Title

sdtest — Variance-comparison tests

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# Description

sdtest performs tests on the equality of standard deviations (variances). In the first form, sdtest tests that the standard deviation of *varname* is #. In the second form, sdtest performs the same test, using the standard deviations of the two groups defined by *groupvar*. In the third form, sdtest tests that *varname*<sub>1</sub> and *varname*<sub>2</sub> have the same standard deviation.

sdtesti is the immediate form of sdtest; see [U] 19 Immediate commands.

Both the traditional F test for the homogeneity of variances and Bartlett's generalization of this test to K samples are sensitive to the assumption that the data are drawn from an underlying Gaussian distribution. See, for example, the cautionary results discussed by Markowski and Markowski (1990). Levene (1960) proposed a test statistic for equality of variance that was found to be robust under nonnormality. Then Brown and Forsythe (1974) proposed alternative formulations of Levene's test statistic that use more robust estimators of central tendency in place of the mean. These reformulations were demonstrated to be more robust than Levene's test when dealing with skewed populations.

robvar reports Levene's robust test statistic  $(W_0)$  for the equality of variances between the groups defined by *groupvar* and the two statistics proposed by Brown and Forsythe that replace the mean in Levene's formula with alternative location estimators. The first alternative  $(W_{50})$  replaces the mean with the median. The second alternative replaces the mean with the 10% trimmed mean  $(W_{10})$ .

## **Quick start**

Test that the standard deviation of v1 is equal to 2 sdtest v1=2

- Equality of standard deviations (variances) test for v1 comparing the two groups defined by catvar1 sdtest v1, by(catvar1)
- Robust equality of variances test for v1 comparing the groups defined by catvar1 robvar v1, by(catvar1)

Compare the variances of v2 and v3 sdtest v2 == v3

Same as above, but with separate tests for each group defined by catvar2 by catvar2, sort: sdtest v2 == v3

```
Test \mathrm{sd}_1=\mathrm{sd}_2 for \mathrm{sd}_1=34,\,\mathrm{sd}_2=45,\,N_1=143,\,\mathrm{and}\,\,N_2=184 sdtesti 143 . 34 184 . 45
```

### Menu

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#### sdtesti

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#### robvar

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > Robust equal-variance test

# Syntax

One-sample variance-comparison test

sdtest varname == # [ if ] [ in ] [, level(#) ]

Two-sample variance-comparison test using groups

sdtest varname [if] [in], by(groupvar)  $[\underline{1}evel(#)]$ 

Two-sample variance-comparison test using variables

sdtest varname1 == varname2 [if] [in] [, level(#)]

Immediate form of one-sample variance-comparison test

sdtesti  $\#_{obs} \{ \#_{mean} | . \} \#_{sd} \#_{val} [, \underline{l}evel(\#)]$ 

Immediate form of two-sample variance-comparison test

 $\texttt{sdtesti} \; \texttt{\#}_{obs,1} \; \big\{ \, \texttt{\#}_{mean,1} \, | \, . \, \big\} \; \texttt{\#}_{sd,1} \; \texttt{\#}_{obs,2} \; \big\{ \, \texttt{\#}_{mean,2} \, | \, . \, \big\} \; \texttt{\#}_{sd,2} \; \big[ \; \texttt{, level(\#)} \; \big]$ 

Robust tests for equality of variances

robvar varname [if] [in], by(groupvar)

by and collect are allowed with sdtest and robvar, and collect is allowed with sdtesti; see [U] 11.1.10 Prefix commands.

# Options

- level(#) specifies the confidence level, as a percentage, for confidence intervals of the means. The default is level(95) or as set by set level; see [U] 20.8 Specifying the width of confidence intervals.
- by(groupvar) specifies the groupvar that defines the groups to be compared. For sdtest, there should be two groups, but for robvar there may be more than two groups. Do not confuse the by() option with the by prefix; both may be specified.

## **Remarks and examples**

#### stata.com

Remarks are presented under the following headings:

Basic form Immediate form Robust test

## **Basic form**

sdtest performs two different statistical tests: one testing equality of variances and the other testing that the standard deviation is equal to a known constant. Which test it performs is determined by whether you type a variable name or a number to the right of the equal sign.

