Title

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ztest — z tests (mean-comparison tests, known variance)

Description

`ztest` performs z tests on the equality of means, assuming known variances. In the first form, `ztest` tests that *varname* has a mean of #. In the second form, `ztest` tests that *varname* has the same mean within the two groups defined by *groupvar*. In the third form, `ztest` tests that $varname_1$ and $varname_2$ have the same mean, assuming *unpaired* data. In the fourth form, `ztest` tests that $varname_1$ and $varname_2$ have the same mean, assuming *paired* data.

`ztesti` is the immediate form of `ztest`; see **[U] 19 Immediate commands**.

For the comparison of means when variances are unknown, use `ttest`; see **[R] ttest**.

Quick start

Unpaired z test that the mean of `v1` is equal between two groups defined by `catvar`

```
ztest v1, by(catvar)
```

Paired test of `v2` and `v3` with standard deviation of the differences between paired observations of 2.4

```
ztest v2 == v3, sddiff(2.4)
```

As above, specified using a common standard deviation of 2 and correlation between observations of 0.28

```
ztest v2 == v3, sd(2) corr(.28)
```

Unpaired test of `v2` and `v3` conducted separately for each level of `catvar`

```
by catvar: ztest v2 == v3, unpaired
```

One-sample test that the mean of `v4` is 3 at the 90% confidence level

```
ztest v4 == 3, level(90)
```

Unpaired test of $\mu_1 = \mu_2$ if $\overline{x}_1 = 3.2$, $\text{sd}_1 = 0.1$, $\overline{x}_2 = 3.4$, and $\text{sd}_2 = 0.15$ with $n_1 = n_2 = 120$

```
ztesti 120 3.2 .1 120 3.4 .15
```

Menu

ztest

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > z test (mean-comparison test, known variance)

ztesti

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > z test calculator

Syntax

One-sample z test

ztest *varname* == # [*if*] [*in*] [, *onesampleopts*]

Two-sample z test using groups

ztest *varname* [*if*] [*in*], by(*groupvar*) [*twosamplegropts*]

Two-sample z test using variables

ztest $varname_1$ == $varname_2$ [*if*] [*in*], unpaired [*twosamplevaropts*]

Paired z test

ztest $varname_1$ == $varname_2$ [*if*] [*in*], sddiff(#) [level(#)]

ztest $varname_1$ == $varname_2$ [*if*] [*in*], corr(#) [*pairedopts*]

Immediate form of one-sample z test

ztesti $\#_{obs}$ $\#_{mean}$ $\#_{sd}$ $\#_{val}$ [, level(#)]

Immediate form of two-sample unpaired z test

ztesti $\#_{obs1}$ $\#_{mean1}$ $\#_{sd1}$ $\#_{obs2}$ $\#_{mean2}$ $\#_{sd2}$ [, level(#)]

onesampleopts	Description
Main	
sd(#)	one-population standard deviation; default is sd(1)
level(#)	set confidence level; default is level(95)

twosamplegropts	Description
Main	
* by(*groupvar*)	variable defining the groups
unpaired	unpaired test; implied when by() is specified
sd(#)	two-population common standard deviation; default is sd(1)
sd1(#)	standard deviation of the first population; requires sd2() and may not be combined with sd()
sd2(#)	standard deviation of the second population; requires sd1() and may not be combined with sd()
level(#)	set confidence level; default is level(95)

*by(*groupvar*) is required.

twosamplevaropts	Description
Main	
* unpaired	unpaired test
sd(#)	two-population common standard deviation; default is sd(1)
sd1(#)	standard deviation of the first population; requires sd2() and may not be combined with sd()
sd2(#)	standard deviation of the second population; requires sd1() and may not be combined with sd()
level(#)	set confidence level; default is level(95)

*unpaired is required.

pairedopts	Description
Main	
* corr(#)	correlation between paired observations
sd(#)	two-population common standard deviation; default is sd(1); may not be combined with sd1(), sd2(), or sddiff()
sd1(#)	standard deviation of the first population; requires corr() and sd2() and may not be combined with sd() or sddiff()
sd2(#)	standard deviation of the second population; requires corr() and sd1() and may not be combined with sd() or sddiff()
level(#)	set confidence level; default is level(95)

*corr(#) is required.

by is allowed with ztest; see [D] **by**.

