Title

eivreg — Errors-in-variables regression

Description

eivreg fits errors-in-variables regression models when one or more of the independent variables are measured with error. To use eivreg, you must have an estimate of each independent variable's reliability or assume it is measured without error.

Quick start

Regression of y on x1, x2, and x3 adjusted for x1 measured with 90% reliability

    eivreg y x1 x2 x3, reliab(x1 .9)

Same as above, but also specify 80% reliability for x2

    eivreg y x1 x2 x3, reliab(x1 .9 x2 .8)

Menu

Statistics > Linear models and related > Errors-in-variables regression

Syntax

> eivreg *depvar* [*indepvars*] [*if*] [*in*] [*weight*] [, *options*]

options	Description
Model	
<u>reliab</u>(*indepvar* # [*indepvar* # [...]])	
	specify measurement reliability for each *indepvar* measured with error
Reporting	
<u>l</u>evel(*#*)	set confidence level; default is level(95)
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
coeflegend	display legend instead of statistics

indepvars may contain factor variables; see [U] **11.4.3 Factor variables**.

bootstrap, by, collect, jackknife, rolling, and statsby are allowed; see [U] **11.1.10 Prefix commands**.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

aweights are not allowed with the jackknife prefix; see [R] **jackknife**.

aweights and fweights are allowed; see [U] **11.1.6 weight**.

coeflegend does not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options

> **Model**

reliab(*indepvar* # [*indepvar* # [...]]) specifies the measurement reliability for each independent variable measured with error. Reliabilities are specified as pairs consisting of an independent variable name (a name that appears in *indepvars*) and the corresponding reliability r, $0 < r \le 1$. Independent variables for which no reliability is specified are assumed to have reliability 1. If the option is not specified, all variables are assumed to have reliability 1, and the result is thus the same as that produced by regress (the ordinary least-squares results).

> **Reporting**

level(*#*); see [R] **Estimation options**.

display_options: noci, nopvalues, <u>noomit</u>ted, vsquish, <u>noempty</u>cells, <u>base</u>levels, <u>allbase</u>levels, <u>nofvlab</u>el, fvwrap(*#*), fvwrapon(*style*), cformat(%*fmt*), pformat(%*fmt*), sformat(%*fmt*), and nolstretch; see [R] **Estimation options**.

The following option is available with eivreg but is not shown in the dialog box:

coeflegend; see [R] **Estimation options**.

Remarks and examples

For an introduction to errors-in-variables regression, see Draper and Smith (1998, 89–91) or Kmenta (1997, 352–357). Treiman (2009, 258–261) compares the results of errors-in-variables regression with conventional regression. Also see Lockwood and McCaffrey (2020) for how to use sem (see [SEM] sem) to fit errors-in-variables regression.

Errors-in-variables regression models are useful when one or more of the independent variables are measured with additive noise. Standard regression (as performed by regress) would underestimate the effect of the variable, and the other coefficients in the model can be biased to the extent that they are correlated with the poorly measured variable. You can adjust for the biases if you know the reliability:

$$r = 1 - \frac{\text{noise variance}}{\text{total variance}}$$

That is, given the model $\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \mathbf{u}$, for some variable $\mathbf{x}_i$ in $\mathbf{X}$, the $\mathbf{x}_i$ is observed with error, $\mathbf{x}_i = \mathbf{x}_i^* + \mathbf{e}$, and the noise variance is the variance of $\mathbf{e}$. The total variance is the variance of $\mathbf{x}_i$.

▷ Example 1

Say that in our automobile data, the weight of cars was measured with error, and the reliability of our measured weight is 0.85. The result of this would be to underestimate the effect of weight in a regression of, say, price on weight and foreign, and it would also bias the estimate of the coefficient on foreign (because being of foreign manufacture is correlated with the weight of cars). We would ignore all of this if we fit the model with regress:

```
. use https://www.stata-press.com/data/r18/auto
(1978 automobile data)

. regress price weight foreign
```

Source	SS	df	MS		
Model	316859273	2	158429637		
Residual	318206123	71	4481776.38		
Total	635065396	73	8699525.97		

