factor variables.

collect is allowed; see [U] 11.1.10 Prefix commands.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Model parameters are regression coefficients {*depvar_q#:indepvars*} and scaling factor {sigma}. Use the dryrun option to see the definitions of model parameters prior to estimation.

For a detailed description of bayesopts, see Options in [BAYES] bayes.

Remarks and examples

For a general introduction to Bayesian analysis, see [BAYES] **Intro**. For a general introduction to Bayesian estimation using an adaptive Metropolis–Hastings algorithm, see [BAYES] **bayesmh**. For remarks and examples specific to the bayes prefix, see [BAYES] **bayes**. For details about the estimation command, see [R] **qreg**.

For a simple example of the bayes prefix, see Introductory example in [BAYES] bayes.

Example 1: Median regression

Consider the following dataset from budget surveys administered to European households in the 19th century, described in Koenker and Bassett (1982). The data are originally from Engel (1857), who argued that as household income increases, food expenditure takes up a smaller share. We have the households' annual income, income, and annual food expenditure, foodexp.

```
. use https://www.stata-press.com/data/r19/engel1857
(European household budget survey)
. describe
Contains data from https://www.stata-press.com/data/r19/engel1857.dta
Observations:
                          235
                                               European household budget survey
   Variables:
                            2
                                               7 Dec 2024 11:11
                                                ( dta has notes)
                        Display
Variable
              Storage
                                    Value
   name
                         format
                                    label
                                                Variable label
                 type
income
                float
                        %9.0g
                                                Annual household income (1,000s
                                                 Belgian francs)
                        %9.0g
                                                Annual household food expenditure
foodexp
                float
                                                  (1,000s Belgian francs)
```

Sorted by:

Below, we fit a Bayesian quantile regression model with outcome variable foodexp and predictor variable income. By default, bayes:qreg fits a median regression model; in other words, we model the 50th percentile of foodexp.

. bayes, rseed Burn-in	1(19): qreg 1	foodexp inco	ome			
Simulation						
Model summary						
Likelihood: foodexp ~ as	symlaplaceq()	cb_foodexp_c	[50,{sigma]	},.5)		
Priors: {foodexp_q50):income _con {sign	ns} ~ normal na} ~ igamma		1)		(1)
(1) Parameters	s are element	ts of the li	near form.	xb_foodexp	_q50.	
Bayesian quant Random-walk Me	0		ing	Burn-in	rations = =	12,500 2,500
					ple size =	10,000
Quantile = $.5$				Number o		235 .3603
				Efficien	ce rate = = = = = = = = = = = = = = = = = = =	.09896
				LIIICICH	avg =	.00050
Log marginal-	likelihood =	186.43947			max =	.2268
					Equal-	tailed
	Mean	Std. dev.	MCSE	Median	-	interval]
foodexp_q50						
income	.5567276	.0159401	.000507	.5562547	.5248025	.587735
_cons	.084986	.0143782	.000403	.0851108	.0575581	.1134264
sigma	.0377533	.0024907	.000052	.0376511	.0331066	.0430957

Using the mean posterior estimates for coefficients, we can express the relationship between the households' annual income and the annual food expenditure as

$$\texttt{foodexp}_{\texttt{median}} = 0.56 \times \texttt{income} + 0.08$$

The median food expenditure is 640 Belgian francs for a household with an income of 1,000 Belgian francs (0.56 + 0.08 = 0.64); note that both income and foodexp are measured in 1,000s of Belgian francs. For this household, food expenditure comprises 64% of income (640/1000 = 0.64). However, the median food expenditure is 2,320 for a household with an income of 4,000 Belgian francs ($0.56 \times 4 + 0.08 = 2.32$); the median food expenditure comprises 58% of household income, as opposed to 64% for a household making 1,000 annually.

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Example 2: Estimating other quantiles

We can check whether the effect of income varies across different quantiles of foodexp by comparing the median regression model from our last example with models for the 25th and 75th percentiles; we will use the quantile() option to specify the quantile levels of the outcome.

