## Negative Binomial Regression Second edition, Cambridge University Press

### Joseph M Hilbe

# ERRATA & COMMENTS

#### as of 12 January, 2012

Note: Data sets and software code can be downloaded from: http://works.bepress.com/joseph\_hilbe/

### ERRATA

P. xv: The inset following the 1st paragraph on page, replace *Stata* bookstore (Stata files/commands) http://www.Stata.com/bookstore/nbr2.html

#### with

Data sets, software code, and electronic *Extensions* to the text can be downloaded from: http://works.bepress.com/joseph\_hilbe/

p 17 Table 2.2. second to last line in table, final term should read, prop.r=FALSE, not pror.r. Read as

```
CrossTable(survived, age, prop.t=FALSE, prop.r=FALSE, prop.c=FALSE, prop.chisq=FALSE)
```

p 18 Equation 2.3. The second or middle term should have (implied) product signs, not division. Read as:

 $\frac{D/(B+D)}{C/(A+C)} = \frac{D(A+C)}{C(B+D)} = \frac{AD+CD}{BC+CD}$ 

p. 20 Table 2.4. The final three lines can be reduced to one line: irr\*rse. Revise Table 2.4 to appear as:

Table 2.4: *R* – *Poisson model with robust standard errors* 

P 22 Table 2.5. The final three lines of the code may be condensed to one line, as it was in Table 2.4 above. Replace

```
ec2 <- c(irr2)
rs2 <- c(rse2)
ec2 * rs2
with
irr2 * rse2</pre>
```

P 27 The interpretation of the model (in italics) can better be expressed as: "*the odds of a child surviving on the Titanic is nearly twice that of adults.*" The line below the italicized interpretation should be amended to that this section in the book should read as:

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The odds of a child surviving on the Titanic is nearly twice that of adults.

Note that the risk of adults surviving compared to children is some 53%.

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p 28. Second to last sentence in the second full paragraph, just above table of joint probabilities. Amend to read:

"... or 25% (0.247) of the passengers are first class.

P 42. Typo. Seventh (-7) line from bottom of page, the word quadrature is misspelled.

p 52 Typo: Table 4.1. One line was inadvertently made into two. Line 9 of the code should appear as  $\eta = X\beta + offset$ . Delete line 10. The middle code should then appear as:

Table 4.1

Dev = 0  $z = \eta + (y - \mu)g' - offset$   $\beta = (X'wX)^{-1}X'wz$   $\eta = X\beta + offset$   $\mu = g^{-1}(\eta)$ Dev0 = Dev  $\vdots$   $\vdots$  P 66: Amend the top of page 66 including the R table, also changing place of INTERCEPT-ONLY MODEL to under Table. The top section of page 66 should appear as:

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INTERCEPT-ONLY MODEL

. poisson los, nolog

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p 66. Typo. Line under the second model output: "... twolog-likelihood..." should read "... two log-likelihood..."

p 69: Amend paragraph above Equation 5.17 to read as follows. Delete Equation 5.19 and substitute the new formula for 5.18 as displayed below. Re-label later equation numbers in the Chapter to adjust for dropping 5.19.. The revision appears as shown between the two lines below. **Substitute what is between lines below for current text**.

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Other AIC statistics have been used in research. The most popular -- other than the two above primary versions – is the finite sample AIC, which may be defined as

or

R

$$AIC_{FS} = -2\{\mathcal{L} - k - k(k+1)/(n-k-1)\}/n$$
(5.17)

$$AIC_{FS} = AIC + \frac{2k(k+1)}{n-k-1}$$

(5.18)

where k is the number of parameters in the model. Note that AIC<sub>FS</sub> employs a greater penalty for added parameters compared to the standard AIC statistic. Note also that AIC<sub>FS</sub>  $\approx$  AIC for models with large numbers of observations. Hurvich and Tsai (1989) first developed the finite sample AIC for use with time series and auto-correlated data, however others have considered it as preferred to AIC for models with non-correlated data as well, particularly for models with many parameters, and/or for models with comparatively few observations. A variety of finite sample AIC statistics have been designed since 1989. We will say no more about it here.