### Example 1: One-sample test of variance

We have a sample of 74 automobiles. For each automobile, we know the mileage rating. We wish to test whether the overall standard deviation is 5 mpg:

```
. use https://www.stata-press.com/data/r18/auto
(1978 automobile data)
. sdtest mpg == 5
One-sample test of variance
```

Variable	Obs	Mean	Std. err.	Std. dev.	[95% conf.	interval]
mpg	74	21.2973	.6725511	5.785503	19.9569	22.63769
sd = s H0: sd = {	sd(mpg) 5			Degrees	c = chi2 = of freedom =	
	sd < 5 c) = 0.9717	2*P	Ha: $sd != !$ r(C > c) = 0	-		sd > 5 ) = 0.0283

### Example 2: Variance ratio test

We are testing the effectiveness of a new fuel additive. We run an experiment on 12 cars, running each without and with the additive. The data can be found in [R] ttest. The results for each car are stored in the variables mpg1 and mpg2:

. use https://www.stata-press.com/data/r18/fuel						
. sdtest r	. sdtest mpg1==mpg2					
Variance 1	ratio test					
Variable	Obs	Mean	Std. err.	Std. dev.	[95% conf.	interval]
mpg1	12	21	.7881701	2.730301	19.26525	22.73475
mpg2	12	22.75	.9384465	3.250874	20.68449	24.81551
Combined	24	21.875	.6264476	3.068954	20.57909	23.17091
ratio = sd(mpg1) / sd(mpg2) f = 0.7054						
HO: ratio = 1				Degrees	of freedom	= 11, 11
Ha: ratio < 1 Pr(F < f) = 0.2862			Ha: ratio != r(F < f) = 0			atio > 1 ) = 0.7138
$11(1 \times 1) = 0.2002$		2.1.		.0120	11(1 > 1	, 0.1100

We cannot reject the hypothesis that the standard deviations are the same.

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In [R] ttest, we draw an important distinction between paired and unpaired data, which, in this example, means whether there are 12 cars in a before-and-after experiment or 24 different cars. For sdtest, on the other hand, there is no distinction. If the data had been unpaired and stored as described in [R] ttest, we could have typed sdtest mpg, by(treated), and the results would have been the same. 4

### Immediate form

Example 3: sdtesti

Immediate commands are used not with data, but with reported summary statistics. For instance, to test whether a variable on which we have 75 observations and a reported standard deviation of 6.5 comes from a population with underlying standard deviation 6, we would type

```
. sdtesti 75 . 6.5 6
One-sample test of variance
```

	Obs	Mean	Std. err.	Std. dev.	[95% conf.	interval]
x	75	•	.7505553	6.5	•	
sd = sd(x) H0: $sd = 6$				Degrees	c = chi2 = of freedom =	
	sd < 6 c) = 0.8542	2*F	Ha: sd $!=$ Pr(C > c) = 0	-	Ha: s Pr(C > c)	

The mean plays no role in the calculation, so it may be omitted.

To test whether the variable comes from a population with the same standard deviation as another for which we have a calculated standard deviation of 7.5 over 65 observations, we would type

```
. sdtesti 75 . 6.5 65 . 7.5
```

Variance ratio test

	Obs	Mean	Std. err.	Std. dev.	[95% conf. in	nterval]
x y	75 65	•	.7505553 .9302605	6.5 7.5	:	•
Combined	140	•	•	•	•	•
<pre>ratio = sd(x) / sd(y) H0: ratio = 1</pre>		y)		Degrees	f = of freedom =	0.7511 74, 64
	atio < 1 f) = 0.1172		Ha: ratio != r(F < f) = 0	-	Ha: rati Pr(F > f) =	

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#### Robust test

#### Example 4: robvar

We wish to test whether the standard deviation of the length of stay for patients hospitalized for a given medical procedure differs by gender. Our data consist of observations on the length of hospital stay for 1778 patients: 884 males and 894 females. Length of stay, lengthstay, is highly skewed (skewness coefficient = 4.912591) and thus violates Bartlett's normality assumption. Therefore, we use robvar to compare the variances.

. use https://www.stata-press.com/data/r18/stay

. robvar lengthstay, by(sex)

Gender		Length of stay Std. dev.	in days Freq.
Male	9.0874434	9.7884747	884
Female	8.800671	9.1081478	894
Total	8.9432508	9.4509466	1,778
WO = 0.5550	05315 df(1,	1776) Pr >	F = 0.45635888
W50 = 0.427	14734 df(1,	1776) Pr >	F = 0.51347664
W10 = 0.445	77674 df(1,	1776) Pr >	F = 0.50443411

For these data, we cannot reject the null hypothesis that the variances are equal. However, Bartlett's test yields a significance probability of 0.0319 because of the pronounced skewness of the data.

