Linear Regression Models with Interaction/Moderation

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Contents

1 Introduction

1.1 Goals

Goals

- Learn how to use factor variable notation when fitting models involving
  - Categorical variables
  - Interactions
  - Polynomial terms
- Learn how to use postestimation tools to interpret interactions
  - Tests for group differences
  - Tests of slopes
  - Graphs

A Linear Model

- We'll use data from the National Health and Nutrition Examination Survey (NHANES) for our examples

```
. webuse nhanes2
```

- We'll start with a basic a model for `bmi` using `age` and sex (`female`).
- Before we fit the model, let's investigate the variables using `codebook`

```
. codebook bmi age female

-----------------------------------------------------------------------------------------------
bmi                                                                       Body Mass Index (BMI)
-----------------------------------------------------------------------------------------------

                  type:  numeric (float)

                 range:  [12.385596,61.129696]        units:  1.000e-07
         unique values:  9,941                    missing .:  0/10,351

                  mean:   25.5376
              std. dev:   4.91497

           percentiles:        10%       25%       50%       75%       90%
                           20.1037    22.142   24.8181   28.0267   31.7259

-----------------------------------------------------------------------------------------------
age                                                                                age in years
-----------------------------------------------------------------------------------------------

                  type:  numeric (byte)

                 range:  [20,74]                      units:  1
         unique values:  55                       missing .:  0/10,351

                  mean:   47.5797
              std. dev:   17.2148

           percentiles:        10%       25%       50%       75%       90%
                                24        31        49        63        69

-----------------------------------------------------------------------------------------------
female                                                                         1=female, 0=male
-----------------------------------------------------------------------------------------------

                  type:  numeric (byte)

                 range:  [0,1]                        units:  1
         unique values:  2                        missing .:  0/10,351

            tabulation:  Freq.  Value
                         4,915  0
                         5,436  1
```

- Now we can fit the model

```
. regress bmi age female

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(2, 10348)     =    156.29
       Model |  7330.98402         2  3665.49201   Prob > F        =    0.0000
    Residual |  242693.178    10,348  23.4531483   R-squared       =    0.0293
-------------+----------------------------------   Adj R-squared   =    0.0291
       Total |  250024.162    10,350  24.1569239   Root MSE        =    4.8428

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0488667   .0027653    17.67   0.000     .0434462    .0542872
      female |   .0380616   .0953249     0.40   0.690    -.1487936    .2249168
       _cons |   23.19255   .1482223   156.47   0.000     22.90201    23.48309
------------------------------------------------------------------------------
```

---

2 Estimation

2.1 Including Categorical Variables

Working with Categorical Variables

- We would now like to include `region` in the model, let's take a look at this variable

```
. codebook region

-----------------------------------------------------------------------------------------------
region                                                                   1=NE, 2=MW, 3=S, 4=W
-----------------------------------------------------------------------------------------------

                  type:  numeric (byte)
                 label:  region

                 range:  [1,4]                        units:  1
         unique values:  4                        missing .:  0/10,351

            tabulation:  Freq.   Numeric  Label
                         2,096         1  NE
                         2,774         2  MW
                         2,853         3  S
                         2,628         4  W
```

  - ◇ It cannot simply be added to the list of covariates because it has 4 categories

- To include a categorical variable, put an `i.` in front of its name—this declares the variable to be a categorical variable, or in Stataese, a *factor variable*

- For example, to add `region` to our model we use

```
. regress bmi age i.female i.region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(5, 10345)     =     63.02
       Model |  7390.19781         5  1478.03956   Prob > F        =    0.0000
    Residual |  242633.964    10,345  23.4542256   R-squared       =    0.0296
-------------+----------------------------------   Adj R-squared   =    0.0291
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.843

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0488851   .0027674    17.66   0.000     .0434605    .0543097
             |
      female |
          0  |          0  (base)
          1  |   .0372717   .0953357     0.39   0.696    -.1496047    .2241481
             |
      region |
         NE  |          0  (base)
         MW  |   .0064779   .1402121     0.05   0.963     -.268365    .2813207
          S  |   .0387957   .1393383     0.28   0.781    -.2343342    .3119256
          W  |  -.1537648   .1418286    -1.08   0.278    -.4317762    .1242466
             |
       _cons |    23.2187   .1760452   131.89   0.000     22.87362    23.56378
------------------------------------------------------------------------------
```

Niceities

- Value labels associated with factor variables are displayed in the regression table
- We can tell Stata to show the base categories for our factor variables

```
. set showbaselevels on
```

Factor Notation as Operators

- The `i.` operator can be applied to many variables at once:

```
. regress bmi age i.(female region)

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(5, 10345)     =     63.02
       Model |  7390.19781         5  1478.03956   Prob > F        =    0.0000
    Residual |  242633.964    10,345  23.4542256   R-squared       =    0.0296
-------------+----------------------------------   Adj R-squared   =    0.0291
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.843

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0488851   .0027674    17.66   0.000     .0434605    .0543097
             |
      female |
          0  |          0  (base)
          1  |   .0372717   .0953357     0.39   0.696    -.1496047    .2241481
             |
      region |
         NE  |          0  (base)
         MW  |   .0064779   .1402121     0.05   0.963     -.268365    .2813207
          S  |   .0387957   .1393383     0.28   0.781    -.2343342    .3119256
          W  |  -.1537648   .1418286    -1.08   0.278    -.4317762    .1242466
             |
       _cons |    23.2187   .1760452   131.89   0.000     22.87362    23.56378
------------------------------------------------------------------------------
```

- In other words, it understands the distributive property
  - ◇ This is useful when using variable ranges, for example
- For the curious, factor variable notation works with wildcards
  - ◇ If there were many variables starting with `u`, then `i.u*` would include them all as factor variables

---

Using Different Base Categories

- By default, the smallest-valued category is the base category
- This can be overridden within commands
  - ◇ `b#.` specifies the value # as the base
  - ◇ `b(##).` specifies the #'th largest value as the base
  - ◇ `b(first).` specifies the smallest value as the base
  - ◇ `b(last).` specifies the largest value as the base
  - ◇ `b(freq).` specifies the most prevalent value as the base
  - ◇ `bn.` specifies there should be no base

Playing with the Base

- We can use `region=3` as the base class on the fly:

```
. regress bmi age i.female b3.region
```

- We can use the most prevalent category as the base

```
. regress bmi age i.female b(freq).region
```

- Factor variables can be distributed across many variables

```
. regress bmi age b(freq).(female region)
```

- The base category can be omitted (with some care here)

```
. regress bmi age i.female bn.region, noconstant
```

- We can also include a term for `region=4` only

```
. regress bmi age i.female 4.region
```

2.2 Including Interactions

Specifying Interactions

- Factor variables are also used for specifying interactions
  - ◇ This is where they really shine
- To include both main effects and interaction terms in a model, put ## between the variables
- To include only the interaction terms, put # between the terms

---

Categorical by Categorical Interactions

- For example, to fit a model that includes main effects for `age`, `female`, and `region`, as well as the interaction of `female`, and `region`

```
. regress bmi age female##region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(8, 10342)     =     40.30
       Model |  7559.19099         8  944.898874   Prob > F        =    0.0000
    Residual |  242464.971    10,342  23.4446888   R-squared       =    0.0302
-------------+----------------------------------   Adj R-squared   =    0.0295
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.842

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0488087   .0027671    17.64   0.000     .0433846    .0542328
             |
      female |
          0  |          0  (base)
          1  |  -.2939562   .2116093    -1.39   0.165    -.7087514    .1208389
             |
      region |
         NE  |          0  (base)
         MW  |  -.1420836   .2023593    -0.70   0.483     -.538747    .2545798
          S  |  -.3347762   .2015721    -1.66   0.097    -.7298965    .0603441
          W  |  -.2694841    .204234    -1.32   0.187    -.6698222    .1308541
             |
female#region |
       1#MW  |   .2897474    .280525     1.03   0.302    -.2601358    .8396306
        1#S  |   .7124639   .2789251     2.55   0.011     .1657169    1.259211
        1#W  |   .2266557   .2837887     0.80   0.424    -.3296251    .7829365
             |
       _cons |   23.39271   .2013939   116.15   0.000     22.99793    23.78748
------------------------------------------------------------------------------
```

- Variables involved in interactions are assumed to be categorical, so no `i.` is needed

- To see all the omitted terms we can add the `allbaselevels` option

```
. regress bmi age female##region, allbaselevels

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(8, 10342)     =     40.30
       Model |  7559.19099         8  944.898874   Prob > F        =    0.0000
    Residual |  242464.971    10,342  23.4446888   R-squared       =    0.0302
-------------+----------------------------------   Adj R-squared   =    0.0295
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.842

------------------------------------------------------------------------------
          bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------+---------------------------------------------------------------
          age |   .0488087   .0027671    17.64   0.000     .0433846    .0542328
              |
       female |
           0  |          0  (base)
           1  |  -.2939562   .2116093    -1.39   0.165    -.7087514    .1208389
              |
       region |
          NE  |          0  (base)
          MW  |  -.1420836   .2023593    -0.70   0.483     -.538747    .2545798
           S  |  -.3347762   .2015721    -1.66   0.097    -.7298965    .0603441
           W  |  -.2694841    .204234    -1.32   0.187    -.6698222    .1308541
              |
female#region |
        0#NE  |          0  (base)
        0#MW  |          0  (base)
         0#S  |          0  (base)
         0#W  |          0  (base)
        1#NE  |          0  (base)
        1#MW  |   .2897474    .280525     1.03   0.302    -.2601358    .8396306
         1#S  |   .7124639   .2789251     2.55   0.011     .1657169    1.259211
         1#W  |   .2266557   .2837887     0.80   0.424    -.3296251    .7829365
              |
        _cons |   23.39271   .2013939   116.15   0.000     22.99793    23.78748
------------------------------------------------------------------------------
```

