xtreg - Fixed-, between-, and random-effects and population-averaged linear models

Description Syntax Options for FE model Remarks and examples Acknowledgments Quick start Options for RE model Options for MLE model Stored results References Menu Options for BE model Options for PA model Methods and formulas Also see

Description

Title

xtreg fits regression models to panel data. In particular, xtreg with the be option fits randomeffects models by using the between regression estimator; with the fe option, it fits fixed-effects models (by using the within regression estimator); and with the re option, it fits random-effects models by using the GLS estimator (producing a matrix-weighted average of the between and within results). See [XT] xtdata for a faster way to fit fixed- and random-effects models.

Quick start

Random-effects linear regression by GLS of y on x1 and xt2 using xtset data xtreg y x1 x2

As above, but estimate by maximum likelihood

xtreg y x1 x2, mle

Fixed-effects model with cluster-robust standard errors for panels nested within cvar xtreg y x1 x2, fe vce(cluster cvar)

Population-averaged model with an exchangeable within-panel correlation structure xtreg y x1 x2, pa

As above, but specify an autoregressive correlation structure of order 1 xtreg y x1 x2, pa corr(ar 1)

Between-effects model

xtreg y x1 x2, be

Menu

Statistics > Longitudinal/panel data > Linear models > Linear regression (FE, RE, PA, BE)

Syntax

```
GLS random-effects (RE) model

xtreg depvar [indepvars] [if] [in] [, re RE_options]
Between-effects (BE) model
xtreg depvar [indepvars] [if] [in], be [BE_options]
```

Fixed-effects (FE) model

 $\texttt{xtreg } \textit{depvar} [\textit{indepvars}] [\textit{if}] [\textit{in}] [\textit{weight}], \texttt{fe} [\textit{FE_options}]$

ML random-effects (MLE) model

xtreg depvar [indepvars] [if] [in] [weight], mle [MLE_options]

Population-averaged (PA) model

```
xtreg depvar [indepvars] [if] [in] [weight], pa [PA\_options]
```

RE_options	Description			
Model				
re	use random-effects estimator; the default			
sa	use Swamy-Arora estimator of the variance components			
SE/Robust				
vce(<i>vcetype</i>)	<i>vcetype</i> may be conventional, <u>r</u> obust, <u>cl</u> uster <i>clustvar</i> , <u>boot</u> strap, or <u>jackknife</u>			
Reporting				
<u>l</u> evel(#)	set confidence level; default is level(95)			
theta	report θ			
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling			
<u>coefl</u> egend	display legend instead of statistics			

BE_options	Description				
Model					
be	use between-effects estimator				
wls	use weighted least squares				
SE					
vce(<i>vcetype</i>)	vcetype may be conventional, <u>boot</u> strap, or <u>jackknife</u>				
Reporting					
<u>l</u> evel(#)	set confidence level; default is level(95)				
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling				
<u>coefl</u> egend	display legend instead of statistics				
FE_options	Description				
Model					
fe	use fixed-effects estimator				
SE/Robust					
vce(<i>vcetype</i>)	<pre>vcetype may be conventional, robust, cluster clustvar, bootstrap, or jackknife</pre>				
Reporting					
<u>l</u> evel(#)	set confidence level; default is level(95)				
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling				
<u>coefl</u> egend	display legend instead of statistics				
MLE_options	Description				
Model					
<u>nocon</u> stant	suppress constant term				
mle	use ML random-effects estimator				
SE					
vce(<i>vcetype</i>)	vcetype may be oim, <u>boot</u> strap, or <u>jackknife</u>				
Reporting					
<u>l</u> evel(#) display_options	set confidence level; default is level(95) control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling				
Maximization					
maximize_options	control the maximization process; seldom used				
<u>coefl</u> egend	display legend instead of statistics				

Description PA_options Model noconstant suppress constant term pa use population-averaged estimator include varname in model with coefficient constrained to 1 offset(varname) Correlation corr(correlation) within-panel correlation structure estimate even if observations unequally spaced in time force SF/Robust vce(vcetype) vcetype may be conventional, robust, bootstrap, or jackknife use divisor N - P instead of the default N nmp rgf multiply the robust variance estimate by (N-1)/(N-P)overrides the default scale parameter; parm may be x2, dev, phi, or # scale(parm) Reporting set confidence level; default is level(95) level(#) control columns and column formats, row spacing, line width, display_options display of omitted variables and base and empty cells, and factor-variable labeling Optimization optimize_options control the optimization process; seldom used coeflegend display legend instead of statistics correlation Description <u>exc</u>hangeable exchangeable independent independent unstructured unstructured user-specified fixed matname

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A panel variable must be specified. For xtreg, pa, correlation structures other than exchangeable and independent require that a time variable also be specified. Use xtset; see [XT] xtset.

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

autoregressive of order #

nonstationary of order #

stationary of order #

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, mi estimate, and statsby are allowed; see [U] 11.1.10 Prefix commands. fp is allowed for the between-effects, fixed-effects, and maximum-likelihood random-effects models.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

aweights, fweights, and pweights are allowed for the fixed-effects model. iweights, fweights, and pweights are allowed for the population-averaged model. iweights are allowed for the maximum-likelihood random-effects (MLE) model. See [U] 11.1.6 weight. Weights must be constant within panel.

coeflegend does not appear in the dialog box.

ar #

stationary #

nonstationary #

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options for RE model

Model

re, the default, requests the GLS random-effects estimator.

sa specifies that the small-sample Swamy-Arora estimator individual-level variance component be used instead of the default consistent estimator. See *xtreg*, *re* in *Methods and formulas* for details.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*); see *xtreg*, *re* in *Methods and formulas*.

Reporting

level(#); see [R] estimation options.

theta specifies that the output include the estimated value of θ used in combining the between and fixed estimators. For balanced data, this is a constant, and for unbalanced data, a summary of the values is presented in the header of the output.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] estimation options.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] estimation options.

Options for BE model

Model

be requests the between regression estimator.

wls specifies that, for unbalanced data, weighted least squares be used rather than the default OLS. Both methods produce consistent estimates. The true variance of the between-effects residual is $\sigma_{\nu}^2 + T_i \sigma_{\epsilon}^2$ (see *xtreg, be* in *Methods and formulas* below). WLS produces a "stabilized" variance of $\sigma_{\nu}^2/T_i + \sigma_{\epsilon}^2$, which is also not constant. Thus the choice between OLS and WLS amounts to which is more stable.

Comment: xtreg, be is rarely used anyway, but between estimates are an ingredient in the randomeffects estimate. Our implementation of xtreg, re uses the OLS estimates for this ingredient, based on our judgment that σ_{ν}^2 is large relative to σ_{ϵ}^2 in most models. Formally, only a consistent estimate of the between estimates is required. SE

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Reporting

level(#); see [R] estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] estimation options.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] estimation options.

Options for FE model

__ Model

fe requests the fixed-effects (within) regression estimator.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*); see *xtreg*, *fe* in *Methods and formulas*.

Reporting

level(#); see [R] estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, notvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] estimation options.

The following option is available with xtreg but is not shown in the dialog box: coeflegend; see [R] estimation options.

Options for MLE model

Model

noconstant; see [R] estimation options.

mle requests the maximum-likelihood random-effects estimator.

SE

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

Reporting

level(#); see [R] estimation options.

```
display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt),
sformat(% fmt), and nolstretch; see [R] estimation options.
```

Maximization

maximize_options: iterate(#), [no]log, trace, tolerance(#), ltolerance(#), and from(init_specs); see [R] maximize. These options are seldom used.

