Title

vec — Vector error-correction models

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Description

`vec` fits a type of vector autoregression in which some of the variables are cointegrated by using Johansen's (1995) maximum likelihood method. Constraints may be placed on the parameters in the cointegrating equations or on the adjustment terms. See [TS] **vec intro** for a list of commands that are used in conjunction with `vec`.

Quick start

Vector error-correction model for `y1`, `y2`, and `y3` using `tsset` data

```
vec y1 y2 y3
```

Use 4 lags for the underlying VAR model

```
vec y1 y2 y3, lags(4)
```

Use 2 cointegrating equations

```
vec y1 y2 y3, lags(4) rank(2)
```

Add a linear trend in the cointegrating equations and a quadratic trend in the undifferenced data

```
vec y1 y2 y3, lags(4) rank(2) trend(trend)
```

As above, but without a trend or a constant

```
vec y1 y2 y3, lags(4) rank(2) trend(none)
```

Menu

Statistics > Multivariate time series > Vector error-correction model (VECM)

Syntax

`vec` *depvarlist* [*if*] [*in*] [, *options*]

options	Description
Model	
`rank(#)`	use # cointegrating equations; default is `rank(1)`
`lags(#)`	use # for the maximum lag in underlying VAR model
`trend(constant)`	include an unrestricted constant in model; the default
`trend(rconstant)`	include a restricted constant in model
`trend(trend)`	include a linear trend in the cointegrating equations and a quadratic trend in the undifferenced data
`trend(rtrend)`	include a restricted trend in model
`trend(none)`	do not include a trend or a constant
`bconstraints(`*constraints*$_{\rm bc}$`)`	place *constraints*$_{\rm bc}$ on cointegrating vectors
`aconstraints(`*constraints*$_{\rm ac}$`)`	place *constraints*$_{\rm ac}$ on adjustment parameters
Adv. model	
`sindicators(`*varlist*$_{\rm si}$`)`	include normalized seasonal indicator variables *varlist*$_{\rm si}$
`noreduce`	do not perform checks and corrections for collinearity among lags of dependent variables
Reporting	
`level(#)`	set confidence level; default is `level(95)`
`nobtable`	do not report parameters in the cointegrating equations
`noidtest`	do not report the likelihood-ratio test of overidentifying restrictions
`alpha`	report adjustment parameters in separate table
`pi`	report parameters in $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$
`noptable`	do not report elements of $\mathbf{\Pi}$ matrix
`mai`	report parameters in the moving-average impact matrix
`noetable`	do not report adjustment and short-run parameters
`dforce`	force reporting of short-run, beta, and alpha parameters when the parameters in beta are not identified; advanced option
`nocnsreport`	do not display constraints
display_options	control columns and column formats, row spacing, and line width
Maximization	
maximize_options	control the maximization process; seldom used
`coeflegend`	display legend instead of statistics

`vec` does not allow gaps in the data.

You must `tsset` your data before using `vec`; see [TS] **tsset**.

varlist must contain at least two variables and may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

`by`, `fp`, `rolling`, `statsby`, and `xi` are allowed; see [U] **11.1.10 Prefix commands**.

`coeflegend` does not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Model

`rank(#)` specifies the number of cointegrating equations; `rank(1)` is the default.

`lags(#)` specifies the maximum lag to be included in the underlying VAR model. The maximum lag in a VECM is one smaller than the maximum lag in the corresponding VAR in levels; the number of lags must be greater than zero but small enough so that the degrees of freedom used up by the model are fewer than the number of observations. The default is `lags(2)`.

`trend(`*trend_spec*`)` specifies which of Johansen's five trend specifications to include in the model. These specifications are discussed in *Specification of constants and trends* below. The default is `trend(constant)`.

`bconstraints(`$constraints_{\mathrm{bc}}$`)` specifies the constraints to be placed on the parameters of the cointegrating equations. When no constraints are placed on the adjustment parameters—that is, when the `aconstraints()` option is not specified—the default is to place the constraints defined by Johansen's normalization on the parameters of the cointegrating equations. When constraints are placed on the adjustment parameters, the default is not to place constraints on the parameters in the cointegrating equations.

`aconstraints(`$constraints_{\mathrm{ac}}$`)` specifies the constraints to be placed on the adjustment parameters. By default, no constraints are placed on the adjustment parameters.

Adv. model

`sindicators(`$varlist_{\mathrm{si}}$`)` specifies the normalized seasonal indicator variables to include in the model. The indicator variables specified in this option must be normalized as discussed in Johansen (1995). If the indicators are not properly normalized, the estimator of the cointegrating vector does not converge to the asymptotic distribution derived by Johansen (1995). More details about how these variables are handled are provided in *Methods and formulas*. `sindicators()` cannot be specified with `trend(none)` or with `trend(rconstant)`.

`noreduce` causes `vec` to skip the checks and corrections for collinearity among the lags of the dependent variables. By default, `vec` checks to see whether the current lag specification causes some of the regressions performed by `vec` to contain perfectly collinear variables; if so, it reduces the maximum lag until the perfect collinearity is removed.

Reporting

`level(#)`; see [R] **estimation options**.

`nobtable` suppresses the estimation table for the parameters in the cointegrating equations. By default, `vec` displays the estimation table for the parameters in the cointegrating equations.

`noidtest` suppresses the likelihood-ratio test of the overidentifying restrictions, which is reported by default when the model is overidentified.

`alpha` displays a separate estimation table for the adjustment parameters, which is not displayed by default.

`pi` displays a separate estimation table for the parameters in $\mathbf{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}'$, which is not displayed by default.

`noptable` suppresses the estimation table for the elements of the $\mathbf{\Pi}$ matrix, which is displayed by default when the parameters in the cointegrating equations are not identified.

`mai` displays a separate estimation table for the parameters in the moving-average impact matrix, which is not displayed by default.

`noetable` suppresses the main estimation table that contains information about the estimated adjustment parameters and the short-run parameters, which is displayed by default.

`dforce` displays the estimation tables for the short-run parameters and $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$—if the last two are requested—when the parameters in $\boldsymbol{\beta}$ are not identified. By default, when the specified constraints do not identify the parameters in the cointegrating equations, estimation tables are displayed only for $\boldsymbol{\Pi}$ and the MAI.

`nocnsreport`; see [R] **estimation options**.

display_options: `noci`, `nopvalues`, `vsquish`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] **estimation options**.

Maximization

maximize_options: `iterate(#)`, `nolog`, `trace`, `toltrace`, `tolerance(#)`, `ltolerance(#)`, `afrom(`*matrix*$_a$`)`, and `bfrom(`*matrix*$_b$`)`; see [R] **maximize**.

`toltrace` displays the relative differences for the log likelihood and the coefficient vector at every iteration. This option cannot be specified if no constraints are defined or if `nolog` is specified.

`afrom(`*matrix*$_a$`)` specifies a $1 \times (K * r)$ row vector with starting values for the adjustment parameters, where K is the number of endogenous variables and r is the number of cointegrating equations specified in the `rank()` option. The starting values should be ordered as they are reported in `e(alpha)`. This option cannot be specified if no constraints are defined.

`bfrom(`*matrix*$_b$`)` specifies a $1 \times (m_1 * r)$ row vector with starting values for the parameters of the cointegrating equations, where m_1 is the number of variables in the trend-augmented system and r is the number of cointegrating equations specified in the `rank()` option. (See *Methods and formulas* for more details about m_1.) The starting values should be ordered as they are reported in `e(betavec)`. As discussed in *Methods and formulas*, for some trend specifications, `e(beta)` contains parameter estimates that are not obtained directly from the optimization algorithm. `bfrom()` should specify only starting values for the parameters reported in `e(betavec)`. This option cannot be specified if no constraints are defined.

The following option is available with `vec` but is not shown in the dialog box:

`coeflegend`; see [R] **estimation options**.

Remarks and examples

stata.com

Remarks are presented under the following headings:

Introduction
Specification of constants and trends
Collinearity

Introduction

VECMs are used to model the stationary relationships between multiple time series that contain unit roots. `vec` implements Johansen's approach for estimating the parameters of a VECM.

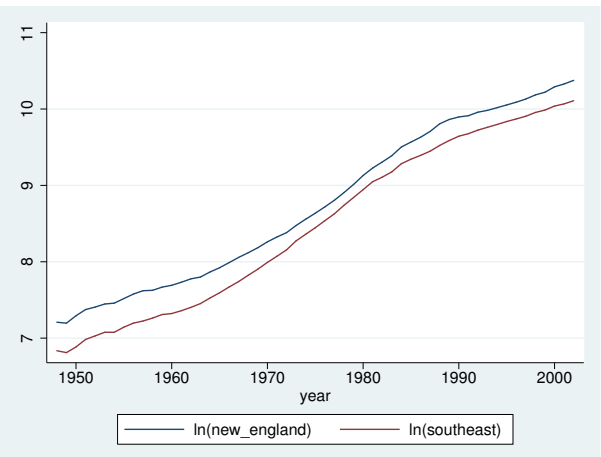
[TS] **vec intro** reviews the basics of integration and cointegration and highlights why we need special methods for modeling the relationships between processes that contain unit roots. This manual entry assumes familiarity with the material in [TS] **vec intro** and provides examples illustrating how to use the `vec` command. See Johansen (1995), Hamilton (1994), and Becketti (2013) for more in-depth introductions to cointegration analysis.