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P 69: paragraph under new equation 5.18, line 3. Change (2007) to (2009)

p 70: Bottom of page. Equation 5.20. Re-label to (5.19)

p 71: Re-label Equation (5.21) to (5.20) Re-label Equation (5.22) to (5.21)

P 71: The first part of Ch 7.2.2.2 have several changes to the text. Please substitute the passage between lines below for the same material in the book.

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#### **5.2.2.2 Bayesian Information Criterion**

The second foremost contemporary comparative fit statistic for likelihood-based statistical models is the Bayesian Information Criterion (BIC). Again, this statistic has undergone a variety of parameterizations. The original formulation is from Gideon Schwarz (1978), who based his reasoning on Bayesian grounds. The statistic is now commonly known as the Schwarz Criterion (SC) in SAS and as BIC in other software. It is formulated as Equation 5.21.

The original formulation of BIC for GLMs was given by the University of Washington's Adrian Raftery in 1986. Based on the deviance statistic, it is given as:

$$BIC_{R} = D - (df) \ln(n)$$
(5.20)

where D is the model deviance statistic and df is the model residual degrees of freedom. The residual degrees of freedom is defined as the number of observations minus the number of predictors in the model including the intercept. Stata's **glm** command provides this parameterization of BIC statistic in its output. Most other applications, including R and other BIC statistics in Stata other than in **glm**, employ the formula in Equation 5.21, or Schwarz Criterion, as the meaning of BIC. The statistic is defined as

$$BIC_{L} = -2\mathcal{L} + k \ln(n) \tag{5.21}$$

with *k* indicating the number of predictors, including intercept, in the model. It is clear that  $BIC_R$  and  $BIC_L$  have very different values. BIC statistics usually give a higher value to the second term of the function than do AIC statistics.

It should be noted that both the AIC and BIC statistics have been designed to be used with non-nested models of the same general distributional type having the same number of observations. However, they are both robust tests that some statisticians have used to compare non-nested models with slightly different numbers of observations. We may engage the statistics accordingly, but only as a rough guide to model preference.

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P 73: Re-label Equation (5.23) to (5.22)

p 110. Comment. The inset unnamed table: The top line, rm(list=ls()), removes all objects in the current workspace. Not everyone may want to do this. If not, delete or comment out, this line. Add comment to top line in table to appear as:

rm(list=ls()) # Warning! Removes all objects in the current workspace.

p 119: Add to the single paragraph above the un-numbered Table at the bottom of the page:

-

Note that if the observed counts have extremely high values compared to the distributional mean, the fit at the extremes will be poor. Recall that for a given distributional mean, values far from it will have increasingly lower *p*-values.

p 127. Typo; Second to last line of code near bottom of page. The comment displays the Greek letter  $\beta$ , which is not Stata text. Replace  $\beta$  with b.

P 129: Second code group on page. The "--" should be "-", a minus sign.

p 135. Typo: Equation 6.59. The equation needs a close parenthesis.

$$\mu_i = \exp(x'_i\beta + \ln(t_i)) \tag{6.59}$$

p 159: top line on page. Amend line to read:

"...with the product of the model standard error and the square root of the dispersion. Scaling by the Pearson..."

p159: Table 7.7: change the final line in the table to read (and delete current comment):  $w = se(\beta) * sqrt(sc)$ 

p 166: The first sentence of the second paragraph, replace the word "scaled" with "quasi-likelihood". It should read as:

"One may calculate the quasi-likelihood standard errors, which also results in a type of quasilikelihood model, by hand using the following formula"

The first sentence in the following paragraph needs to have the word "quasi-likelihood" inserted between the beginning words 'The same" and "may be obtained..." :

"The same quasi-likelihood model may be obtained by employing the Pearson dispersion statistic as an importance weight."

p. 166: missing term. Following final text on page, the first line of code should read:

```
. gen pearsond = 6.260391
```

p. 172: top line of code on page, there is a space between "saving" and (bsmedpar) which needs to be closed. "rep()" should be "reps()". Code to read:

```
. glm los hmo white type2 type3, fam(poi) vce(bootstrap) reps(1000) saving(bsmedpar))
```

Page 178: last line on page: The "di" was excluded from the start of code, and 0 should be displayed under the code. Should read:

