Title

nlcom — Nonlinear combinations of estimators

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stata.com

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Also see

Description

nlcom computes point estimates, standard errors, test statistics, significance levels, and confidence intervals for (possibly) nonlinear combinations of parameter estimates after any Stata estimation command, including survey estimation. Results are displayed in the usual table format used for displaying estimation results. Calculations are based on the "delta method", an approximation appropriate in large samples.

Quick start

```
Estimate the ratio of the coefficient of x2 to the coefficient of x1 nlcom _b[x2]/_b[x1]
```

```
Also estimate the ratio of the coefficient of x3 to coefficient of x1 nlcom (b[x2]/b[x1]) (b[x3]/b[x1])
```

Add labels to the ratios

```
nlcom (r21:_b[x2]/_b[x1]) (r31:_b[x3]/_b[x1])
```

```
As above, but post estimates and use the test command to test that both ratios are equal to 1 nlcom (r21:_b[x2]/_b[x1]) (r31:_b[x3]/_b[x1]), post test (r21 = 1) (r31 = 1)
```

```
Estimate the ratio of the coefficients of factor indicators 2.a and 3.a nlcom _b[2.a]/_b[3.a]
```

```
Estimate the ratio of the coefficients of x1 in the equations for y1 and y2 in a multiequation model nlcom _b[y1:x1]/_b[y2:x1]
```

Menu

Statistics > Postestimation

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Syntax

Nonlinear combination of estimators—one expression

```
nlcom [name:]exp [, options]
```

Nonlinear combinations of estimators—more than one expression

```
nlcom ([name: ]exp) [([name: ]exp) ...] [, options]
```

options	Description
<pre>level(#) iterate(#) post display_options</pre>	set confidence level; default is level(95) maximum number of iterations post estimation results control column formats and line width
<pre>noheader df(#)</pre>	suppress output header use t distribution with # degrees of freedom for computing p -values and confidence intervals

noheader and df(#) do not appear in the dialog box.

The second syntax means that if more than one expression is specified, each must be surrounded by parentheses. The optional *name* is any valid Stata name and labels the transformations.

exp is a possibly nonlinear expression containing

_b[coef]
_b[eqno:coef]
[eqno]coef
[eqno]_b[coef]

eqno is

name

coef identifies a coefficient in the model. coef is typically a variable name, a level indicator, an interaction indicator, or an interaction involving continuous variables. Level indicators identify one level of a factor variable and interaction indicators identify one combination of levels of an interaction; see [U] 11.4.3 Factor variables. coef may contain time-series operators; see [U] 11.4.4 Time-series varlists.

Distinguish between [], which are to be typed, and [], which indicate optional arguments.

Options

level(#) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] 20.7 Specifying the width of confidence intervals.

iterate(#) specifies the maximum number of iterations used to find the optimal step size in calculating numerical derivatives of the transformation(s) with respect to the original parameters. By default, the maximum number of iterations is 100, but convergence is usually achieved after only a few iterations. You should rarely have to use this option.

post causes nlcom to behave like a Stata estimation (eclass) command. When post is specified, nlcom will post the vector of transformed estimators and its estimated variance-covariance matrix to e(). This option, in essence, makes the transformation permanent. Thus you could, after posting, treat the transformed estimation results in the same way as you would treat results from other Stata estimation commands. For example, after posting, you could redisplay the results by typing nlcom without any arguments, or use test to perform simultaneous tests of hypotheses on linear combinations of the transformed estimators; see [R] test.

Specifying post clears out the previous estimation results, which can be recovered only by refitting the original model or by storing the estimation results before running nlcom and then restoring them; see [R] estimates store.

display_options: cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] estimation options.

The following options are available with nlcom but are not shown in the dialog box:

noheader suppresses the output header.

df(#) specifies that the t distribution with # degrees of freedom be used for computing p-values and confidence intervals.