Options

Main

by(*groupvar*) specifies the *groupvar* that defines the two groups that ztest will use to test the hypothesis that their means are equal. Specifying by(*groupvar*) implies an unpaired (two-sample) z test. Do not confuse the by() option with the by prefix; you can specify both.

unpaired specifies that the data be treated as unpaired. The unpaired option is used when the two sets of values to be compared are in different variables.

sddiff(#) specifies the population standard deviation of the differences between paired observations for a paired z test. For this kind of test, either sddiff() or corr() must be specified.

corr(#) specifies the correlation between paired observations for a paired z test. This option along with sd1() and sd2() or with sd() is used to compute the standard deviation of the differences between paired observations unless that standard deviation is supplied directly in the sddiff() option. For a paired z test, either sddiff() or corr() must be specified.

sd(#) specifies the population standard deviation for a one-sample z test or the common population standard deviation for a two-sample z test. The default is sd(1). sd() may not be combined with sd1(), sd2(), or sddiff().

sd1(#) specifies the standard deviation of the first population or group. When sd1() is specified with by(*groupvar*), the first group is defined by the first category of the sorted *groupvar*. sd1() requires sd2() and may not be combined with sd() or sddiff().

sd2(#) specifies the standard deviation of the second population or group. When sd2() is specified with by(*groupvar*), the second group is defined by the second category of the sorted *groupvar*. sd2() requires sd1() and may not be combined with sd() or sddiff().

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see **[U] 20.7 Specifying the width of confidence intervals**.

When by() is used, sd1() and sd2() or sd() is used to specify the population standard deviations of the two groups defined by *groupvar* for an unpaired two-sample z test (using groups). By default, a common standard deviation of one, sd(1), is assumed.

When unpaired is used, sd1() and sd2() or sd() is used to specify the population standard deviations of $varname_1$ and $varname_2$ for an unpaired two-sample z test (using variables). By default, a common standard deviation of one, sd(1), is assumed.

Options corr(), sd1(), and sd2() or corr() and sd() are used for a paired z test to compute the standard deviation of the differences between paired observations. By default, a common standard deviation of one, sd(1), is assumed for both populations. Alternatively, the standard deviation of the differences between paired observations may be supplied directly with the sddiff() option.

Remarks and examples

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Remarks are presented under the following headings:

One-sample z test
Two-sample z test
Paired z test
Immediate form

For the purpose of illustration, we assume that variances are known in all the examples below.

One-sample z test

▷ Example 1

In the first form, ztest tests whether the mean of the sample is equal to a known constant under the assumption of known variance. Assume that we have a sample of 74 automobiles. We know each automobile's average mileage rating and wish to test whether the overall average for the sample is 20 miles per gallon. We also assume that the population standard deviation is 6.

```
. use http://www.stata-press.com/data/r14/auto
(1978 Automobile Data)

. ztest mpg==20, sd(6)

One-sample z test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
     mpg |      74     21.2973    .6974858           6    19.93025    22.66434
------------------------------------------------------------------------------
    mean = mean(mpg)                                              z =   1.8600
Ho: mean = 20

    Ha: mean < 20               Ha: mean != 20                Ha: mean > 20
 Pr(Z < z) = 0.9686        Pr(|Z| > |z|) = 0.0629          Pr(Z > z) = 0.0314
```

The p-value for the two-sided test is 0.0629, so we do not have statistical evidence to reject the null hypothesis that the mean equals 20 at a 5% significance level, but we would reject the null hypothesis at a 10% level.

◁

Two-sample z test

▷ Example 2: Two-sample z test using groups

We are testing the effectiveness of a new fuel additive. We run an experiment in which 12 cars are given the fuel treatment and 12 cars are not. The results of the experiment are as follows:

treated	mpg
0	20
0	23
0	21
0	25
0	18
0	17
0	18
0	24
0	20
0	24
0	23
0	19
1	24
1	25
1	21
1	22
1	23
1	18
1	17
1	28
1	24
1	27
1	21
1	23

The treated variable is coded as 1 if the car received the fuel treatment and 0 otherwise.