▷ Example 1

This example uses annual data on the average per-capita disposable personal income in the eight U.S. Bureau of Economic Analysis (BEA) regions of the United States. We use data from 1948–2002 in logarithms. Unit-root tests on these series fail to reject the null hypothesis that per-capita disposable income in each region contains a unit root. Because capital and labor can move easily between the different regions of the United States, we would expect that no one series will diverge from all the remaining series and that cointegrating relationships exist.

Below we graph the natural logs of average disposal income in the New England and the Southeast regions.

```
. use http://www.stata-press.com/data/r14/rdinc
. line ln_ne ln_se year
```



The graph indicates a differential between the two series that shrinks between 1960 and about 1980 and then grows until it stabilizes around 1990. We next estimate the parameters of a bivariate VECM with one cointegrating relationship.

```
. vec ln_ne ln_se
Vector error-correction model
Sample:  1950 - 2002                            Number of obs     =         53
                                                AIC               =  -11.00462
Log likelihood =  300.6224                      HQIC              =  -10.87595
Det(Sigma_ml)  =  4.06e-08                      SBIC              =  -10.67004
Equation           Parms      RMSE     R-sq      chi2     P>chi2
----------------------------------------------------------------
D_ln_ne                4     .017896   0.9313   664.4668   0.0000
D_ln_se                4     .018723   0.9292   642.7179   0.0000
----------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
D_ln_ne      |
        _ce1 |
         L1. |  -.4337524   .0721365    -6.01   0.000    -.5751373   -.2923675
             |
       ln_ne |
         LD. |   .7168658   .1889085     3.79   0.000     .3466119     1.08712
             |
       ln_se |
         LD. |  -.6748754   .2117975    -3.19   0.001    -1.089991   -.2597599
             |
       _cons |  -.0019846   .0080291    -0.25   0.805    -.0177214    .0137521
-------------+----------------------------------------------------------------
D_ln_se      |
        _ce1 |
         L1. |  -.3543935   .0754725    -4.70   0.000    -.5023168   -.2064701
             |
       ln_ne |
         LD. |   .3366786   .1976448     1.70   0.088     -.050698    .7240553
             |
       ln_se |
         LD. |  -.1605811   .2215922    -0.72   0.469    -.5948939    .2737317
             |
       _cons |    .002429   .0084004     0.29   0.772    -.0140355    .0188936
------------------------------------------------------------------------------
Cointegrating equations
Equation           Parms    chi2     P>chi2
----------------------------------------------
_ce1                   1   29805.02   0.0000
----------------------------------------------
Identification:  beta is exactly identified
                 Johansen normalization restriction imposed

------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
       ln_ne |          1          .        .       .            .           .
       ln_se |  -.9433708   .0054643  -172.64   0.000    -.9540807   -.9326609
       _cons |  -.8964065          .        .       .            .           .
------------------------------------------------------------------------------
```

The default output has three parts. The header provides information about the sample, the model fit, and the identification of the parameters in the cointegrating equation. The main estimation table contains the estimates of the short-run parameters, along with their standard errors and confidence intervals. The second estimation table reports the estimates of the parameters in the cointegrating equation, along with their standard errors and confidence intervals.

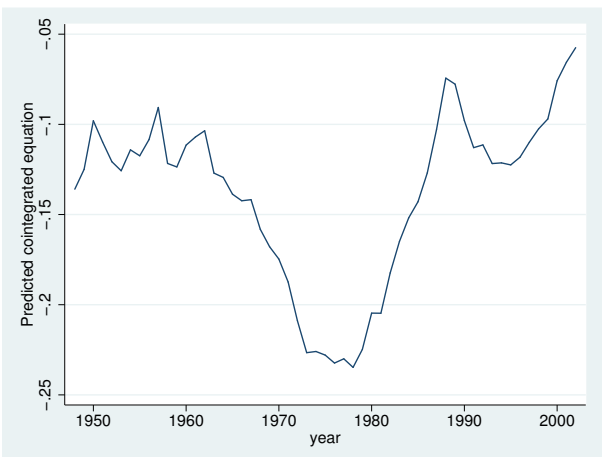
The results indicate strong support for a cointegrating equation such that

$$\texttt{ln_ne} - 0.943\,\texttt{ln_se} - 0.896$$

should be a stationary series. Identification of the parameters in the cointegrating equation is achieved by constraining some of them to be fixed, and fixed parameters do not have standard errors. In this example, the coefficient on `ln_ne` has been normalized to 1, so its standard error is missing. As discussed in *Methods and formulas*, the constant term in the cointegrating equation is not directly estimated in this trend specification but rather is backed out from other estimates. Not all the elements of the VCE that correspond to this parameter are readily available, so the standard error for the `_cons` parameter is missing.

To get a better idea of how our model fits, we predict the cointegrating equation and graph it over time:

```
. predict ce, ce
. line ce year
```



Although the predicted cointegrating equation has the right appearance for the time before the mid-1960s, afterward the predicted cointegrating equation does not look like a stationary series. A better model would account for the trends in the size of the differential.

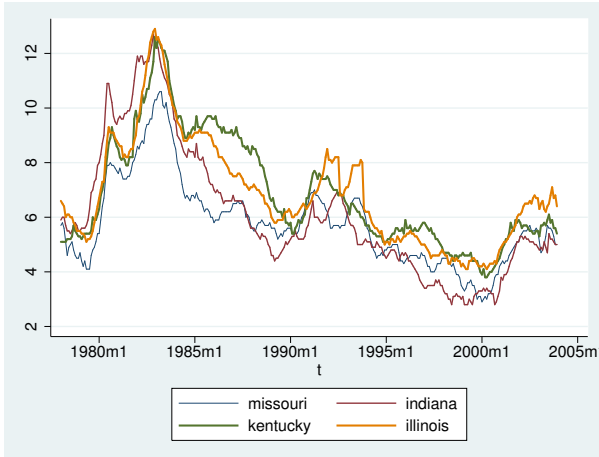
◁

As discussed in [TS] **vec intro**, simply normalizing one of the coefficients to be one is sufficient to identify the parameters of the single cointegrating vector. When there is more than one cointegrating equation, more restrictions are required.

▷ Example 2

We have data on monthly unemployment rates in Indiana, Illinois, Kentucky, and Missouri from January 1978 through December 2003. We suspect that factor mobility will keep the unemployment rates in equilibrium. The following graph plots the data.

```
. use http://www.stata-press.com/data/r14/urates, clear
. line missouri indiana kentucky illinois t
```



The graph shows that although the series do appear to move together, the relationship is not as clear as in the previous example. There are periods when Indiana has the highest rate and others when Indiana has the lowest rate. Although the Kentucky rate moves closely with the other series for most of the sample, there is a period in the mid-1980s when the unemployment rate in Kentucky does not fall at the same rate as the other series.

We will model the series with two cointegrating equations and no linear or quadratic time trends in the original series. Because we are focusing on the cointegrating vectors, we use the `noetable` option to suppress displaying the short-run estimation table.

```
. vec missouri indiana kentucky illinois, trend(rconstant) rank(2) lags(4)
> noetable

Vector error-correction model

Sample:  1978m5 - 2003m12                       Number of obs     =        308
                                                AIC               =  -2.306048
Log likelihood =  417.1314                      HQIC              =  -2.005818
Det(Sigma_ml)  =  7.83e-07                      SBIC              =  -1.555184

Cointegrating equations

Equation           Parms    chi2     P>chi2
----------------------------------------------
_ce1                   2   133.3885   0.0000
_ce2                   2   195.6324   0.0000
----------------------------------------------

Identification:  beta is exactly identified
                 Johansen normalization restrictions imposed
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
    missouri |          1          .        .       .            .           .
     indiana |          0  (omitted)
    kentucky |   .3493902   .2005537     1.74   0.081    -.0436879    .7424683
    illinois |  -1.135152   .2069063    -5.49   0.000    -1.540681   -.7296235
       _cons |  -.3880707   .4974323    -0.78   0.435     -1.36302    .5868787
-------------+----------------------------------------------------------------
_ce2         |
    missouri |  -1.11e-16          .        .       .            .           .
     indiana |          1          .        .       .            .           .
    kentucky |   .2059473   .2718678     0.76   0.449    -.3269038    .7387985
    illinois |   -1.51962   .2804792    -5.42   0.000    -2.069349   -.9698907
       _cons |    2.92857   .6743122     4.34   0.000     1.606942    4.250197
------------------------------------------------------------------------------
```

