ineqrbd:
Regression-based inequality decomposition, following Fields (2003)

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Motivation of RB decomposition

- Decomposition analysis of inequality is important for understanding the main determinants of inequality and for policy analysis.
- The “traditional” approach to the subject was based purely on the analysis of the mathematical properties of inequality indices and is open to the criticism that the formal requirements for exact decomposition are perhaps too demanding for some practical applications.
  - It allows inequality accounting but not a causal analysis.
- Recent applied work has reawakened interest in inequality decomposition by focusing on the use of regression-based (RB) approaches to avoid some of the restrictions of the traditional methods.
The Fields’ approach to RB decomposition of inequality

Assuming that the income DGP is

\[ y = X\beta + \epsilon \]  \hspace{1cm} (1)

where:

- \( y \) is an \( n \times 1 \) vector of incomes;
- \( X \) is an \( n \times (K + 1) \) matrix of individual and household characteristics (age, education, household size, residence, etc.) including the constant;
- \( \beta \) is a \((K + 1) \times 1\) vector of coefficients and \( \epsilon \) is an \( n \times 1 \) vector of residuals.

a sample of observations \( \{y_i, x_i, i = 1, 2, \ldots, n\} \) can be used to estimate the model.
The Fields’ approach to RB decomposition of inequality

The linear model (1) can be rewritten as:

\[ y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 \ldots + \beta_K x_K + \epsilon \]  
\[ = \beta_0 + z_1 + z_2 \ldots + z_K + \epsilon \]

where:

- each \( z_k \) is a “composite” variable, equal to the product of a regression coefficient and its variable \( (z_k = \beta_k x_k) \), with \( k = 0, 1, \ldots, K \) and \( x_0 = 1 \).

- NB: For inequality decomposition calculations, the value of \( \beta_0 \) is irrelevant as it is constant for every observation.
The Fields’ approach to RB decomposition of inequality

- Following Fields suggestion, the OLS estimate of (3) can be used for inequality decomposition:

\[ y = b_0 + \hat{z}_1 + \hat{z}_2 + \ldots + \hat{z}_K + \hat{\epsilon}_i \]  

(4)

- Alternatively, one may look at the predicted income:

\[ \hat{y} = b_0 + \hat{z}_1 + \hat{z}_2 + \ldots + \hat{z}_K \]  

(5)

in which case there is no residual term.

- \[ \hat{z}_k = b_k x_k \] and \( b_k \) is the OLS estimate of \( \beta_k \), \( k = 0, 1, \ldots, K \).
Our focus

- Neglecting the constant, equations (4) and (5) are of exactly the same form as the equation used by Shorrocks (1982) when deriving rules for inequality decomposition by factor components (e.g. total income is the sum of labour earnings, income from savings and other assets, private and public transfers. How much inequality in total income is attributable to each of these factors?)

- Shorrocks proved that a set of arguably persuasive axioms led to a unique additive and exact decomposition rule, with one term for each factor.
  - The decomposition rule did not depend on the choice of measure summarizing inequality in total income.
Fields (2003) exploited the parallel with the factor decomposition case, and applied the Shorrocks decomposition rule to relate inequality in $\hat{y}$ to contributions from each of the RHS variables ($x_k$).

There are two main issues to notice about Fields decomposition:

1. One can only relate inequality in $y$ to contributions from each of the composite variables $z_k$, not $x_k$.
2. Decomposing $\hat{y}$ instead of $y$ does make a difference!
Our focus

- `ineqrbd` uses code from `ineqfac` by S.P. Jenkins, which performs Shorrocks’ factor decomposition.
- `ineqrbd` provides a regression-based Shorrocks-type decomposition of a variable labelled `Total`, where `Total` is defined as `yvar (y)`, unless the `fields` option is used, in which case `Total` refers to `yhat ˆy`.
- In either case, the contribution to inequality in `Total` of each term is labelled `x_k` in the output.
An example: wage inequality

- Use LIS sample dataset (US, year 2000. Not a random sample of the original!): how relevant is the contribution of individual characteristics to explain the inequality of log-wages?
- We used gross individual wage of working-age people. A very simple model (i.e. no sample selection considered) is assumed and finally estimated with OLS.

\[ \ln(gross\text{wage}) = \beta_0 + \beta_1 \text{age} + \beta_2 \text{age}^2 + \beta_3 \text{female} + \sum_{k=4}^{7} \beta_k(\text{educ}_k) + \epsilon \]