Categorical by Continuous Interactions

- To include continuous variables in interactions use `c.` to specify that a variable is continuous
  - Otherwise it will be assumed to be categorical
- Here is our model with an interaction between `age` and `region`

```
. regress bmi c.age##region i.female

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(8, 10342)     =     40.35
       Model |  7568.54189         8  946.067737   Prob > F        =    0.0000
    Residual |   242455.62    10,342  23.4437846   R-squared       =    0.0303
-------------+----------------------------------   Adj R-squared   =    0.0295
       Total |  250024.162    10,350  24.1569239   Root MSE        =    4.8419

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0607829   .0062164     9.78   0.000     .0485975    .0729683
             |
      region |
         NE  |          0  (base)
         MW  |   .3951518   .4106204     0.96   0.336    -.4097436    1.200047
          S  |   1.051668   .4181868     2.51   0.012     .2319407    1.871395
          W  |   .5921285   .4181932     1.42   0.157     -.227611    1.411868
             |
region#c.age |
         MW  |  -.0080245   .0081638    -0.98   0.326    -.0240272    .0079782
          S  |  -.0211109    .008219    -2.57   0.010    -.0372217   -.0050002
          W  |  -.0155977   .0082261    -1.90   0.058    -.0317225     .000527
             |
      female |
          0  |          0  (base)
          1  |    .038259   .0953259     0.40   0.688    -.1485982    .2251161
             |
       _cons |   22.64929   .3193208    70.93   0.000     22.02336    23.27522
------------------------------------------------------------------------------
```

Continuous by Continuous Interactions

- Prefix both variables in the interaction with `c.` to fit models with continuous by continuous variable interactions
- For example, we can interact age with serum vitamin c levels (`vitaminc`)

```
. regress bmi c.age##c.vitaminc i.female i.region

      Source |       SS           df       MS      Number of obs   =     9,973
-------------+----------------------------------   F(7, 9965)      =     63.61
       Model |  10298.9223         7  1471.27461   Prob > F        =    0.0000
    Residual |  230479.207     9,965  23.1288718   R-squared       =    0.0428
-------------+----------------------------------   Adj R-squared   =    0.0421
       Total |   240778.13     9,972  24.1454201   Root MSE        =    4.8092

----------------------------------------------------------------------------------
             bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----------------+----------------------------------------------------------------
             age |   .0220407   .0059366     3.71   0.000     .0104038    .0336777
        vitaminc |  -2.331426   .2717928    -8.58   0.000    -2.864194   -1.798657
                 |
c.age#c.vitaminc |    .029107   .0050017     5.82   0.000     .0193026    .0389115
                 |
          female |
              0  |          0  (base)
              1  |   .1858965   .0982311     1.89   0.058    -.0066564    .3784494
                 |
          region |
             NE  |          0  (base)
             MW  |  -.0936871   .1412331    -0.66   0.507    -.3705326    .1831584
              S  |  -.2137082   .1431247    -1.49   0.135    -.4942615    .0668451
              W  |  -.1626738   .1430181    -1.14   0.255    -.4430182    .1176706
                 |
           _cons |   25.45695   .3293507    77.29   0.000     24.81136    26.10255
----------------------------------------------------------------------------------
```

- To include polynomial terms, interact a variable with itself
- For example, a model that includes both age and age^2

```
. regress bmi c.age##c.age i.female i.region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(6, 10344)     =     73.84
       Model |  10269.3919         6  1711.56532   Prob > F        =    0.0000
    Residual |   239754.77    10,344  23.1781487   R-squared       =    0.0411
-------------+----------------------------------   Adj R-squared   =    0.0405
       Total |  250024.162    10,350  24.1569239   Root MSE        =    4.8144

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .2731368   .0203077    13.45   0.000     .2333297    .3129439
             |
 c.age#c.age |  -.0024099   .0002162   -11.15   0.000    -.0028337   -.0019861
             |
      female |
          0  |          0  (base)
          1  |   .0462855   .0947764     0.49   0.625    -.1394945    .2320656
             |
      region |
         NE  |          0  (base)
         MW  |   .0322091   .1394036     0.23   0.817    -.2410489    .3054671
          S  |   .0289346   .1385186     0.21   0.835    -.2425886    .3004579
          W  |  -.1105093   .1410448    -0.78   0.433    -.3869844    .1659657
             |
       _cons |    18.6987   .4416971    42.33   0.000     17.83289    19.56451
------------------------------------------------------------------------------
```

  - ◇ The coefficient for age-squared is next to `c.age#c.age`

Higher Order Interactions

- Factor variable syntax can be used to specify higher order interactions
- If the interactions are specified using ## all lower order terms are included

- For example, here we fit a model for bmi using a model that includes the three-way interaction of continuous variables age and vitaminc and categorical variable female

```
. regress bmi c.age##c.vitaminc##female

      Source |       SS           df       MS      Number of obs   =     9,973
-------------+----------------------------------   F(7, 9965)      =     76.60
       Model |  12294.4386         7  1756.34837   Prob > F        =    0.0000
    Residual |  228483.691     9,965  22.9286193   R-squared       =    0.0511
-------------+----------------------------------   Adj R-squared   =    0.0504
       Total |   240778.13     9,972  24.1454201   Root MSE        =    4.7884

------------------------------------------------------------------------------------------
                       bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
---------------------------+--------------------------------------------------------------
                       age |  -.0038595   .0084263    -0.46   0.647    -.0203767    .0126578
                  vitaminc |  -2.008713   .4231851    -4.75   0.000    -2.838241   -1.179185
                           |
           c.age#c.vitaminc |   .0313728   .0078481     4.00   0.000     .0159889    .0467566
                           |
                    female |
                        0  |          0  (base)
                        1  |  -2.098183   .6208318    -3.38   0.001    -3.315138   -.8812268
                           |
              female#c.age |
                        1  |   .0646392   .0119517     5.41   0.000     .0412115    .0880668
                           |
         female#c.vitaminc |
                        1  |   .0314475   .5539279     0.06   0.955    -1.054363    1.117258
                           |
   female#c.age#c.vitaminc |
                        1  |  -.0166002   .0102645    -1.62   0.106    -.0367206    .0035203
                           |
                     _cons |   26.16464   .4416624    59.24   0.000     25.29889    27.03039
------------------------------------------------------------------------------------------
```

Some Factor Variable Notes

- If you plan to look at marginal effects of any kind, it is best to
  - Explicitly mark all categorical variables with i.
  - Specify all interactions using # or ##
  - Specify powers of a variable as interactions of the variable with itself
- There can be up to 8 categorical and 8 continuous interactions in one expression
  - Have fun with the interpretation

---

3 Postestimation

3.1 About Postestimation

Introduction to Postestimation

- In Stata jargon, postestimation commands are commands that can be run after a model is fit, for example

- ◇ Predictions
- ◇ Additional hypothesis tests
- ◇ Checks of assumptions

- We'll explore postestimation tools that can be used to help interpret the results of models that include interactions
- The usefulness of specific tools will depend on the types of hypotheses you wish to examine

---

3.2 Investigating Categorical by Categorical Interactions

Estimating a Model

- Lets begin by running a model with main effects for `age`, `female` and `region`, and the interaction of `female` and `region`

```
. regress bmi age female##region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(8, 10342)     =     40.30
       Model |  7559.19099         8  944.898874   Prob > F        =    0.0000
    Residual |  242464.971    10,342  23.4446888   R-squared       =    0.0302
-------------+----------------------------------   Adj R-squared   =    0.0295
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.842

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0488087   .0027671    17.64   0.000     .0433846    .0542328
             |
      female |
          0  |          0  (base)
          1  |  -.2939562   .2116093    -1.39   0.165    -.7087514    .1208389
             |
      region |
         NE  |          0  (base)
         MW  |  -.1420836   .2023593    -0.70   0.483     -.538747    .2545798
          S  |  -.3347762   .2015721    -1.66   0.097    -.7298965    .0603441
          W  |  -.2694841    .204234    -1.32   0.187    -.6698222    .1308541
             |
female#region |
       1#MW  |   .2897474    .280525     1.03   0.302    -.2601358    .8396306
        1#S  |   .7124639   .2789251     2.55   0.011     .1657169    1.259211
        1#W  |   .2266557   .2837887     0.80   0.424    -.3296251    .7829365
             |
       _cons |   23.39271   .2013939   116.15   0.000     22.99793    23.78748
------------------------------------------------------------------------------
```

- How might we begin?
  - ◇ Perform joint tests of coefficients
  - ◇ Estimate and test hypotheses about group differences