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### Technical note

robvar implements both the conventional Levene's test centered at the mean and a median-centered test. In a simulation study, Conover, Johnson, and Johnson (1981) compare the properties of the two tests and recommend using the median test for asymmetric data, although for small sample sizes the test is somewhat conservative. See Carroll and Schneider (1985) for an explanation of why both mean- and median-centered tests have approximately the same level for symmetric distributions, but for asymmetric distributions the median test is closer to the correct level.

# **Stored results**

sdtest and sdtesti store the following in r():

Scalars

r(N)	number of observations
r(p_1)	lower one-sided p-value
r(p_u)	upper one-sided p-value
r(p)	two-sided p-value
r(F)	F statistic
r(sd)	standard deviation
r(sd_1)	standard deviation for first variable
r(sd_2)	standard deviation for second variable
r(df)	degrees of freedom
r(df_1)	numerator degrees of freedom
r(df_2)	denominator degrees of freedom
r(chi2)	$\chi^2$

robvar stores the following in r():

Scalars

r(N)	number of observations
r(w50)	Brown and Forsythe's F statistic (median)
r(p_w50)	Brown and Forsythe's p-value
r(w0)	Levene's F statistic
r(p_w0)	Levene's p-value
r(w10)	Brown and Forsythe's F statistic (trimmed mean)
r(p_w10)	Brown and Forsythe's <i>p</i> -value (trimmed mean)
$r(df_1)$	numerator degrees of freedom
r(df_2)	denominator degrees of freedom

## Methods and formulas

See Armitage et al. (2002, 149–153) or Bland (2015, 144–145) for an introduction and explanation of the calculation of these tests.

The test for  $\sigma = \sigma_0$  is given by

$$\chi^2 = \frac{(n-1)s^2}{\sigma_0^2}$$

which is distributed as  $\chi^2$  with n-1 degrees of freedom.

The test for  $\sigma_x^2 = \sigma_y^2$  is given by

$$F = \frac{s_x^2}{s_y^2}$$

which is distributed as F with  $n_x - 1$  and  $n_y - 1$  degrees of freedom.

Let  $X_{ij}$  be the *j*th observation of X for the *i*th group. Let  $Z_{ij} = |X_{ij} - \overline{X}_i|$ , where  $\overline{X}_i$  is the mean of X in the *i*th group. Levene's test statistic is

$$W_0 = \frac{\sum_i n_i (\overline{Z}_i - \overline{Z})^2 / (g-1)}{\sum_i \sum_j (Z_{ij} - \overline{Z}_i)^2 / \sum_i (n_i - 1)}$$

where  $n_i$  is the number of observations in group *i* and *g* is the number of groups.  $W_{50}$  is obtained by replacing  $\overline{X}_i$  with the *i*th group median of  $X_{ij}$ , whereas  $W_{10}$  is obtained by replacing  $\overline{X}_i$  with the 10% trimmed mean for group *i*.

### References

- Armitage, P., G. Berry, and J. N. S. Matthews. 2002. Statistical Methods in Medical Research. 4th ed. Oxford: Blackwell.
- Bland, M. 2015. An Introduction to Medical Statistics. 4th ed. Oxford: Oxford University Press.
- Brown, M. B., and A. B. Forsythe. 1974. Robust tests for the equality of variances. Journal of the American Statistical Association 69: 364–367. https://www.jstor.org/stable/2285659.
- Carroll, R. J., and H. Schneider. 1985. A note on Levene's tests for equality of variances. *Statistics and Probability Letters* 3: 191–194. https://doi.org/10.1016/0167-7152(85)90016-1.
- Conover, W. J., M. E. Johnson, and M. M. Johnson. 1981. A comparative study of tests for homogeneity of variances, with applications to the outer continental shelf bidding data. *Technometrics* 23: 351–361. https://doi.org/10.2307/1268225.
- Gastwirth, J. L., Y. R. Gel, and W. Miao. 2009. The impact of Levene's test of equality of variances on statistical theory and practice. *Statistical Science* 24: 343–360. https://doi.org/10.1214/09-STS301.

- Levene, H. 1960. Robust tests for equality of variances. In Contributions to Probability and Statistics: Essays in Honor of Harold Hotelling, ed. I. Olkin, S. G. Ghurye, W. Hoeffding, W. G. Madow, and H. B. Mann, 278–292. Menlo Park, CA: Stanford University Press.
- Markowski, C. A., and E. P. Markowski. 1990. Conditions for the effectiveness of a preliminary test of variance. American Statistician 44: 322–326. https://doi.org/10.2307/2684360.

## Also see

[R] **ttest** — t tests (mean-comparison tests)

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