Except for the coefficients on `kentucky` in the two cointegrating equations and the constant term in the first, all the parameters are significant at the 5% level. We can refit the model with the Johansen normalization and the overidentifying constraint that the coefficient on `kentucky` in the second cointegrating equation is zero.

```
. constraint define 1 [_ce1]missouri = 1
. constraint define 2 [_ce1]indiana  = 0
. constraint define 3 [_ce2]missouri = 0
. constraint define 4 [_ce2]indiana  = 1
. constraint define 5 [_ce2]kentucky = 0
```

```
. vec missouri indiana kentucky illinois, trend(rconstant) rank(2)
> lags(4) noetable bconstraints(1/5)
Iteration 1:     log likelihood = 416.97177
  (output omitted)
Iteration 20:    log likelihood = 416.9744
Vector error-correction model
Sample:  1978m5 - 2003m12                       Number of obs     =        308
                                                AIC               =  -2.311522
Log likelihood =  416.9744                      HQIC              =  -2.016134
Det(Sigma_ml)  =  7.84e-07                      SBIC              =  -1.572769
Cointegrating equations
Equation           Parms    chi2     P>chi2
----------------------------------------------
_ce1                   2    145.233   0.0000
_ce2                   1   209.9344   0.0000
----------------------------------------------
Identification:  beta is overidentified
 ( 1)  [_ce1]missouri = 1
 ( 2)  [_ce1]indiana = 0
 ( 3)  [_ce2]missouri = 0
 ( 4)  [_ce2]indiana = 1
 ( 5)  [_ce2]kentucky = 0
------------------------------------------------------------------------------
        beta |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
_ce1         |
    missouri |          1          .        .       .            .           .
     indiana |          0  (omitted)
    kentucky |   .2521685   .1649653     1.53   0.126    -.0711576    .5754946
    illinois |  -1.037453   .1734165    -5.98   0.000    -1.377343   -.6975626
       _cons |  -.3891102   .4726968    -0.82   0.410    -1.315579    .5373586
-------------+----------------------------------------------------------------
_ce2         |
    missouri |          0  (omitted)
     indiana |          1          .        .       .            .           .
    kentucky |          0  (omitted)
    illinois |  -1.314265   .0907071   -14.49   0.000    -1.492048   -1.136483
       _cons |   2.937016   .6448924     4.55   0.000      1.67305    4.200982
------------------------------------------------------------------------------
LR test of identifying restrictions: chi2(1) = .3139       Prob > chi2 = 0.575
```

The test of the overidentifying restriction does not reject the null hypothesis that the restriction is valid, and the *p*-value on the coefficient on `kentucky` in the first cointegrating equation indicates that it is not significant. We will leave the variable in the model and attribute the lack of significance to whatever caused the `kentucky` series to temporarily rise above the others from 1985 until 1990, though we could instead consider removing `kentucky` from the model.

Next, we look at the estimates of the adjustment parameters. In the output below, we `replay` the previous results. We specify the `alpha` option so that `vec` will display an estimation table for the estimates of the adjustment parameters, and we specify `nobtable` to suppress the table for the parameters of the cointegrating equations because we have already looked at those.

```
. vec, alpha nobtable noetable
Vector error-correction model
Sample:  1978m5 - 2003m12                       Number of obs     =        308
                                                AIC               =  -2.311522
Log likelihood =  416.9744                      HQIC              =  -2.016134
Det(Sigma_ml)  =  7.84e-07                      SBIC              =  -1.572769
Adjustment parameters
Equation           Parms    chi2     P>chi2
----------------------------------------------
D_missouri             2   19.39607   0.0001
D_indiana              2   6.426086   0.0402
D_kentucky             2   8.524901   0.0141
D_illinois             2   22.32893   0.0000
----------------------------------------------
```

alpha	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
D_missouri						
_ce1						
L1.	-.0683152	.0185763	-3.68	0.000	-.1047242	-.0319063
_ce2						
L1.	.0405613	.0112417	3.61	0.000	.018528	.0625946
D_indiana						
_ce1						
L1.	-.0342096	.0220955	-1.55	0.122	-.0775159	.0090967
_ce2						
L1.	.0325804	.0133713	2.44	0.015	.0063732	.0587877
D_kentucky						
_ce1						
L1.	-.0482012	.0231633	-2.08	0.037	-.0936004	-.0028021
_ce2						
L1.	.0374395	.0140175	2.67	0.008	.0099657	.0649133
D_illinois						
_ce1						
L1.	.0138224	.0227041	0.61	0.543	-.0306768	.0583215
_ce2						
L1.	.0567664	.0137396	4.13	0.000	.0298373	.0836955

```
LR test of identifying restrictions: chi2(1) = .3139       Prob > chi2 = 0.575
```

All the coefficients are significant at the 5% level, except those on Indiana and Illinois in the first cointegrating equation. From an economic perspective, the issue is whether the unemployment rates in Indiana and Illinois adjust when the first cointegrating equation is out of equilibrium. We could impose restrictions on one or both of those parameters and refit the model, or we could just decide to use the current results.

◁

❑ Technical note

`vec` can be used to fit models in which the parameters in $\boldsymbol{\beta}$ are not identified, in which case only the parameters in $\mathbf{\Pi}$ and the moving-average impact matrix $\mathbf{C}$ are identified. When the parameters in $\boldsymbol{\beta}$ are not identified, the values of $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\alpha}}$ can vary depending on the starting values. However, the estimates of $\mathbf{\Pi}$ and $\mathbf{C}$ are identified and have known asymptotic distributions. This method is valid because these additional normalization restrictions impose no restriction on $\mathbf{\Pi}$ or $\mathbf{C}$.

❑

Specification of constants and trends

As discussed in [TS] **vec intro**, allowing for a constant term and linear time trend allow us to write the VECM as

$$\Delta\mathbf{y}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}\mathbf{y}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\rho}t) + \sum_{i=1}^{p-1}\mathbf{\Gamma}_i\Delta\mathbf{y}_{t-i} + \boldsymbol{\gamma} + \boldsymbol{\tau}t + \epsilon_t$$

Five different trend specifications are available:

Option in `trend()`	Parameter restrictions	Johansen (1995) notation
`trend`	none	$H(r)$
`rtrend`	$\boldsymbol{\tau} = \mathbf{0}$	$H^*(r)$
`constant`	$\boldsymbol{\rho} = \mathbf{0}$, and $\boldsymbol{\tau} = \mathbf{0}$	$H_1(r)$
`rconstant`	$\boldsymbol{\rho} = \mathbf{0}$, $\boldsymbol{\gamma} = \mathbf{0}$ and $\boldsymbol{\tau} = \mathbf{0}$	$H_1^*(r)$
`none`	$\boldsymbol{\mu} = \mathbf{0}$, $\boldsymbol{\rho} = \mathbf{0}$, $\boldsymbol{\gamma} = \mathbf{0}$, and $\boldsymbol{\tau} = \mathbf{0}$	$H_2(r)$

`trend(trend)` allows for a linear trend in the cointegrating equations and a quadratic trend in the undifferenced data. A linear trend in the cointegrating equations implies that the cointegrating equations are assumed to be trend stationary.

`trend(rtrend)` defines a restricted trend model that excludes linear trends in the differenced data but allows for linear trends in the cointegrating equations. As in the previous case, a linear trend in a cointegrating equation implies that the cointegrating equation is trend stationary.

`trend(constant)` defines a model with an unrestricted constant. This allows for a linear trend in the undifferenced data and cointegrating equations that are stationary around a nonzero mean. This is the default.

`trend(rconstant)` defines a model with a restricted constant in which there is no linear or quadratic trend in the undifferenced data. A nonzero $\boldsymbol{\mu}$ allows for the cointegrating equations to be stationary around nonzero means, which provide the only intercepts for differenced data. Seasonal indicators are not allowed with this specification.

`trend(none)` defines a model that does not include a trend or a constant. When there is no trend or constant, the cointegrating equations are restricted to being stationary with zero means. Also, after adjusting for the effects of lagged endogenous variables, the differenced data are modeled as having mean zero. Seasonal indicators are not allowed with this specification.

❑ Technical note

vec uses a switching algorithm developed by Boswijk (1995) to maximize the log-likelihood function when constraints are placed on the parameters. The starting values affect both the ability of the algorithm to find a maximum and its speed in finding that maximum. By default, vec uses the parameter estimates that correspond to Johansen's normalization. Sometimes, other starting values will cause the algorithm to find a maximum faster.

To specify starting values for the parameters in $\boldsymbol{\alpha}$, we specify a $1 \times (K * r)$ matrix in the afrom() option. Specifying starting values for the parameters in $\boldsymbol{\beta}$ is slightly more complicated. As explained in *Methods and formulas*, specifying trend(constant), trend(rtrend), or trend(trend) causes some of the estimates of the trend parameters appearing in $\widehat{\boldsymbol{\beta}}$ to be "backed out". The switching algorithm estimates only the parameters of the cointegrating equations whose estimates are stored in e(betavec). For this reason, only the parameters stored in e(betavec) can have their initial values set via bfrom().

The table below describes which trend parameters in the cointegrating equations are estimated by the switching algorithm for each of the five specifications.

Trend specification	Trend parameters in cointegrating equations	Trend parameter estimated via switching algorithm
none	none	none
rconstant	_cons	_cons
constant	_cons	none
rtrend	_cons, _trend	_trend
trend	_cons, _trend	none

❑

Collinearity

As expected, collinearity among variables causes some parameters to be unidentified numerically. If vec encounters perfect collinearity among the dependent variables, it exits with an error.