Finding the Coefficient Names

- Some postestimation commands require that you know the names used to store the coefficients
- To see these names we can replay the model and showing the *coefficient legend*

```
. regress, coeflegend

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(8, 10342)     =     40.30
       Model |  7559.19099         8  944.898874   Prob > F        =    0.0000
    Residual |  242464.971    10,342  23.4446888   R-squared       =    0.0302
-------------+----------------------------------   Adj R-squared   =    0.0295
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.842

------------------------------------------------------------------------------
         bmi |      Coef.  Legend
-------------+----------------------------------------------------------------
         age |   .0488087  _b[age]
             |
      female |
          0  |          0  _b[0b.female]
          1  |  -.2939562  _b[1.female]
             |
      region |
         NE  |          0  _b[1b.region]
         MW  |  -.1420836  _b[2.region]
          S  |  -.3347762  _b[3.region]
          W  |  -.2694841  _b[4.region]
             |
female#region |
       1#MW  |   .2897474  _b[1.female#2.region]
        1#S  |   .7124639  _b[1.female#3.region]
        1#W  |   .2266557  _b[1.female#4.region]
             |
       _cons |   23.39271  _b[_cons]
------------------------------------------------------------------------------
```

- From here, we can see the full specification of the factor levels:

  `_b[2.region]` corresponds to `region=2` which is “MW” or midwest

  `_b[3.region]` corresponds to `region=3` which is “S” or south

- We can also see the terms for the interaction:

  `_b[1.female#2.region]` corresponds to the term for the interaction of `region=2` and `female=1`

  `_b[1.female#3.region]` corresponds to the term for the interaction of `region=3` and `female=1`

Joint Tests

- The `test` command performs a Wald test of the specified null hypothesis
  - ◇ The default test is that the listed terms are equal to 0
- `test` takes a list of terms, which may be variable names, but can also be terms associated with factor variables

- To perform a joint test of the null hypothesis that the coefficients for the levels of `region` are all equal to 0

```
. test 2.region 3.region 4.region

 ( 1)  2.region = 0
 ( 2)  3.region = 0
 ( 3)  4.region = 0

       F(  3, 10342) =    1.07
            Prob > F =    0.3600
```

  ◇ Since the model contains an interaction, this is a test of the effect of `region` when `female=0`

Testing Sets of Coefficients

- To test that all of the coefficients associated with the interaction of `female` and `region` we would need to give the full name of all the coefficients

```
. test 1.female#2.region 1.female#3.region 1.female#4.region
```

- `testparm` also performs Wald tests, but it accepts lists of variables, rather than coefficients in the model
- So we can perform joint tests with less typing, for example

```
. testparm i.region#i.female

 ( 1)  1.female#2.region = 0
 ( 2)  1.female#3.region = 0
 ( 3)  1.female#4.region = 0

       F(  3, 10342) =    2.40
            Prob > F =    0.0656
```

An Alternative Test

- Likelihood ratio tests provide an alternative method of testing sets of coefficients
- To test the coefficients associated with the interaction of `female` and `region` we need to store our model results. The name is arbitrary, we'll call them `m1`

```
. estimates store m1
```

- Now we can rerun our model without region

```
. regress bmi age i.female i.region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(5, 10345)     =     63.02
       Model |  7390.19781         5  1478.03956   Prob > F        =    0.0000
    Residual |  242633.964    10,345  23.4542256   R-squared       =    0.0296
-------------+----------------------------------   Adj R-squared   =    0.0291
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.843

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0488851   .0027674    17.66   0.000     .0434605    .0543097
             |
      female |
          0  |          0  (base)
          1  |   .0372717   .0953357     0.39   0.696    -.1496047    .2241481
             |
      region |
         NE  |          0  (base)
         MW  |   .0064779   .1402121     0.05   0.963     -.268365    .2813207
          S  |   .0387957   .1393383     0.28   0.781    -.2343342    .3119256
          W  |  -.1537648   .1418286    -1.08   0.278    -.4317762    .1242466
             |
       _cons |    23.2187   .1760452   131.89   0.000     22.87362    23.56378
------------------------------------------------------------------------------
```

- If we were removing one of these variables entirely, we would want to add `if e(sample)` to makes sure the same sample, what Stata calls the *estimation sample*, is used for both models

---

Likelihood Ratio Tests (Continued)

- Now we store the second set of estimates

```
. estimates store m2
```

- And use the `lrtest` command to perform the likelihood ratio test

```
. lrtest m1 m2

Likelihood-ratio test                                 LR chi2(3)  =      7.21
(Assumption: m2 nested in m1)                         Prob > chi2 =    0.0654
```

- We'll restore the results from `m1`

```
. estimates restore m1

(results m1 are active now)
```

- Now it's as if we just ran the model stored in `m1`

Tests of Differences

- `test` can also be used to the equality of coefficients

```
. test 3.region#1.female = 4.region#1.female

 ( 1)  1.female#3.region - 1.female#4.region = 0

       F(  1, 10342) =    3.43
            Prob > F =    0.0640
```

- A likelihood ratio test can also be used; see `help constraint` for information on setting the necessary constraints
- The `lincom` command can be used to calculate linear combinations of coefficients, along with standard errors, hypothesis tests, and confidence intervals
- For example, to obtain the difference in coefficients

```
. lincom 3.region#1.female - 4.region#1.female

 ( 1)  1.female#3.region - 1.female#4.region = 0

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .4858082   .2622654     1.85   0.064    -.0282827    .9998991
------------------------------------------------------------------------------
```

Contrasts

- The `contrast` command allows us to test a wide variety of comparisons across groups
- For example comparing regions separately for men and women

```
. contrast region@female, effects

Contrasts of marginal linear predictions

Margins      : asbalanced

-------------------------------------------------
              |         df           F        P>F
--------------+----------------------------------
region@female |
           0  |          3        1.07     0.3600
           1  |          3        2.17     0.0890
       Joint  |          6        1.62     0.1364
              |
  Denominator |      10342
-------------------------------------------------

--------------------------------------------------------------------------------
                |   Contrast   Std. Err.      t    P>|t|     [95% Conf. Interval]
----------------+---------------------------------------------------------------
  region@female |
(MW vs base) 0  |  -.1420836   .2023593    -0.70   0.483     -.538747    .2545798
(MW vs base) 1  |   .1476637   .1943419     0.76   0.447    -.2332839    .5286114
 (S vs base) 0  |  -.3347762   .2015721    -1.66   0.097    -.7298965    .0603441
 (S vs base) 1  |   .3776878   .1927872     1.96   0.050    -.0002125     .755588
 (W vs base) 0  |  -.2694841    .204234    -1.32   0.187    -.6698222    .1308541
 (W vs base) 1  |  -.0428284   .1970381    -0.22   0.828    -.4290612    .3434044
--------------------------------------------------------------------------------
```

- The `@` symbol requests comparisons of the levels of `region` at each value of `female`
- The `effects` option requests that individual contrasts be displayed along with their standard errors, hypothesis tests, and confidence intervals

---

Adjusting for Multiple Comparisons

- Use of `contrast` can result in a large number of hypothesis tests
- The `mcompare()` option can be used to adjust p-values and confidence intervals for multiple comparisons within factor variable terms
- The available methods are
  - `noadjust`
  - `bonferroni`
  - `sidak`
  - `scheffe`
- To apply Bonferroni's adjustment to our previous `contrast`

```
. contrast region@female, effects mcompare(bonferroni)

Contrasts of marginal linear predictions

Margins      : asbalanced

-------------------------------------------------
              |         df           F        P>F
--------------+----------------------------------
region@female |
           0  |          3        1.07     0.3600
           1  |          3        2.17     0.0890
       Joint  |          6        1.62     0.1364
              |
  Denominator |      10342
-------------------------------------------------
Note: Bonferroni-adjusted p-values are reported
      for tests on individual contrasts only.

----------------------------
              |    Number of
              |  Comparisons
--------------+-------------
region@female |            6
----------------------------

---------------------------------------------------------------------------------
                |                             Bonferroni            Bonferroni
                |   Contrast   Std. Err.       t    P>|t|     [95% Conf. Interval]
----------------+----------------------------------------------------------------
  region@female |
(MW vs base) 0  |  -.1420836   .2023593    -0.70   1.000    -.6760623     .391895
(MW vs base) 1  |   .1476637   .1943419     0.76   1.000    -.3651588    .6604863
 (S vs base) 0  |  -.3347762   .2015721    -1.66   0.581    -.8666776    .1971252
 (S vs base) 1  |   .3776878   .1927872     1.96   0.301    -.1310325     .886408
 (W vs base) 0  |  -.2694841    .204234    -1.32   1.000    -.8084096    .2694415
 (W vs base) 1  |  -.0428284   .1970381    -0.22   1.000    -.5627657    .4771089
---------------------------------------------------------------------------------
```

Average Predicted Values

- We might want to explore predictions based on our model and data
- Predictions for individual observations can be made using the `predict` command, see `help predict`
- To find out about our model more generally, we may be more interested in average predicted values
  - Also known as predictive margins or recycled predictions
- To obtain the average predicted value of `bmi`

```
. margins

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
       _cons |    25.5376   .0475917   536.60   0.000     25.44431    25.63089
------------------------------------------------------------------------------
```