We use the collect prefix to collect results from each model, to be displayed in a table, and we store regression coefficients as scalars for later use.

```
. collect get: bayes, rseed(19): qreg foodexp income, quantile(0.25)
Burn-in ...
Simulation ...
Model summary
Likelihood:
  foodexp ~ asymlaplaceq(xb_foodexp_q25,{sigma},.25)
Priors:
  {foodexp_q25:income _cons} ~ normal(0,10000)
                                                                              (1)
                     {sigma} ~ igamma(0.01,0.01)
(1) Parameters are elements of the linear form xb_foodexp_q25.
Bayesian quantile regression
                                                   MCMC iterations =
                                                                           12,500
Random-walk Metropolis-Hastings sampling
                                                   Burn-in
                                                                    =
                                                                           2,500
                                                   MCMC sample size =
                                                                           10,000
Quantile = .25
                                                                              235
                                                   Number of obs
                                                                  =
                                                   Acceptance rate =
                                                                            .3423
                                                   Efficiency: min =
                                                                            .1436
                                                                avg =
                                                                            .1765
Log marginal-likelihood =
                           169.18624
                                                                max =
                                                                            .2421
                                                                Equal-tailed
                    Mean
                            Std. dev.
                                          MCSE
                                                    Median
                                                            [95% cred. interval]
foodexp_q25
      income
                 .4718604
                            .0140225
                                        .00037
                                                  .4735463
                                                             .4414884
                                                                         .4948657
       _cons
                 .0962851
                            .0116976
                                       .000308
                                                  .0957929
                                                             .0742573
                                                                         .1196877
                            .0020364
                                       .000041
                                                  .0303373
                                                                         .0347907
       sigma
                 .0304463
                                                             .0266857
```

. scalar bqr1_b1 = e(mean)[1,1]

. scalar $bqr1_b0 = e(mean)[1,2]$

(1)

. collect get: bayes, rseed(19): qreg foodexp income, quantile(0.5)
Burn-in ...
Simulation ...
Model summary
Likelihood:
 foodexp ~ asymlaplaceq(xb_foodexp_q50,{sigma},.5)
Priors:
 {foodexp_q50:income _cons} ~ normal(0,10000)
 {sigma} ~ igamma(0.01,0.01)

(1) Parameters are elements of the linear form	xb_foodexp_q50.	
Bayesian quantile regression	MCMC iterations =	12,500
Random-walk Metropolis-Hastings sampling	Burn-in =	2,500
	MCMC sample size =	10,000
Quantile = .5	Number of obs =	235
	Acceptance rate =	.3603
	Efficiency: min =	.09896
	avg =	.151
Log marginal-likelihood = 186.43947	max =	.2268

	Mean	Std. dev.	MCSE	Median	Equal- [95% cred.	
foodexp_q50 income _cons	.5567276 .084986	.0159401 .0143782	.000507 .000403	.5562547 .0851108	.5248025 .0575581	.587735 .1134264
sigma	.0377533	.0024907	.000052	.0376511	.0331066	.0430957