In contrast, if vec encounters perfect collinearity that appears to be due to too many lags in the model, vec displays a warning message and reduces the maximum lag included in the model in an effort to find a model with fewer lags in which all the parameters are identified by the data. Specifying the noreduce option causes vec to skip over these additional checks and corrections for collinearity. Thus the noreduce option can be used to force the estimation to proceed when not all the parameters are identified by the data. When some parameters are not identified because of collinearity, the results cannot be interpreted but can be used to find the source of the collinearity.

Stored results

vec stores the following in e():

Scalars

e(N)	number of observations
e(k_rank)	number of unconstrained parameters
e(k_eq)	number of equations in e(b)
e(k_dv)	number of dependent variables
e(k_ce)	number of cointegrating equations
e(n_lags)	number of lags
e(df_m)	model degrees of freedom
e(ll)	log likelihood
e(chi2_res)	value of test of overidentifying restrictions
e(df_lr)	degrees of freedom of the test of overidentifying restrictions
e(beta_iden)	1 if the parameters in $\boldsymbol{\beta}$ are identified and 0 otherwise
e(beta_icnt)	number of independent restrictions placed on $\boldsymbol{\beta}$
e(k_#)	number of variables in equation #
e(df_m#)	model degrees of freedom in equation #
e(r2_#)	R^2 of equation #
e(chi2_#)	$\chi 2$ statistic for equation #
e(rmse_#)	RMSE of equation #
e(aic)	value of AIC
e(hqic)	value of HQIC
e(sbic)	value of SBIC
e(tmin)	minimum time
e(tmax)	maximum time
e(detsig_ml)	determinant of the estimated covariance matrix
e(rank)	rank of e(V)
e(converge)	1 if the switching algorithm converged, 0 if it did not converge

Macros

e(cmd)	vec
e(cmdline)	command as typed
e(trend)	trend specified
e(tsfmt)	format of the time variable
e(tvar)	variable denoting time within groups
e(endog)	endogenous variables
e(covariates)	list of covariates
e(eqnames)	equation names
e(cenames)	names of cointegrating equations
e(reduce_opt)	noreduce, if noreduce is specified
e(reduce_lags)	list of maximum lags to which the model has been reduced
e(title)	title in estimation output
e(aconstraints)	constraints placed on $\boldsymbol{\alpha}$
e(bconstraints)	constraints placed on $\boldsymbol{\beta}$
e(sindicators)	sindicators, if specified
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(marginsdefault)	default predict() specification for margins

Matrices

`e(b)`	estimates of short-run parameters
`e(V)`	VCE of short-run parameter estimates
`e(beta)`	estimates of $\boldsymbol{\beta}$
`e(V_beta)`	VCE of $\widehat{\boldsymbol{\beta}}$
`e(betavec)`	directly obtained estimates of $\boldsymbol{\beta}$
`e(pi)`	estimates of $\widehat{\boldsymbol{\Pi}}$
`e(V_pi)`	VCE of $\widehat{\boldsymbol{\Pi}}$
`e(alpha)`	estimates of $\boldsymbol{\alpha}$
`e(V_alpha)`	VCE of $\widehat{\boldsymbol{\alpha}}$
`e(omega)`	estimates of $\widehat{\boldsymbol{\Omega}}$
`e(mai)`	estimates of $\mathbf{C}$
`e(V_mai)`	VCE of $\widehat{\mathbf{C}}$

Functions

`e(sample)`	marks estimation sample

Methods and formulas

Methods and formulas are presented under the following headings:

General specification of the VECM
The log-likelihood function
- *Unrestricted trend*
- *Restricted trend*
- *Unrestricted constant*
- *Restricted constant*
- *No trend*

Estimation with Johansen identification
Estimation with constraints: β identified
Estimation with constraints: β not identified
Formulas for the information criteria
Formulas for predict

General specification of the VECM

`vec` estimates the parameters of a VECM that can be written as

$$\Delta\mathbf{y}_t = \boldsymbol{\alpha}\boldsymbol{\beta}'\mathbf{y}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\mathbf{y}_{t-i} + \mathbf{v} + \boldsymbol{\delta}t + \mathbf{w}_1 s_1 + \cdots + \mathbf{w}_m s_m + \boldsymbol{\epsilon}_t \tag{1}$$

where

$\mathbf{y}_t$ is a $K \times 1$ vector of endogenous variables,

$\boldsymbol{\alpha}$ is a $K \times r$ matrix of parameters,

$\boldsymbol{\beta}$ is a $K \times r$ matrix of parameters,

$\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{p-1}$ are $K \times K$ matrices of parameters,

$\mathbf{v}$ is a $K \times 1$ vector of parameters,

$\boldsymbol{\delta}$ is a $K \times 1$ vector of trend coefficients,

t is a linear time trend,

$s_1, \ldots, s_m$ are orthogonalized seasonal indicators specified in the `sindicators()` option, and

$\mathbf{w}_1, \ldots, \mathbf{w}_m$ are $K \times 1$ vectors of coefficients on the orthogonalized seasonal indicators.

There are two types of deterministic elements in (1): the trend, $\mathbf{v} + \boldsymbol{\delta} t$, and the orthogonalized seasonal terms, $\mathbf{w}_1 s_1 + \cdots + \mathbf{w}_m s_m$. Johansen (1995, chap. 11) shows that inference about the number of cointegrating equations is based on nonstandard distributions and that the addition of any term that generalizes the deterministic specification in (1) changes the asymptotic distributions of the statistics used for inference on the number of cointegrating equations and the asymptotic distribution of the ML estimator of the cointegrating equations. In fact, Johansen (1995, 84) notes that including event indicators causes the statistics used for inference on the number of cointegrating equations to have asymptotic distributions that must be computed case by case. For this reason, event indicators may not be specified in the present version of `vec`.

If seasonal indicators are included in the model, they cannot be collinear with a constant term. If they are collinear with a constant term, one of the indicator variables is omitted.

As discussed in *Specification of constants and trends*, we can reparameterize the model as

$$\Delta \mathbf{y}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}\mathbf{y}_{t-1} + \boldsymbol{\mu} + \boldsymbol{\rho} t) + \sum_{i=1}^{p-1} \boldsymbol{\Gamma}_i \Delta \mathbf{y}_{t-i} + \boldsymbol{\gamma} + \boldsymbol{\tau} t + \epsilon_t \tag{2}$$

The log-likelihood function

We can maximize the log-likelihood function much more easily by writing it in concentrated form. In fact, as discussed below, in the simple case with the Johansen normalization on $\boldsymbol{\beta}$ and no constraints on $\boldsymbol{\alpha}$, concentrating the log-likelihood function produces an analytical solution for the parameter estimates.

To concentrate the log likelihood, rewrite (2) as

$$\mathbf{Z}_{0t} = \boldsymbol{\alpha}\widetilde{\boldsymbol{\beta}}'\mathbf{Z}_{1t} + \boldsymbol{\Psi}\mathbf{Z}_{2t} + \boldsymbol{\epsilon}_t \tag{3}$$

where $\mathbf{Z}_{0t}$ is a $K \times 1$ vector of variables $\Delta \mathbf{y}_t$, $\boldsymbol{\alpha}$ is the $K \times r$ matrix of adjustment coefficients, and $\boldsymbol{\epsilon}_t$ is a $K \times 1$ vector of independent and identically distributed normal vectors with mean 0 and contemporaneous covariance matrix $\boldsymbol{\Omega}$. $\mathbf{Z}_{1t}$, $\mathbf{Z}_{2t}$, $\widetilde{\boldsymbol{\beta}}$, and $\boldsymbol{\Psi}$ depend on the trend specification and are defined below.

The log-likelihood function for the model in (3) is

$$\begin{aligned} L = -\frac{1}{2}\Big\{ & TK\ln(2\pi) + T\ln(|\boldsymbol{\Omega}|) \\ & + \sum_{t=1}^{T} (\mathbf{Z}_{0t} - \boldsymbol{\alpha}\widetilde{\boldsymbol{\beta}}'\mathbf{Z}_{1t} - \boldsymbol{\Psi}\mathbf{Z}_{2t})'\boldsymbol{\Omega}^{-1}(\mathbf{Z}_{0t} - \boldsymbol{\alpha}\widetilde{\boldsymbol{\beta}}'\mathbf{Z}_{1t} - \boldsymbol{\Psi}\mathbf{Z}_{2t}) \Big\} \end{aligned} \tag{4}$$

with the constraints that $\boldsymbol{\alpha}$ and $\widetilde{\boldsymbol{\beta}}$ have rank r.