(education=no title, high school, some college, college, postgrad)
Motivation

Model

ineqrbd

Example: wage inequality

An example: wage inequality

. use http://www.lisproject.org/dataaccess/sample/us00sanwpp.dta
. keep if page>25 & page<65 & pgwage>0
. gen page2=page*page
. gen lpgwage=log(pgwage)
. recode peduc (-1/8=0)(9=1)(10=2)(11/13=3)(14/16=4)
. xi: ineqrbd lpgwage page page2 i.psex i.peduc
The `ineqrbd` output. OLS regression

<table>
<thead>
<tr>
<th>Source</th>
<th>SS</th>
<th>df</th>
<th>MS</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Model</td>
<td>214.068896</td>
<td>7</td>
<td>30.5812709</td>
<td>Number of obs  = 1180</td>
</tr>
<tr>
<td>Residual</td>
<td>892.370392</td>
<td>1172</td>
<td>0.761408184</td>
<td>F( 7, 1172) = 40.16</td>
</tr>
<tr>
<td>Total</td>
<td>1106.43929</td>
<td>1179</td>
<td>0.938455715</td>
<td>Prob &gt; F       = 0.0000</td>
</tr>
</tbody>
</table>

Regression of `lpgwage` on RHS variables (analytic weights assumed) (sum of wgt is 1.1800e+03)

| `lpgwage`  | Coef.   | Std. Err. | t    | P>|t| | [95% Conf. Interval] |
|------------|---------|-----------|------|------|---------------------|
| page       | 0.0866839 | 0.0229005 | 3.79 | 0.000 | 0.0417534 - 0.1316144 |
| page2      | -0.000963 | 0.0002609 | -3.69 | 0.000 | -0.0014747 - 0.0004512 |
| _Ipsex_2   | -0.5136189 | 0.050975 | -10.08 | 0.000 | -0.6136314 - 0.4136064 |
| _Ipeduc_1  | 0.520429  | 0.1311271 | 3.97 | 0.000 | 0.2631589 - 0.7776991 |
| _Ipeduc_2  | 0.7096313 | 0.133643 | 5.31 | 0.000 | 0.4474251 - 0.9718376 |
| _Ipeduc_3  | 0.933268  | 0.1306687 | 7.14 | 0.000 | 0.6768973 - 1.189639 |
| _Ipeduc_4  | 1.423139  | 0.1403917 | 10.14 | 0.000 | 1.147692 - 1.698586 |
| _cons      | 7.897521  | 0.4979311 | 15.86 | 0.000 | 6.920585 - 8.874456 |
The `ineqrbd` output. Default choice of LHS variable: $y$

<table>
<thead>
<tr>
<th>Decomp.</th>
<th>100*s_f</th>
<th>s_f</th>
<th>100*m_f/m</th>
<th>cv_f</th>
<th>cv_f/cv(total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>residual</td>
<td>80.6524</td>
<td>0.0759</td>
<td>0.0000</td>
<td>6.64e+14</td>
<td>7.05e+15</td>
</tr>
<tr>
<td>page</td>
<td>2.6546</td>
<td>0.0025</td>
<td>36.3223</td>
<td>0.2250</td>
<td>2.3909</td>
</tr>
<tr>
<td>page2</td>
<td>-1.6647</td>
<td>-0.0016</td>
<td>-18.2834</td>
<td>-0.4356</td>
<td>-4.6281</td>
</tr>
<tr>
<td>_Ipsex_2</td>
<td>6.7031</td>
<td>0.0063</td>
<td>-2.4484</td>
<td>-1.0193</td>
<td>-10.8299</td>
</tr>
<tr>
<td>_Ipeduc_1</td>
<td>-4.4383</td>
<td>-0.0042</td>
<td>1.4697</td>
<td>1.5628</td>
<td>16.6051</td>
</tr>
<tr>
<td>_Ipeduc_2</td>
<td>-0.9357</td>
<td>-0.0009</td>
<td>1.5191</td>
<td>1.8819</td>
<td>19.9956</td>
</tr>
<tr>
<td>_Ipeduc_3</td>
<td>3.7284</td>
<td>0.0035</td>
<td>2.7969</td>
<td>1.4979</td>
<td>15.9155</td>
</tr>
<tr>
<td>_Ipeduc_4</td>
<td>13.3002</td>
<td>0.0125</td>
<td>1.8982</td>
<td>2.5078</td>
<td>26.6467</td>
</tr>
<tr>
<td>Total</td>
<td>100.0000</td>
<td>0.0941</td>
<td>100.0000</td>
<td>0.0941</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: proportionate contribution of composite var f to inequality of Total,

$s_f = \rho_f*sd(f)/sd(Total)$.  $s_f = s_f*CV(Total)$.