Remarks and examples

stata.com

Remarks are presented under the following headings:

Introduction Basics Using the post option Reparameterizing ML estimators for univariate data nlcom versus eform

Introduction

nlcom and predictnl both use the delta method. They take nonlinear transformations of the estimated parameter vector from some fitted model and apply the delta method to calculate the variance, standard error, Wald test statistic, etc., of the transformations. nlcom is designed for functions of the parameters, and predictnl is designed for functions of the parameters and of the data, that is, for predictions.

nlcom generalizes lincom (see [R] lincom) in two ways. First, nlcom allows the transformations to be nonlinear. Second, nlcom can be used to simultaneously estimate many transformations (whether linear or nonlinear) and to obtain the estimated variance—covariance matrix of these transformations.

Basics

In [R] lincom, the following regression was performed:

- . use http://www.stata-press.com/data/r14/regress
- . regress y x1 x2 x3

Source	SS	df	MS	Number of F(3, 144)	obs =	148 96.12
Model Residual	3259.3561 1627.56282	3 144	1086.45203 11.3025196	Prob > F R-squared	=	0.0000 0.6670 0.6600
Total	4886.91892	147	33.2443464	- Adj R-squ Root MSE	=	3.3619
у	Coef.	Std. Err.	t	P> t [9:	5% Conf.	Interval]
x1 x2 x3 _cons	1.457113 2.221682 006139 36.10135	1.07461 .8610358 .0005543 4.382693	2.58 -11.08	0.011 .5 0.0000	666934 197797 072345 .43863	3.581161 3.923583 0050435 44.76407

Then lincom was used to estimate the difference between the coefficients of x1 and x2:

```
. lincom _b[x2] - _b[x1]
```

$$(1) - x1 + x2 = 0$$

у	Coef.	Std. Err.	t	P> t	[95% Conf.	Interval]
(1)	.7645682	.9950282	0.77	0.444	-1.20218	2.731316

It was noted, however, that nonlinear expressions are not allowed with lincom:

```
. lincom _b[x2]/_b[x1]
not possible with test
r(131);
```

Nonlinear transformations are instead estimated using nlcom:

у	Coef.	Std. Err.	z	P> z	[95% Conf	. Interval]
_nl_1	1.524714	.9812848	1.55	0.120	3985686	3.447997

□ Technical note

The notation _b[name] is the standard way in Stata to refer to regression coefficients; see [U] 13.5 Accessing coefficients and standard errors. Some commands, such as lincom and test, allow you to drop the _b[] and just refer to the coefficients by name. nlcom, however, requires the full specification _b[name].

Returning to our linear regression example, nlcom also allows simultaneous estimation of more than one combination:

```
. nlcom (_b[x2]/_b[x1]) (_b[x3]/_b[x1]) (_b[x3]/_b[x2])
```

_nl_1: _b[x2]/_b[x1] _nl_2: _b[x3]/_b[x1] _nl_3: _b[x3]/_b[x2]

у	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
_nl_1	1.524714	.9812848	-1.26	0.120	3985686	3.447997
_nl_2	0042131	.0033483		0.208	0107756	.0023494
_nl_3	0027632	.0010695		0.010	0048594	000667

We can also label the transformations to produce more informative names in the estimation table:

```
. nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1]) (ratio32:_b[x3]/_b[x2])
```

ratio21: _b[x2]/_b[x1] ratio31: _b[x3]/_b[x1] ratio32: _b[x3]/_b[x2]

у	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
ratio21	1.524714	.9812848	1.55	0.120	3985686	3.447997
ratio31	0042131	.0033483	-1.26	0.208	0107756	.0023494
ratio32	0027632	.0010695	-2.58	0.010	0048594	000667

nlcom stores the vector of estimated combinations and its estimated variance-covariance matrix in r().

```
. matrix list r(b)
r(b)[1,3]
       ratio21
                   ratio31
                               ratio32
     1.5247143 -.00421315 -.00276324
. matrix list r(V)
symmetric r(V)[3,3]
            ratio21
                        ratio31
                                    ratio32
ratio21
          .96291982
ratio31
        -.00287781
                      .00001121
ratio32 -.00014234
                      2.137e-06
                                  1.144e-06
```