The following option is available with xtreg but is not shown in the dialog box: coeflegend; see [R] estimation options.

Options for PA model

(Model)

noconstant; see [R] estimation options.

pa requests the population-averaged estimator. For linear regression, this is the same as a random-effects estimator (both interpretations hold).

xtreg, pa is equivalent to xtgee, family(gaussian) link(id) corr(exchangeable), which are the defaults for the xtgee command. xtreg, pa allows all the relevant xtgee options such as vce(robust). Whether you use xtreg, pa or xtgee makes no difference. See [XT] xtgee.

offset(varname); see [R] estimation options.

Correlation

corr(*correlation*) specifies the within-panel correlation structure; the default corresponds to the equal-correlation model, corr(exchangeable).

When you specify a correlation structure that requires a lag, you indicate the lag after the structure's name with or without a blank; for example, corr(ar 1) or corr(ar1).

If you specify the fixed correlation structure, you specify the name of the matrix containing the assumed correlations following the word fixed, for example, corr(fixed myr).

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force specifies that estimation be forced even though the time variable is not equally spaced. This is relevant only for correlation structures that require knowledge of the time variable. These correlation structures require that observations be equally spaced so that calculations based on lags correspond to a constant time change. If you specify a time variable indicating that observations are not equally spaced, the (time dependent) model will not be fit. If you also specify force, the model will be fit, and it will be assumed that the lags based on the data ordered by the time variable are appropriate.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

nmp; see [XT] vce_options.

rgf specifies that the robust variance estimate is multiplied by (N-1)/(N-P), where N is the total number of observations and P is the number of coefficients estimated. This option can be used with family(gaussian) only when vce(robust) is either specified or implied by the use of pweights. Using this option implies that the robust variance estimate is not invariant to the scale of any weights used.

scale(x2|dev|phi|#); see [XT] vce_options.

Reporting

level(#); see [R] estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] estimation options.

Optimization

optimize_options control the iterative optimization process. These options are seldom used.

iterate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimization stops and presents the current results, even if convergence has not been reached. The default is iterate(100).

tolerance(#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

nolog suppresses display of the iteration log.

trace specifies that the current estimates be printed at each iteration.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] estimation options.

Remarks and examples

stata.com

If you have not read [XT] xt, please do so.

See Baltagi (2013, chap. 2) and Wooldridge (2016, chap. 14) for good overviews of fixed-effects and random-effects models. Allison (2009) provides perspective on the use of fixed- versus random-effects estimators and provides many examples using Stata.

Consider fitting models of the form

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it} \tag{1}$$

In this model, $\nu_i + \epsilon_{it}$ is the error term that we have little interest in; we want estimates of β . ν_i is the unit-specific error term; it differs between units, but for any particular unit, its value is constant. In the pulmonary data of [XT] **xt**, a person who exercises less would presumably have a lower forced expiratory volume (FEV) year after year and so would have a negative ν_i .

 ϵ_{it} is the "usual" error term with the usual properties (mean 0, uncorrelated with itself, uncorrelated with x, uncorrelated with ν , and homoskedastic), although in a more thorough development, we could decompose $\epsilon_{it} = v_t + \omega_{it}$, assume that ω_{it} is a conventional error term, and better describe v_t .

Before making the assumptions necessary for estimation, let's perform some useful algebra on (1). Whatever the properties of ν_i and ϵ_{it} , if (1) is true, it must also be true that

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta} + \nu_i + \overline{\epsilon}_i \tag{2}$$

where $\overline{y}_i = \sum_t y_{it}/T_i$, $\overline{\mathbf{x}}_i = \sum_t \mathbf{x}_{it}/T_i$, and $\overline{\epsilon}_i = \sum_t \epsilon_{it}/T_i$. Subtracting (2) from (1), it must be equally true that

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta} + (\epsilon_{it} - \overline{\epsilon}_i)$$
(3)

These three equations provide the basis for estimating β . In particular, xtreg, fe provides what is known as the fixed-effects estimator—also known as the within estimator—and amounts to using OLS to perform the estimation of (3). xtreg, be provides what is known as the between estimator and amounts to using OLS to perform the estimation of (2). xtreg, re provides the random-effects estimator and is a (matrix) weighted average of the estimates produced by the between and within estimators. In particular, the random-effects estimator turns out to be equivalent to estimation of

$$(y_{it} - \theta \overline{y}_i) = (1 - \theta)\alpha + (\mathbf{x}_{it} - \theta \overline{\mathbf{x}}_i)\beta + \{(1 - \theta)\nu_i + (\epsilon_{it} - \theta \overline{\epsilon}_i)\}$$
(4)

where θ is a function of σ_{ν}^2 and σ_{ϵ}^2 . If $\sigma_{\nu}^2 = 0$, meaning that ν_i is always 0, $\theta = 0$ and (1) can be estimated by OLS directly. Alternatively, if $\sigma_{\epsilon}^2 = 0$, meaning that ϵ_{it} is 0, $\theta = 1$ and the within estimator returns all the information available (which will, in fact, be a regression with an R^2 of 1).

For more reasonable cases, few assumptions are required to justify the fixed-effects estimator of (3). The estimates are, however, conditional on the sample in that the ν_i are not assumed to have a distribution but are instead treated as fixed and estimable. This statistical fine point can lead to difficulty when making out-of-sample predictions, but that aside, the fixed-effects estimator has much to recommend it.

More is required to justify the between estimator of (2), but the conditioning on the sample is not assumed because $\nu_i + \overline{\epsilon}_i$ is treated as an error term. Newly required is that we assume that ν_i and $\overline{\mathbf{x}}_i$ are uncorrelated. This follows from the assumptions of the OLS estimator but is also transparent: were ν_i and $\overline{\mathbf{x}}_i$ correlated, the estimator could not determine how much of the change in \overline{y}_i , associated with an increase in $\overline{\mathbf{x}}_i$ to assign to β versus how much to attribute to the unknown correlation. (This, of course, suggests the use of an instrumental-variable estimator, $\overline{\mathbf{z}}_i$, which is correlated with $\overline{\mathbf{x}}_i$ but uncorrelated with ν_i , though that approach is not implemented here.) The random-effects estimator of (4) requires the same no-correlation assumption. In comparison with the between estimator, the random-effects estimator produces more efficient results, albeit ones with unknown small-sample properties. The between estimator is less efficient because it discards the over-time information in the data in favor of simple means; the random-effects estimator uses both the within and the between information.

All of this would seem to leave the between estimator of (2) with no role (except for a minor, technical part it plays in helping to estimate σ_{ν}^2 and σ_{ϵ}^2 , which are used in the calculation of θ , on which the random-effects estimates depend). Let's, however, consider a variation on (1):

$$y_{it} = \alpha + \overline{\mathbf{x}}_i \beta_1 + (\mathbf{x}_{it} - \overline{\mathbf{x}}_i) \beta_2 + \nu_i + \epsilon_{it} \tag{1'}$$

In this model, we postulate that changes in the average value of \mathbf{x} for an individual have a different effect from temporary departures from the average. In an economic situation, y might be purchases of some item and \mathbf{x} income; a change in average income should have more effect than a transitory change. In a clinical situation, y might be a physical response and \mathbf{x} the level of a chemical in the brain; the model allows a different response to permanent rather than transitory changes.