Predictions at Specified Values of Factor Variables

- Stata calls the list of variables that follow the `margins` command the *marginslist*
  - To appear in the *marginslist* a variable must have been specified as factor variable in the model
- To obtain the average predicted value of `bmi` at different values of `region`

```
. margins region

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      region |
         NE  |   25.56063   .1057882   241.62   0.000     25.35327      25.768
         MW  |   25.57071     .09198   278.00   0.000     25.39042    25.75101
          S  |   25.60002   .0906777   282.32   0.000     25.42227    25.77776
          W  |   25.41018   .0944557   269.02   0.000     25.22503    25.59533
------------------------------------------------------------------------------
```

- How were these values generated?
  1. Calculate the predicted value of `bmi` setting `region=1` and using each case's observed values of `female` and `age`
  2. Find the mean of the predicted values
  3. Repeat steps 1 and 2 for each value of `region`

Predicted Values with Multiple Factor Variables

- We can obtain `margins` for multiple variables

```
. margins region female

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      region |
         NE  |   25.56063   .1057882   241.62   0.000     25.35327      25.768
         MW  |   25.57071     .09198   278.00   0.000     25.39042    25.75101
          S  |   25.60002   .0906777   282.32   0.000     25.42227    25.77776
          W  |   25.41018   .0944557   269.02   0.000     25.22503    25.59533
             |
      female |
          0  |   25.51624   .0690736   369.41   0.000     25.38084    25.65164
          1  |   25.55385   .0656788   389.07   0.000     25.42511    25.68259
------------------------------------------------------------------------------
```

- Or we can oobtain predicted values of `bmi` at each combination of `region` and `female`

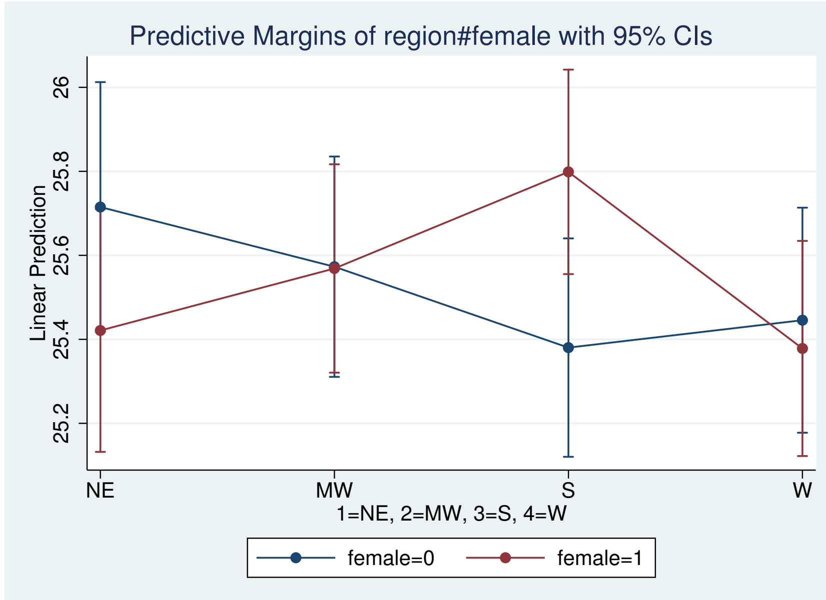
```
. margins region#female

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()

-------------------------------------------------------------------------------
              |            Delta-method
              |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
--------------+----------------------------------------------------------------
region#female |
        NE#0  |   25.71501   .1517587   169.45   0.000     25.41753    26.01248
        NE#1  |   25.42105   .1474742   172.38   0.000     25.13197    25.71013
        MW#0  |   25.57292   .1338383   191.07   0.000     25.31058    25.83527
        MW#1  |   25.56872   .1265618   202.03   0.000     25.32063     25.8168
         S#0  |   25.38023   .1326702   191.30   0.000     25.12017    25.64029
         S#1  |   25.79874   .1241829   207.75   0.000     25.55532    26.04216
         W#0  |   25.44552   .1366851   186.16   0.000     25.17759    25.71345
         W#1  |   25.37822   .1306734   194.21   0.000     25.12208    25.63437
-------------------------------------------------------------------------------
```

- We might prefer to graph these results, we can do so using the `marginsplot` command

---

Graphing Predicted Values

```
. marginsplot
```



◇ If our model did not include a `region` by `female` interaction, the lines would be parallel

---

Predicted Values for Specific Groups

- When we specify the variables in the *marginslist* Stata calculates predicted values treating each case as though it belonged to each group
- The `over()` option allows us to obtain predictions separately for each group, for example

```
. margins, over(female)

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
over         : female

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      female |
          0  |   25.50999   .0690654   369.36   0.000     25.37461    25.64538
          1  |   25.56256   .0656723   389.24   0.000     25.43383    25.69129
------------------------------------------------------------------------------
```

- This time the table shows
  - ◇ The average predicted value of `bmi` for cases where `female=0` using each case's observed values of `age` and `region`
  - ◇ The average predicted value of `bmi` for cases where `female=1` using each case's observed values of `age` and `region`
- This can be useful when we want to compare groups

3.3 Investigating Categorical by Continuous Interactions

A Categorical by Continuous Interaction

- For this set of examples, we'll fit a model that includes an interaction between the continuous variable age and the categorical variable `region`

```
. regress bmi c.age##region i.female

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(8, 10342)     =     40.35
       Model |  7568.54189         8  946.067737   Prob > F        =    0.0000
    Residual |   242455.62    10,342  23.4437846   R-squared       =    0.0303
-------------+----------------------------------   Adj R-squared   =    0.0295
       Total |  250024.162    10,350  24.1569239   Root MSE        =    4.8419

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .0607829   .0062164     9.78   0.000     .0485975    .0729683
             |
      region |
         NE  |          0  (base)
         MW  |   .3951518   .4106204     0.96   0.336    -.4097436    1.200047
         S   |   1.051668   .4181868     2.51   0.012     .2319407    1.871395
         W   |   .5921285   .4181932     1.42   0.157     -.227611    1.411868
             |
region#c.age |
         MW  |  -.0080245   .0081638    -0.98   0.326    -.0240272    .0079782
         S   |  -.0211109    .008219    -2.57   0.010    -.0372217   -.0050002
         W   |  -.0155977   .0082261    -1.90   0.058    -.0317225     .000527
             |
      female |
          0  |          0  (base)
          1  |    .038259   .0953259     0.40   0.688    -.1485982    .2251161
             |
       _cons |   22.64929   .3193208    70.93   0.000     22.02336    23.27522
------------------------------------------------------------------------------
```

- Let's take a look at how the coefficients are stored

```
. regress, coeflegend

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(8, 10342)     =     40.35
       Model |  7568.54189         8  946.067737   Prob > F        =    0.0000
    Residual |   242455.62    10,342  23.4437846   R-squared       =    0.0303
-------------+----------------------------------   Adj R-squared   =    0.0295
       Total |  250024.162    10,350  24.1569239   Root MSE        =    4.8419

------------------------------------------------------------------------------
         bmi |      Coef.  Legend
-------------+----------------------------------------------------------------
         age |   .0607829  _b[age]
             |
      region |
         NE  |          0  _b[1b.region]
         MW  |   .3951518  _b[2.region]
          S  |   1.051668  _b[3.region]
          W  |   .5921285  _b[4.region]
             |
region#c.age |
         MW  |  -.0080245  _b[2.region#c.age]
          S  |  -.0211109  _b[3.region#c.age]
          W  |  -.0155977  _b[4.region#c.age]
             |
      female |
          0  |          0  _b[0b.female]
          1  |    .038259  _b[1.female]
             |
       _cons |   22.64929  _b[_cons]
------------------------------------------------------------------------------
```

---

test and testparm

- As before, we can test the null hypothesis that all of the coefficients associated with the interaction of age and region are equal to 0 using testparm

```
. testparm c.age#i.region

 ( 1)  2.region#c.age = 0
 ( 2)  3.region#c.age = 0
 ( 3)  4.region#c.age = 0

       F(  3, 10342) =    2.54
            Prob > F =    0.0549
```

- We could also use lrtest
- We can test specific hypotheses about the slopes
- For example we might want to test whether the slope of age is significantly different in the south (region=3) versus the west (region=4)

```
. test 3.region#c.age = 4.region#c.age

 ( 1)  3.region#c.age - 4.region#c.age = 0

       F(  1, 10342) =    0.52
            Prob > F =    0.4689
```

Estimated Slopes

- We can use `lincom` to estimate the slope of `age` for the south (`region=3`)

```
. lincom c.age + 3.region#c.age

 ( 1)  age + 3.region#c.age = 0

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .0396719   .0053765     7.38   0.000     .0291329    .0502109
------------------------------------------------------------------------------
```

- We can also use `margins` with the `dydx()` option to calculate the slope of `age` for each `region`

```
. margins region, dydx(age)

Average marginal effects                        Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
dy/dx w.r.t. : age

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
age          |
      region |
         NE  |   .0607829   .0062164     9.78   0.000     .0485975    .0729683
         MW  |   .0527584   .0052919     9.97   0.000     .0423853    .0631315
          S  |   .0396719   .0053765     7.38   0.000     .0291329    .0502109
          W  |   .0451852   .0053875     8.39   0.000     .0346246    .0557457
------------------------------------------------------------------------------
```