. di chiprob(1, 4830)/2 0

Page 181: both the R code and Stata code are missing a line, which causes a problem in subsequent output. For the R code at top of page, after "xb <- 3" add line: exb <- exp(xb). In the Stata code, add a line after "gen xb = 3" to read: . gen exb = exp(xb) Lines should appear as:

```
R
```

. dl exp 20.085537

P 231: Table 9.8 add the line . tab x1, gen(x1) before line beginning with py

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Table 9.8: Synthetic Poisson and NB2 with categorical predictors

```
_____
. clear
. set obs 50000
. set seed 12345
. gen byte x1 = irecode(runiform(), 0, 0.5, 0.8, 1)
. gen byte x2 = irecode(runiform(), 0.6, 1)
. tab x1, gen(x1)
. gen py = rpoisson(exp(1 + 2*x1 2 + 3*x1 3 - 2.5*x2))
. gen a = .5
. gen ia = 1/.5
. gen xg = rgamma(ia, a)
\cdot gen xbg = exb * xg
. gen nby = rpoisson(xbg)
                     /// NB2 variates
        _____
_____
```

**P** 303: Equation 10.31: The left side of the equation should read:  $\mathcal{L}(\mu, \alpha; y)$ 

p 319: Re-write the paragraph above section 10.4, which begins with, "As shall be..." The new paragraph is to read:

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Cytel's **LogXact** software, Stata's **expoisson** command, and SAS's exact Poisson procedure employ exact statistical methods for determining Poisson parameter estimates and standard errors. *Exact Poisson regression* conditions on the number of events in each panel or stratum of counts, which is similar to fixed-effects Poisson regression. Exact statistical models are based on the canonical link of the distribution, therefore an exact negative binomial model would be based on the canonical link, not the traditional log link, No exact negative binomial model yet exists.

P 324: Line 2 -- add a sentence to end of the sentence ending on the 2nd line of page. "The model may be estimated using Limdep or by using the user-authored Stata command, *nbregp*, in Hardin and Hilbe (2011)"

P 324: First two sentences after Equation 10.45 should read:

"The NB-P distribution takes the same form as NB2, as expressed in equations 10.41 through 10.43. However, for each value of  $\theta$  in equations 10.41 and 10.42, we substitute the value  $\theta \mu^{2-P}$ ."

p 324: Equation 10.47 should read:

$$\mathcal{L}(\lambda; y, \theta, Q) = \sum_{i=1}^{n} \left\{ \theta \lambda_{i}^{Q} \ln\left(\frac{\theta \lambda_{i}^{Q}}{\theta \lambda_{i}^{Q} + \lambda_{i}}\right) + y_{i} \ln\left(1 - \frac{\theta \lambda_{i}^{Q}}{\theta \lambda_{i}^{Q} + \lambda_{i}}\right) + \ln\Gamma\left(y_{i} + \theta \lambda_{i}^{Q}\right) - \ln\Gamma\left(\theta \lambda_{i}^{Q}\right) - \ln\Gamma(y_{i} + 1) \right\}$$

P 328: line 2: SE<sub>NB2</sub> should read instead: SE<sub>NB-P</sub>; i.e. (NB-P – NB2)/SE<sub>NB-P</sub>

**P** 338: Equation 10.63 should read (note that y is not an exponent of  $\lambda$ )

$$f(y; \lambda, \theta) = \theta(\theta + \lambda_i y_i)^{y_i - 1} \frac{\exp(-\theta - \lambda_i y_i)}{y_i!}$$

p 338. The line directly above the final R table at the bottom of the page should be amended to read (I display the entire paragraph):

The value of  $\theta$  serves as a second parameter, analogous to the negative binomial model. It reflects the amount of extra Poisson overdispersion in the data. We use the **azpro** data as an example of how it can be used. The value of  $\theta$ , the heterogeneity parameter, is displayed as (Intercept):2 in the statistical output. In this instance,  $\theta$ =1.02352.

p 338: R table at bottom of page. Amend first and second lines to read:

```
rm(list=ls())  # caution - function drops all objects from memory
library(COUNT); library(VGAM)
```

P 339: Amend the paragraph and code and continued text to the end of the page starting directly below the Stata statistical output in the middle of the page to read as.