The variations of (2) and (3) corresponding to (1') are

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \beta_1 + \nu_i + \overline{\epsilon}_i \tag{2'}$$

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta}_2 + (\epsilon_{it} - \overline{\epsilon}_i)$$
(3')

That is, the between estimator estimates β_1 and the within β_2 , and neither estimates the other. Thus even when estimating equations like (1), it is worth comparing the within and between estimators. Differences in results can suggest models like (1'), or at the least some other specification error.

Finally, it is worth understanding the role of the between and within estimators with regressors that are constant over time or constant over units. Consider the model

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta}_1 + \mathbf{s}_i\boldsymbol{\beta}_2 + \mathbf{z}_t\boldsymbol{\beta}_3 + \nu_i + \epsilon_{it} \tag{1"}$$

This model is the same as (1), except that we explicitly identify the variables that vary over both time and i (\mathbf{x}_{it} , such as output or FEV); variables that are constant over time (\mathbf{s}_i , such as race or sex); and variables that vary solely over time (\mathbf{z}_t , such as the consumer price index or age in a cohort study). The corresponding between and within equations are

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \beta_1 + \mathbf{s}_i \beta_2 + \overline{\mathbf{z}} \beta_3 + \nu_i + \overline{\epsilon}_i \tag{2"}$$

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\beta_1 + (\mathbf{z}_t - \overline{\mathbf{z}})\beta_3 + (\epsilon_{it} - \overline{\epsilon}_i)$$
(3")

In the between estimator of (2''), no estimate of β_3 is possible because \overline{z} is a constant across the *i* observations; the regression-estimated intercept will be an estimate of $\alpha + \overline{z}\beta_3$. On the other hand, it can provide estimates of β_1 and β_2 . It can estimate effects of factors that are constant over time, such as race and sex, but to do so it must assume that ν_i is uncorrelated with those factors.

The within estimator of (3''), like the between estimator, provides an estimate of β_1 but provides no estimate of β_2 for time-invariant factors. Instead, it provides an estimate of β_3 , the effects of the time-varying factors. The within estimator can also provide estimates u_i for ν_i . More correctly, the estimator u_i is an estimator of $\nu_i + s_i\beta_2$. Thus u_i is an estimator of ν_i only if there are no time-invariant variables in the model. If there are time-invariant variables, u_i is an estimate of ν_i plus the effects of the time-invariant variables.

Remarks are presented under the following headings:

Assessing goodness of fit xtreg and associated commands

Assessing goodness of fit

 R^2 is a popular measure of goodness of fit in ordinary regression. In our case, given $\hat{\alpha}$ and β estimates of α and β , we can assess the goodness of fit with respect to (1), (2), or (3). The prediction equations are, respectively,

$$\widehat{y}_{it} = \widehat{\alpha} + \mathbf{x}_{it}\widehat{\boldsymbol{\beta}} \tag{1'''}$$

$$\widehat{\overline{y}}_i = \widehat{\alpha} + \overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}} \tag{2'''}$$

$$\widehat{\widetilde{y}}_{it} = (\widehat{y}_{it} - \overline{\overline{y}}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}$$

$$(3''')$$

xtreg reports "*R*-squares" corresponding to these three equations. *R*-squares is in quotes because the *R*-squares reported do not have all the properties of the OLS R^2 .

The ordinary properties of R^2 include being equal to the squared correlation between \hat{y} and y and being equal to the fraction of the variation in y explained by \hat{y} —formally defined as $Var(\hat{y})/Var(y)$. The identity of the definitions is from a special property of the OLS estimates; in general, given a prediction \hat{y} for y, the squared correlation is not equal to the ratio of the variances, and the ratio of the variances is not required to be less than 1.

xtreg reports R^2 values calculated as correlations squared, calling them R^2 overall, corresponding to (1'''); R^2 between, corresponding to (2'''); and R^2 within, corresponding to (3'''). In fact, you can think of each of these three numbers as having all the properties of ordinary R^2 's, if you bear in mind that the prediction being judged is not \hat{y}_{it} , $\hat{\overline{y}}_i$, and $\hat{\overline{y}}_{it}$, but $\gamma_1 \hat{y}_{it}$ from the regression $y_{it} = \gamma_1 \hat{y}_{it}$; $\gamma_2 \hat{\overline{y}}_i$ from the regression $\overline{y}_i = \gamma_2 \hat{\overline{y}}_i$; and $\gamma_3 \hat{\overline{y}}_{it}$ from $\tilde{y}_{it} = \gamma_3 \hat{\overline{y}}_{it}$.

In particular, xtreg, be obtains its estimates by performing OLS on (2), and therefore its reported R^2 between is an ordinary R^2 . The other two reported R^2 's are merely correlations squared, or, if you prefer, R^2 's from the second-round regressions $y_{it} = \gamma_{11}\hat{y}_{it}$ and $\tilde{y}_{it} = \gamma_{13}\hat{y}_{it}$.

xtreg, fe obtains its estimates by performing OLS on (3), so its reported R^2 within is an ordinary R^2 . As with be, the other R^2 's are correlations squared, or, if you prefer, R^2 's from the second-round regressions $\overline{y_i} = \gamma_{22} \hat{\overline{y}}_i$ and, as with be, $\hat{\overline{y}}_{it} = \gamma_{23} \hat{\overline{y}}_{it}$.

xtreg, re obtains its estimates by performing OLS on (4); none of the R^2 's corresponding to (1'''), (2'''), or (3''') correspond directly to this estimator (the "relevant" R^2 is the one corresponding to (4)). All three reported R^2 's are correlations squared, or, if you prefer, from second-round regressions.

xtreg and associated commands

Example 1: Between-effects model

Using nlswork.dta described in [XT] xt, we will model ln_wage in terms of completed years of schooling (grade), current age and age squared, current years worked (experience) and experience squared, current years of tenure on the current job and tenure squared, whether black (race = 2), whether residing in an area not designated a standard metropolitan statistical area (SMSA), and whether residing in the South.

```
. use http://www.stata-press.com/data/r14/nlswork
(National Longitudinal Survey. Young Women 14-26 years of age in 1968)
```

To obtain the between-effects estimates, we use xtreg, be. nlswork.dta has previously been xtset idcode year because that is what is true of the data, but for running xtreg, it would have been sufficient to have xtset idcode by itself.

. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure > c.tenure#c.tenure 2.race not_smsa south, be						
Between regres Group variable		sion on grou	p means)	Number Number	of obs = of groups =	28,091 4,697
R-sq:				Obs per	group:	
within =	= 0.1591			-	min =	1
between =	= 0.4900				avg =	6.0
overall =	= 0.3695				max =	15
				F(10,46	86) =	450.23
sd(u_i + avg(e	e_i.))= .3036	5114		Prob >	F =	0.0000
	r					
ln_wage	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
grade	.0607602	.0020006	30.37	0.000	.0568382	.0646822
age	.0323158	.0087251	3.70	0.000	.0152105	.0494211
_						
c.age#c.age	0005997	.0001429	-4.20	0.000	0008799	0003194
			o 15			
ttl_exp	.0138853	.0056749	2.45	0.014	.0027598	.0250108
c.ttl_exp#						
c.ttl_exp	.0007342	.0003267	2.25	0.025	.0000936	.0013747
or our_onp		10000201	2120	01020		
tenure	.0698419	.0060729	11.50	0.000	.0579361	.0817476
c.tenure#						
c.tenure	0028756	.0004098	-7.02	0.000	0036789	0020722
race	0504405					
black	0564167	.0105131	-5.37	0.000	0770272	0358061
not_smsa	1860406	.0112495	-16.54 -9.80	0.000	2080949	1639862
south _cons	0993378 .3339113	.010136 .1210434	-9.80 2.76	0.000	1192091 .0966093	0794665 .5712133
	.3339113	.1210404	2.10	0.000	.0300033	.0/12133

The between-effects regression is estimated on person-averages, so the "n = 4697" result is relevant. xtreg, be reports the "number of observations" and group-size information: describe in [XT] xt showed that we have 28,534 "observations"—person-years, really—of data. If we take the subsample that has no missing values in ln_wage , grade, ..., south leaves us with 28,091 observations on person-years, reflecting 4,697 persons, each observed for an average of 6.0 years.