- The `dydx()` option calculates derivative of the predicted values with respect to the specified variable, also known as the marginal effect

---

Predictions at Specified Values

- To obtain margins at set values of continuous variables use the `at()` option

- For example, the predicted value of `bmi` at each level of `region` setting `age=20`

```
. margins region, at(age=20) vsquish

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
at           : age             =          20

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      region |
         NE  |   23.88504   .2026955   117.84   0.000     23.48772    24.28236
         MW  |    24.1197   .1678019   143.74   0.000     23.79078    24.44862
          S  |   24.51449   .1766004   138.81   0.000     24.16832    24.86066
          W  |   24.16521   .1772397   136.34   0.000     23.81779    24.51264
------------------------------------------------------------------------------
```

  - The `vsquish` option reduces the vertical space in the output

- The `at()` option accepts *numlists* so we aren't restricted to a single value of `age`

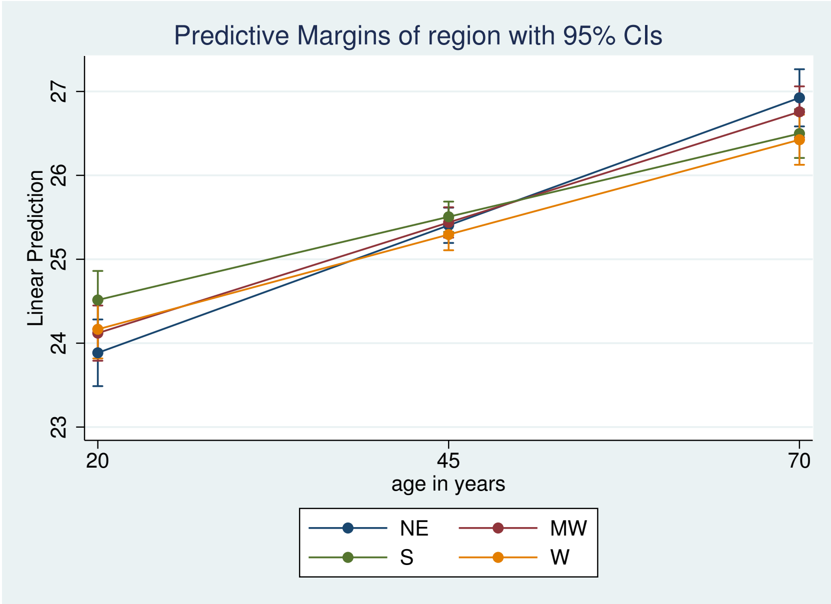
```
. margins region, at(age=(20(25)70)) vsquish

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
1._at        : age             =          20
2._at        : age             =          45
3._at        : age             =          70

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
  _at#region |
       1#NE  |   23.88504   .2026955   117.84   0.000     23.48772    24.28236
       1#MW  |    24.1197   .1678019   143.74   0.000     23.79078    24.44862
        1#S  |   24.51449   .1766004   138.81   0.000     24.16832    24.86066
        1#W  |   24.16521   .1772397   136.34   0.000     23.81779    24.51264
       2#NE  |   25.40461   .1072029   236.98   0.000     25.19447    25.61475
       2#MW  |   25.43866   .0922856   275.65   0.000     25.25776    25.61956
        2#S  |   25.50629   .0922593   276.46   0.000     25.32544    25.68713
        2#W  |   25.29484   .0956797   264.37   0.000     25.10729    25.48239
       3#NE  |   26.92418   .1737943   154.92   0.000     26.58351    27.26485
       3#MW  |   26.75762   .1545335   173.15   0.000      26.4547    27.06054
        3#S  |   26.49809    .148221   178.77   0.000     26.20754    26.78863
        3#W  |   26.42447   .1522388   173.57   0.000     26.12605    26.72289
------------------------------------------------------------------------------
```

  - The observed values of `age` are from 20 to 74

Graphing Predicted Values

- And we can plot the results

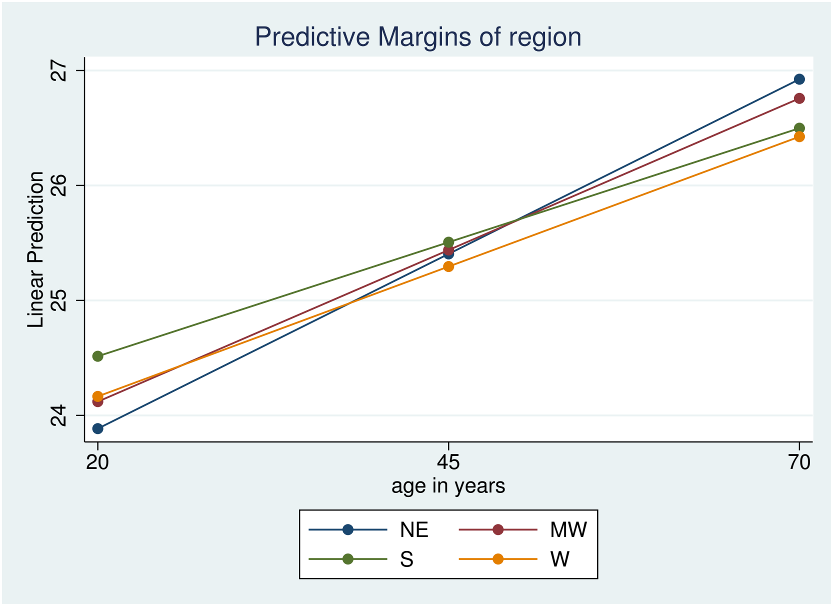
```
. marginsplot
```



Suppressing Confidence Intervals

- The confidence intervals can make the graph appear messy; we can suppress them

```
. marginsplot, noci
```



  - This is dangerous because it makes the predictions look more precise than they are

Testing for Differences

- We might want to perform tests of differences at different levels of the continuous variable
- To obtain tests of differences between levels of `region` at each level of `age`

```
. margins region, at(age=(20(10)70)) vsquish contrast

Contrasts of predictive margins
Model VCE    : OLS

Expression   : Linear prediction, predict()
1._at        : age             =          20
2._at        : age             =          30
3._at        : age             =          40
4._at        : age             =          50
5._at        : age             =          60
6._at        : age             =          70

------------------------------------------------
             |         df           F        P>F
-------------+----------------------------------
  region@_at |
          1  |          3        1.94     0.1200
          2  |          3        1.59     0.1884
          3  |          3        1.06     0.3642
          4  |          3        0.93     0.4251
          5  |          3        1.56     0.1974
          6  |          3        2.05     0.1041
      Joint  |          6        1.69     0.1193
             |
 Denominator |      10342
------------------------------------------------
```

Predicted Values Over Groups

- As with *marginslist*, when we specify `at()` Stata calculates predicted values treating each case as though they belong to each group or combination of values
- As before, we can use the `over()` option after models with categorical by continuous interactions
- For example, to obtain predicted values for each `region` using the observed values of `female` and `age` in that `region`

```
. margins, over(region)

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
over         : region

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      region |
         NE  |   25.57535   .1057592   241.83   0.000     25.36804    25.78266
         MW  |   25.51936   .0919307   277.59   0.000     25.33916    25.69956
          S  |   25.63317    .090649   282.77   0.000     25.45548    25.81086
          W  |   25.42299   .0944498   269.17   0.000     25.23785    25.60813
------------------------------------------------------------------------------
```

3.4 Investigating Continuous by Continuous Interactions

A Continuous by Continuous Interaction

- For this example we'll use a similar model for `bmi` but we'll add a main effect of serum vitamin c (`vitaminc`), and an interaction between `age` and `vitaminc`
- Before we fit the model, let's take a closer look at `vitaminc`

```
. summm vitaminc, detail

                   serum vitamin C (mg/dL)
-------------------------------------------------------------
      Percentiles      Smallest
 1%           .2             .1
 5%           .3             .1
10%           .3             .1       Obs               9,973
25%           .6             .1       Sum of Wgt.       9,973

50%            1                      Mean           1.034814
                        Largest       Std. Dev.      .5813791
75%          1.4            8.3
90%          1.7            9.4       Variance       .3380017
95%          1.9           13.9       Skewness       4.539869
99%          2.4           18.1       Kurtosis       108.2617
```

  ◇ The distribution has a long tail, but most observations are between .2 and 2.