Using the post option

When used with the post option, nlcom stores the estimation vector and variance-covariance matrix in e(), making the transformation permanent:

```
. quietly nlcom (ratio21:_b[x2]/_b[x1]) (ratio31:_b[x3]/_b[x1])
> (ratio32:_b[x3]/_b[x2]), post
. matrix list e(b)
e(b)[1,3]
                 ratio31
      ratio21
                             ratio32
     1.5247143 -.00421315 -.00276324
y1
. matrix list e(V)
symmetric e(V)[3,3]
           ratio21
                       ratio31
                                   ratio32
        .96291982
ratio21
ratio31 -.00287781
                     .00001121
ratio32 -.00014234
                     2.137e-06
                                 1.144e-06
```

After posting, we can proceed as if we had just run a Stata estimation (eclass) command. For instance, we can replay the results,

. nlcom

у	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
ratio21	1.524714	.9812848	1.55	0.120	3985686	3.447997
ratio31	0042131	.0033483	-1.26	0.208	0107756	.0023494
ratio32	0027632	.0010695	-2.58	0.010	0048594	000667

or perform other postestimation tasks in the transformed metric, this time making reference to the new "coefficients":

- . display _b[ratio31]
- -.00421315
- . estat vce. correlation

Correlation matrix of coefficients of nlcom model

e(V)	ratio21	ratio31	ratio32				
ratio21 ratio31 ratio32	1.0000 -0.8759 -0.1356	1.0000 0.5969	1.0000				
. test _b[rati	1021] = 1						
(1) ratio21	(1) ratio21 = 1						
chi Prob							

We see that testing _b[ratio21]=1 in the transformed metric is equivalent to testing using testnl $_b[x2]/_b[x1]=1$ in the original metric:

```
. quietly regress y x1 x2 x3
. testnl b[x2]/b[x1] = 1
 (1) _b[x2]/_b[x1] = 1
              chi2(1) =
                               0.29
          Prob > chi2 =
                               0.5928
```

We needed to refit the regression model to recover the original parameter estimates.

 \Box

Technical note

In a previous technical note, we mentioned that commands such as lincom and test permit reference to *name* instead of _b[*name*]. This is not the case when lincom and test are used after nlcom, post. In the above, we used

. test $_b[ratio21] = 1$

rather than

. test ratio21 = 1

which would have returned an error. Consider this a limitation of Stata. For the shorthand notation to work, you need a variable named *name* in the data. In nlcom, however, *name* is just a coefficient label that does not necessarily correspond to any variable in the data.

Reparameterizing ML estimators for univariate data

When run using only a response and no covariates, Stata's maximum likelihood (ML) estimation commands will produce ML estimates of the parameters of some assumed univariate distribution for the response. The parameterization, however, is usually not one we are used to dealing with in a nonregression setting. In such cases, nlcom can be used to transform the estimation results from a regression model to those from a maximum likelihood estimation of the parameters of a univariate probability distribution in a more familiar metric.

Example 1

Consider the following univariate data on Y=# of traffic accidents at a certain intersection in a given year:

- . use http://www.stata-press.com/data/r14/trafint
- . summarize accidents

Variable	Obs	Mean	Std. Dev.	Min	Max
accidents	12	13.83333	14.47778	0	41

A quick glance of the output from summarize leads us to quickly reject the assumption that Y is distributed as Poisson because the estimated variance of Y is much greater than the estimated mean of Y.

Instead, we choose to model the data as univariate negative binomial, of which a common parameterization is

$$\Pr(Y = y) = \frac{\Gamma(r + y)}{\Gamma(r)\Gamma(y + 1)} p^r (1 - p)^y \qquad 0 \le p \le 1, \quad r > 0, \quad y = 0, 1, \dots$$

with

$$E(Y) = \frac{r(1-p)}{p} \qquad \text{Var}(Y) = \frac{r(1-p)}{p^2}$$

There exist no closed-form solutions for the maximum likelihood estimates of p and r, yet they may be estimated by the iterative method of Newton-Raphson. One way to get these estimates would be to write our own Newton-Raphson program for the negative binomial. Another way would be to write our own ML evaluator; see [R] ml.