We can test the equality of means of the treated and untreated group by typing

```
. use http://www.stata-press.com/data/r14/fuel3
. ztest mpg, by(treated) sd(3)
Two-sample z test
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |      12          21    .8660254           3    19.30262    22.69738
       1 |      12       22.75    .8660254           3    21.05262    24.44738
---------+--------------------------------------------------------------------
    diff |              -1.75    1.224745               -4.150456    .6504558
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      z =  -1.4289
Ho: diff = 0

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(Z < z) = 0.0765         Pr(|Z| > |z|) = 0.1530          Pr(Z > z) = 0.9235
```

We do not have evidence to reject the null hypothesis that the means of the two groups are equal at a 5% significance level.

In the above, we assumed that the two groups have the same standard deviation of 3. If the standard deviations for the two groups are different, we can specify group-specific standard deviations in options `sd1()` and `sd2()`:

```
. ztest mpg, by(treated) sd1(2.7) sd2(3.2)
Two-sample z test
------------------------------------------------------------------------------
   Group |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       0 |      12          21    .7794229         2.7    19.47236    22.52764
       1 |      12       22.75    .9237604         3.2    20.93946    24.56054
---------+--------------------------------------------------------------------
    diff |               -1.75    1.208649               -4.118909    .6189093
------------------------------------------------------------------------------
    diff = mean(0) - mean(1)                                      z =  -1.4479
Ho: diff = 0

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(Z < z) = 0.0738         Pr(|Z| > |z|) = 0.1476          Pr(Z > z) = 0.9262
```

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❑ Technical note

In two-sample randomized designs, subjects will sometimes refuse the assigned treatment but still be measured for an outcome. In this case, take care to specify the group properly. You might be tempted to let *varname* contain missing where the subject refused and thus let `ztest` drop such observations from the analysis. Zelen (1979) argues that it would be better to specify that the subject belongs to the group in which he or she was randomized, even though such inclusion will dilute the measured effect.

❑

▷ Example 3: Two-sample z test using variables

There is a second, inferior way to organize the data in the preceding example. We ran a test on 24 cars, 12 without the additive and 12 with. We now create two new variables, `mpg1` and `mpg2`.

mpg1	mpg2
20	24
23	25
21	21
25	22
18	23
17	18
18	17
24	28
20	24
24	27
23	21
19	23

This method is inferior because it suggests a connection that is not there. There is no link between the car with 20 mpg and the car with 24 mpg in the first row of the data. Each column of data could be arranged in any order. Nevertheless, if our data are organized like this, `ztest` can accommodate us.

```
. use http://www.stata-press.com/data/r14/fuel
. ztest mpg1==mpg2, unpaired sd(3)
Two-sample z test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    mpg1 |      12          21    .8660254           3    19.30262    22.69738
    mpg2 |      12       22.75    .8660254           3    21.05262    24.44738
---------+--------------------------------------------------------------------
    diff |               -1.75    1.224745               -4.150456    .6504558
------------------------------------------------------------------------------
    diff = mean(mpg1) - mean(mpg2)                                z =  -1.4289
Ho: diff = 0

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(Z < z) = 0.0765         Pr(|Z| > |z|) = 0.1530          Pr(Z > z) = 0.9235
```

◁

Paired z test

▷ Example 4

Suppose that the preceding data were actually collected by running a test on 12 cars. Each car was run once with the fuel additive and once without. Our data are stored in the same manner as in example 3, but this time, there is most certainly a connection between the mpg values that appear in the same row. These come from the same car. The variables `mpg1` and `mpg2` represent mileage without and with the treatment, respectively. Suppose that the two variables have a common standard deviation of 2 and the correlation between them is 0.4.

```
. use http://www.stata-press.com/data/r14/fuel
. ztest mpg1==mpg2, sd(2) corr(0.4)
Paired z test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    mpg1 |      12          21    .5773503           2    19.86841    22.13159
    mpg2 |      12       22.75    .5773503           2    21.61841    23.88159
---------+--------------------------------------------------------------------
    diff |      12       -1.75    .6324555     2.19089    -2.98959   -.5104099
------------------------------------------------------------------------------
     mean(diff) = mean(mpg1 - mpg2)                               z =  -2.7670
 Ho: mean(diff) = 0

 Ha: mean(diff) < 0           Ha: mean(diff) != 0           Ha: mean(diff) > 0
 Pr(Z < z) = 0.0028         Pr(|Z| > |z|) = 0.0057          Pr(Z > z) = 0.9972
```

The p-value for the two-sided test is 0.0057, so we reject, for example, the null hypothesis that the two means are equal at a 5% significance level.