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The estimates are near identical, but not the heterogeneity parameter. The Stata parameter is calculated in terms of a hyperbolic tangent; the R parameter is terms of an extended logit, defined in R as  $[\exp(lambda)-1]/[\exp(lambda)+1]$ .

Given the R output, *lambda* is found as the value of intercept:1, which is 0.76482. Applying the formula for extended logit, we have a calculated value of .3648, which is the same as *delta* in the Stata model.

```
. (exp(.76482)-1) / (exp(.76482)+1)
.36479858
```

.3648 is the estimated value of the Stata dispersion parameter.

Another well-known parameterization....

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p 340: The equation of V(y), as the second equation following equation 10.64, is mistaken. The sign within the parenthesis is reversed. It should read:

$$E(y) = \mu_i$$
  $V(y) = \mu_i (1+\alpha\mu_i)^2$ 

P 340: Insert to end of paragraph before final paragraph on page; i.e. add after "... with R and Stata." the following:

"New *gpoisson* and zero-inflated generalized Poisson (*zigp*) Stata commands can be found with Hardin and Hilbe (2011)"

P 343: midpage - add at end of section (new information not available at book's writing)

User-authored PIG (*pigreg*) and ZIPIG (*zipig*) Stata commands are available with Hardin & Hilbe (2011)

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P 354: Add to end of section, before Section 11.2:

"Note that new user-authored Stata commands for zero-inflated generalized Poisson (*zigp*) and ZI Poisson-inverse Gaussian (*zipig*) are available with Hardin and Hilbe (2011)."

P 375: amend the top lines on the page, which is part of a section on R code. Change to read:

\_\_\_\_\_

```
p 377: table at top of page. Replace second line:
> source("c://source/ZANBI.r")
with
> data(mdvis)
```

P 387 Add the following to the final paragraph above section 12.1 Note, however, that recently developed censored Poisson and negative binomial models have been developed for which the likelihood functions allow both parameterizations to be estimated using the same model. Interval censoring is also provided. Refer to Hardin and Hilbe (2012).

P 396 Add the following (in red) to the end of the inset for "Censored" at the top of the page. "Any response in the data that is greater than 15 is also considered to be greater than or equal to 15. Left: P(Y<=C), Right: P(Y>=C) "

P 396 Amend the equations for censored Poisson and negative binomial in mid page, Changes are in red. Note the added close parenthesis for the Left Poisson.

#### CENSORED POISSON

LEFT : ln(poisson(exp(xb)),C) or ln(1-gammap(C+1,exp(xb))) RIGHT: ln(1-poissontail(exp(xb)),C)) or ln(1-gammaptail(C, exp(xb)))

#### CENSORED NEGATIVE BINOMIAL

LEFT : ln(ibeta(C, 1/alpha, 1/(1+exp(-xb-ln(alpha))))) RIGHT: ln(ibetatail(C-1, 1/alpha, 1/(1+exp(-xb-ln(alpha)))))

P 400 First sentence of final paragraph on page. replace "derived" with "employed". Read: "First employed by Hilbe and published in SAS by Hilbe and Johnston (1995), ..."

P 401 Amend the material in the text with the text between lines below. Changes are in red for identification.

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- $\delta$ : 1 if observation not censored; 0 otherwise
- $\zeta$  : 1 if observation is left censored; 0 otherwise
- $\tau$ : 1 if observation is right censored; 0 otherwise

and  $\ln\Gamma_I$  is the 2 parameter incomplete gamma function. Note that the final term of Equation 12.12 is a modification of the Poisson survival function, with  $\Gamma(y+1, \mu)$  being the numerator of the Poisson cumulative distribution function (CDF). The censored Poison log-likelihood function can also be expressed in the same manner as given for the econometric or cut point parameterization above, with C indexed by observations, C<sub>i</sub>, In this manner both parameterizations may be estimated using the same algorithm or function.