Johansen (1995, chap. 6), building on Anderson (1951), shows how the $\boldsymbol{\Psi}$ parameters can be expressed as analytic functions of $\boldsymbol{\alpha}$, $\widetilde{\boldsymbol{\beta}}$, and the data, yielding the concentrated log-likelihood function

$$\begin{aligned} L_c = -\frac{1}{2}\Big\{ & TK\ln(2\pi) + T\ln(|\boldsymbol{\Omega}|) \\ & + \sum_{t=1}^{T} (\mathbf{R}_{0t} - \boldsymbol{\alpha}\widetilde{\boldsymbol{\beta}}'\mathbf{R}_{1t})'\boldsymbol{\Omega}^{-1}(\mathbf{R}_{0t} - \boldsymbol{\alpha}\widetilde{\boldsymbol{\beta}}'\mathbf{R}_{1t}) \Big\} \end{aligned} \tag{5}$$

where

$$\mathbf{M}_{ij} = T^{-1}\sum_{t=1}^{T}\mathbf{Z}_{it}\mathbf{Z}_{jt}', \qquad i,j \in \{0,1,2\};$$

$$\mathbf{R}_{0t} = \mathbf{Z}_{0t} - \mathbf{M}_{02}\mathbf{M}_{22}^{-1}\mathbf{Z}_{2t}; \text{ and}$$

$$\mathbf{R}_{1t} = \mathbf{Z}_{1t} - \mathbf{M}_{12}\mathbf{M}_{22}^{-1}\mathbf{Z}_{2t}.$$

The definitions of $\mathbf{Z}_{1t}$, $\mathbf{Z}_{2t}$, $\widetilde{\boldsymbol{\beta}}$, and $\boldsymbol{\Psi}$ change with the trend specifications, although some of their components stay the same.

Unrestricted trend

When the trend in the VECM is unrestricted, we can define the variables in (3) directly in terms of the variables in (1):

$\mathbf{Z}_{1t} = \mathbf{y}_{t-1}$ is $K \times 1$

$\mathbf{Z}_{2t} = (\Delta\mathbf{y}_{t-1}', \ldots, \Delta\mathbf{y}_{t-p+1}', 1, t, s_1, \ldots, s_m)'$ is $\{K(p-1)+2+m\} \times 1$;

$\boldsymbol{\Psi} = (\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{p-1}, \mathbf{v}, \boldsymbol{\delta}, \mathbf{w}_1, \ldots, \mathbf{w}_m)$ is $K \times \{K(p-1)+2+m\}$

$\widetilde{\boldsymbol{\beta}} = \boldsymbol{\beta}$ is the $K \times r$ matrix composed of the r cointegrating vectors.

In the unrestricted trend specification, $m_1 = K$, $m_2 = K(p-1)+2+m$, and there are $n_{\text{parms}} = Kr + Kr + K\{K(p-1)+2+m\}$ parameters in (3).

Restricted trend

When there is a restricted trend in the VECM in (2), $\boldsymbol{\tau} = 0$, but the intercept $\mathbf{v} = \boldsymbol{\alpha}\boldsymbol{\mu} + \boldsymbol{\gamma}$ is unrestricted. The VECM with the restricted trend can be written as

$$\Delta\mathbf{y}_t = \boldsymbol{\alpha}(\boldsymbol{\beta}', \boldsymbol{\rho})\begin{pmatrix}\mathbf{y}_{t-1}\\ t\end{pmatrix} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\mathbf{y}_{t-i} + \mathbf{v} + \mathbf{w}_1 s_1 + \cdots + \mathbf{w}_m s_m + \epsilon_t$$

This equation can be written in the form of (3) by defining

$\mathbf{Z}_{1t} = \left(\mathbf{y}_{t-1}', t\right)'$ is $(K+1) \times 1$

$\mathbf{Z}_{2t} = (\Delta\mathbf{y}_{t-1}', \ldots, \Delta\mathbf{y}_{t-p+1}', 1, s_1, \ldots, s_m)'$ is $\{K(p-1)+1+m\} \times 1$

$\boldsymbol{\Psi} = (\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{p-1}, \mathbf{v}, \mathbf{w}_1, \ldots, \mathbf{w}_m)$ is $K \times \{K(p-1)+1+m\}$

$\widetilde{\boldsymbol{\beta}} = \left(\boldsymbol{\beta}', \boldsymbol{\rho}\right)'$ is the $(K+1) \times r$ matrix composed of the r cointegrating vectors and the r trend coefficients $\boldsymbol{\rho}$

In the restricted trend specification, $m_1 = K+1$, $m_2 = \{K(p-1)+1+m\}$, and there are $n_{\text{parms}} = Kr + (K+1)r + K\{K(p-1)+1+m\}$ parameters in (3).

Unrestricted constant

An unrestricted constant in the VECM in (2) is equivalent to setting $\boldsymbol{\delta} = 0$ in (1), which can be written in the form of (3) by defining

$\mathbf{Z}_{1t} = \mathbf{y}_{t-1}$ is $(K \times 1)$

$\mathbf{Z}_{2t} = (\Delta\mathbf{y}_{t-1}', \ldots, \Delta\mathbf{y}_{t-p+1}', 1, s_1, \ldots, s_m)'$ is $\{K(p-1)+1+m\} \times 1$;

$\boldsymbol{\Psi} = (\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{p-1}, \mathbf{v}, \mathbf{w}_1, \ldots, \mathbf{w}_m)$ is $K \times \{K(p-1)+1+m\}$

$\widetilde{\boldsymbol{\beta}} = \boldsymbol{\beta}$ is the $K \times r$ matrix composed of the r cointegrating vectors

In the unrestricted constant specification, $m_1 = K$, $m_2 = \{K(p-1)+1+m\}$, and there are $n_{\text{parms}} = Kr + Kr + K\{K(p-1)+1+m\}$ parameters in (3).

Restricted constant

When there is a restricted constant in the VECM in (2), it can be written in the form of (3) by defining

$\mathbf{Z}_{1t} = \left(\mathbf{y}'_{t-1}, 1\right)'$ is $(K+1)\times 1$

$\mathbf{Z}_{2t} = (\Delta\mathbf{y}'_{t-1}, \ldots, \Delta\mathbf{y}'_{t-p+1})'$ is $K(p-1)\times 1$

$\boldsymbol{\Psi} = (\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{p-1})$ is $K \times K(p-1)$

$\widetilde{\boldsymbol{\beta}} = \left(\boldsymbol{\beta}', \boldsymbol{\mu}\right)'$ is the $(K+1)\times r$ matrix composed of the r cointegrating vectors and the r constants in the cointegrating relations.

In the restricted trend specification, $m_1 = K+1$, $m_2 = K(p-1)$, and there are $n_{\text{parms}} = Kr + (K+1)r + K\{K(p-1)\}$ parameters in (3).

No trend

When there is no trend in the VECM in (2), it can be written in the form of (3) by defining

$\mathbf{Z}_{1t} = \mathbf{y}_{t-1}$ is $K \times 1$

$\mathbf{Z}_{2t} = (\Delta\mathbf{y}'_{t-1}, \ldots, \Delta\mathbf{y}'_{t-p+1})'$ is $K(p-1)+m \times 1$

$\boldsymbol{\Psi} = (\boldsymbol{\Gamma}_1, \ldots, \boldsymbol{\Gamma}_{p-1})$ is $K \times K(p-1)$

$\widetilde{\boldsymbol{\beta}} = \boldsymbol{\beta}$ is $K \times r$ matrix of r cointegrating vectors

In the no-trend specification, $m_1 = K$, $m_2 = K(p-1)$, and there are $n_{\text{parms}} = Kr + Kr + K\{K(p-1)\}$ parameters in (3).

Estimation with Johansen identification

Not all the parameters in $\boldsymbol{\alpha}$ and $\widetilde{\boldsymbol{\beta}}$ are identified. Consider the simple case in which $\widetilde{\boldsymbol{\beta}}$ is $K \times r$ and let $\mathbf{Q}$ be a nonsingular $r \times r$ matrix. Then

$$\boldsymbol{\alpha}\widetilde{\boldsymbol{\beta}}' = \boldsymbol{\alpha}\mathbf{Q}\mathbf{Q}^{-1}\widetilde{\boldsymbol{\beta}}' = \boldsymbol{\alpha}\mathbf{Q}(\widetilde{\boldsymbol{\beta}}\mathbf{Q}'^{-1})' = \dot{\boldsymbol{\alpha}}\dot{\boldsymbol{\beta}}'$$

Substituting $\dot{\boldsymbol{\alpha}}\dot{\boldsymbol{\beta}}'$ into the log likelihood in (5) for $\boldsymbol{\alpha}\widetilde{\boldsymbol{\beta}}'$ would not change the value of the log likelihood, so some a priori identification restrictions must be found to identify $\boldsymbol{\alpha}$ and $\widetilde{\boldsymbol{\beta}}$. As discussed in Johansen (1995, chap. 5 and 6) and Boswijk (1995), if the restrictions exactly identify or overidentify $\widetilde{\boldsymbol{\beta}}$, the estimates of the unconstrained parameters in $\widetilde{\boldsymbol{\beta}}$ will be superconsistent, meaning that the estimates of the free parameters in $\widetilde{\boldsymbol{\beta}}$ will converge at a faster rate than estimates of the short-run parameters in $\boldsymbol{\alpha}$ and $\boldsymbol{\Gamma}_i$. This allows the distribution of the estimator of the short-run parameters to be derived conditional on the estimated $\widetilde{\boldsymbol{\beta}}$.

Johansen (1995, chap. 6) has proposed a normalization method for use when theory does not provide sufficient a priori restrictions to identify the cointegrating vector. This method has become widely adopted by researchers. Johansen's identification scheme is

$$\widetilde{\boldsymbol{\beta}}' = (\mathbf{I}_r, \breve{\boldsymbol{\beta}}') \tag{6}$$

where $\mathbf{I}_r$ is the $r \times r$ identity matrix and $\breve{\boldsymbol{\beta}}$ is a $(m_1 - r) \times r$ matrix of identified parameters.