For goodness of fit, the R^2 between is directly relevant; our R^2 is 0.4900. If, however, we use these estimates to predict the within model, we have an R^2 of 0.1591. If we use these estimates to fit the overall data, our R^2 is 0.3695.

The F statistic tests that the coefficients on the regressors grade, age, ..., south are all jointly zero. Our model is significant.

The root mean squared error of the fitted regression, which is an estimate of the standard deviation of $\nu_i + \overline{\epsilon}_i$, is 0.3036.

For our coefficients, each year of schooling increases hourly wages by 6.1%; age increases wages up to age 26.9 and thereafter decreases them (because the quadratic $ax^2 + bx + c$ turns over at x = -b/2a, which for our age and c.age#c.age coefficients is $0.0323158/(2 \times 0.0005997) \approx 26.9$); total experience increases wages at an increasing rate (which is surprising and bothersome); tenure on the current job increases wages up to a tenure of 12.1 years and thereafter decreases them; wages of blacks are, these things held constant, (approximately) 5.6% below that of nonblacks (approximately because 2.race is an indicator variable); residing in a non-SMSA (rural area) reduces wages by 18.6%; and residing in the South reduces wages by 9.9%.

Example 2: Fixed-effects model

To fit the same model with the fixed-effects estimator, we specify the fe option.

<pre>. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure > c.tenure#c.tenure 2.race not_smsa south, fe note: grade omitted because of collinearity note: 2.race omitted because of collinearity</pre>						
Fixed-effects Group variable	0	ression		Number Number	of obs = of groups =	28,091 4,697
R-sq:				Obs per	group:	
within =	= 0.1727				min =	1
between =	= 0.3505				avg =	6.0
overall =	= 0.2625				max =	15
				F(8,233	(86) =	610.12
corr(u_i, Xb)	= 0.1936			Prob >		0.0000
0011(u_1, mb)	0.1000			1100 /	•	0.0000
ln_wage	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
grade	0	(omitted)				
age	.0359987	.0033864	10.63	0.000	.0293611	.0426362
ugo			10.00	0.000	.0200011	.0120002
c.age#c.age	000723	.0000533	-13.58	0.000	0008274	0006186
0 0						
ttl_exp	.0334668	.0029653	11.29	0.000	.0276545	.039279
c.ttl_exp#						
c.ttl_exp	.0002163	.0001277	1.69	0.090	0000341	.0004666
tenure	.0357539	.0018487	19.34	0.000	.0321303	.0393775
c.tenure# c.tenure	0019701	.000125	-15.76	0.000	0022151	0017251
c.tenure	0019701	.000125	-15.76	0.000	0022151	0017251
race						
black	0	(omitted)				
not_smsa	0890108	.0095316	-9.34	0.000	1076933	0703282
south	0606309	.0109319	-5.55	0.000	0820582	0392036
_cons	1.03732	.0485546	21.36	0.000	.9421496	1.13249
sigma_u	.35562203					
sigma_e	.29068923					
rho	.59946283	(fraction	of varia	nce due t	o u_i)	
	l					
F test that all $u_i=0$: F(4696, 23386) = 6.65 Prob > F = 0.0000						

The observation summary at the top is the same as for the between-effects model, although this time it is the "Number of obs" that is relevant.

Our three R^2 's are not too different from those reported previously; the R^2 within is slightly higher (0.1727 versus 0.1591), and the R^2 between is a little lower (0.3505 versus 0.4900), as expected, because the between estimator maximizes R^2 between and the within estimator R^2 within. In terms of overall fit, these estimates are somewhat worse (0.2625 versus 0.3695).

If the unobserved time-invariant component ν is not correlated with the regressors, estimates from the fixed-effects model are consistent but inefficient relative to estimates from the random-effects model. In this case, the interpretation of sigma_u in the coefficient table is the same for the fixed-effects and random-effects models. However, sigma_u is a nuisance parameter when ν is correlated with the covariates. Here both grade and 2.race were omitted from the model because they do not vary over time. Because grade and 2.race are time invariant, our estimate u_i is an estimate of ν_i plus the effects of grade and 2.race, so our estimate of the standard deviation is based on the variation in ν_i , grade, and 2.race. On the other hand, had 2.race and grade been omitted merely because they were collinear with the other regressors in our model, u_i would be an estimate of ν_i , and 0.355622 would be an estimate of σ_{ν} . (xtsum and xttab allow you to determine whether a variable is time invariant; see [XT] xtsum and [XT] xttab.)

Regardless of the status of u_i , our estimate of the standard deviation of ϵ_{it} is valid (and, in fact, is the estimate that would be used by the random-effects estimator to produce its results).

Our estimate of the correlation of u_i with \mathbf{x}_{it} suffers from the problem of what u_i measures. We find correlation but cannot say whether this is correlation of ν_i with \mathbf{x}_{it} or merely correlation of grade and 2.race with \mathbf{x}_{it} . In any case, the fixed-effects estimator is robust to such a correlation, and the other estimates it produces are unbiased.

So, although this estimator produces no estimates of the effects of grade and 2.race, it does predict that age has a positive effect on wages up to age 24.9 years (compared with 26.9 years estimated by the between estimator); that total experience still increases wages at an increasing rate (which is still bothersome); that tenure increases wages up to 9.1 years (compared with 12.1); that living in a non-SMSA reduces wages by 8.9% (compared with a more drastic 18.6%); and that living in the South reduces wages by 6.1% (as compared with 9.9%).

Example 3: Fixed-effects models with robust standard errors

If we suspect that there is heteroskedasticity or within-panel serial correlation in the idiosyncratic error term ϵ_{it} , we could specify the vce(robust) option:

<pre>. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure > c.tenure#c.tenure 2.race not_smsa south, fe vce(robust) note: grade omitted because of collinearity note: 2.race omitted because of collinearity</pre>						
Fixed-effects Group variable	0	ression		Number Number	of obs = of groups =	28,091 4,697
R-sq:				Obs per	group:	
within =					min =	1
between =					avg =	6.0
overall =	= 0.2625				max =	15
corr(u_i, Xb)	= 0.1936			F(8,469 Prob >	-	273.86 0.0000
0011(u_1,)	012000	(Std Err	- odiust		- 697 clusters	
		(Std. EI)	. aujuste	ed 101 4,	697 Clusters	
		Robust				
ln_wage	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
grade	0	(omitted)				
age	.0359987	.0052407	6.87	0.000	.0257243	.046273
	000700	0000045	0 50	0 000	000007	0005570
c.age#c.age	000723	.0000845	-8.56	0.000	0008887	0005573
ttl_exp	.0334668	.004069	8.22	0.000	.0254896	.0414439
ccr_evb	.0334000	.004003	0.22	0.000	.0204030	.0414433
c.ttl_exp#						
c.ttl_exp	.0002163	.0001763	1.23	0.220	0001294	.0005619
- 1						
tenure	.0357539	.0024683	14.49	0.000	.0309148	.040593
c.tenure#						
c.tenure	0019701	.0001696	-11.62	0.000	0023026	0016376
race	0	(
black	0 0890108	(omitted) .0137629	-6.47	0.000	1159926	062029
not_smsa south	0606309	.0163366	-0.47	0.000	0926583	0286035
_cons	1.03732	.0739644	14.02	0.000	.8923149	1.182325
	1.00702	.0100011	11.02	0.000	.0020140	1.102020
sigma_u	.35562203					
sigma_e	.29068923					
rho	.59946283	(fraction	of varia	nce due t	o u_i)	
	L					