- Now lets fit the model

```
. regress bmi c.age##c.vitaminc i.female i.region

      Source |       SS           df       MS      Number of obs   =     9,973
-------------+----------------------------------   F(7, 9965)      =     63.61
       Model |  10298.9223         7  1471.27461   Prob > F        =    0.0000
    Residual |  230479.207     9,965  23.1288718   R-squared       =    0.0428
-------------+----------------------------------   Adj R-squared   =    0.0421
       Total |   240778.13     9,972  24.1454201   Root MSE        =    4.8092

----------------------------------------------------------------------------------
             bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-----------------+----------------------------------------------------------------
             age |   .0220407   .0059366     3.71   0.000     .0104038    .0336777
        vitaminc |  -2.331426   .2717928    -8.58   0.000    -2.864194   -1.798657
                 |
c.age#c.vitaminc |    .029107   .0050017     5.82   0.000     .0193026    .0389115
                 |
          female |
              0  |          0  (base)
              1  |   .1858965   .0982311     1.89   0.058    -.0066564    .3784494
                 |
          region |
             NE  |          0  (base)
             MW  |  -.0936871   .1412331    -0.66   0.507    -.3705326    .1831584
              S  |  -.2137082   .1431247    -1.49   0.135    -.4942615    .0668451
              W  |  -.1626738   .1430181    -1.14   0.255    -.4430182    .1176706
                 |
           _cons |   25.45695   .3293507    77.29   0.000     24.81136    26.10255
----------------------------------------------------------------------------------
```

- We can replay the model using `coeflegend`

```
. regress, coeflegend
```

Estimating Slopes

- We can use `lincom` to calculate the slope for `vitaminc` when `age=49` (it's median)

```
. lincom vitaminc + c.vitaminc#c.age*49

 ( 1)  vitaminc + 49*c.age#c.vitaminc = 0

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |  -.9051806   .0870624   -10.40   0.000    -1.075841   -.7345206
------------------------------------------------------------------------------
```

- We could also calculate the slope of `age` when `vitaminc=1` (it's median)

```
. lincom age + c.vitaminc#c.age*1

 ( 1)  age + c.age#c.vitaminc = 0

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         (1) |   .0511478   .0028212    18.13   0.000     .0456176    .0566779
------------------------------------------------------------------------------
```

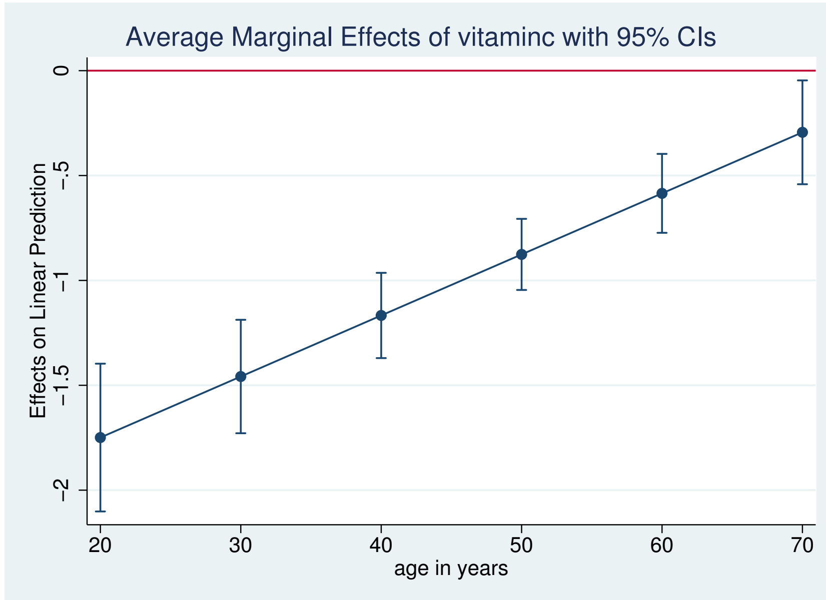
- `margins` can produce estimates of the slopes for a range of values

```
. margins, dydx(vitaminc) at(age=(20(10)70)) vsquish

Average marginal effects                        Number of obs     =      9,973
Model VCE    : OLS

Expression   : Linear prediction, predict()
dy/dx w.r.t. : vitaminc
1._at        : age             =          20
2._at        : age             =          30
3._at        : age             =          40
4._at        : age             =          50
5._at        : age             =          60
6._at        : age             =          70

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
vitaminc     |
         _at |
          1  |  -1.749285   .1797317    -9.73   0.000    -2.101595   -1.396974
          2  |  -1.458214   .1379304   -10.57   0.000    -1.728586   -1.187843
          3  |  -1.167144   .1036802   -11.26   0.000    -1.370378   -.9639098
          4  |  -.8760735   .0864746   -10.13   0.000    -1.045581   -.7065659
          5  |  -.5850031   .0959667    -6.10   0.000    -.7731173   -.3968889
          6  |  -.2939327    .126273    -2.33   0.020    -.5414532   -.0464122
------------------------------------------------------------------------------
```

Graphing Slopes

- We can graph the slopes of `vitaminc` across `age`

```
. marginsplot, yline(0)
```

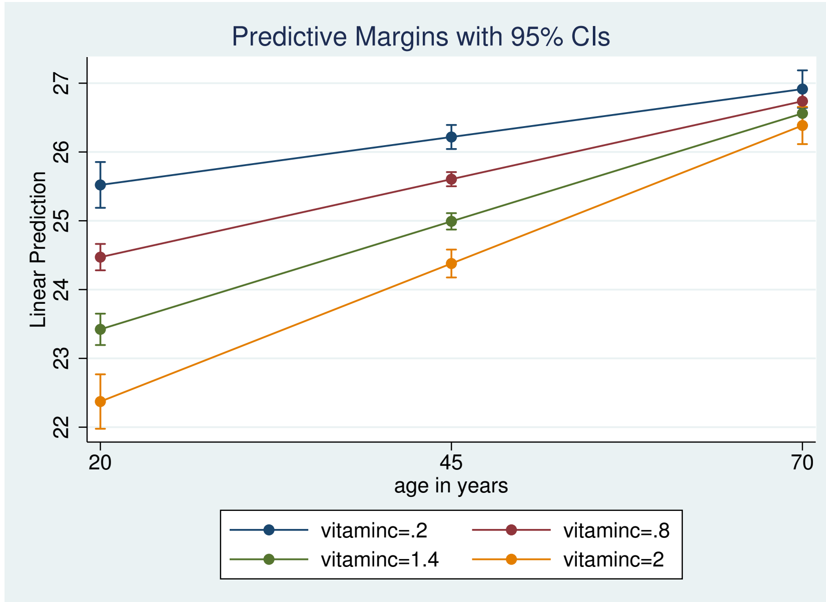


Predicted Values

- Specifying multiple variables in the at() option results in predictions at each combination of values

```
. margins , at(age=(20(25)70) vitaminc=(.2(.6)2)) vsquish

Predictive margins                              Number of obs     =      9,973
Model VCE    : OLS

Expression   : Linear prediction, predict()
1._at        : age             =          20
               vitaminc        =          .2
2._at        : age             =          20
               vitaminc        =          .8
3._at        : age             =          20
               vitaminc        =         1.4
4._at        : age             =          20
               vitaminc        =           2
5._at        : age             =          45
               vitaminc        =          .2
6._at        : age             =          45
               vitaminc        =          .8
7._at        : age             =          45
               vitaminc        =         1.4
8._at        : age             =          45
               vitaminc        =           2
9._at        : age             =          70
               vitaminc        =          .2
10._at       : age             =          70
               vitaminc        =          .8
11._at       : age             =          70
               vitaminc        =         1.4
12._at       : age             =          70
               vitaminc        =           2

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   25.52113   .1698638   150.24   0.000     25.18816     25.8541
          2  |   24.47156    .097744   250.36   0.000     24.27996    24.66316
          3  |   23.42199   .1162436   201.49   0.000     23.19413    23.64985
          4  |   22.37242   .2018162   110.86   0.000     21.97682    22.76802
          5  |   26.21768   .0891689   294.02   0.000     26.04289    26.39247
          6  |   25.60472   .0525344   487.39   0.000     25.50174    25.70769
          7  |   24.99175   .0606647   411.97   0.000     24.87284    25.11067
          8  |   24.37879   .1034993   235.55   0.000     24.17591    24.58167
          9  |   26.91423   .1388456   193.84   0.000     26.64207     27.1864
         10  |   26.73788   .0879343   304.07   0.000     26.56551    26.91024
         11  |   26.56152   .0875619   303.35   0.000     26.38988    26.73315
         12  |   26.38516   .1381377   191.01   0.000     26.11438    26.65593
------------------------------------------------------------------------------
```

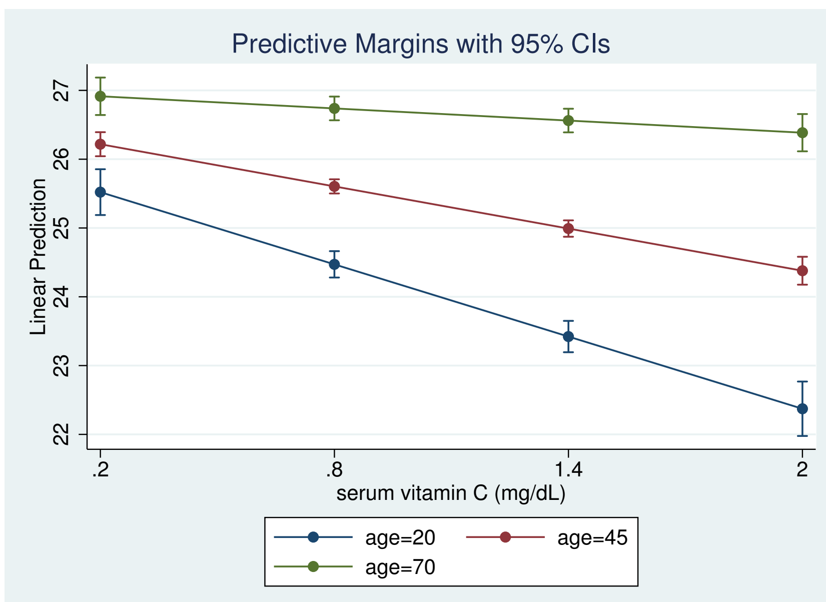
```
. marginsplot
```



Changing the X-axis Variable

- We can select which variable appears on the x-axis using the `xdimension()` option

```
. marginsplot, xdimension(vitaminc)
```