Johansen's identification method places r^2 linearly independent constraints on the parameters in $\widetilde{\boldsymbol{\beta}}$, thereby defining an exactly identified model. The total number of freely estimated parameters is $n_{\text{parms}} - r^2 = \{K + m_2 + (K + m_1 - r)r\}$, and the degrees of freedom d is calculated as the integer part of $(n_{\text{parms}} - r^2)/K$.

When only the rank and the Johansen identification restrictions are placed on the model, we can further manipulate the log likelihood in (5) to obtain analytic formulas for the parameters in $\widetilde{\boldsymbol{\beta}}$, $\boldsymbol{\alpha}$, and $\boldsymbol{\Omega}$. For a given value of $\widetilde{\boldsymbol{\beta}}$, $\boldsymbol{\alpha}$ and $\boldsymbol{\Omega}$ can be found by regressing $\mathbf{R}_{0t}$ on $\widetilde{\boldsymbol{\beta}}'\mathbf{R}_{1t}$. This allows a further simplification of the problem in which

$$\boldsymbol{\alpha}(\widetilde{\boldsymbol{\beta}}) = \mathbf{S}_{01}\widetilde{\boldsymbol{\beta}}(\widetilde{\boldsymbol{\beta}}'\mathbf{S}_{11}\widetilde{\boldsymbol{\beta}})^{-1}$$

$$\boldsymbol{\Omega}(\widetilde{\boldsymbol{\beta}}) = \mathbf{S}_{00} - \mathbf{S}_{01}\widetilde{\boldsymbol{\beta}}(\widetilde{\boldsymbol{\beta}}'\mathbf{S}_{11}\widetilde{\boldsymbol{\beta}})^{-1}\widetilde{\boldsymbol{\beta}}'\mathbf{S}_{10}$$

$$\mathbf{S}_{ij} = (1/T)\textstyle\sum_{t=1}^{T} R_{it}R'_{jt} \qquad i,j \in \{0,1\}$$

Johansen (1995) shows that by inserting these solutions into equation (5), $\widehat{\boldsymbol{\beta}}$ is given by the r eigenvectors $\mathbf{v}_1, \ldots, \mathbf{v}_r$ corresponding to the r largest eigenvalues $\lambda_1, \ldots, \lambda_r$ that solve the generalized eigenvalue problem

$$|\lambda_i\mathbf{S}_{11} - \mathbf{S}_{10}\mathbf{S}_{00}^{-1}\mathbf{S}_{01}| = 0 \tag{7}$$

The eigenvectors corresponding to $\lambda_1, \ldots, \lambda_r$ that solve (7) are the unidentified parameter estimates. To impose the identification restrictions in (6), we normalize the eigenvectors such that

$$\lambda_i\mathbf{S}_{11}\mathbf{v}_i = \mathbf{S}_{01}\mathbf{S}_{00}^{-1}\mathbf{S}_{01}\mathbf{v}_i \tag{8}$$

and

$$\mathbf{v}_i'\mathbf{S}_{11}\mathbf{v}_j = \begin{cases} 1 \text{ if } i = j \\ 0 \text{ otherwise} \end{cases} \tag{9}$$

At the optimum the log-likelihood function with the Johansen identification restrictions can be expressed in terms of T, K, $\mathbf{S}_{00}$, and the r largest eigenvalues

$$L_c = -\frac{1}{2}T\Big\{K\ln(2\pi) + K + \ln(|\mathbf{S}_{00}|) + \sum_{i=1}^{r}\ln(1 - \widehat{\lambda}_i)\Big\}$$

where the $\widehat{\lambda}_i$ are the eigenvalues that solve (7), (8), and (9).

Using the normalized $\widehat{\boldsymbol{\beta}}$, we can then obtain the estimates

$$\widehat{\boldsymbol{\alpha}} = \mathbf{S}_{01}\widehat{\boldsymbol{\beta}}(\widehat{\boldsymbol{\beta}}'S_{11}\widehat{\boldsymbol{\beta}})^{-1} \tag{10}$$

and

$$\widehat{\boldsymbol{\Omega}} = \mathbf{S}_{00} - \widehat{\boldsymbol{\alpha}}\widehat{\boldsymbol{\beta}}'\mathbf{S}_{10}$$

Let $\widehat{\boldsymbol{\beta}}_y$ be a $K \times r$ matrix that contains the estimates of the parameters in $\boldsymbol{\beta}$ in (1). $\widehat{\boldsymbol{\beta}}_y$ differs from $\widehat{\boldsymbol{\beta}}$ in that any trend parameter estimates are omitted from $\widehat{\boldsymbol{\beta}}$. We can then use $\widehat{\boldsymbol{\beta}}_y$ to obtain predicted values for the r nondemeaned cointegrating equations

$$\widehat{\widetilde{\mathbf{E}}}_t = \widehat{\boldsymbol{\beta}}_y'\mathbf{y}_t$$

The r series in $\widehat{\widetilde{E}}_t$ are called the predicted, nondemeaned cointegrating equations because they still contain the terms $\boldsymbol{\mu}$ and $\boldsymbol{\rho}$. We want to work with the predicted, demeaned cointegrating equations. Thus we need estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\rho}$. In the `trend(rconstant)` specification, the algorithm directly produces the estimator $\widehat{\boldsymbol{\mu}}$. Similarly, in the `trend(rtrend)` specification, the algorithm directly produces the estimator $\widehat{\boldsymbol{\rho}}$. In the remaining cases, to back out estimates of $\boldsymbol{\mu}$ and $\boldsymbol{\rho}$, we need estimates of $\mathbf{v}$ and $\boldsymbol{\delta}$, which we can obtain by estimating the parameters of the following VAR:

$$\Delta\mathbf{y}_t = \boldsymbol{\alpha}\widehat{\widetilde{\mathbf{E}}}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\mathbf{y}_{t-i} + \mathbf{v} + \boldsymbol{\delta}t + \mathbf{w}_1 s_1 + \cdots + \mathbf{w}_m s_m + \boldsymbol{\epsilon}_t \tag{11}$$

Depending on the trend specification, we use $\widehat{\boldsymbol{\alpha}}$ to back out the estimates of

$$\widehat{\boldsymbol{\mu}} = (\widehat{\boldsymbol{\alpha}}'\widehat{\boldsymbol{\alpha}})^{-1}\widehat{\boldsymbol{\alpha}}'\widehat{\mathbf{v}} \tag{12}$$

$$\widehat{\boldsymbol{\rho}} = (\widehat{\boldsymbol{\alpha}}'\widehat{\boldsymbol{\alpha}})^{-1}\widehat{\boldsymbol{\alpha}}'\widehat{\boldsymbol{\delta}} \tag{13}$$

if they are not already in $\widehat{\boldsymbol{\beta}}$ and are included in the trend specification.

We then augment $\widehat{\boldsymbol{\beta}}_y$ to

$$\widehat{\boldsymbol{\beta}}_f' = (\widehat{\boldsymbol{\beta}}_y', \widehat{\boldsymbol{\mu}}, \widehat{\boldsymbol{\rho}})$$

where the estimates of $\widehat{\boldsymbol{\mu}}$ and $\widehat{\boldsymbol{\rho}}$ are either obtained from $\widehat{\boldsymbol{\beta}}$ or backed out using (12) and (13). We next use $\widehat{\boldsymbol{\beta}}_f$ to obtain the r predicted, demeaned cointegrating equations, $\widehat{\mathbf{E}}_t$, via

$$\widehat{\mathbf{E}}_t = \widehat{\boldsymbol{\beta}}_f'\,(\mathbf{y}_t', 1, t)'$$

We last obtain estimates of all the short-run parameters from the VAR:

$$\Delta\mathbf{y}_t = \boldsymbol{\alpha}\widehat{\mathbf{E}}_{t-1} + \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i\Delta\mathbf{y}_{t-i} + \boldsymbol{\gamma} + \boldsymbol{\tau}t + \mathbf{w}_1 s_1 + \cdots + \mathbf{w}_m s_m + \boldsymbol{\epsilon}_t \tag{14}$$

Because the estimator $\widehat{\boldsymbol{\beta}}_f$ converges in probability to its true value at a rate faster than $T^{-\frac{1}{2}}$, we can take our estimated $\widehat{\mathbf{E}}_{t-1}$ as given data in (14). This allows us to estimate the variance–covariance (VCE) matrix of the estimates of the parameters in (14) by using the standard VAR VCE estimator. Equation (11) can be used to obtain consistent estimates of all the parameters and of the VCE of all the parameters, except $\mathbf{v}$ and $\boldsymbol{\delta}$. The standard VAR VCE of $\widehat{\mathbf{v}}$ and $\widehat{\boldsymbol{\delta}}$ is incorrect because these estimates converge at a faster rate. This is why it is important to use the predicted, demeaned cointegrating equations, $\widehat{\mathbf{E}}_{t-1}$, when estimating the short-run parameters and trend terms. In keeping with the cointegration literature, `vec` makes a small-sample adjustment to the VCE estimator so that the divisor is $(T-d)$ instead of T, where d represents the degrees of freedom of the model. d is calculated as the integer part of n_{parms}/K, where n_{parms} is the total number of freely estimated parameters in the model.

