

hotelling — Hotelling's T-squared generalized means test

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Description

`hotelling` performs Hotelling's T -squared test of whether a set of means is zero or, alternatively, equal between two groups.

See [\[MV\] mvtest means](#) for generalizations of Hotelling's one-sample test with more general hypotheses, two-sample tests that do not assume that the covariance matrices are the same in the two groups, and tests with more than two groups.

Quick start

Hotelling's T -squared test that the means of `v1`, `v2`, and `v3` are jointly zero

```
hotelling v1 v2 v3
```

As above, but suppress the table of means

```
hotelling v1 v2 v3, notable
```

Hotelling's T -squared test that the means of `v1`, `v2`, and `v3` are the same in the two groups defined by `catvar`

```
hotelling v1 v2 v3, by(catvar)
```

Menu

Statistics > Multivariate analysis > MANOVA, multivariate regression, and related > Hotelling's generalized means test

Syntax

```
hotelling varlist [if] [in] [weight] [, by(varname) notable]
```

`aweight`s and `fweight`s are allowed; see [U] 11.1.6 [weight](#).

Note: `hotel` is a synonym for `hotelling`.

Options

Main

`by(varname)` specifies a variable identifying two groups; the test of equality of means between groups is performed. If `by()` is not specified, a test of means being jointly zero is performed.

`notable` suppresses printing a table of the means being compared.

Remarks and examples

[stata.com](http://www.stata.com)

`hotelling` performs Hotelling's T -squared test of whether a set of means is zero or two sets of means are equal. It is a multivariate test that reduces to a standard t test if only one variable is specified.

► Example 1

You wish to test whether a new fuel additive improves gas mileage in both stop-and-go and highway situations. Taking 12 cars, you fill them with gas and run them on a highway-style track, recording their gas mileage. You then refill them and run them on a stop-and-go style track. Finally, you repeat the two runs, but this time you use fuel with the additive. Your dataset is

```
. use http://www.stata-press.com/data/r14/gasexp
. describe
```

Contains data from <http://www.stata-press.com/data/r14/gasexp.dta>

```
obs:           12
vars:           5                21 Nov 2014 12:56
size:           96
```

variable name	storage type	display format	value label	variable label
<code>id</code>	byte	%9.0g		car ID
<code>bmpg1</code>	byte	%9.0g		track 1 before additive
<code>ampg1</code>	byte	%9.0g		track 1 after additive
<code>bmpg2</code>	byte	%9.0g		track 2 before additive
<code>ampg2</code>	float	%9.0g		track 2 after additive

Sorted by:

To perform the statistical test, you jointly test whether the differences in before-and-after results are zero:

```
. generate diff1 = ampg1 - bmpg1
. generate diff2 = ampg2 - bmpg2
. hotelling diff1 diff2
```

Variable	Obs	Mean	Std. Dev.	Min	Max
diff1	12	1.75	2.70101	-3	5
diff2	12	2.083333	2.906367	-3.5	5.5

```
1-group Hotelling's T-squared = 9.6980676
F test statistic: ((12-2)/(12-1)(2)) x 9.6980676 = 4.4082126
H0: Vector of means is equal to a vector of zeros
      F(2,10) = 4.4082
      Prob > F(2,10) = 0.0424
```

The means are different at the 4.24% significance level.

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□ Technical note

We used Hotelling's *T*-squared test because we were testing two differences jointly. Had there been only one difference, we could have used a standard *t* test, which would have yielded the same results as Hotelling's test:

```
. ttest ampg1 = bmpg1
```

Paired t test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
ampg1	12	22.75	.9384465	3.250874	20.68449 24.81551
bmpg1	12	21	.7881701	2.730301	19.26525 22.73475
diff	12	1.75	.7797144	2.70101	.0338602 3.46614

```
      mean(diff) = mean(ampg1 - bmpg1)          t = 2.2444
Ho: mean(diff) = 0                          degrees of freedom = 11
Ha: mean(diff) < 0                          Ha: mean(diff) != 0      Ha: mean(diff) > 0
Pr(T < t) = 0.9768                          Pr(|T| > |t|) = 0.0463      Pr(T > t) = 0.0232
```

```
. ttest diff1 = 0
```