Although the estimated coefficients are the same with and without the vce(robust) option, the robust estimator produced larger standard errors and a *p*-value for c.ttl_exp#c.ttl_exp above the conventional 10%. The *F* test of $\nu_i = 0$ is suppressed because it is too difficult to compute the robust form of the statistic when there are more than a few panels.

4

Technical note

The robust standard errors reported above are identical to those obtained by clustering on the panel variable idcode. Clustering on the panel variable produces an estimator of the VCE that is robust to cross-sectional heteroskedasticity and within-panel (serial) correlation that is asymptotically equivalent to that proposed by Arellano (1987). Although the example above applies the fixed-effects estimator, the robust and cluster–robust VCE estimators are also available for the random-effects estimator. Wooldridge (2016) and Arellano (2003) discuss these robust and cluster–robust VCE estimators. More details are available in *Methods and formulas*.

Example 4: Random-effects model

Refitting our log-wage model with the random-effects estimator, we obtain

<pre>. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure > c.tenure#c.tenure 2.race not_smsa south, re theta</pre>						
Random-effects	GLS regressi	Number	of obs =	28,091		
Group variable	e: idcode			Number	of groups =	4,697
R-sq:				Obs per	group:	
within =	= 0.1715				min =	1
between =					avg =	6.0
overall =	= 0.3708				max =	15
				Wald ch		9244.74
corr(u_i, X)	= 0 (assumed	.)		Prob >	chi2 =	0.0000
	theta -					
min 5%	median	95%	max			
0.2520 0.252	0.5499	0.7016	0.7206			
ln_wage	Coef.	Std. Err.	Z	P> z	[95% Conf.	Interval]
grade	.0646499	.0017812	36.30	0.000	.0611589	.0681409
age	.0368059	.0031195	11.80	0.000	.0306918	.0429201
0						
c.age#c.age	0007133	.00005	-14.27	0.000	0008113	0006153
ttl_exp	.0290208	.002422	11.98	0.000	.0242739	.0337678
ccr_exb	.0290208	.002422	11.90	0.000	.0242739	.0337078
c.ttl_exp#						
c.ttl_exp	.0003049	.0001162	2.62	0.009	.000077	.0005327
-						
tenure	.0392519	.0017554	22.36	0.000	.0358113	.0426925
c.tenure#						
c.tenure	0020035	.0001193	-16.80	0.000	0022373	0017697
010011410			10.00		10022010	
race						
black	053053	.0099926	-5.31	0.000	0726381	0334679
not_smsa	1308252	.0071751	-18.23	0.000	1448881	1167622
south	0868922	.0073032	-11.90	0.000	1012062	0725781
_cons	.2387207	.049469	4.83	0.000	.1417633	.3356781
sigma_u	.25790526					
sigma_e	.29068923					
rho	.44045273	(fraction	of varian	ce due t	o u_i)	
	L					

According to the R^2 's, this estimator performs worse within than the within fixed-effects estimator and worse between than the between estimator, as it must, and slightly better overall.

We estimate that σ_{ν} is 0.2579 and σ_{ϵ} is 0.2907 and, by assertion, assume that the correlation of ν and x is zero.

All that is known about the random-effects estimator is its asymptotic properties, so rather than reporting an F statistic for overall significance, xtreg, re reports a χ^2 . Taken jointly, our coefficients are significant.

xtreg, re also reports a summary of the distribution of θ_i , an ingredient in the estimation of (4). θ is not a constant here because we observe women for unequal periods.

We estimate that schooling has a rate of return of 6.5% (compared with 6.1% between and no estimate within); that the increase of wages with age turns around at 25.8 years (compared with 26.9 between and 24.9 within); that total experience yet again increases wages increasingly; that the effect of job tenure turns around at 9.8 years (compared with 12.1 between and 9.1 within); that being black reduces wages by 5.3% (compared with 5.6% between and no estimate within); that living in a non-SMSA reduces wages 13.1% (compared with 18.6% between and 8.9% within); and that living in the South reduces wages 8.7% (compared with 9.9% between and 6.1% within).

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Example 5: Random-effects model fit using ML

We could also have fit this random-effects model with the maximum likelihood estimator:

<pre>. xtreg ln_w ; > c.tenure#c.;</pre>			-	tl_exp#c.	ttl_exp ter	nure
Fitting consta Iteration 0: Iteration 1: Iteration 2: Iteration 3:		1: pod = -12663 pod = -12643 pod = -12643	3.954 9.756 9.614			
Fitting full r Iteration 0: Iteration 1: Iteration 2: Iteration 3:	nodel: log likeliho log likeliho log likeliho log likeliho	pod = -8853 pod = -8853	.6409 .4255			
Random-effect:	s ML regressio			Number		= 28,091
Group variable	e: idcode			Number	of groups	= 4,697
0						= 6.0
Log likelihoo	d = -8853.42	54		LR chi2 Prob >		= 7592.38 = 0.0000
ln_wage	Coef.	Std. Err.	z	P> z	[95% Co	nf. Interval]
grade age	.0646093 .0368531	.0017372 .0031226	37.19 11.80	0.000	.061204 .03073	
c.age#c.age	0007132	.0000501	-14.24	0.000	0008113	3000615
ttl_exp	.0288196	.0024143	11.94	0.000	.024087	7.0335515
c.ttl_exp# c.ttl_exp	.000309	.0001163	2.66	0.008	.000081	1.0005369
tenure	.0394371	.0017604	22.40	0.000	.0359868	.0428875
c.tenure# c.tenure	0020052	.0001195	-16.77	0.000	002239	50017709
race black	0533394	.0097338	-5.48	0.000	0724172	
not_smsa	1323433	.0071322 .0072143	-18.56 -12.14	0.000	146322:	
	- 0875599		12.17	0.000	. 1010396	.0104201
south	0875599 .2390837	.0491902	4.86	0.000	.142672	7 .3354947

LR test of sigma_u=0: chibar2(01) = 7339.84

Prob >= chibar2 = 0.000

The estimates are nearly the same as those produced by xtreg, re—the GLS estimator. For instance, xtreg, re estimated the coefficient on grade to be 0.0646499, xtreg, mle estimated 0.0646093, and the ratio is 0.0646499/0.0646093 = 1.001 to three decimal places. Similarly, the standard errors are nearly equal: 0.0017811/0.0017372 = 1.025. Below we compare all 11 coefficients:

	Co	efficient	t ratio	S	E ratio	
Estimator	mean	min.	max.	mean	min.	max.
xtreg, mle (ML) xtreg, re (GLS)	1. .997		1. 1.007	1. 1.006	1. .997	1. 1.027

