

New methods for filtering time series in Stata 12

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Outline

- 1 Filters and methods
- 2 An introduction to linear filters
- 3 Filters and examples

Filters, broadly interpreted

- Consider a time series that can be decomposed into two additively separable parts

$$y_t = \tau_t + c_t$$

where τ_t is the trend component that we want to remove and c_t is the stationary component that we wish to analyze

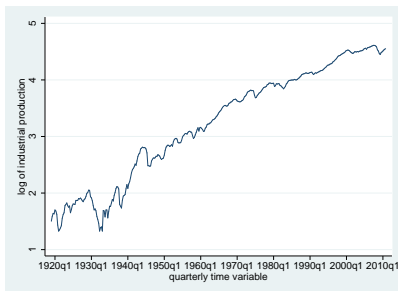
- I am going to discuss some new methods in Stata 12 that can be used to filter the trend component out of the series y_t and leave behind the stationary component c_t
 - One we have c_t , we can calculate $\tau_t = y_t - c_t$

New methods

- This talk will discuss the linear filters
 - Baxter-King band-pass filter
 - Butterworth high-pass filter
 - Christiano-Fitzgerald band-pass filter
 - Hodrick-Prescott high-pass filter
- Stata 12 has also new commands for estimation and postestimation of
 - the unobserved-components model, and
 - the autoregressive fractionally integrated moving average modelwhich can also be used as filters

Filtering industrial production

- Here a plot of quarterly observations on the natural log of industrial production in the United States



- This series is trending over time
- We want to remove the trend and isolate the business-cycle component that is composed of stochastic cycles at periods between 6 and 32 periods

From time domain to frequency domain

- We present some results for stationary processes and deal with trending process as we proceed
- The autocovariances $\gamma_j, j \in \{0, 1, \dots, \infty\}$ of a covariance-stationary process y_t specify its variance and dependence structure
- In the frequency-domain approach to time-series analysis, y_t and the autocovariances are specified in terms of independent stochastic cycles that occur at frequencies $\omega \in [-\pi, \pi]$.
- The spectral-density function $f_y(\omega)$ specifies the contribution of stochastic cycles at each frequency ω relative to the variance of y_t , which is denoted by σ_y^2

From time domain to frequency domain II

- The variance and the autocovariances can be expressed as an integral of the spectral-density function

$$\gamma_j = \int_{-\pi}^{\pi} e^{i\omega j} f_y(\omega) d\omega \quad (1)$$

where i is the imaginary number $i = \sqrt{-1}$,

- Equation (1) can be manipulated to show what fraction of the variance of y_t is attributable to stochastic cycles in a specified range of frequencies
- Equation (1) implies that if $f_y(\omega) = 0$ for $\omega \in [\omega_1, \omega_2]$, stochastic cycles at these frequencies contribute zero to the variance and autocovariances of y_t

From time domain to frequency domain III

- Time-series filters attempt to transform the original series into a new series y_t^* for which the spectral-density function of the filtered series $f_{y^*}(\omega)$ is 0 for unwanted frequencies and equal to $f_y(\omega)$ for desired frequencies

From time domain to frequency domain IV

- A linear filter of y_t can be written as

$$y_t^* = \sum_{j=-\infty}^{\infty} \alpha_j y_{t-j} = \sum_{j=-\infty}^{\infty} \alpha(L) y_t$$
- By expressing the spectral density of the filtered series $f_{y^*}(\omega)$ in terms of the spectral density of the original series $f_y(\omega)$ and the filter weights α_j

$$f_{y^*}(\omega) = |\alpha(e^{i\omega})|^2 f_y(\omega) \quad (2)$$

we see the impact of the filter on the components of y_t at each frequency ω ,

- $|\alpha(e^{i\omega})|$ is known as the gain of the filter
- Equation (2) makes explicit that it is the squared gain function $|a(e^{i\omega})|^2$ that converts $f_y(\omega)$ into $f_{y^*}(\omega)$
- Equation (2) says that, for each frequency ω , the spectral density of the filtered series is the product of the square of the gain of the filter and the spectral density of the original series.