One-sample t test

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
diff1	12	1.75	.7797144	2.70101	.0338602 3.46614

```
      mean = mean(diff1)                      t = 2.2444
Ho: mean = 0                                degrees of freedom = 11
Ha: mean < 0                                Ha: mean != 0          Ha: mean > 0
Pr(T < t) = 0.9768                          Pr(|T| > |t|) = 0.0463      Pr(T > t) = 0.0232
```

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```
. hotelling diff1
```

Variable	Obs	Mean	Std. Dev.	Min	Max
diff1	12	1.75	2.70101	-3	5

```
1-group Hotelling's T-squared = 5.0373832  
F test statistic: ((12-1)/(12-1)(1)) x 5.0373832 = 5.0373832  
H0: Vector of means is equal to a vector of zeros  
      F(1,11) = 5.0374  
      Prob > F(1,11) = 0.0463
```



► Example 2

Now consider a variation on the experiment: rather than using 12 cars and running each car with and without the fuel additive, you run 24 cars, 12 with the additive and 12 without. You have the following dataset:

```
. use http://www.stata-press.com/data/r14/gasexp2, clear  
. describe  
Contains data from http://www.stata-press.com/data/r14/gasexp2.dta  
obs:      24  
vars:      4                21 Nov 2014 13:01  
size:      168
```

variable name	storage type	display format	value label	variable label
id	byte	%9.0g		car ID
mpg1	byte	%9.0g		track 1
mpg2	float	%9.0g		track 2
additive	byte	%9.0g	yesno	additive?

Sorted by:

```
. tabulate additive
```

additive?	Freq.	Percent	Cum.
no	12	50.00	50.00
yes	12	50.00	100.00
Total	24	100.00	

This is an unpaired experiment because there is no natural pairing of the cars; you want to test that the means of mpg1 are equal for the two groups specified by additive, as are the means of mpg2:

```
. hotelling mpg1 mpg2, by(additive)
```

```
-> additive = no
```

Variable	Obs	Mean	Std. Dev.	Min	Max
mpg1	12	21	2.730301	17	25
mpg2	12	19.91667	2.644319	16	24

```
-> additive = yes
```

Variable	Obs	Mean	Std. Dev.	Min	Max
mpg1	12	22.75	3.250874	17	28
mpg2	12	22	3.316625	16.5	27.5

2-group Hotelling's T-squared = 7.1347584

F test statistic: $((24-2-1)/(24-2)(2)) \times 7.1347584 = 3.4052256$

H0: Vectors of means are equal for the two groups

F(2,21) = 3.4052

Prob > F(2,21) = 0.0524

◀

□ Technical note

As in the paired experiment, had there been only one test track, the *t* test would have yielded the same results as Hotelling's test:

```
. hotelling mpg1, by(additive)
```

```
-> additive = no
```

Variable	Obs	Mean	Std. Dev.	Min	Max
mpg1	12	21	2.730301	17	25

```
-> additive = yes
```

Variable	Obs	Mean	Std. Dev.	Min	Max
mpg1	12	22.75	3.250874	17	28

2-group Hotelling's T-squared = 2.0390921

F test statistic: $((24-1-1)/(24-2)(1)) \times 2.0390921 = 2.0390921$

H0: Vectors of means are equal for the two groups

F(1,22) = 2.0391

Prob > F(1,22) = 0.1673

```
. ttest mpg1, by(additive)
```

Two-sample t test with equal variances

Group	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]	
no	12	21	.7881701	2.730301	19.26525	22.73475
yes	12	22.75	.9384465	3.250874	20.68449	24.81551
combined	24	21.875	.6264476	3.068954	20.57909	23.17091
diff		-1.75	1.225518		-4.291568	.7915684

diff = mean(no) - mean(yes)

t = -1.4280

Ho: diff = 0

degrees of freedom = 22

Ha: diff < 0

Ha: diff != 0

Ha: diff > 0

Pr(T < t) = 0.0837

Pr(|T| > |t|) = 0.1673

Pr(T > t) = 0.9163

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With more than one pair of means, however, there is no t test equivalent to Hotelling's test, although there are other logically (but not practically) equivalent solutions. One is the discriminant function: if the means of mpg1 and mpg2 are different, the discriminant function should separate the groups along that dimension.