Models with Polynomial Terms

- We'll start by fitting a model that includes age and age^2

```
. regress bmi c.age##c.age i.female i.region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(6, 10344)     =     73.84
       Model |  10269.3919         6  1711.56532   Prob > F        =    0.0000
    Residual |   239754.77    10,344  23.1781487   R-squared       =    0.0411
-------------+----------------------------------   Adj R-squared   =    0.0405
       Total |  250024.162    10,350  24.1569239   Root MSE        =    4.8144

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         age |   .2731368   .0203077    13.45   0.000     .2333297    .3129439
             |
 c.age#c.age |  -.0024099   .0002162   -11.15   0.000    -.0028337   -.0019861
             |
      female |
          0  |          0  (base)
          1  |   .0462855   .0947764     0.49   0.625    -.1394945    .2320656
             |
      region |
         NE  |          0  (base)
         MW  |   .0322091   .1394036     0.23   0.817    -.2410489    .3054671
          S  |   .0289346   .1385186     0.21   0.835    -.2425886    .3004579
          W  |  -.1105093   .1410448    -0.78   0.433    -.3869844    .1659657
             |
       _cons |    18.6987   .4416971    42.33   0.000     17.83289    19.56451
------------------------------------------------------------------------------
```

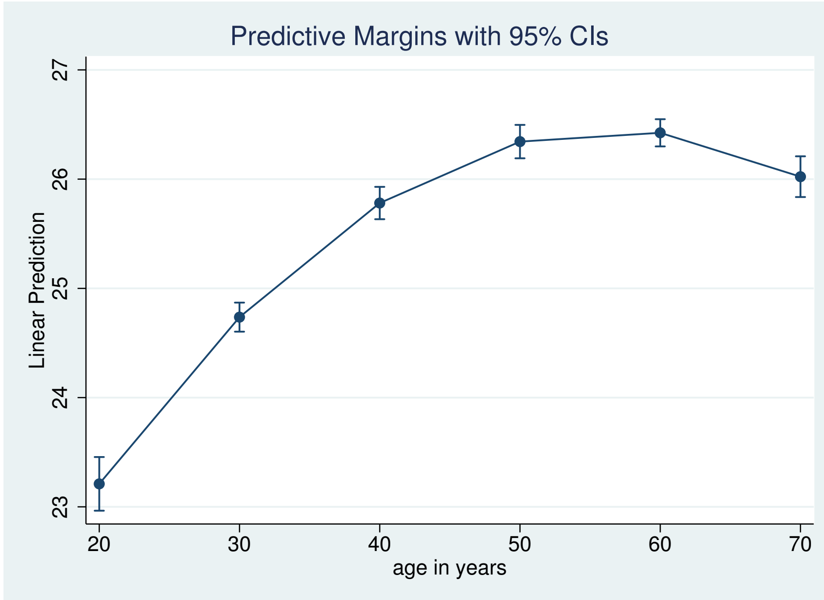
- Graphs can be particularly useful in understanding models with polynomial terms
- Here we predict values of `bmi` at different values of age

```
. margins, at(age=(20(10)70)) vsquish

Predictive margins                              Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
1._at        : age             =          20
2._at        : age             =          30
3._at        : age             =          40
4._at        : age             =          50
5._at        : age             =          60
6._at        : age             =          70

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   23.21033   .1253478   185.17   0.000     22.96462    23.45604
          2  |   24.73675   .0678653   364.50   0.000     24.60372    24.86977
          3  |   25.78118   .0755647   341.18   0.000     25.63306     25.9293
          4  |   26.34363   .0780441   337.55   0.000     26.19065    26.49661
          5  |    26.4241   .0635204   415.99   0.000     26.29959    26.54861
          6  |   26.02259   .0951272   273.56   0.000     25.83612    26.20905
------------------------------------------------------------------------------
```

Graphing Predicted Values

- And graph the predictions

```
. marginsplot
```



Slopes

- We can also obtain estimates of the slope of age across its range
- To do so we'll include age in both the dyed() and at() options

```
. margins, dydx(age) at(age=(20(10)70)) vsquish

Average marginal effects                          Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
dy/dx w.r.t. : age
1._at        : age             =          20
2._at        : age             =          30
3._at        : age             =          40
4._at        : age             =          50
5._at        : age             =          60
6._at        : age             =          70

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
age          |
         _at |
          1  |   .1767405   .0117968    14.98   0.000     .1536164    .1998646
          2  |   .1285424   .0076583    16.78   0.000     .1135307     .143554
          3  |   .0803442   .0039415    20.38   0.000     .0726181    .0880703
          4  |    .032146   .0031343    10.26   0.000     .0260022    .0382899
          5  |  -.0160521   .0064432    -2.49   0.013     -.028682   -.0034222
          6  |  -.0642503    .010517    -6.11   0.000    -.0848657   -.0436349
------------------------------------------------------------------------------
```

Adding a Cubic Term

- The same process can be used with higher order polynomials, here we add a cubic term for age

```
. regress bmi c.age##c.age##c.age i.female i.region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(7, 10343)     =     64.27
       Model |  10422.3157         7  1488.90224   Prob > F        =    0.0000
    Residual |  239601.846    10,343  23.1656044   R-squared       =    0.0417
-------------+----------------------------------   Adj R-squared   =    0.0410
       Total |  250024.162    10,350  24.1569239   Root MSE        =    4.8131

------------------------------------------------------------------------------------
               bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------------+----------------------------------------------------------------
               age |   .5056311   .0927387     5.45   0.000     .3238453    .6874169
                   |
       c.age#c.age |  -.0077683   .0020967    -3.70   0.000    -.0118782   -.0036583
                   |
 c.age#c.age#c.age |   .0000383   .0000149     2.57   0.010     9.07e-06    .0000675
                   |
            female |
                0  |          0  (base)
                1  |   .0449127   .0947522     0.47   0.636    -.1408201    .2306454
                   |
            region |
               NE  |          0  (base)
               MW  |   .0274302   .1393783     0.20   0.844    -.2457782    .3006386
                S  |    .025305   .1384883     0.18   0.855    -.2461589    .2967689
                W  |  -.1172832   .1410312    -0.83   0.406    -.3937317    .1591653
                   |
             _cons |    15.6426   1.268785    12.33   0.000     13.15554    18.12967
------------------------------------------------------------------------------------
```

- As before we can predict slopes at specified values of age

```
. margins, dydx(age) at(age=(20(10)70)) vsquish

Average marginal effects                          Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
dy/dx w.r.t. : age
1._at        : age             =          20
2._at        : age             =          30
3._at        : age             =          40
4._at        : age             =          50
5._at        : age             =          60
6._at        : age             =          70

------------------------------------------------------------------------------
             |            Delta-method
             |      dy/dx   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
age          |
         _at |
          1  |   .2408252   .0275901     8.73   0.000     .1867432    .2949071
          2  |   .1428662   .0094709    15.08   0.000     .1243014     .161431
          3  |     .06787   .0062529    10.85   0.000      .055613    .0801269
          4  |   .0158363   .0070791     2.24   0.025     .0019598    .0297128
          5  |  -.0132346   .0065341    -2.03   0.043    -.0260428   -.0004265
          6  |  -.0193429   .0203971    -0.95   0.343    -.0593252    .0206395
------------------------------------------------------------------------------
```

- Or predict `bmi` at different values of age

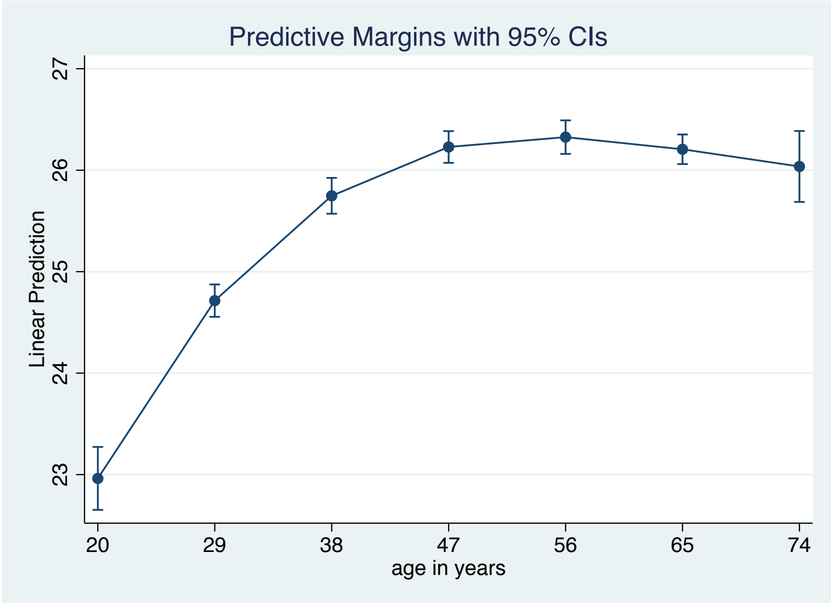
```
. margins, at(age=(20(9)74)) vsquish

Predictive margins                                Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()
1._at        : age             =          20
2._at        : age             =          29
3._at        : age             =          38
4._at        : age             =          47
5._at        : age             =          56
6._at        : age             =          65
7._at        : age             =          74

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
         _at |
          1  |   22.96222   .1582057   145.14   0.000      22.6521    23.27233
          2  |   24.71431   .0814708   303.35   0.000     24.55461    24.87401
          3  |   25.74733   .0900762   285.84   0.000     25.57077     25.9239
          4  |   26.22869   .0798098   328.64   0.000     26.07225    26.38513
          5  |   26.32577   .0843705   312.03   0.000     26.16039    26.49115
          6  |   26.20598   .0744024   352.22   0.000     26.06013    26.35182
          7  |   26.03671   .1783785   145.96   0.000     25.68705    26.38636
------------------------------------------------------------------------------
```