Number of obs	=	74
F(2, 71)	=	35.35
Prob > F	=	0.0000
R-squared	=	0.4989
Adj R-squared	=	0.4848
Root MSE	=	2117

price	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]
weight	3.320737	.3958784	8.39	0.000	2.531378 4.110096
foreign	3637.001	668.583	5.44	0.000	2303.885 4970.118
_cons	-4942.844	1345.591	-3.67	0.000	-7625.876 -2259.812

With `eivreg`, we can account for our measurement error:

```
. eivreg price weight foreign, reliab(weight .85)

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```

Variable	Assumed reliability
weight	0.8500
*	1.0000

Number of obs	=	74
F(2, 71)	=	18.46
Prob > F	=	0.0000
R-squared	=	0.6483
Root MSE	=	1773.54

price	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]
weight	4.31985	.7134251	6.06	0.000	2.89732	5.742379
foreign	4637.32	849.0221	5.46	0.000	2944.418	6330.222
_cons	-8257.017	2390.337	-3.45	0.001	-13023.21	-3490.821

The effect of weight is increased—as we knew it would be—and here the effect of foreign manufacture is also increased. A priori, we knew only that the estimate of `foreign` might be biased; we did not know the direction.

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❑ Technical note

Swept under the rug in our example is how we would determine the reliability, r. We can easily see that a variable is measured with error, but we may not know the reliability because the ingredients for calculating r depend on the unobserved noise.

For our example, we made up a value for r, and in fact we do not believe that weight is measured with error at all, so the reported `eivreg` results have no validity. The `regress` results were the statistically correct results here.

But let's say that we do suspect that weight is measured with error and that we do not know r. We could then experiment with various values of r to describe the sensitivity of our estimates to possible error levels. We may not know r, but r does have a simple interpretation, and we could probably produce a sensible range for r by thinking about how the data were collected.

If the reliability, r, is less than the R^2 from a regression of the poorly measured variable on all the other variables, including the dependent variable, the information might as well not have been collected; no adjustment to the final results is possible. For our automobile data, running a regression of `weight` on `foreign` and `price` would result in an R^2 of 0.6743. Thus, the reliability must be at least 0.6743 here. If we specify a reliability that is too small, `eivreg` will inform us and refuse to fit the model:

```
. eivreg price weight foreign, reliab(weight .6742)
reliability r() too small
r(399);
```

Returning to our problem of how to estimate r, too small or not, if the measurements are summaries of scaled items, the reliability may be estimated using the `alpha` command; see [MV] **alpha**. If the score is computed from factor analysis and the data are scored using `predict`'s default options (see [MV] **factor postestimation**), the square of the standard deviation of the score is an estimate of the reliability.

❑

▷ Example 2

Consider a model with more than one variable measured with error. For instance, say that our model is that price is a function of weight, foreign, and mpg and that both weight and mpg are measured with error.

```
. eivreg price weight foreign mpg, reliab(weight .85 mpg .95)
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```

Variable	Assumed reliability
weight	0.8500
mpg	0.9500
*	1.0000

Number of obs	=	74
F(3, 70)	=	9.58
Prob > F	=	0.0000
R-squared	=	0.8522
Root MSE	=	1158.04

price	Coefficient	Std. err.	t	P>|t|	[95% conf. interval]	
weight	9.69903	3.768985	2.57	0.012	2.182027	17.21603
foreign	6918.624	2259.531	3.06	0.003	2412.132	11425.12
mpg	627.6764	431.0284	1.46	0.150	-231.9826	1487.335
_cons	-38545.27	20960.72	-1.84	0.070	-80350.11	3259.564

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Stored results

eivreg stores the following in e():

Scalars
    e(N)   number of observations
    e(df_m)   model degrees of freedom
    e(df_r)   residual degrees of freedom
    e(r2) R^2
    e(F) F statistic
    e(rmse)   root mean squared error
    e(rank)   rank of e(V)

Macros
    e(cmd)   eivreg
    e(cmdline)   command as typed
    e(depvar)   name of dependent variable
    e(rellist)   *indepvars* and associated reliabilities
    e(wtype)   weight type
    e(wexp)   weight expression
    e(properties)   b V
    e(predict)   program used to implement predict
    e(marginsok)   predictions allowed by margins
    e(asbalanced)   factor variables fvset as asbalanced
    e(asobserved)   factor variables fvset as asobserved

Matrices
    e(b)   coefficient vector
    e(V)   variance–covariance matrix of the estimators

Functions
    e(sample)   marks estimation sample

In addition to the above, the following is stored in `r()`:

Matrices
r(table)   matrix containing the coefficients with their standard errors, test statistics, p-values,
                              and confidence intervals

Note that results stored in `r()` are updated when the command is replayed and will be replaced when
any r-class command is run after the estimation command.