```
. regress additive mpg1 mpg2
```

Source	SS	df	MS	Number of obs	=	24
Model	1.46932917	2	.734664585	F(2, 21)	=	3.41
Residual	4.53067083	21	.21574623	Prob > F	=	0.0524
				R-squared	=	0.2449
				Adj R-squared	=	0.1730
Total	6	23	.260869565	Root MSE	=	.46448

additive	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
mpg1	-.4570407	.2416657	-1.89	0.072	-.959612 .0455306
mpg2	.5014605	.2376762	2.11	0.047	.0071859 .9957352
_cons	-.0120115	.7437049	-0.02	0.987	-1.55863 1.534607

This test would declare the means different at the 5.24% level. You could also have fit this model by using logistic regression:

```
. logit additive mpg1 mpg2
```

```
Iteration 0: log likelihood = -16.635532
Iteration 1: log likelihood = -13.395178
Iteration 2: log likelihood = -13.371201
Iteration 3: log likelihood = -13.371143
Iteration 4: log likelihood = -13.371143
```

```
Logistic regression
```

Number of obs	=	24
LR chi2(2)	=	6.53
Prob > chi2	=	0.0382
Pseudo R2	=	0.1962

```
Log likelihood = -13.371143
```

additive	Coef.	Std. Err.	z	P> z	[95% Conf. Interval]
mpg1	-2.306844	1.36139	-1.69	0.090	-4.975119 .3614307
mpg2	2.524477	1.367373	1.85	0.065	-.1555257 5.20448
_cons	-2.446527	3.689821	-0.66	0.507	-9.678443 4.78539

This test would have declared the means different at the 3.82% level.

Are the means different? Hotelling's T -squared and the discriminant function reject equality at the 5.24% level. The logistic regression rejects equality at the 3.82% level.

□

Stored results

hotelling stores the following in `r()`:

Scalars

<code>r(N)</code>	number of observations	<code>r(T2)</code>	Hotelling's T -squared
<code>r(k)</code>	number of variables	<code>r(df)</code>	degrees of freedom

Methods and formulas

See [Wilks \(1962, 556–561\)](#) for a general discussion. The original formulation was by [Hotelling \(1931\)](#) and [Mahalanobis \(1930, 1936\)](#).

For the test that the means of k variables are 0, let $\bar{\mathbf{x}}$ be a $1 \times k$ matrix of the means and \mathbf{S} be the estimated covariance matrix. Then $T^2 = \bar{\mathbf{x}}\mathbf{S}^{-1}\bar{\mathbf{x}}'$.

For two groups, the test of equality is $T^2 = (\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)\mathbf{S}^{-1}(\bar{\mathbf{x}}_1 - \bar{\mathbf{x}}_2)'$.

Harold Hotelling (1895–1973) was an American economist and statistician who made many important contributions to mathematical economics, multivariate analysis, and statistical inference. After obtaining degrees in journalism and mathematics, he taught and researched at Stanford, Columbia, and the University of North Carolina. His work generalizing Student's t ratio and on principal components, canonical correlation, multivariate analysis of variance, and correlation continues to be widely used.

Prasanta Chandra Mahalanobis (1893–1972) studied physics and mathematics at Calcutta and Cambridge. He became interested in statistics and on his return to India worked on applications in anthropology, meteorology, hydrology, and agriculture. Mahalanobis became the leader in Indian statistics, specializing in multivariate problems (including what is now called the Mahalanobis distance), the design of large-scale sample surveys, and the contribution of statistics to national planning.

References

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Also see

- [MV] [manova](#) — Multivariate analysis of variance and covariance
- [MV] [mvtest means](#) — Multivariate tests of means
- [R] [regress](#) — Linear regression
- [R] [ttest](#) — t tests (mean-comparison tests)