. nbreg accidents Fitting Poisson model:

/lnalpha

alpha

The easiest solution, however, would be to use Stata's existing negative binomial ML regression command, nbreg. The only problem with this solution is that nbreg estimates a different parameterization of the negative binomial, but we can worry about that later.

```
Iteration 0:
               log\ likelihood = -105.05361
Iteration 1:
               log\ likelihood = -105.05361
Fitting constant-only model:
Iteration 0:
               log likelihood = -43.948619
Iteration 1:
               log likelihood = -43.891483
Iteration 2:
               log\ likelihood = -43.89144
Iteration 3:
               log\ likelihood = -43.89144
Fitting full model:
Iteration 0:
               log\ likelihood = -43.89144
Iteration 1:
               log likelihood = -43.89144
Negative binomial regression
                                                  Number of obs
                                                                               12
                                                  LR chi2(0)
                                                                             0.00
                                                  Prob > chi2
Dispersion
               = mean
                                                                           0.0000
                                                  Pseudo R2
Log likelihood =
                  -43.89144
                    Coef.
                             Std. Err.
                                                  P>|z|
                                                            [95% Conf. Interval]
   accidents
       _cons
                  2.627081
                             .3192233
                                          8.23
                                                  0.000
                                                            2.001415
                                                                        3.252747
```

LR	test	of	alpha=0:	chibar2(01)	=	122.32
. :	nbreg	. с	eflegend			

.1402425

1.150553

Prob >= ch	ibar2 =	0.000
------------	---------	-------

.9609083

2.61407

-.6804233

.5064026

. Herog, coorregend			
Negative binomial regression	Number of obs	=	12
	LR chi2(0)	=	0.00
Dispersion = mean	Prob > chi2	=	
Log likelihood = -43.89144	Pseudo R2	=	0.0000

.4187147

.4817534

accidents	Coef.	Legend
_cons	2.627081	_b[accidents:_cons]
/lnalpha	.1402425	_b[lnalpha:_cons]
alpha	1.150553	

LR test of alpha=0: chibar2(01) = 122.32

Prob >= chibar2 = 0.000

From this output, we see that, when used with univariate data, nbreg estimates a regression intercept, β_0 , and the logarithm of some parameter α . This parameterization is useful in regression models: β_0 is the intercept meant to be augmented with other terms of the linear predictor, and α is an overdispersion parameter used for comparison with the Poisson regression model.

However, we need to transform $(\beta_0, \ln \alpha)$ to (p, r). Examining Methods and formulas of [R] **nbreg** reveals the transformation as

$$p = \{1 + \alpha \exp(\beta_0)\}^{-1}$$
 $r = \alpha^{-1}$

which we apply using nlcom:

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accidents	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
p	.0591157	.0292857	2.02	0.044	.0017168	.1165146
r	.8691474	.3639248		0.017	.1558679	1.582427

Given the invariance of maximum likelihood estimators and the properties of the delta method, the above parameter estimates, standard errors, etc., are precisely those we would have obtained had we instead performed the Newton-Raphson optimization in the (p, r) metric.

□ Technical note

Note how we referred to the estimate of $\ln \alpha$ above as [lnalpha]_b[_cons]. This is not entirely evident from the output of nbreg, which is why we redisplayed the results using the coeflegend option so that we would know how to refer to the coefficients; [U] 13.5 Accessing coefficients and standard errors.

nlcom versus eform

Many Stata estimation commands allow you to display exponentiated regression coefficients, some by default, some optionally. Known as "eform" in Stata terminology, this reparameterization serves many uses: it gives odds ratios for logistic models, hazard ratios in survival models, incidence-rate ratios in Poisson models, and relative-risk ratios in multinomial logit models, to name a few.

For example, consider the following estimation taken directly from the technical note in [R] **poisson**:

```
. use http://www.stata-press.com/data/r14/airline
```

- . generate lnN = ln(n)
- . poisson injuries ${\tt XYZowned\ lnN}$

Iteration 0: log likelihood = -22.333875
Iteration 1: log likelihood = -22.332276
Iteration 2: log likelihood = -22.332276

Poisson regression

Number of obs	_	J
LR chi2(2)	=	19.15
Prob > chi2	=	0.0001
Pseudo R2	=	0.3001

Number of obs

Log	likelihood	=	-22.332276
-----	------------	---	------------

injuries	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
XYZowned	.6840667	.3895877	1.76	0.079	0795111	1.447645
lnN	1.424169	.3725155	3.82	0.000	.6940517	2.154285
_cons	4.863891	.7090501	6.86	0.000	3.474178	6.253603