In the `trend(rconstant)` specification, the estimation procedure directly estimates $\boldsymbol{\mu}$. For `trend(constant)`, `trend(rtrend)`, and `trend(trend)`, the estimates of $\boldsymbol{\mu}$ are backed out using (12). In the `trend(rtrend)` specification, the estimation procedure directly estimates $\boldsymbol{\rho}$. In the `trend(trend)` specification, the estimates of $\boldsymbol{\rho}$ are backed out using (13). Because the elements of the estimated VCE are readily available only when the estimates are obtained directly, when the trend parameter estimates are backed out, their elements in the VCE for $\widehat{\boldsymbol{\beta}}_f$ are missing.

Under the Johansen identification restrictions, `vec` obtains $\widehat{\boldsymbol{\beta}}$, the estimates of the parameters in the $r \times m_1$ matrix $\widetilde{\boldsymbol{\beta}}'$ in (5). The VCE of $\mathrm{vec}(\widehat{\boldsymbol{\beta}})$ is $rm_1 \times rm_1$. Per Johansen (1995), the asymptotic distribution of $\widehat{\boldsymbol{\beta}}$ is mixed Gaussian, and its VCE is consistently estimated by

$$\left(\frac{1}{T-d}\right)(\mathbf{I}_r \otimes \mathbf{H}_J)\left\{(\widehat{\boldsymbol{\alpha}}'\boldsymbol{\Omega}^{-1}\widehat{\boldsymbol{\alpha}}) \otimes (\mathbf{H}_J'\mathbf{S}_{11}\mathbf{H}_J)\right\}^{-1}(\mathbf{I}_r \otimes \mathbf{H}_J)' \tag{15}$$

where $\mathbf{H}_J$ is the $m_1 \times (m_1 - r)$ matrix given by $\mathbf{H}_J = (\mathbf{0}'_{r\times(m_1-r)}, \mathbf{I}_{m_1-r})'$. The VCE reported in `e(V_beta)` is the estimated VCE in (15) augmented with missing values to account for any backed-out estimates of $\boldsymbol{\mu}$ or $\boldsymbol{\rho}$.

The parameter estimates $\widehat{\boldsymbol{\alpha}}$ can be found either as a function of $\widehat{\boldsymbol{\beta}}$, using (10) or from the VAR in (14). The estimated VCE of $\widehat{\boldsymbol{\alpha}}$ reported in `e(V_alpha)` is given by

$$\frac{1}{(T-d)}\widehat{\boldsymbol{\Omega}} \otimes \widehat{\boldsymbol{\Sigma}}_B$$

where $\widehat{\boldsymbol{\Sigma}}_B = (\widehat{\boldsymbol{\beta}}'\mathbf{S}_{11}\widehat{\boldsymbol{\beta}})^{-1}$.

As we would expect, the estimator of $\Pi = \boldsymbol{\alpha}\boldsymbol{\beta}'$ is

$$\widehat{\Pi} = \widehat{\boldsymbol{\alpha}}\widehat{\boldsymbol{\beta}}'$$

and its estimated VCE is given by

$$\frac{1}{(T-d)}\widehat{\boldsymbol{\Omega}} \otimes (\widehat{\boldsymbol{\beta}}\widehat{\boldsymbol{\Sigma}}_B\widehat{\boldsymbol{\beta}}')$$

The moving-average impact matrix $\mathbf{C}$ is estimated by

$$\widehat{\mathbf{C}} = \widehat{\boldsymbol{\beta}}_\perp(\widehat{\boldsymbol{\alpha}}_\perp\widehat{\boldsymbol{\Gamma}}\widehat{\boldsymbol{\beta}}_\perp)^{-1}\widehat{\boldsymbol{\alpha}}'_\perp$$

where $\widehat{\boldsymbol{\beta}}_\perp$ is the orthogonal complement of $\widehat{\boldsymbol{\beta}}_y$, $\widehat{\boldsymbol{\alpha}}_\perp$ is the orthogonal complement of $\widehat{\boldsymbol{\alpha}}$, and $\widehat{\boldsymbol{\Gamma}} = \mathbf{I}_\mathbf{K} - \sum_{i=1}^{p-1}\boldsymbol{\Gamma}_i$. The orthogonal complement of a $K \times r$ matrix $\mathbf{Q}$ that has rank r is a matrix $\mathbf{Q}_\perp$ of rank $K - r$, such that $\mathbf{Q}'\mathbf{Q}_\perp = \mathbf{0}$. Although this operation is not uniquely defined, the results used by `vec` do not depend on the method of obtaining the orthogonal complement. `vec` uses the following method: the orthogonal complement of $\mathbf{Q}$ is given by the r eigenvectors with the highest eigenvalues from the matrix $\mathbf{Q}'(\mathbf{Q}'\mathbf{Q})^{-1}\mathbf{Q}'$.

Per Johansen (1995, chap. 13) and Drukker (2004), the VCE of $\widehat{\mathbf{C}}$ is estimated by

$$\frac{T-d}{T}\widehat{\mathbf{S}}_q\widehat{\mathbf{V}}_{\widehat{\boldsymbol{\nu}}}\widehat{\mathbf{S}}'_q \tag{16}$$

where

$$\widehat{\mathbf{S}}_q = \widehat{\mathbf{C}} \otimes \widehat{\boldsymbol{\xi}}$$

$$\widehat{\boldsymbol{\xi}} = \begin{cases} (\widehat{\boldsymbol{\xi}}_1, \widehat{\boldsymbol{\xi}}_2) & \text{if } p > 1 \\ \widehat{\boldsymbol{\xi}}_1 & \text{if } p = 1 \end{cases}$$

$$\widehat{\boldsymbol{\xi}}_1 = (\widehat{\mathbf{C}}'\widehat{\boldsymbol{\Gamma}}' - \mathbf{I}_K)\bar{\boldsymbol{\alpha}}$$

$$\bar{\boldsymbol{\alpha}} = \widehat{\boldsymbol{\alpha}}(\widehat{\boldsymbol{\alpha}}'\widehat{\boldsymbol{\alpha}})^{-1}$$

$$\widehat{\boldsymbol{\xi}}_2 = \boldsymbol{\iota}_{p-1} \otimes \widehat{\mathbf{C}}$$

$\boldsymbol{\iota}_{p-1}$ is a $(p-1) \times 1$ vector of ones

$\widehat{\mathbf{V}}_{\widehat{\boldsymbol{\nu}}}$ is the estimated VCE of $\widehat{\boldsymbol{\nu}} = (\widehat{\boldsymbol{\alpha}}, \widehat{\boldsymbol{\Gamma}}_1, \ldots \widehat{\boldsymbol{\Gamma}}_{p-1})$

Estimation with constraints: β identified

`vec` can also fit models in which the adjustment parameters are subject to homogeneous linear constraints and the cointegrating vectors are subject to general linear restrictions. Mathematically, `vec` allows for constraints of the form

$$\mathbf{R}'_{\boldsymbol{\alpha}}\mathrm{vec}(\boldsymbol{\alpha}) = \mathbf{0} \tag{17}$$

where $\mathbf{R}_{\boldsymbol{\alpha}}$ is a known $Kr \times n_{\boldsymbol{\alpha}}$ constraint matrix, and

$$\mathbf{R}'_{\widetilde{\boldsymbol{\beta}}}\mathrm{vec}(\widetilde{\boldsymbol{\beta}}) = \mathbf{b} \tag{18}$$

where $\mathbf{R}_{\widetilde{\boldsymbol{\beta}}}$ is a known $m_1 r \times n_{\boldsymbol{\beta}}$ constraint matrix and $\mathbf{b}$ is a known $n_{\boldsymbol{\beta}} \times 1$ vector of constants. Although (17) and (18) are intuitive, they can be rewritten in a form to facilitate computation. Specifically, (17) can be written as

$$\mathrm{vec}(\boldsymbol{\alpha}') = \mathbf{Ga} \tag{19}$$

where $\mathbf{G}$ is $Kr \times n_{\boldsymbol{\alpha}}$ and $\mathbf{a}$ is $n_{\boldsymbol{\alpha}} \times 1$. Equation (18) can be rewritten as

$$\mathrm{vec}(\widetilde{\boldsymbol{\beta}}) = \mathbf{Hb} + \mathbf{h}_0 \tag{20}$$

where $\mathbf{H}$ is a known $n_1 r \times n_{\boldsymbol{\beta}}$ matrix, $\mathbf{b}$ is an $n_{\boldsymbol{\beta}} \times 1$ matrix of parameters, and $\mathbf{h}_0$ is a known $n_1 r \times 1$ matrix. See [P] **makecns** for a discussion of the different ways of specifying the constraints.

When constraints are specified via the `aconstraints()` and `bconstraints()` options, the Boswijk (1995) rank method determines whether the parameters in $\widetilde{\boldsymbol{\beta}}$ are underidentified, exactly identified, or overidentified.

