Estimating dynamic stochastic general equilibrium models in Stata

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Motivation

- Models used in macroeconomics for policy analysis

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- Stochastic
- General equilibrium

Here's a model in words

- Households
  - Consume and save output
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  - Produce output and set prices
  - Take demand as given
- Central bank
  - Sets interest rate
  - Adjusts interest rate in response to inflation
- In equilibrium, this is a model that simultaneously determines output, inflation, and the interest rate

Here's a model in equations I

- Households demand output, given inflation and interest rates:

$$x_t = E_t(x_{t+1}) - (r_t - E_t(\pi_{t+1}) - z_t)$$

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$$\pi_t = \beta E_t(\pi_{t+1}) + \kappa x_t$$

- Central bank sets interest rate, given inflation

$$r_t = \frac{1}{\beta}\pi_t + u_t$$

Here's a model in equations II

- The model's endogenous variables are characterized by equations:

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\begin{aligned}
x_t &= E_t(x_{t+1}) - (r_t - E_t(\pi_{t+1}) - z_t) \\
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- The model is completed by adding equations for the state variables:

$$\begin{aligned} z_{t+1} &= \rho_z z_t + \xi_{t+1} \\ u_{t+1} &= \rho_u u_t + \varepsilon_{t+1} \end{aligned}$$

Here's a model in Stata I

- The model equations:

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- In Stata:

```
. dsge    (x = E(F.x) - (r - E(F.p) - z), unobserved) ///
          (p = {beta}*E(F.p) + {kappa}*x)             ///
          (r = 1/{beta}*p + u)                        ///
          (F.z = {rhoz}*z, state)                     ///
          (F.u = {rhou}*u, state)
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- There are some rules
    - Equations are bound in parentheses.
    - Parameters are bound in braces.
    - Each variable appears on the left-hand side of one equation.
    - State equations are written in terms of their one-period-ahead value.

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  - Equations are bound in parentheses.
  - Parameters are bound in braces.
  - Each variable appears on the left-hand side of one equation.
  - State equations are written in terms of their one-period-ahead value.
- Data: US inflation rate and nominal interest rate, quarterly

Parameter estimation

```
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DSGE model

Sample: 1955q1 - 2015q4                        Number of obs     =        244
Log likelihood = -753.57131
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.5146675	.0783489	6.57	0.000	.3611065	.6682284
kappa	.1659054	.0474072	3.50	0.000	.0729889	.2588218
rhoz	.9545256	.0186424	51.20	0.000	.9179872	.991064
rhou	.7005486	.0452604	15.48	0.000	.6118398	.7892573
sd(e.z)	.6211211	.101508			.4221692	.8200731
sd(e.u)	2.318202	.3047436			1.720916	2.915489

Policy questions

What is the effect of an unexpected increase in interest rates?

Estimated DSGE model provides an answer to this question. We can subject the model to a shock, then see how that shock feeds through the rest of the system.

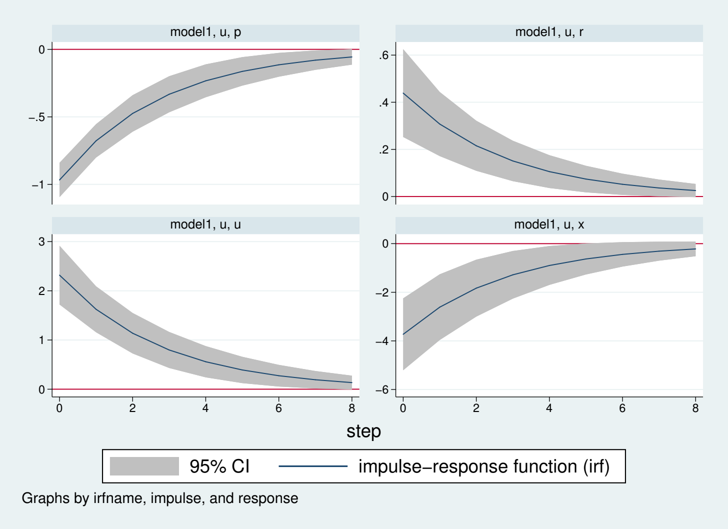
Impulse response functions

Q: What is the effect of a shock to u_t on the model variables?