. scalar $bqr2_b1 = e(mean)[1,1]$

. scalar $bqr2_b0 = e(mean)[1,2]$

. collect get: bayes, rseed(19): greg foodexp income, quantile(0.75) Burn-in ... Simulation ... Model summary Likelihood: foodexp ~ asymlaplaceq(xb foodexp q75,{sigma},.75) Priors: {foodexp_q75:income _cons} ~ normal(0,10000) (1){sigma} ~ igamma(0.01,0.01) (1) Parameters are elements of the linear form xb_foodexp_q75. Bayesian quantile regression MCMC iterations = 12,500 Random-walk Metropolis-Hastings sampling Burn-in 2,500 MCMC sample size = 10,000 Quantile = .75Number of obs 235 = Acceptance rate = .3103 Efficiency: min = .1421 .1704 avg = Log marginal-likelihood = 188.25668 .2262 max = Equal-tailed Std. dev. MCSE [95% cred. interval] Mean Median foodexp q75 income .6456717 .0170002 .000451 .6461026 .6089782 .6757706 .0606789 .014418 .000381 .060412 .034519 .0924086 cons sigma .0280768 .0018942 .00004 .0279643 .0245888 .0321131 . scalar bqr3_b1 = e(mean)[1,1] . scalar $bqr3_b0 = e(mean)[1,2]$. collect label levels colname income "Annual household income", modify . collect label levels cmdset 1 "25th" 2 "50th" 3 "75th" . collect layout (colname[income]#result[mean sd]) (cmdset) Collection: default Rows: colname[income]#result[mean sd] Columns: cmdset Table 1: 3 x 3

	25th	50th	75th
Annual household income Posterior means Std. dev.		.5567276 .0159401	

Before we lay out our table, we shorten the label for income, and we label the results with the quantile being estimated. To learn more about modifying labels in a collection and laying out a table, see [TABLES] **collect label** and [TABLES] **collect layout**. The table shows that the coefficient of income increases across the quantiles, from 0.472 for the 25th quantile to 0.646 for the 75th quantile. Below, we plot the posterior mean quantile lines corresponding to the three models.

```
. twoway (scatter foodexp income, mcolor(%30)) ||
> (function y = bqr3_b1 * x + bqr3_b0, range(0.2 5) lcolor(black)) ||
> (function y = bqr2_b1 * x + bqr2_b0,
                                           range(0.2 5) lcolor(black)) ||
> (function y = bqr1_b1 * x + bqr1_b0, range(0.2 5) lcolor(black)),
> legend(off) xtitle("Annual household income")
> ytitle("Annual household food expenditure") aspect(1)
> text(3.3 5.3 "75th" 2.9 5.3 "50th" 2.4 5.3 "25th")
                           4
                         Annual household food expenditure
                                                                   75th
                           3
                                                                   50th
                                                                   25th
                           2
                           0
                                                                  5
                              Ó
                                            2
                                                   3
                                                           4
```

The above plot of foodexp versus income (and the fitted quantile lines) indicates the potential presence of heteroskedasticity, although this inference may require further verification.

Annual household income

In contrast to quantile regression, the linear regression model assumes homoskedasticity of the outcome with respect to each predictor variable, meaning that the residual variance is uniform throughout the range of predicted values. A formal comparison between quantile and linear regression models will show which one provides a better fit for the data.

We first run the linear and the median regression models and store the estimation results in memory with estimates store. Then, we use the bayestest model command to compute and compare the posterior model probabilities.

- . quietly bayes, rseed(19) saving(meanreg_sim, replace): regress foodexp income
- . estimates store meanreg
- . quietly bayes, rseed(19) saving(medianreg_sim, replace): qreg foodexp income
- . estimates store medianreg
- . bayestest model meanreg medianreg

Bayesian model tests

	log(ML)	P(M)	P(M y)
meanreg	152.5311	0.5000	0.0000
medianreg	186.4395	0.5000	1.0000

Note: Marginal likelihood (ML) is computed using Laplace-Metropolis approximation.

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The median regression model, with an estimated posterior model probability of 1, provides an overwhelmingly better fit than the simple linear regression, which is consistent with the noted heteroskedasticity of the outcome foodexp.

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Stored results

See Stored results in [BAYES] bayes. In addition, bayes: qreg stores the following in e():

Scalars e(q) e(q_v)

quantile requested value of the quantile

Methods and formulas

In the context of quantile regression, it is instructive to consider the optimization process as outlined in *Methods and formulas* of [R] qreg.

Let τ be the target estimation quantile of the outcome. For the *i*th observation, let \mathbf{x}_i be the vector of independent variables and y_i be the outcome value. The *i*th residual is $\varepsilon_i = y_i - \mathbf{x}'_i \boldsymbol{\beta}_{\tau}$, where $\boldsymbol{\beta}_{\tau}$ is a quantile-specific vector of coefficients that is subject to estimation.