Boswijk (1995) uses the Rothenberg (1971) method to determine whether the parameters in $\widetilde{\boldsymbol{\beta}}$ are identified. Thus the parameters in $\widetilde{\boldsymbol{\beta}}$ are exactly identified if $\rho_{\boldsymbol{\beta}} = r^2$, and the parameters in $\widetilde{\boldsymbol{\beta}}$ are overidentified if $\rho_{\boldsymbol{\beta}} > r^2$, where

$$\rho_{\boldsymbol{\beta}} = \mathrm{rank}\left\{\mathbf{R}_{\widetilde{\boldsymbol{\beta}}}(\mathbf{I}_r \otimes \ddot{\boldsymbol{\beta}})\right\}$$

and $\ddot{\boldsymbol{\beta}}$ is a full-rank matrix with the same dimensions as $\widetilde{\boldsymbol{\beta}}$. The computed $\rho_{\boldsymbol{\beta}}$ is stored in `e(beta_icnt)`.

Similarly, the number of freely estimated parameters in $\boldsymbol{\alpha}$ and $\widetilde{\boldsymbol{\beta}}$ is given by ρ_{jacob}, where

$$\rho_{\mathrm{jacob}} = \mathrm{rank}\left\{(\widehat{\boldsymbol{\alpha}} \otimes \mathbf{I}_{m_1})\mathbf{H}, (\mathbf{I}_K \otimes \widehat{\boldsymbol{\beta}})\mathbf{G}\right\}$$

Using ρ_{jacob}, we can calculate several other parameter counts of interest. In particular, the degrees of freedom of the overidentifying test are given by $(K + m_1 - r)r - \rho_{\mathrm{jacob}}$, and the number of freely estimated parameters in the model is $n_{\mathrm{parms}} = Km_2 + \rho_{\mathrm{jacob}}$.

Although the problem of maximizing the log-likelihood function in (4), subject to the constraints in (17) and (18), could be handled by the algorithms in [R] **ml**, the switching algorithm of Boswijk (1995) has proven to be more convergent. For this reason, `vec` uses the Boswijk (1995) switching algorithm to perform the optimization.

Given starting values $(\widehat{\mathbf{b}}_0, \widehat{\mathbf{a}}_0, \widehat{\boldsymbol{\Omega}}_0)$, the algorithm iteratively updates the estimates until convergence is achieved, as follows:

$\widehat{\boldsymbol{\alpha}}_j$ is constructed from (19) and $\widehat{\mathbf{a}}_j$

$\widehat{\boldsymbol{\beta}}_j$ is constructed from (20) and $\widehat{\mathbf{b}}_j$

$$\widehat{\mathbf{b}}_{j+1} = \{\mathbf{H}'(\widehat{\boldsymbol{\alpha}}_j'\widehat{\boldsymbol{\Omega}}_j^{-1}\widehat{\boldsymbol{\alpha}}_j \otimes \mathbf{S}_{11})\mathbf{H}\}^{-1}\mathbf{H}'(\widehat{\boldsymbol{\alpha}}_j\widehat{\boldsymbol{\Omega}}_j^{-1} \otimes \mathbf{S}_{11})\{\mathrm{vec}(\widehat{\mathbf{P}}) - (\widehat{\boldsymbol{\alpha}}_j \otimes \mathbf{I}_{nz1})\mathbf{h}_0\}$$

$$\widehat{\mathbf{a}}_{j+1} = \{\mathbf{G}(\widehat{\boldsymbol{\Omega}}_j^{-1} \otimes \widehat{\boldsymbol{\beta}}_j\mathbf{S}_{11}\widehat{\boldsymbol{\beta}}_j)\mathbf{G}\}^{-1}\mathbf{G}'(\widehat{\boldsymbol{\Omega}}_j^{-1} \otimes \widehat{\boldsymbol{\beta}}_j\mathbf{S}_{11})\mathrm{vec}(\widehat{\mathbf{P}})$$

$$\widehat{\Omega}_{j+1} = \mathbf{S}_{00} - \mathbf{S}_{01}\widehat{\boldsymbol{\beta}}_j\widehat{\boldsymbol{\alpha}}_j' - \widehat{\boldsymbol{\alpha}}_j\widehat{\boldsymbol{\beta}}_j'\mathbf{S}_{10} + \widehat{\boldsymbol{\alpha}}_j\widehat{\boldsymbol{\beta}}_j'\mathbf{S}_{11}\widehat{\boldsymbol{\beta}}_j\widehat{\boldsymbol{\alpha}}_j'$$

The estimated VCE of $\widehat{\boldsymbol{\beta}}$ is given by

$$\frac{1}{(T-d)}\mathbf{H}\{\mathbf{H}'(\mathbf{W} \otimes \mathbf{S}_{11})\mathbf{H}\}^{-1}\mathbf{H}'$$

where $\mathbf{W}$ is $\widehat{\boldsymbol{\alpha}}'\widehat{\boldsymbol{\Omega}}^{-1}\widehat{\boldsymbol{\alpha}}$. As in the case without constraints, the estimated VCE of $\widehat{\boldsymbol{\alpha}}$ can be obtained either from the VCE of the short-run parameters, as described below, or via the formula

$$\widehat{V}_{\widehat{\boldsymbol{\alpha}}} = \frac{1}{(T-d)}\mathbf{G}\left[\mathbf{G}'\left\{\widehat{\boldsymbol{\Omega}}^{-1} \otimes (\widehat{\boldsymbol{\beta}}'\mathbf{S}_{11}\widehat{\boldsymbol{\beta}})\mathbf{G}\right\}^{-1}\right]\mathbf{G}'$$

Boswijk (1995) notes that, as long as the parameters of the cointegrating equations are exactly identified or overidentified, the constrained ML estimator produces superconsistent estimates of $\widetilde{\boldsymbol{\beta}}$. This implies that the method of estimating the short-run parameters described above applies in the presence of constraints, as well, albeit with a caveat: when there are constraints placed on $\boldsymbol{\alpha}$, the VARs must be estimated subject to these constraints.

With these estimates and the estimated VCE of the short-run parameter matrix $\widehat{\mathbf{V}}_{\widehat{\boldsymbol{\nu}}}$, Drukker (2004) shows that the estimated VCE for $\widehat{\Pi}$ is given by

$$(\widehat{\boldsymbol{\beta}} \otimes \mathbf{I}_K)\widehat{V}_{\widehat{\boldsymbol{\alpha}}}(\widehat{\boldsymbol{\beta}} \otimes \mathbf{I}_K)'$$

Drukker (2004) also shows that the estimated VCE of $\widehat{\mathbf{C}}$ can be obtained from (16) with the extension that $\widehat{V}_{\widehat{\boldsymbol{\nu}}}$ is the estimated VCE of $\widehat{\nu}$ that takes into account any constraints on $\widehat{\boldsymbol{\alpha}}$.

Estimation with constraints: β not identified

When the parameters in $\boldsymbol{\beta}$ are not identified, only the parameters in $\boldsymbol{\Pi} = \boldsymbol{\alpha}\boldsymbol{\beta}$ and $\mathbf{C}$ are identified. The estimates of $\boldsymbol{\Pi}$ and $\mathbf{C}$ would not change if more identification restrictions were imposed to achieve exact identification. Thus the VCE matrices for $\widehat{\boldsymbol{\Pi}}$ and $\widehat{\mathbf{C}}$ can be derived as if the model exactly identified $\boldsymbol{\beta}$.

Formulas for the information criteria

The AIC, SBIC, and HQIC are calculated according to their standard definitions, which include the constant term from the log likelihood; that is,

$$\text{AIC} = -2\left(\frac{L}{T}\right) + \frac{2n_{\text{parms}}}{T}$$

$$\text{SBIC} = -2\left(\frac{L}{T}\right) + \frac{\ln(T)}{T}n_{\text{parms}}$$

$$\text{HQIC} = -2\left(\frac{L}{T}\right) + \frac{2\ln\{\ln(T)\}}{T}n_{\text{parms}}$$

where n_{parms} is the total number of parameters in the model and L is the value of the log likelihood at the optimum.

Formulas for predict

`xb`, `residuals` and `stdp` are standard and are documented in [R] **predict**. `ce` causes predict to compute $\widehat{E}_t = \widehat{\boldsymbol{\beta}}_f \mathbf{y}_t$ for the requested cointegrating equation.

`levels` causes `predict` to compute the predictions for the levels of the data. Let $\widehat{y}_t^d$ be the predicted value of Δy_t. Because the computations are performed for a given equation, y_t is a scalar. Using $\widehat{y}_t^d$, we can predict the level by $\widehat{y}_t = \widehat{y}_t^d + y_{t-1}$.

Because the residuals from the VECM for the differences and the residuals from the corresponding VAR in levels are identical, there is no need for an option for predicting the residuals in levels.

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Also see

[TS] **vec postestimation** — Postestimation tools for vec

[TS] **tsset** — Declare data to be time-series data

[TS] **var** — Vector autoregressive models

[TS] **var svar** — Structural vector autoregressive models

[U] 20 Estimation and postestimation commands

[TS] **vec intro** — Introduction to vector error-correction models