$m_f = mean(f)$.  $sd(f) = std.dev. of f$.  $CV_f = sd(f)/m_f$.

Total = $lpwage$
The `ineqrbd` output. **Fields** option of LHS variable, $\hat{y}$

![Results](image)

<table>
<thead>
<tr>
<th>Decomp.</th>
<th>100*s_f</th>
<th>s_f</th>
<th>100*m_f/m</th>
<th>CV_f</th>
<th>CV_f/CV(total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>page</td>
<td>13.7208</td>
<td>0.0057</td>
<td>36.3223</td>
<td>0.2250</td>
<td>5.4356</td>
</tr>
<tr>
<td>page2</td>
<td>-8.6042</td>
<td>-0.0036</td>
<td>-18.2834</td>
<td>-0.4356</td>
<td>-10.5217</td>
</tr>
<tr>
<td>_Ipsex_2</td>
<td>34.6455</td>
<td>0.0143</td>
<td>-2.4484</td>
<td>-1.0193</td>
<td>-24.6214</td>
</tr>
<tr>
<td>_Ipeduc_1</td>
<td>-22.9398</td>
<td>-0.0095</td>
<td>1.4697</td>
<td>1.5628</td>
<td>37.7511</td>
</tr>
<tr>
<td>_Ipeduc_2</td>
<td>-4.8361</td>
<td>-0.0020</td>
<td>1.5191</td>
<td>1.8819</td>
<td>45.4591</td>
</tr>
<tr>
<td>_Ipeduc_3</td>
<td>19.2704</td>
<td>0.0080</td>
<td>2.7969</td>
<td>1.4979</td>
<td>36.1833</td>
</tr>
<tr>
<td>_Ipeduc_4</td>
<td>68.7433</td>
<td>0.0285</td>
<td>1.8982</td>
<td>2.5078</td>
<td>60.5801</td>
</tr>
<tr>
<td>Total</td>
<td>100.0000</td>
<td>0.0414</td>
<td>100.0000</td>
<td>0.0414</td>
<td>1.0000</td>
</tr>
</tbody>
</table>

Note: proportionate contribution of composite var f to inequality of Total, $s_f = \rho_f \cdot sd(f) / sd(Total)$. $s_f = s_f \cdot CV(f)$. $m_f = mean(f)$. $sd(f) = std.dev. \ of \ f$. $CV_f = sd(f) / m_f$. Total = predicted lpgwage

C.V. Fiorio & S.P. Jenkins  
`ineqrbd.ado`
Options

- **fields** implies decomposition of predicted *yvar* ($\hat{y}$) rather than of *yvar* ($y$).
- **noregression** suppresses reporting of the OLS regression equation used to derive the composite variables and residual.
- **noconstant** excludes the intercept term from the regression.
- **stats** provides the means, sd, and $\rho$, of Total, the residual (unless the fields option is used), and the composite variables.
- **i2** summarises inequality using half the squared coefficient of variation (the Generalized Entropy measure I2), rather than the coefficient of variation (CV).
Saved results

- $r(\text{total})$ contains *predicted yvar* ($\hat{y}$) if fields used; else contains yvar ($y$)
- $r(\text{mean\_tot})$, $r(\text{sd\_tot})$, $r(\text{cv\_tot})$ mean, standard deviation, CV for Total
- $r(\text{sf\_Z0})$, $r(\text{mean\_Z0})$, proportionate inequality contribution, mean,
- $r(\text{sd\_Z0})$, $r(\text{cv\_Z0})$ standard deviation, CV for the residual.
- $r(\text{sf\_Z0})$ is not reported if fields option used.
- $r(\text{sf\_Zf})$, $r(\text{mean\_Zf})$, proportionate inequality contribution, mean,
- $r(\text{sd\_Zf})$, $r(\text{cv\_Zf})$ standard deviation, CV for each of variables in rhsvars, where "f" is an integer 1,\ldots, K, indicating the order in which entered in rhsvars.