The objective function under consideration seeks to minimize a specific criterion:

$$\min_{\boldsymbol{\beta}_{\tau}} \sum_{i} c_{\tau}(\varepsilon_{i}) \tag{1}$$

Here $c_{\tau}(\varepsilon_i)$ is defined as $c_{\tau}(\varepsilon_i) = \{\tau - \mathbf{1}(\varepsilon_i < 0)\}\varepsilon_i$, where $\mathbf{1}(\cdot)$ is an indicator function.

Yu and Moyeed (2001) proposed an alternative representation of (1), wherein the optimization problem was reformulated as the maximization of a likelihood function employing the asymmetric Laplace distribution (ALD).

The probability density function of ALD can be defined as

$$f_\tau(x;\mu,\sigma) = \frac{\tau(1-\tau)}{\sigma} \exp\left\{-c_\tau\left(\frac{x-\mu}{\sigma}\right)\right\}, \ \sigma > 0$$

where μ is a location parameter and σ is a scale parameter.

The likelihood function of a quantile regression with outcome observations y_i and covariates \mathbf{x}_i , i = 1, ..., n, is a product of ALDs with location parameters $\mu_i = \mathbf{x}'_i \boldsymbol{\beta}_{\tau}$,

$$L(\mathbf{y}|\boldsymbol{\beta}_{\tau},\sigma) = \prod_{i=1}^{n} f_{\tau}(y_{i};\mathbf{x}_{i}^{\prime}\boldsymbol{\beta}_{\tau},\sigma) = \frac{\tau^{n}(1-\tau)^{n}}{\sigma^{n}} \exp\left\{-\sum_{i} c_{\tau}\left(\frac{y_{i}-\mathbf{x}_{i}^{\prime}\boldsymbol{\beta}_{\tau}}{\sigma}\right)\right\}$$

Bayesian quantile regression considers a posterior distribution of β_{τ} and σ , denoted as $p(\beta_{\tau}, \sigma | \mathbf{y})$, which is proportional to the product of the likelihood function and a prior distribution for β_{τ} and σ , $\pi(\beta_{\tau}, \sigma)$,

$$p(\boldsymbol{\beta}_{\tau},\sigma|\mathbf{y}) \propto L(\mathbf{y}|\boldsymbol{\beta}_{\tau},\sigma)\pi(\boldsymbol{\beta}_{\tau},\sigma)$$

The default prior distribution choices are independent normal with mean 0 and variance of 10,000 for β_{τ} and inverse-gamma with shape 0.01 and scale of 0.01 for σ . The bayes: qreg command performs estimation using adaptive Metropolis–Hastings sampling.

See Methods and formulas in [BAYES] bayesmh.

References

- Engel, E. 1857. Die productions-und consumtionsverhältnisse des königreichs sachsen. Zeitschrift des Statistischen Bureaus des Königlich Sächsischen Ministeriums des Innern 8: 1–54.
- Koenker, R., and G. Bassett, Jr. 1982. Robust tests for heteroscedasticity based on regression quantiles. *Econometrica* 50: 43–61. https://doi.org/10.2307/1912528.
- Yu, K., and R. A. Moyeed. 2001. Bayesian quantile regression. *Statistics and Probability Letters* 54: 437–447. https://doi.org/10.1016/S0167-7152(01)00124-9.

Also see

- [BAYES] bayes Bayesian regression models using the bayes prefix
- [R] **qreg** Quantile regression
- [BAYES] Bayesian postestimation Postestimation tools after Bayesian estimation
- [BAYES] Bayesian estimation Bayesian estimation commands
- [BAYES] Bayesian commands Introduction to commands for Bayesian analysis
- [BAYES] Intro Introduction to Bayesian analysis
- [BAYES] Glossary

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