1.981921

4.154402

129.5272

When we replay results and specify the irr (incidence-rate ratios) option,

. poisson, irr

XYZowned

lnN

_cons

Poisson regression					Number of obs			9
•					LR chi2(2)			19.15
					Prob >	chi2	=	0.0001
Log likelihood = -22.332276				Pseudo	R2	=	0.3001	
injuries	IRR	Std.	Err.	z	P> z	[95%	Conf.	Interval]

we obtain the exponentiated regression coefficients and their estimated standard errors.

.7721322

1.547579

91.84126

Contrast this with what we obtain if we exponentiate the coefficients manually by using nlcom:

1.76

3.82

6.86

0.079

0.000

0.000

.9235678

2.00181

32.2713

4.253085

8.621728

519.8828

. nlcom (E_XYZowned:exp(_b[XYZowned])) (E_lnN:exp(_b[lnN]))

E_XYZowned: exp(_b[XYZowned])
 E_lnN: exp(_b[lnN])

injuries	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
E_XYZowned	1.981921	.7721322	2.57	0.010	.4685701	3.495273
E_lnN	4.154402	1.547579	2.68	0.007	1.121203	7.187602

There are three things to note when comparing poisson, irr (and eform in general) with nlcom:

- 1. The exponentiated coefficients and standard errors are identical. This is certainly good news.
- 2. The Wald test statistic (z) and level of significance are different. When using poisson, irr and other related eform options, the Wald test does not change from what you would have obtained without the eform option, and you can see this by comparing both versions of the poisson output given previously.

When you use eform, Stata knows that what is usually desired is a test of

$$H_0$$
: $\exp(\beta) = 1$

and not the uninformative-by-comparison

$$H_0$$
: $\exp(\beta) = 0$

The test of H_0 : $\exp(\beta) = 1$ is asymptotically equivalent to a test of H_0 : $\beta = 0$, the Wald test in the original metric, but the latter has better small-sample properties. Thus if you specify eform, you get a test of H_0 : $\beta = 0$.

nlcom, however, is general. It does not attempt to infer the test of greatest interest for a given transformation, and so a test of

$$H_0$$
: transformed coefficient = 0

is always given, regardless of the transformation.

3. You may be surprised to see that, even though the coefficients and standard errors are identical, the confidence intervals (both 95%) are different.

eform confidence intervals are standard confidence intervals with the endpoints transformed. For example, the confidence interval for the coefficient on lnN is [0.694, 2.154], whereas the confidence interval for the incidence-rate ratio due to lnN is [exp(0.694), exp(2.154)] = [2.002, 8.619], which, except for some roundoff error, is what we see from the output of poisson, irr. For exponentiated coefficients, confidence intervals based on transform-the-endpoints methodology generally have better small-sample properties than their asymptotically equivalent counterparts.

The transform-the-endpoints method, however, gives valid coverage only when the transformation is monotonic. nlcom uses a more general and asymptotically equivalent method for calculating confidence intervals, as described in *Methods and formulas*.

Stored results

nlcom stores the following in r():

```
Scalars
r(N)
number of observations
r(df_r)
residual degrees of freedom

Matrices
r(b)
vector of transformed coefficients
r(V)
estimated variance—covariance matrix of the transformed coefficients
```

If post is specified, nlcom also stores the following in e():

```
Scalars
    e(N)
                       number of observations
    e(df_r)
                       residual degrees of freedom
    e(N_strata)
                       number of strata L, if used after svy
    e(N_psu)
                       number of sampled PSUs n, if used after svy
    e(rank)
                       rank of e(V)
Macros
    e(cmd)
                       nlcom
    e(predict)
                       program used to implement predict
    e(properties)
Matrices
                       vector of transformed coefficients
    e(b)
                       estimated variance-covariance matrix of the transformed coefficients
    e(V)
    e(V_srs)
                       simple-random-sampling-without-replacement (co)variance \widehat{V}_{srswor}, if svy
    e(V_srswr)
                       simple-random-sampling-with-replacement (co)variance \widehat{V}_{\text{srswr}}, if svy and fpc()
                       misspecification (co)variance \widehat{V}_{\mathrm{msp}}, if svy and available
    e(V_msp)
Functions
    e(sample)
                       marks estimation sample
```