First derived and published in Hilbe (2005), the survival parameterized censored negative binomial log-likelihood function is given as:

#### CENSORED NEGATIVE BINOMIAL LOG-LIKELIHOOD FUNCTION

#### $\mathcal{L}(\mathbf{x}\boldsymbol{\beta};\mathbf{y},\boldsymbol{\alpha}) =$

```
\delta\{y_i \ln(\exp(x_i\beta)/(1+\exp(x_i\beta))) - \ln(1+\exp(x_i\beta))/\alpha + \ln\Gamma(y_i+1/\alpha) - \ln\Gamma(y_i+1) - \ln\Gamma(1/\alpha)\} + \zeta\{\ln(\beta_I(C_i,1/\alpha,1/(1+\exp(x'_i\beta - \ln(\alpha))))\} + \tau\{\ln(\beta_I(C_i-1,1/\alpha,1/(1+\exp(x'_i\beta - \ln(\alpha))))\}\}
```

(12.13)

Other terms are  $\alpha$  = heterogeneity parameter and  $\beta_{I}$  = incomplete beta function. The 3 parameter  $\beta_{I}$  function returns the cumulative Beta distribution, or incomplete beta function, for censored responses. The function is identical to the econometric parameterization given in the last section, but with C indexed.

Using the medpar data used earlier in the text, we model length of hospital stay (los) on

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p 411/412. The R code in Table 13.1 does not work for a gamma-Poisson finite mixture model due to zero values in the response variable, The gamma portion of the mixture distribution does not allow zero values. Therefore, amend the code and explanation for estimating a Gaussian-Poisson finite mixture. The first line of the bottom paragraph on page 411 should read Gaussian-Poisson mixture distribution. The amended paragraph and Table 13.1 should now appear as (new and amended items in red).

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It may be of interest to use R's **flexmix** command to construct a **Gaussian-Poisson** mixture distribution. Notably the distributions will not be easy to pull apart. The output is not displayed here, but is simple to recreate.

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Table 13.1 R Gaussian--Poisson finite mixture model

P 412 Table 13.2: Minor correction needed in middle of code. These lines to read:

exb1 <- exp(xb1) exb2 <- exp(xb2)

P 414 Table 13.3: Add line and amend 2nd to last line. Last 3 lines to appear as:

fmmnb <- data.frame(nbxnb, x1, x2)
nxn <- gamlssNP(nbxnb~x1+x2, random=~1,family=NBI, K=2, data=fmmnb)
summary(nxn)</pre>

P500: Final paragraph in section 14.7. Amend the second sentence to read: "However, with respect to ..., there are currently few implementations of multilevel..."

Amend the 4th sentence to the end of the section in same paragraph to read (new in red): "Stata, R, LIMDEP, and SAS are capable of estimating these models. Discussions of them can be found in Skrondal and Rabe-Hesketh (2004), Lee, Nelder and Pawitan (2006), Zuur, A. et al (2009), and Rabe-Hesketh and Skrondal (2008). Only Zuur et al discuss the negative binomial deeper than the random intercept model."

p 531 Add Stata commands to table

Bottom table on page 531, right hand side: "Selected Specialized Author Commands". Add the following to the list. All three commands are in the zip file containing Stata ado and do files.

negbin 8.4.1 ztcnb 11.1 hnblogit\_p 11.2

STATA do files				Selected Specialized Author Commands			
poi_rng.do poio_rng.do nb2_rng.do cgeo_rng.do nb1_rng.do nb2logit_hurdle.do	Table         6.2           Table         6.19           Table         9.4           Ch 10.1.2         Ch 10.2           Table         11.5	poi_sim.ado nb2ml_sim.ado nb2o_rng.do geo_rng.do nbc_rng.do nbr2_f11.11.do	Table 6.3 Table 9.6 Table 9.10 Ch 10.1.3 Ch 10.3.2 Table 11.13	abic.ado negbin cnbreg.ado chnbreg.ado hnbreg1.ado ztcnb	5.2.1.1 8.4.1 10.1.2 10.4 10.4 11.1	hnblogit.ado hnblogit_p cpoissone.ado cpoisson.ado censornb.ado	11.2 11.2 12.1.2 12.2 12.2

P 535: mid page: amend Hardin and Hilbe (2007) to appear as follows Hardin, J.W. and J.M Hilbe (2007), *Generalized Linear Models & Extensions*, second edition, College Station, TC: Stata Press. Third edition (2012).

P 536: add to list of references (if possible)

Hurvich, C. M. and C.L Tsai (1989), Regression and time series model selection in small samples, *Biometrika*, **76**: 297–307.

P 539: add to the list of references (if possible)

Schwarz, G.E. (1978), Estimating the dimension of a model, Annals of Statistics 6 (2): 461–464.