Equivalently, we could specify directly the standard deviation of the differences between paired observations with the `sddiff()` option:

```
. ztest mpg1==mpg2, sddiff(2.191)
Paired z test
------------------------------------------------------------------------------
Variable |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
    diff |      12       -1.75    .6324872       2.191   -2.989652   -.5103478
------------------------------------------------------------------------------
     mean(diff) = mean(mpg1 - mpg2)                              z =  -2.7669
 Ho: mean(diff) = 0
 Ha: mean(diff) < 0           Ha: mean(diff) != 0           Ha: mean(diff) > 0
 Pr(Z < z) = 0.0028         Pr(|Z| > |z|) = 0.0057          Pr(Z > z) = 0.9972
```

◁

Immediate form

▷ Example 5: One-sample z test

ztesti is like ztest, except that we specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the mean number of sunspots per month as 62.6 with a standard deviation of 15.8. We assume this standard deviation is the population standard deviation. There are 24 months of data. We wish to test whether the mean is 75:

```
. ztesti 24 62.6 15.8 75
One-sample z test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      24        62.6    3.225161        15.8     56.2788     68.9212
------------------------------------------------------------------------------
    mean = mean(x)                                               z =  -3.8448
Ho: mean = 75
    Ha: mean < 75               Ha: mean != 75                 Ha: mean > 75
 Pr(Z < z) = 0.0001         Pr(|Z| > |z|) = 0.0001          Pr(Z > z) = 0.9999
```

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▷ Example 6: Two-sample z test

There is no immediate form of ztest with paired data because the test is also a function of the covariance, a number unlikely to be reported in any published source. For unpaired data, however, we might type

```
. ztesti 20 20 5  32 15 4
Two-sample z test
------------------------------------------------------------------------------
         |     Obs        Mean    Std. Err.   Std. Dev.   [95% Conf. Interval]
---------+--------------------------------------------------------------------
       x |      20          20    1.118034           5    17.80869    22.19131
       y |      32          15    .7071068           4     13.6141     16.3859
---------+--------------------------------------------------------------------
    diff |                   5    1.322876                2.407211    7.592789
------------------------------------------------------------------------------
    diff = mean(x) - mean(y)                                     z =   3.7796
Ho: diff = 0
    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(Z < z) = 0.9999         Pr(|Z| > |z|) = 0.0002          Pr(Z > z) = 0.0001
```

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Stored results

`ztest` and `ztesti` store the following in `r()`:

Scalars

`r(N_1)`	sample size n_1	`r(sd_d)`	standard deviation of the differences between paired observations
`r(N_2)`	sample size n_2	`r(corr)`	correlation between paired observations
`r(p_l)`	lower one-sided p-value	`r(sd_1)`	standard deviation for group one
`r(p_u)`	upper one-sided p-value	`r(sd_2)`	standard deviation for group two
`r(p)`	two-sided p-value	`r(mu_1)`	$\overline{x}_1$, sample mean for group one
`r(se)`	estimate of standard error	`r(mu_2)`	$\overline{x}_2$, sample mean for group two
`r(z)`	z statistic	`r(level)`	confidence level
`r(sd)`	one-population standard deviation or two-population common standard deviation		

Methods and formulas

Methods and formulas are presented under the following headings:

One-sample z test
Two-sample unpaired z test
Paired z test

One-sample z test

Suppose that we observe a random sample $x_1, x_2, \ldots, x_n$ of size n, which follows a normal distribution with mean μ and standard deviation σ. We are interested in testing the null hypothesis H_0: $\mu = \mu_0$ versus the two-sided alternative hypothesis H_a: $\mu \neq \mu_0$, the upper one-sided alternative H_a: $\mu > \mu_0$, or the lower one-sided alternative H_a: $\mu < \mu_0$. Assuming a known standard deviation σ, we use the following test statistic,

$$z = \frac{(\overline{x} - \mu_0)\sqrt{n}}{\sigma}$$

where $\overline{x} = (\sum_{i=1}^{n} x_i)/n$ is the sample mean.