From time domain to frequency domain V

- Recall that

$$f_{y^*}(\omega) = |\alpha(e^{i\omega})|^2 f_y(\omega) \quad (2)$$

- We want a filter for which $f_{y^*}(\omega) = 0$ for unwanted frequencies and for which $f_{y^*}(\omega) = f_y(\omega)$ for desired frequencies
- So we seek a filter for which the gain is 0 for unwanted frequencies and for which the gain is 1 for desired frequencies

From time domain to frequency domain VI

- In practice, we cannot find such an ideal filter exactly
 - The constraints an ideal filter places on filter coefficients cannot be satisfied for time-series with only a finite number of observations
- The large literature on filters is a result of the trade offs involved in designing implementable filters that approximate the ideal filter

From time domain to frequency domain VII

- Ideally, filters pass or block the components caused by stochastic cycles of specified frequencies by having a gain of 1 or 0
- Band-pass filters, such as the Baxter-King (BK) and the Christiano–Fitzgerald (CF) filters, allow the components in the specified range of frequencies to pass through and they block all the other components
- High-pass filters such as the Hodrick–Prescott (HP) and Butterworth filters, only allow the components of stochastic cycles at or above a specified frequency to pass through and they block the components corresponding to the lower-frequency stochastic cycles

tsfilter

- New command `tsfilter` implements the Baxter-King, Butterworth, Chistiano-Fitzgerald, and Hodrick-Prescott filters
- We thank Kit Baum for his previous work in writing commands !!

Baxter-King filter

- Symmetric moving-average (SMA) filters with coefficients that sum to 0 remove trends from a series
 - linear or quadratic deterministic trends
 - first-order or second-order integration stochastic trends
(first-order integration requires differencing once for stationarity)
(second-order integration requires differencing twice for stationarity)
- There is an ideal SMA band-pass filter with coefficients that sum to 0
- The Baxter-King (BK) filter uses $2q + 1$ coefficients to approximate the infinite-order ideal filter, see [Baxter and King(1995), Baxter and King(1999)]
- There is a trade off in choosing q
Larger values of q cause the gain of the BK filter to be closer to the gain of the ideal filter, but they also increase the number of missing observations in the filtered series

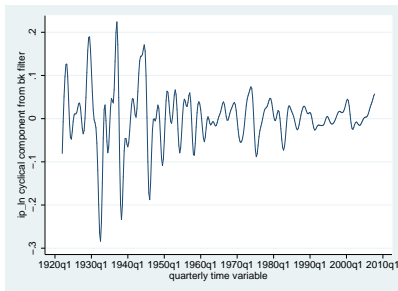
Periods and frequencies

- The mathematics of the frequency-domain approach to time-series analysis is in terms of components at frequencies $\omega \in [-\pi, \pi]$
- Applied work is generally in terms of periods p , where $p = 2\pi/\omega$
- The options for `tsfilter` are in terms of periods

BK filter on industrial production data

- Below we use `tsfilter bk`, which implements the BK filter, to estimate the business-cycle component composed of shocks between 6 and 32 periods and then we graph the estimated component.

```
. tsfilter bk ip_bk = ip_ln, minperiod(6) maxperiod(32)  
. tsline ip_bk
```

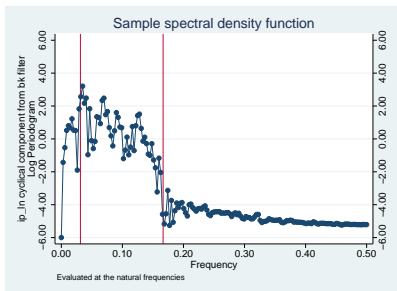


Periodogram specification view

- The graph of the filtered series tells us what the estimated business-cycle component looks like, but it presents no evidence as to how well we have estimated the component
- A periodogram is an estimator of a transform of the spectral density function
- Below we use `pergram` to plot the periodogram for the BK estimate of the business-cycle component.
- `pergram` displays the results in natural frequencies, which are the standard frequencies divided by 2π .
- The lower natural-frequency cutoff ($1/32 = 0.03125$) and the upper natural-frequency cutoff ($1/6 \approx 0.16667$).