Example 6: Population-averaged model

We could also have fit this model with the population-averaged estimator:

. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure > c.tenure#c.tenure 2.race not_smsa south, pa							
Iteration 2: 1 Iteration 3: 1	Iteration 1: tolerance = .0310561 Iteration 2: tolerance = .00074898 Iteration 3: tolerance = .0000147 Iteration 4: tolerance = 2.880e-07						
GEE population Group variable Link: Family: Correlation:	k: identity ily: Gaussian				of groups = c group: min = avg = max =	28,091 4,697 1 6.0 15 9598.89	
Scale paramete	er:	. 143	36709	Prob >	,	0.0000	
ln_wage	Coef.	Std. Err.	Z	P> z	[95% Conf.	Interval]	
grade age	.0645427 .036932	.0016829 .0031509	38.35 11.72	0.000 0.000	.0612442 .0307564	.0678412 .0431076	
c.age#c.age	0007129	.0000506	-14.10	0.000	0008121	0006138	
ttl_exp	.0284878	.0024169	11.79	0.000	.0237508	.0332248	
c.ttl_exp# c.ttl_exp	.0003158	.0001172	2.69	0.007	.000086	.0005456	
tenure	.0397468	.0017779	22.36	0.000	.0362621	.0432315	
c.tenure# c.tenure	002008	.0001209	-16.61	0.000	0022449	0017711	
race black not_smsa south _cons	0538314 1347788 0885969 .2396286	.0094086 .0070543 .0071132 .0491465	-5.72 -19.11 -12.46 4.88	0.000 0.000 0.000 0.000	072272 1486049 1025386 .1433034	0353909 1209526 0746552 .3359539	

4

These results differ from those produced by xtreg, re and xtreg, mle. Coefficients are larger and standard errors smaller. xtreg, pa is simply another way to run the xtgee command. That is, we would have obtained the same output had we typed

. xtgee ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp > tenure c.tenure#c.tenure 2.race not_smsa south (output omitted because it is the same as above)

See [XT] **xtgee**. In the language of **xtgee**, the random-effects model corresponds to an **exchangeable** correlation structure and identity link, and **xtgee** also allows other correlation structures. Let's stay with the random-effects model, however. **xtgee** will also produce robust estimates of variance, and we refit this model that way by typing

. xtgee ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp > tenure c.tenure#c.tenure 2.race not_smsa south, vce(robust) (output omitted, coefficients the same, standard errors different)

In the previous example, we presented a table comparing xtreg, re with xtreg, mle. Below we add the results from the estimates shown and the ones we did with xtgee, vce(robust):

		С	oefficier	nt ratio		SE ratio	
Estimator		mean	min.	max.	mea	an min.	max.
xtreg, mle	(ML)	1.	1.	1.	1.	1.	1.
xtreg, re	(GLS)	.997	.987	1.007	1.00	.997	1.027
xtreg, pa	(PA)	1.060	.847	1.317	.85	.626 .53	.986
xtgee, vce(robust)	(PA)	1.060	.847	1.317	1.30	.957	1.545

So, which are right? This is a real dataset, and we do not know. However, in example 2 in [XT] **xtreg postestimation**, we will present evidence that the assumptions underlying the **xtreg**, **re** and **xtreg**, **mle** results are not met.

Stored results

xtreg, re stores the following in e():

e(N)number of observationse(N_g)number of groupse(df_m)model degrees of freedome(g_min)smallest group sizee(g_avg)average group sizee(g_max)largest group sizee(Tcon)1 if T is constante(sigma)ancillary parameter (gamma, lnormal)e(sigma_u)panel-level standard deviation
e(df_m)model degrees of freedome(g_min)smallest group sizee(g_avg)average group sizee(g_max)largest group sizee(Tcon)1 if T is constante(sigma)ancillary parameter (gamma, lnormal)e(sigma_u)panel-level standard deviation
e(g_min)smallest group sizee(g_avg)average group sizee(g_max)largest group sizee(Tcon)1 if T is constante(sigma)ancillary parameter (gamma, lnormal)e(sigma_u)panel-level standard deviation
e(g_avg)average group sizee(g_max)largest group sizee(Tcon)1 if T is constante(sigma)ancillary parameter (gamma, lnormal)e(sigma_u)panel-level standard deviation
e(g_max)largest group sizee(Tcon)1 if T is constante(sigma)ancillary parameter (gamma, lnormal)e(sigma_u)panel-level standard deviation
e(g_max)largest group sizee(Tcon)1 if T is constante(sigma)ancillary parameter (gamma, lnormal)e(sigma_u)panel-level standard deviation
e(Tcon)1 if T is constante(sigma)ancillary parameter (gamma, lnormal)e(sigma_u)panel-level standard deviation
e(sigma_u) panel-level standard deviation
e(sigma_u) panel-level standard deviation
$e(sigma_e)$ standard deviation of ϵ_{it}
e(r2_w) R-squared for within model
e(r2_o) R-squared for overall model
e(r2_b) R-squared for between model
e(N_clust) number of clusters
e(chi2) χ^2
e(p) significance
e (rho) ρ
e(thta_min) minimum θ
e(thta_5) θ , 5th percentile
$e(thta_50)$ θ , 50th percentile
e(thta_95) θ , 95th percentile
e(thta_max) maximum θ
e(rmse) root mean squared error of GLS regression
e(Tbar) harmonic mean of group sizes
e(rank) rank of e(V)
Macros
e(cmd) xtreg
e(cmdline) command as typed
e(depvar) name of dependent variable
e(ivar) variable denoting groups
e(model) re
e(clustvar) name of cluster variable
e(chi2type) Wald; type of model χ^2 test
e(vce) vcetype specified in vce()
e(vcetype) title used to label Std. Err.
e(sa) Swamy-Arora estimator of the variance components (sa only)
e(properties) b V
e(predict) program used to implement predict
e(marginsnotok) predictions disallowed by margins
e(asbalanced) factor variables fvset as asbalanced
e(asobserved) factor variables fvset as asobserved
Matrices
e(b) coefficient vector
e(bf) coefficient vector for fixed-effects model
e(theta) $ heta$
e(V) variance-covariance matrix of the estimators
e(VCEf) VCE for fixed-effects model
Functions
e(sample) marks estimation sample

xtreg, be stores the following in e():

Scal	ars	
	e(N)	number of observations
	e(N_g)	number of groups
	e(typ)	WLS, if wls specified
	e(mss)	model sum of squares
	e(df_m)	model degrees of freedom
	e(rss)	residual sum of squares
	e(df_r)	residual degrees of freedom
	e(11)	log likelihood
	e(11_0)	log likelihood, constant-only model
	e(g_min)	smallest group size
	e(g_avg)	average group size
	e(g_max)	largest group size
	e(Tcon)	1 if T is constant
	e(r2)	R-squared
	e(r2_a)	adjusted R-squared
	e(r2_w)	R-squared for within model
	e(r2_o)	R-squared for overall model
	e(r2_b)	R-squared for between model
	e(F)	F statistic
	e(rmse)	root mean squared error
	e(Tbar)	harmonic mean of group sizes
	e(rank)	rank of e(V)
Mac	cros	
	e(cmd)	xtreg
	e(cmdline)	command as typed
	e(depvar)	name of dependent variable
	e(ivar)	variable denoting groups
	e(model)	be
	e(title)	title in estimation output
	e(vce)	vcetype specified in vce()
	e(vcetype)	title used to label Std. Err.
	e(properties)	b V
	e(predict)	program used to implement predict
	e(marginsok)	predictions allowed by margins
	e(marginsnotok)	predictions disallowed by margins
	e(asbalanced)	factor variables fvset as asbalanced
	e(asobserved)	factor variables fvset as asobserved
Mat	rices	
wiat	e(b)	coefficient vector
	e(V)	variance-covariance matrix of the estimators
-		variance-covariance matrix of the estimators
Fune	ctions	
	e(sample)	marks estimation sample

xtreg, fe stores the following in e():