Two-sample unpaired z test

Suppose that we observe a random sample $x_{11}, x_{12}, \ldots, x_{1n_1}$ of size n_1, which follows a normal distribution with mean μ_1 and standard deviation σ_1, and another random sample $x_{21}, x_{22}, \ldots, x_{2n_2}$ of size n_2, which follows a normal distribution with mean μ_2 and standard deviation σ_2. We are interested in testing the null hypothesis H_0: $\mu_2 = \mu_1$ versus the two-sided alternative hypothesis H_a: $\mu_2 \neq \mu_1$, the upper one-sided alternative H_a: $\mu_2 > \mu_1$, or the lower one-sided alternative H_a: $\mu_2 < \mu_1$. Assuming known standard deviations σ_1 and σ_2, we use the following test statistic,

$$z = \frac{\overline{x}_2 - \overline{x}_1}{\left(\frac{\sigma_1^2}{n_1} + \frac{\sigma_2^2}{n_2}\right)^{1/2}}$$

where $\overline{x}_1 = (\sum_{i=1}^{n_1} x_{1i})/n_1$ and $\overline{x}_2 = (\sum_{i=1}^{n_2} x_{2i})/n_2$ are the two sample means.

Paired z test

Some experiments have paired observations (also known as matched observations, correlated pairs, or permanent components). Consider a sequence of n paired observations denoted by x_{ij} for subjects $i = 1, 2, \ldots, n$ and groups $j = 1, 2$. An individual observation corresponds to the pair (x_{i1}, x_{i2}), and inference is made on the differences within the pairs. Let $\mu_d = \mu_2 - \mu_1$ denote the mean difference, where μ_j is the population mean of group j, and let $D_i = x_{i2} - x_{i1}$ denote the difference between individual observations. D_i follows a normal distribution with mean $\mu_2 - \mu_1$ and standard deviation σ_d, where $\sigma_d = \sqrt{{\sigma_1}^2 + {\sigma_2}^2 - 2\rho\sigma_1\sigma_2}$, σ_j is the population standard deviation of group j, and ρ is the correlation between paired observations.

We are interested in testing the null hypothesis $H_0\colon \mu_2 = \mu_1$ versus the two-sided alternative hypothesis $H_a\colon \mu_2 \neq \mu_1$, the upper one-sided alternative $H_a\colon \mu_2 > \mu_1$, or the lower one-sided alternative $H_a\colon \mu_2 < \mu_1$. Assuming the standard deviation of the differences σ_d is known, we use the following test statistic,

$$z = \frac{\overline{d}\sqrt{n}}{\sigma_d}$$

where $\overline{d} = (\sum_{i=1}^{n} D_i)/n$ is the sample mean of the differences between paired observations.

For all the tests above, the test statistic z is distributed as standard normal, and the p-value is computed as

$$p = \begin{cases} 1 - \Phi(z) & \text{for an upper one-sided test} \\ \Phi(z) & \text{for a lower one-sided test} \\ 2\Big(1 - \Phi(|z|)\Big) & \text{for a two-sided test} \end{cases}$$

where $\Phi(\cdot)$ is the cdf of a standard normal distribution, and $|z|$ is an absolute value of z.

Also see, for instance, Hoel (1984, 140–161), Dixon and Massey (1983, 100–130), and Tamhane and Dunlop (2000, 237–290) for more information about z tests.

References

Dixon, W. J., and F. J. Massey, Jr. 1983. *Introduction to Statistical Analysis*. 4th ed. New York: McGraw–Hill.

Hoel, P. G. 1984. *Introduction to Mathematical Statistics*. 5th ed. New York: Wiley.

Tamhane, A. C., and D. D. Dunlop. 2000. *Statistics and Data Analysis: From Elementary to Intermediate*. Upper Saddle River, NJ: Prentice Hall.

Zelen, M. 1979. A new design for randomized clinical trials. *New England Journal of Medicine* 300: 1242–1245.

Also see

[R] **ci** — Confidence intervals for means, proportions, and variances

[R] **esize** — Effect size based on mean comparison

[R] **mean** — Estimate means

[R] **oneway** — One-way analysis of variance

[R] **ttest** — t tests (mean-comparison tests)

[MV] **hotelling** — Hotelling's T-squared generalized means test

[PSS] **power onemean** — Power analysis for a one-sample mean test

[PSS] **power twomeans** — Power analysis for a two-sample means test

[PSS] **power pairedmeans** — Power analysis for a two-sample paired-means test