Methods and formulas

Let the model to be fit be

$$\mathbf{y} = \mathbf{X}^* \boldsymbol{\beta} + \boldsymbol{\epsilon}$$

$$\mathbf{X} = \mathbf{X}^* + \mathbf{U}$$

where $\mathbf{X}^*$ are the true values and $\mathbf{X}$ are the observed values. $\boldsymbol{\epsilon}$ and $\mathbf{U}$ are assumed to be independent
and have zero means and finite fourth moments. $\text{Var}(\mathbf{U})$ is assumed to be diagonal.

Let $\mathbf{W}$ be the user-specified weights. If no weights are specified, $\mathbf{W} = \mathbf{I}$. If weights are specified,
let $\mathbf{v}$ be the specified weights. If `fweight` frequency weights are specified, then $\mathbf{W} = \text{diag}(\mathbf{v})$. If
`aweight` analytic weights are specified, then $\mathbf{W} = \text{diag}\{\mathbf{v}/(\mathbf{1}'\mathbf{v})(\mathbf{1}'\mathbf{1})\}$, meaning that the weights
are normalized to sum to the number of observations.

The estimates $\mathbf{b}$ of $\boldsymbol{\beta}$ are obtained as $\mathbf{A}^{-1}\mathbf{X}'\mathbf{W}\mathbf{y}$, where $\mathbf{A} = \mathbf{X}'\mathbf{W}\mathbf{X} - \mathbf{S}$. $\mathbf{S}$ is a diagonal matrix
with elements $(1 - r_j)s_j^2$. Here r_j is the user-specified reliability coefficient for the jth explanatory
variable (or 1 if not specified), and s_j^2 is the (appropriately weighted) sample variance of the variable.

The root mean squared error is $(\mathbf{y}'\mathbf{W}\mathbf{y} - \mathbf{b}\mathbf{A}\mathbf{b}')/(n - p)$, where n is the number of observations
and p is the number of estimated parameters. The variance–covariance matrix of the estimators is
obtained based on the formulas provided in Stefanski and Boos (2002), Buonaccorsi (2010), and
Fuller (1987). For each $i = 1, 2, \ldots, n$, let residual $e_i = y_i - \mathbf{x}_i\mathbf{b}$, where $\mathbf{x}_i$ is the ith row of $\mathbf{X}$.
Consider matrix $\mathbf{H}$, where the ith row of $\mathbf{H}$, $\mathbf{h}_i$, is

$$\mathbf{h}_i' = \begin{pmatrix} e_i x_{i1} + (x_{i1} - \overline{x}_1)^2(1 - r_1)b_1 \\ e_i x_{i2} + (x_{i2} - \overline{x}_2)^2(1 - r_2)b_2 \\ \vdots \\ e_i x_{ip} + (x_{ip} - \overline{x}_p)^2(1 - r_p)b_p \end{pmatrix}$$

where $\overline{x}_j$ is the weighted mean of the jth variable.

If analytic weights, `aweights`, are specified, the variance–covariance matrix is $\mathbf{A}^{-1}$
$\mathbf{H}'\mathbf{W}\mathbf{W}\mathbf{H}\mathbf{A}^{-1}$; otherwise, it is $\mathbf{A}^{-1}\mathbf{H}'\mathbf{W}\mathbf{H}\mathbf{A}^{-1}$.

References

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Lee, Y. J., and D. Wilhelm. 2020. Testing for the presence of measurement error in Stata. *Stata Journal* 20: 382–404.

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Also see

[R] **eivreg postestimation** — Postestimation tools for eivreg

[R] **regress** — Linear regression

[SEM] **Example 24** — Reliability

[U] **20 Estimation and postestimation commands**