Periodogram of BK filtered series

```
. pergram ip_bk, xline(0.03125 0.16667)
```



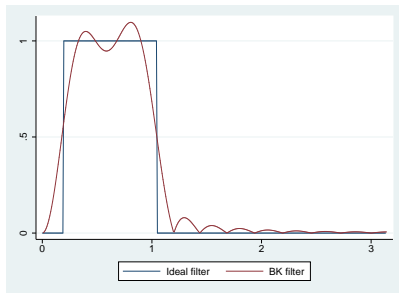
- If the filter completely removed the stochastic cycles at the unwanted frequencies, the periodogram would be a flat line at the minimum value of -6 outside the range identified by the vertical lines
- That the periodogram takes on values greater than -6 outside the specified range indicates the inability of the BK filter to pass through only components of frequencies inside the specified band

Gains of BK and ideal filters

- We can also evaluate the BK filter by plotting its gain function against the gain function of an ideal filter.
 - The coefficients and the gain of the BK filter are completely determined by the specified minimum period, maximum period, and the order of the SMA filter

Gains of BK and ideal filters II

```
. drop ip_bk
. tsfilter bk ip_bk = ip_ln, minperiod(6) maxperiod(32) gain(bkgain abk)
. label variable bkgain "BK filter"
```

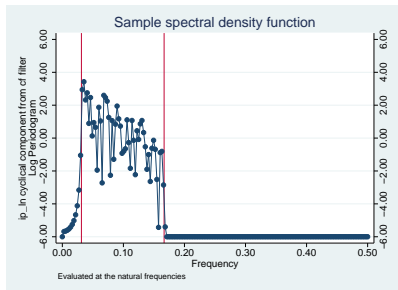


- The gain of the BK filter deviates markedly from the square-wave gain of the ideal filter
- Using option `smaorder()` to increase the order will cause the gain of the BK filter to more closely approximate the gain of the ideal filter at the cost of lost observations in the filtered series

- Recall that the Baxter-King filter minimizes the error between the filter coefficients and the ideal band-pass filter coefficients
- The Christiano-Fitzgerald (CF) filter minimizes the mean squared error between the estimated component and the true component, assuming that the raw series is a random-walk process, see [Christiano and Fitzgerald(2003)]
- The CF filter obtains its optimality properties at the cost of an additional parameter that must be estimated and a loss of robustness
 - The CF filter is optimal for a random-walk process
 - If the true process is a random walk with drift, the drift term must be estimated and removed
 - The CF filter also works well for processes that are close to being random walks or random walks plus drift

CF filtered series

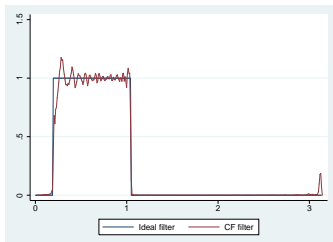
```
. tsfilter cf ip_cf = ip_ln, minperiod(6) maxperiod(32) drift
. pergram ip_cf, xline(0.03125 0.16667)
```



- The periodogram indicates that the CF filter did a better job than the BK filter of passing through only the business-cycle components
- Given that `ip_ln` is well approximated by a random-walk-plus-drift process, the relative performance of the CF filter is not surprising

Gains of CF and ideal filters

```
. drop ip_cf
. tsfilter cf ip_cf = ip_ln, minperiod(6) maxperiod(32) drift gain(cfgain acf)
. label variable cfgain "CF filter"
```



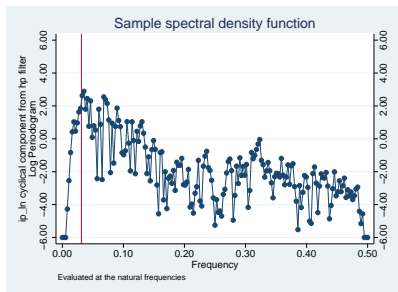
- Comparing this graph with the graph of the BK gain function, reveals that the CF filter is closer to the gain of the ideal filter than the BK filter
- The graph also reveals that the gain of the CF filter oscillates above and below 1 for desired frequencies
 - Some of the desired stochastic cycles are given too much weight and some too little