P 540: Add to bottom of references:

Zuur, A.F., E.N. Ieno, N.J. Walker, A.A. Saveliev, and G.M. Smith (2009), *Mixed Effects Models and Extensions in Ecology with R*, New York: Springer.

Zuur, A.F., A.A. Saveliev, and E.N. Ieno (2012). Zero-Inflated Models and Generalized Linear Mixed Models, To be published

## COMMENTS

p. 19/20 – comment. The relative risk ratios displayed in the Poisson output under Table 2.4 are identical to the values we created by hand on page 19. If there were interactions in the model, or if there were more than a single predictor, regardless if it is a categorical variable, the identity would no longer hold. This is implied in the book, but probably should be made explicit.

P 326-328: NB-P. Stata code for constructing a NB-P model is in Hardin & Hilbe (2011). See comment for pages 341-343 below for details. I created synthetic NB2 and NB1 data and models using the algorithms described in this book. I then ran Limdep's NB-P facility on each data, expecting that the NB-P procedure would produce values of P equal to 2 and 1 respectively. It did not, although the other parameter estimates were fine. It appears that Limdep has used Q=P-2 rather than Q=2-P in its estimating algorithm, giving incorrect values for P. I can replicate Limdep output if I use P-2, but doing so results in failure to obtain the correct values of P for "true" synthetically produced data.

P 338-339: Fully working Stata commands for generalized Poisson and zero-inflated generalized Poisson models is in Hardin & Hilbe (2011). See below for details.

P 341-343: Fully working Stata commands for PIG and zero-inflated PIG have been written and will be available in the data sets file for Hardin, JW and JM Hilbe (2011), *Generalized Linear Models and Extensions, 3rd edition*, Stata Press. Chapman & Hall/CRC, owned by Taylor & Francis, markets Stata Press books, as does Stata Corp. The new book should be in print in March, 2012.

Chapters 11-12 (p 346-386). The 3rd edition of James Hardin and Joseph Hilbe, *Generalized Linear Models and Extensions* (Stata Press) is due to be published about the first of December, 2011. We developed complete Stata commands for a new version of generalized Poisson (*gpoisson*), and new commands for zero-inflated generalized Poisson (*zigp*), Poisson inverse-Gaussian (*pigreg*), zero-inflated Poisson inverse-Gaussian (*zipig*), and generalized negative binomial or NB-P (*nbregp*). These will be available on the book's web site, as well as on a partition for the book in *http://works.bepress.com/joseph\_hilbe/*. Other Stata count model

commands that are also used in the book will be posted as well, although they may already be posted on the web site for this book. Synthetic models are also displayed in the book, including the following synthetic count models: Poisson, NB2, zero-inflated Poisson, Poisson-logit hurdle, NB-P, 2-component Poisson finite mixture, and a 3-component NB2 finite mixture model.

p 365-366 Comment: I replaced the Stata command *hnblogit.ado* in the zip file containing the Stata ado and do files used in the book on June 1, 2011. I also added *hnblogit\_p.ado* to the list at the same time. An older version of *hnblogit* was inadvertently placed in the initial list of commands. The changes have been made to the list located on my BePress *Selected Works* site. *http://works.bepress.com/joseph\_hilbe/* Changes to other sites may take may take longer.

p 370: Comment: Subsequent to writing the book I developed code to create synthetic zero-inflated Poisson and zero--inflated negative binomial models. The code is given in the COUNT package and displayed in *Negative Binomial Regression Extensions*, which can be downloaded from my Selected Works web site, http://works.bepress.com/joseph\_hilbe/

Thank you to Andreas Krause, Director, Lead Scientist Modeling and Simulation, Dept. of Clinical Pharmacology, Actelion Pharmaceuticals Ltd, Allschwil, Switzerland, for identifying several of the typo, amend, and comment items reflected in the above "Errata and Comments". My thanks also to Wouter Vahl of the Dept of Biosciences, Helsinki University, for identifying typos in Chapter 2 and Reginald Jordan, who spotted many of the remaining typos listed here. These should all be corrected in the next printing. I encourage readers to send me any typos or suspected errors that they may discover in the book. Send to: hilbe@asu.edu