Scalars	
e(N)	number of observations
e(N_g)	number of groups
e(mss)	model sum of squares
e(df_m)	model degrees of freedom
e(rss)	residual sum of squares
e(df_r)	residual degrees of freedom
e(tss)	total sum of squares
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(Tcon)	1 if T is constant
e(sigma)	ancillary parameter (gamma, lnormal)
e(corr)	$\operatorname{corr}(u_i, Xb)$
e(sigma_u)	panel-level standard deviation
e(sigma_e)	standard deviation of ϵ_{it}
e(r2)	<i>R</i> -squared
e(r2_a)	adjusted R-squared
e(r2_w)	<i>R</i> -squared for within model
e(r2_0)	R-squared for overall model
e(r2_b)	<i>R</i> -squared for between model
e(11)	log likelihood
e(11_0)	log likelihood, constant-only model
e(N_clust)	number of clusters
e(rho)	ρ
e(F)	<i>F</i> statistic
e(F_f)	F for $u_i=0$
e(df_a)	degrees of freedom for absorbed effect
e(df_b)	numerator degrees of freedom for F statistic
e(rmse)	root mean squared error
e(Tbar)	harmonic mean of group sizes
e(rank)	rank of e(V)
Macros	
e(cmd)	xtreg
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups
e(nodel)	fe
e(wtype)	weight type
e(wexp)	weight expression
e(clustvar)	name of cluster variable
e(vce)	<i>vcetype</i> specified in vce()
e(vcetype)	title used to label Std. Err.
e(properties)	b V
e(predict)	program used to implement predict
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fyset as asbalanced
e(asobserved)	factor variables fyset as asobserved
	actor variables ryber as abobberved
Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(V)	variance-covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

xtreg, mle stores the following in e():

Scalars	
e(N)	number of observations
e(N_g)	number of groups
e(df_m)	model degrees of freedom
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(sigma_u)	panel-level standard deviation
e(sigma_e)	standard deviation of ϵ_{it}
e(11)	log likelihood
e(11_0)	log likelihood, constant-only model
e(ll_c)	log likelihood, comparison model
e(chi2)	χ^2
e(chi2_c)	χ^2 for comparison test
e(rho)	ρ
e(rank)	rank of e(V)
Macros	
e(cmd)	xtreg
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups
e(model)	ml
e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(vce)	vcetype specified in vce()
e(vcetype)	title used to label Std. Err.
e(chi2type)	Wald or LR; type of model χ^2 test
e(chi2_ct)	Wald or LR; type of model χ^2 test corresponding to e(chi2_c)
e(distrib)	Gaussian; the distribution of the RE
e(properties)	b V
e(predict)	program used to implement predict
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(V)	variance-covariance matrix of the estimators
Functions	
e(sample)	marks estimation sample

xtreg, pa stores the following in e():

Scalars	
e(N)	number of observations
e(N_g)	number of groups
e(df_m)	model degrees of freedom
e(chi2)	χ^2
e(p)	x significance
e(df_pear)	degrees of freedom for Pearson χ^2
e(chi2_dev)	χ^2 test of deviance
e(chi2_dis)	χ^2 test of deviance dispersion
e(deviance)	deviance
e(dispers)	deviance dispersion
e(phi)	scale parameter
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(g_max) e(rank)	rank of e(V)
e(tol)	target tolerance
e(dif)	achieved tolerance
e(uii) e(rc)	return code
Macros	
e(cmd)	xtgee
e(cmd2)	xtreg
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups
e(tvar)	variable denoting time within groups
e(model)	pa Courseise
e(family)	Gaussian
e(link)	identity; link function
e(corr)	correlation structure
e(scale)	x2, dev, phi, or #; scale parameter
e(wtype)	weight type
e(wexp)	weight expression
e(offset)	linear offset variable
e(chi2type)	Wald; type of model χ^2 test
e(vce)	vcetype specified in vce()
e(vcetype)	title used to label Std. Err.
e(rgf)	rgf, if rgf specified
e(nmp)	nmp, if specified b V
e(properties)	
e(predict) e(marginsnotok)	program used to implement predict predictions disallowed by margins
e(asbalanced)	factor variables fyset as asbalanced
e(asobserved)	factor variables fyset as asobserved
	factor variables ivset as asobserved
Matrices	
e(b)	coefficient vector
e(R)	estimated working correlation matrix
e(V)	variance-covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample

Methods and formulas

The model to be fit is

 $y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it}$

for i = 1, ..., n and, for each i, t = 1, ..., T, of which T_i periods are actually observed.

Methods and formulas are presented under the following headings:

xtreg, fe xtreg, be xtreg, re xtreg, mle xtreg, pa

xtreg, fe

xtreg, fe produces estimates by running OLS on

$$(y_{it} - \overline{y}_i + \overline{\overline{y}}) = \alpha + (\mathbf{x}_{it} - \overline{\mathbf{x}}_i + \overline{\overline{\mathbf{x}}})\boldsymbol{\beta} + (\epsilon_{it} - \overline{\epsilon}_i + \overline{\nu}) + \overline{\overline{\epsilon}}$$

where $\overline{y}_i = \sum_{t=1}^{T_i} y_{it}/T_i$, and similarly, $\overline{\overline{y}} = \sum_i \sum_t y_{it}/(nT_i)$. The conventional covariance matrix of the estimators is adjusted for the extra n-1 estimated means, so results are the same as using OLS on (1) to estimate ν_i directly. Specifying vce(robust) or vce(cluster *clustvar*) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] _robust, particularly *Introduction* and *Methods and formulas*. Wooldridge (2016) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2016), Stock and Watson (2008), and Arellano (2003), specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*), where *panelvar* is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in ϵ_{it} .

The cluster–robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation induced by the within transform.

From the estimates $\hat{\alpha}$ and $\hat{\beta}$, estimates u_i of ν_i are obtained as $u_i = \overline{y}_i - \hat{\alpha} - \overline{\mathbf{x}}_i \hat{\beta}$. Reported from the calculated u_i are its standard deviation and its correlation with $\overline{\mathbf{x}}_i \hat{\beta}$. Reported as the standard deviation of e_{it} is the regression's estimated root mean squared error, s, which is adjusted (as previously stated) for the n-1 estimated means.

Reported as R^2 within is the R^2 from the mean-deviated regression.

Reported as R^2 between is corr $(\overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}, \overline{y}_i)^2$.

Reported as R^2 overall is corr $(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$.

xtreg, be

xtreg, be fits the following model:

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta} + \nu_i + \overline{\epsilon}_i$$

Estimation is via OLS unless T_i is not constant and the wls option is specified. Otherwise, the estimation is performed via WLS. The estimates and conventional VCE are obtained from regress for both cases, but for WLS, [aweight= T_i] is specified.

Reported as R^2 between is the R^2 from the fitted regression.

Reported as R^2 within is corr $\{(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}, y_{it} - \overline{y}_i\}^2$.