  - ◇ Here, we get predictions across the full range of ages in the dataset (i.e. 20-74)

Graphing the Cubic Term

- And we can easily graph this as well

```
. marginsplot
```



---

4 Conclusion

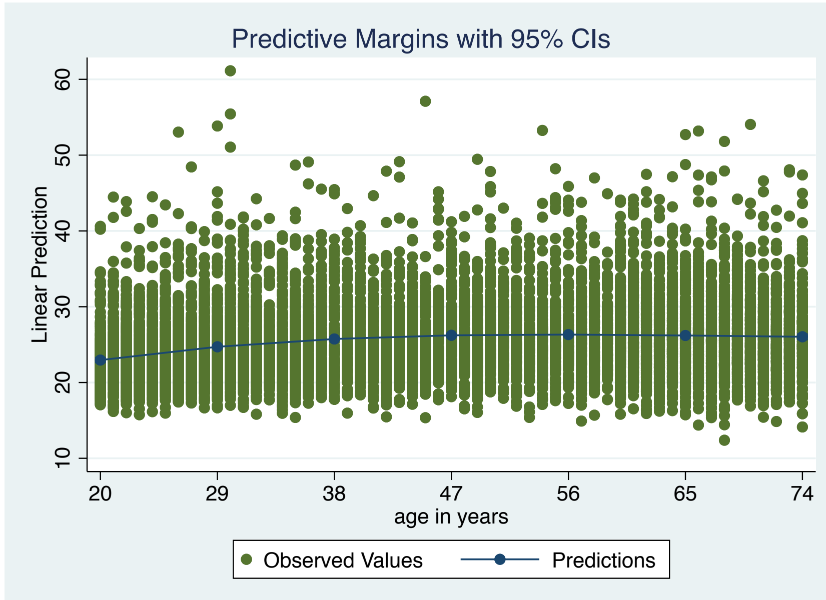
4.1 Graphing Extras

Adding Additional Plots

- We can add other types of `twoway` plots to the plots drawn by `marginsplots`
  - ◇ Continuing with our cubic example
- The `addplot` option allows us to add additional plots to our `marginsplots`
- We do want to be careful about the order in which graphs are drawn, we usually want the most dense graphs, for example individual data points, drawn first
  - ◇ Specifying `addplot(..., below)` draws the added plot below the `marginsplot`

Adding Observed Data

```
. marginsplot, addplot(scatter bmi age, below ///
    legend(order(3 "Observed Values" 2 "Predictions")) ///
    xlabel(20(9)74))
```



- Note: The confidence intervals are in the plot, they're just small relative to the scale of the y-axis, so they're hard to see.

Changing the Plot Type

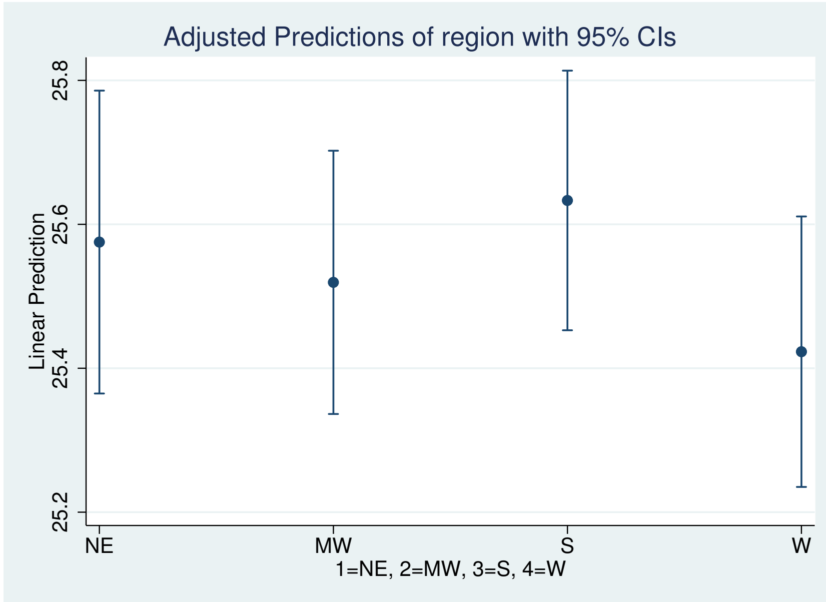
- We can change the plots drawn by `marginsplot` to another `twoway` plot type
  - ◇ See `help twoway` for a list
- The `recast()` option changes the plot for the predictions
  - ◇ `recastci()` changes how the CIs are plotted

- Let's run a simple model to demonstrate

```
. regress bmi i.region
. margins region

      Source |       SS           df       MS      Number of obs   =    10,351
-------------+----------------------------------   F(3, 10347)     =      0.89
       Model |  64.491028         3  21.4970093   Prob > F        =    0.4455
    Residual |  249959.671    10,347  24.1576951   R-squared       =    0.0003
-------------+----------------------------------   Adj R-squared   =   -0.0000
       Total |  250024.162    10,350  24.1569239   Root MSE        =     4.915

------------------------------------------------------------------------------
         bmi |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      region |
         NE  |          0  (base)
         MW  |   -.055989   .1422471    -0.39   0.694    -.3348208    .2228428
          S  |   .0578207   .1413969     0.41   0.683    -.2193446     .334986
          W  |  -.1523645   .1439376    -1.06   0.290    -.4345101    .1297811
             |
       _cons |   25.57535   .1073574   238.23   0.000     25.36491    25.78579
------------------------------------------------------------------------------

Adjusted predictions                            Number of obs     =     10,351
Model VCE    : OLS

Expression   : Linear prediction, predict()

------------------------------------------------------------------------------
             |            Delta-method
             |     Margin   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      region |
         NE  |   25.57535   .1073574   238.23   0.000     25.36491    25.78579
         MW  |   25.51936     .09332   273.46   0.000     25.33644    25.70229
          S  |   25.63317   .0920189   278.56   0.000      25.4528    25.81355
          W  |   25.42299   .0958771   265.16   0.000     25.23505    25.61092
------------------------------------------------------------------------------
```

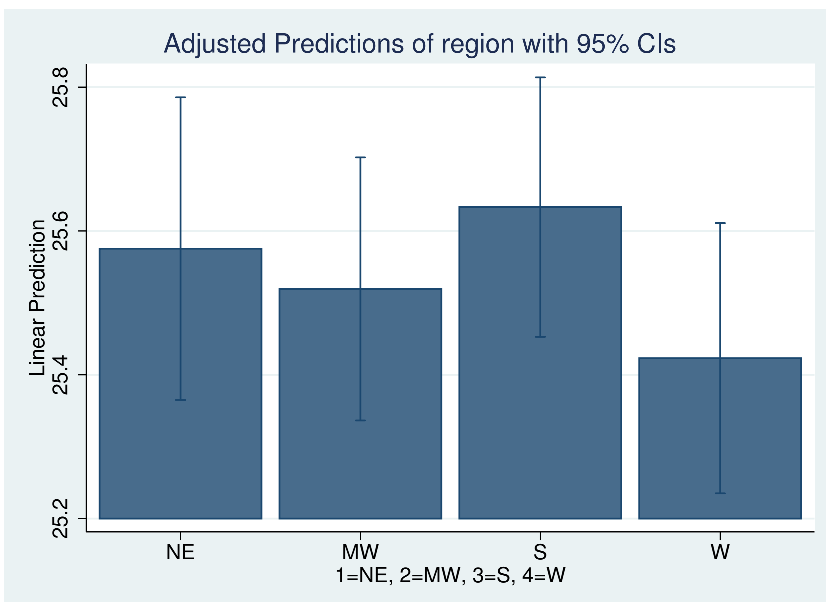
Estimates as a Scatterplot

```
. marginsplot, recast(scatter)
```



Estimates as a Bar plot

```
. marginsplot, recast(bar) plotopts(barwidth(.9))
```



- ◇ The `plotopts()` option allows you to specify options for the plots
- ◇ `barwidth()` specifies the width of the bars in units of the x variable

---

4.2 Conclusion

Conclusion

- We've seen how to fit models that include interactions
- We've learned how to use Stata's postestimation tools to explore the resulting models
- We've learned how to graph predictions and how to modify those graphs

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