Recall our model:

$$
\begin{aligned}
x_t &= E_t(x_{t+1}) - (r_t - E_t(\pi_{t+1}) - z_t) && \text{(Demand)} \\
\pi_t &= \beta E_t(\pi_{t+1}) + \kappa x_t && \text{(Pricing)} \\
r_t &= \frac{1}{\beta}\pi_t + u_t && \text{(Interest rate)} \\
z_{t+1} &= \rho_z z_t + \xi_{t+1} && \text{(Natural rate of interest)} \\
u_{t+1} &= \rho_u u_t + \varepsilon_{t+1} && \text{(Monetary policy)}
\end{aligned}
$$

Impulse responses from the estimated model



Solving a DSGE Model I

- Solution to a model is the key to estimation and generating impulse responses

Solving a DSGE Model I

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- Solution expresses endogenous variables as a function of state variables alone

Solving a DSGE model II

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- The reduced form for output is:

$$x_t = g_1 z_t + g_2 u_t$$

g_1 and g_2 are coefficients whose values are functions of the structural parameters

Solving a DSGE model III

- What about expectations?

  Roll the solution forward one period,

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- And take expectations,

$$E_t(x_{t+1}) = g_1 E_t(z_{t+1}) + g_2 E_t(u_{t+1})$$

- Then roll the state variables back one period

$$E_t(x_{t+1}) = g_1 \rho_z z_t + g_2 \rho_u u_t$$

- Compactly, the solution to a model is

$$\begin{aligned}\mathbf{y}_t &= \mathbf{G}\mathbf{z}_t \\ \mathbf{z}_{t+1} &= \mathbf{H}\mathbf{z}_t + \mathbf{M}\mathbf{e}_{t+1}\end{aligned}$$

where $\mathbf{y}_t$ is a vector of control variables, $\mathbf{z}_t$ is a vector of state variables, and $\mathbf{e}_t$ is a vector of shocks.

Solving and Estimating a DSGE Model

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- This is a state-space model whose parameters can be estimated by maximum likelihood.

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- Each entry in **G** and **H** is a function of the model's structural parameters.

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  where $\mathbf{y}_t$ is a vector of control variables, $\mathbf{z}_t$ is a vector of state variables, and $\mathbf{e}_t$ is a vector of shocks.
- This is a state-space model whose parameters can be estimated by maximum likelihood.
- Each entry in **G** and **H** is a function of the model's structural parameters.
- Diagonal elements of matrix **M** hold the standard deviations of the shocks.

Impulse Responses, again

$$\mathbf{y}_t = \mathbf{G}\mathbf{z}_t$$
$$\mathbf{z}_{t+1} = \mathbf{H}\mathbf{z}_t + \mathbf{M}\mathbf{e}_{t+1}$$

- An impulse is a specific sequence of values $(1, 0, 0, 0, \dots)$ given to one shock.

Impulse Responses, again

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- An impulse is a specific sequence of values $(1, 0, 0, 0, \dots)$ given to one shock.
- From the sequence of shocks and the state transition equation, you can obtain the sequence of state variables.
- From the sequence of state variables and the policy matrix, you can obtain the sequence of control variables.

Using constraints to fix some parameters

You might want to fix some parameters and estimate others.

$$
\begin{aligned}
x_t &= E_t(x_{t+1}) - (r_t - E_t(\pi_{t+1}) - z_t) && \text{(Demand)}\\
\pi_t &= \beta E_t(\pi_{t+1}) + \kappa x_t && \text{(Pricing)}\\
r_t &= \psi \pi_t + u_t && \text{(Interest rate)}\\
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u_{t+1} &= \rho_u u_t + \varepsilon_{t+1} && \text{(Monetary policy)}
\end{aligned}
$$

New: the ψ parameter in the third equation.

New: β no longer appears in multiple equations.

Assume: you wish to fix β and estimate the remaining parameters conditional on your choice of β.

Constrained model in Stata

```
. constraint 1 _b[beta]=0.96

. dsge    (x = E(F.x) - (r - E(F.p) - z), unobserved)   ///
          (p = {beta}*E(F.p) + {kappa}*x)                ///
          (r = {psi=1.5}*p + u)                          ///
          (F.z = {rhoz}*z, state)                        ///
          (F.u = {rhou}*u, state), constraint(1) nolog
```

Parameter estimation

```
DSGE model

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 ( 1)  [/structural]beta = .96
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.96	(constrained)				
kappa	.0849632	.0287693	2.95	0.003	.0285764	.1413501
psi	1.943004	.2957865	6.57	0.000	1.363273	2.522734
rhoz	.9545257	.0186424	51.20	0.000	.9179873	.991064
rhou	.7005482	.0452603	15.48	0.000	.6118396	.7892568
sd(e.z)	.568989	.0982973			.3763299	.7616482
sd(e.u)	2.318204	.3047431			1.720918	2.915489

Conclusion

- `dsge` estimates the parameters of DSGE models
- Impulse response functions trace the effect of a shock on the model
- Other features: `estat` commands to view the model's state-space matrices, predictions of latent states, identification and stability diagnostics

Thank You!