Reported as R^2 overall is $\operatorname{corr}(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$.

xtreg, re

The key to the random-effects estimator is the GLS transform. Given estimates of the idiosyncratic component, $\hat{\sigma}_e^2$, and the individual component, $\hat{\sigma}_u^2$, the GLS transform of a variable z for the random-effects model is

$$z_{it}^* = z_{it} - \widehat{\theta}_i \overline{z}_i$$

where $\overline{z}_i = \frac{1}{T_i} \sum_t^{T_i} z_{it}$ and

$$\widehat{\theta}_i = 1 - \sqrt{\frac{\widehat{\sigma}_e^2}{T_i \widehat{\sigma}_u^2 + \widehat{\sigma}_e^2}}$$

Given an estimate of $\hat{\theta}_i$, one transforms the dependent and independent variables, and then the coefficient estimates and the conventional variance-covariance matrix come from an OLS regression of y_{it}^* on \mathbf{x}_{it}^* and the transformed constant $1 - \hat{\theta}_i$. Specifying vce(robust) or vce(cluster *clustvar*) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] _robust; in particular, see *Introduction* and *Methods and formulas*. Wooldridge (2016) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2016), Stock and Watson (2008), and Arellano (2003), specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*), where *panelvar* is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in ϵ_{it} .

The cluster–robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation that is generally induced by the random-effects transform when there is heteroskedasticity or within-panel serial correlation in the idiosyncratic errors.

Stata has two implementations of the Swamy–Arora method for estimating the variance components. They produce the same results in balanced panels and share the same estimator of σ_e^2 . However, the two methods differ in their estimator of σ_u^2 in unbalanced panels. We call the first $\hat{\sigma}_{u\overline{T}}^2$ and the second $\hat{\sigma}_{uSA}^2$. Both estimators are consistent; however, $\hat{\sigma}_{uSA}^2$ has a more elaborate adjustment for small samples than $\hat{\sigma}_{u\overline{T}}^2$. (See Baltagi [2013], Baltagi and Chang [1994], and Swamy and Arora [1972] for derivations of these methods.)

Both methods use the same function of within residuals to estimate the idiosyncratic error component σ_e . Specifically,

$$\widehat{\sigma}_e^2 = \frac{\sum_i^n \sum_t^{T_i} e_{it}^2}{N - n - K + 1}$$

where

$$e_{it} = (y_{it} - \overline{y}_i + \overline{\overline{y}}) - \widehat{\alpha}_w - (\mathbf{x}_{it} - \overline{\mathbf{x}}_i + \overline{\overline{\mathbf{x}}})\widehat{\boldsymbol{\beta}}_w$$

and $\hat{\alpha}_w$ and $\hat{\beta}_w$ are the within estimates of the coefficients and $N = \sum_i^n T_i$. After passing the within residuals through the within transform, only the idiosyncratic errors are left.

The default method for estimating σ_u^2 is

$$\widehat{\sigma}_{u\overline{T}}^2 = \max\left\{0, \frac{SSR_b}{n-K} - \frac{\widehat{\sigma}_e^2}{\overline{T}}\right\}$$

where

$$SSR_b = \sum_{i}^{n} \left(\overline{y}_i - \widehat{\alpha}_b - \overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}_b \right)^2$$

 $\widehat{\alpha}_b$ and $\widehat{\beta}_b$ are coefficient estimates from the between regression and \overline{T} is the harmonic mean of T_i :

$$\overline{T} = \frac{n}{\sum_{i=\frac{1}{T_i}}^n \frac{1}{T_i}}$$

This estimator is consistent for σ_u^2 and is computationally less expensive than the second method. The sum of squared residuals from the between model estimate a function of both the idiosyncratic component and the individual component. Using our estimator of σ_e^2 , we can remove the idiosyncratic component, leaving only the desired individual component.

The second method is the Swamy–Arora method for unbalanced panels derived by Baltagi and Chang (1994), which has a more precise small-sample adjustment. Using this method,

$$\widehat{\sigma}_{\mathrm{uSA}}^2 = \max\left\{0, \frac{SSR_b - (n - K)\widehat{\sigma}_e^2}{N - tr}\right\}$$

where

$$tr = \operatorname{trace}\left\{ (\mathbf{X'PX})^{-1}\mathbf{X'ZZ'X} \right\}$$

$$\mathbf{P} = \operatorname{diag}\left\{ \left(\frac{1}{T_i}\right) \boldsymbol{\iota}_{T_i} \boldsymbol{\iota}_{T_i}' \right\}$$

$$\mathbf{Z} = \operatorname{diag}\left[\boldsymbol{\iota}_{T_i}\right]$$

X is the $N \times K$ matrix of covariates, including the constant, and ι_{T_i} is a $T_i \times 1$ vector of ones.

The estimated coefficients $(\hat{\alpha}_r, \hat{\beta}_r)$ and their covariance matrix \mathbf{V}_r are reported together with the previously calculated quantities $\hat{\sigma}_e$ and $\hat{\sigma}_u$. The standard deviation of $\nu_i + e_{it}$ is calculated as $\sqrt{\hat{\sigma}_e^2 + \hat{\sigma}_u^2}$.

Reported as R^2 between is $\operatorname{corr}(\overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}, \overline{y}_i)^2$. Reported as R^2 within is $\operatorname{corr}\{(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}, y_{it} - \overline{y}_i\}^2$. Reported as R^2 overall is $\operatorname{corr}(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$.

xtreg, mle

The log likelihood for the *i*th unit is

$$l_{i} = -\frac{1}{2} \left(\frac{1}{\sigma_{e}^{2}} \left[\sum_{t=1}^{T_{i}} (y_{it} - \mathbf{x}_{it}\beta)^{2} - \frac{\sigma_{u}^{2}}{T_{i}\sigma_{u}^{2} + \sigma_{e}^{2}} \left\{ \sum_{t=1}^{T_{i}} (y_{it} - \mathbf{x}_{it}\beta) \right\}^{2} \right] + \ln \left(T_{i} \frac{\sigma_{u}^{2}}{\sigma_{e}^{2}} + 1 \right) + T_{i} \ln(2\pi\sigma_{e}^{2}) \right)$$

The mle and re options yield essentially the same results, except when total $N = \sum_i T_i$ is small (200 or less) and the data are unbalanced.

xtreg, pa

See [XT] **xtgee** for details on the methods and formulas used to calculate the population-averaged model using a generalized estimating equations approach.

Acknowledgments

We thank Richard Goldstein, who wrote the first draft of the routine that fits random-effects regressions, Badi Baltagi of the Department of Economics at Syracuse University, and Manuelita Ureta of the Department of Economics at Texas A&M University, who assisted us in working our way through the literature.

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Also see

- [XT] xtreg postestimation Postestimation tools for xtreg
- [XT] xtgee Fit population-averaged panel-data models by using GEE
- [XT] xtgls Fit panel-data models by using GLS
- [XT] xtivreg Instrumental variables and two-stage least squares for panel-data models
- [XT] xtregar Fixed- and random-effects linear models with an AR(1) disturbance
- [XT] **xtset** Declare data to be panel data
- [ME] mixed Multilevel mixed-effects linear regression
- [MI] estimation Estimation commands for use with mi estimate
- [R] areg Linear regression with a large dummy-variable set
- [R] regress Linear regression
- [TS] forecast Econometric model forecasting
- [TS] prais Prais-Winsten and Cochrane-Orcutt regression
- [U] 20 Estimation and postestimation commands