Hodrick-Prescott filter

- [Hodrick and Prescott(1981)] and [Hodrick and Prescott(1997)] motivated the Hodrick-Prescott (HP) filter as a trend-removal technique that could be applied to data that came from a wide class of data generating processes.
- In their view, the technique specified a trend in the data and the data were filtered by removing the trend
 - The smoothness of the trend depends on a parameter λ
 - The trend becomes smoother as $\lambda \rightarrow \infty$ and [Hodrick and Prescott(1997)] recommended setting λ to 1600 for quarterly data.
- [King and Rebelo(1993)] showed that removing a trend estimated by the HP filter is equivalent to a high-pass filter
- They derived the gain function of this high-pass filter and showed that the filter would make integrated processes of order 4 or less stationary

HP filtered series

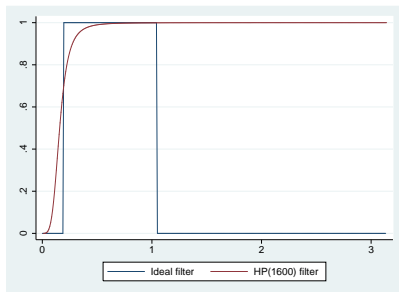
```
. tsfilter hp ip_hp = ip_ln, gain(hpg1600 ahp1600)
. label variable hpg1600 "HP(1600) filter"
. pergram ip_hp, xline(0.03125)
```



- Because the HP filter is a high-pass filter, the high-frequency stochastic components remain in the estimated component
- The presence of the low-frequency shocks that the filter should remove is concerning

Gains of HP and ideal filters

```
. twoway line ideal f || line hpg1600 ahp1600
```



- The HP gain function deviates markedly from the ideal gain function
- In [TS] tsfilter, we use results from [Ravn and Uhlig(2002)] to pick a better choice for λ , but the HP filter still performs poorly
- One explanation is that this high-pass filter only has one parameter

Butterworth filter

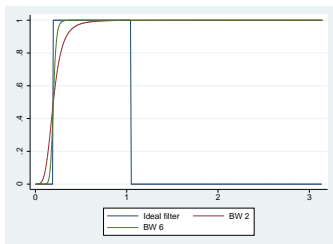
- Engineers have used Butterworth filters for a long time because they are “maximally flat”
 - The gain functions of these filters are as close as possible to being a flat line at 0 for the unwanted periods and a flat line at 1 for the desired periods, see [Butterworth(1930)] and [Bianchi and Sorrentino(2007)] (pp 17–20)
- [Pollock(2000)] showed that Butterworth filters can be derived from some axioms that specify properties we would like a filter to have. While the Butterworth and BK filters share the properties of symmetry and phase neutrality, the coefficients of Butterworth filters do not need to sum to 0
 - Phase-neutral filters do not shift the signal forward or backward in time, see [Pollock(1999)].
- While the BK filter relies on the detrending properties of SMA filters with coefficients that sum to 0, [Pollock(2000)] shows that Butterworth filters have detrending properties which depend on the filters’ parameters

Butterworth filter

- `tsfilter` `bw` implements the high-pass Butterworth filter using the computational method that [Pollock(2000)] derived
- This filter has two parameters, the cut-off period and the order of the filter denoted by m
 - The cut-off period sets the location where the gain function starts to filter out the high-period (low-frequency) components and m sets the slope of the gain function for a given cut-off period
 - For a given cut-off period, the slope of the gain function at the cut-off period increases with m
 - For a given m , the slope of the gain function at the cut-off period increases with the cut-off period
 - We cannot obtain a vertical slope at the cut-off frequency, which is the ideal, because the computation becomes unstable, see [Pollock(2000)]
 - The m for which the computation becomes unstable depends on the cut-off period

Gains of Butterworth and ideal filters







```
. tsfilter bw ip_bw1 = ip_ln, gain(bwgain1 abw1) maxperiod(32) order(2)
. label variable bwgain1 "BW 2"
. tsfilter bw ip_bw6 = ip_ln, gain(bwgain6 abw6) maxperiod(32) order(6)
. label variable bwgain6 "BW 6"
. twoway line ideal f || line bwgain1 abw1 || line bwgain6 abw6
```







- We use `tsfilter bw` to estimate the components driven by shocks greater than 32 periods using Butterworth filters of order 1 and order 6, and plot the computed the gain functions for each filter
- Butterworth filters can approximate the ideal gain function quite well
- **[TS]** `tsfilter` shows how make a high-pass filter work like a band-pass filter by running it twice

Conclusion

- This talk has provided a quick introduction to some popular linear filters and how to use them in Stata 12

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