Methods and formulas

Given a $1 \times k$ vector of parameter estimates, $\widehat{\boldsymbol{\theta}} = (\widehat{\theta}_1, \dots, \widehat{\theta}_k)$, consider the estimated p-dimensional transformation

$$g(\widehat{\boldsymbol{\theta}}) = [g_1(\widehat{\boldsymbol{\theta}}), g_2(\widehat{\boldsymbol{\theta}}), \dots, g_p(\widehat{\boldsymbol{\theta}})]$$

The estimated variance-covariance of $g(\widehat{\boldsymbol{\theta}})$ is given by

$$\widehat{\operatorname{Var}}\left\{g(\widehat{\boldsymbol{ heta}})
ight\} = \mathbf{GVG}'$$

12 IIICOIII — NOIIII

where G is the $p \times k$ matrix of derivatives for which

$$\mathbf{G}_{ij} = \frac{\partial g_i(\boldsymbol{\theta})}{\partial \theta_j} \bigg|_{\boldsymbol{\theta} = \widehat{\boldsymbol{\theta}}} \qquad i = 1, \dots, p \qquad j = 1, \dots, k$$

and V is the estimated variance-covariance matrix of $\widehat{\theta}$. Standard errors are obtained as the square roots of the variances.

The Wald test statistic for testing

$$H_0: g_i(\boldsymbol{\theta}) = 0$$

versus the two-sided alternative is given by

$$Z_{i} = \frac{g_{i}(\widehat{\boldsymbol{\theta}})}{\left[\widehat{\operatorname{Var}}_{ii}\left\{g(\widehat{\boldsymbol{\theta}})\right\}\right]^{1/2}}$$

When the variance-covariance matrix of $\hat{\theta}$ is an asymptotic covariance matrix, Z_i is approximately distributed as Gaussian. For linear regression, Z_i is taken to be approximately distributed as $t_{1,r}$ where r is the residual degrees of freedom from the original fitted model.

A $(1-\alpha) \times 100\%$ confidence interval for $g_i(\theta)$ is given by

$$g_i(\widehat{\boldsymbol{\theta}}) \pm z_{\alpha/2} \Big[\widehat{\mathrm{Var}}_{ii} \left\{ g(\widehat{\boldsymbol{\theta}}) \right\} \Big]^{1/2}$$

for those cases where Z_i is Gaussian and

$$g_i(\widehat{\boldsymbol{\theta}}) \pm t_{\alpha/2,r} \Big[\widehat{\mathrm{Var}}_{ii} \left\{ g(\widehat{\boldsymbol{\theta}}) \right\} \Big]^{1/2}$$

for those cases where Z_i is t distributed. z_p is the 1-p quantile of the standard normal distribution, and $t_{p,r}$ is the 1-p quantile of the t distribution with r degrees of freedom.

References

Feiveson, A. H. 1999. FAQ: What is the delta method and how is it used to estimate the standard error of a transformed parameter? http://www.stata.com/support/faqs/stat/deltam.html.

Gould, W. W. 1996. crc43: Wald test of nonlinear hypotheses after model estimation. Stata Technical Bulletin 29: 2–4. Reprinted in Stata Technical Bulletin Reprints, vol. 5, pp. 15–18. College Station, TX: Stata Press.

Oehlert, G. W. 1992. A note on the delta method. American Statistician 46: 27-29.

Phillips, P. C. B., and J. Y. Park. 1988. On the formulation of Wald tests of nonlinear restrictions. *Econometrica* 56: 1065–1083.

Also see

- [R] **lincom** Linear combinations of parameters
- [R] **predictnl** Obtain nonlinear predictions, standard errors, etc., after estimation
- [R] test Test linear hypotheses after estimation
- [R] testnl Test nonlinear hypotheses after estimation

[SVY] svy postestimation — Postestimation tools for svy

[U] 20 Estimation and postestimation commands