Title

> **meqrpoisson postestimation** — Postestimation tools for meqrpoisson

Postestimation commands

The following postestimation commands are of special interest after meqrpoisson:

Command	Description
estat group	summarize the composition of the nested groups
estat recovariance	display the estimated random-effects covariance matrix (or matrices)

The following standard postestimation commands are also available:

Command	Description
contrast	contrasts and ANOVA-style joint tests of estimates
estat ic	Akaike's and Schwarz's Bayesian information criteria (AIC and BIC)
estat summarize	summary statistics for the estimation sample
estat vce	variance–covariance matrix of the estimators (VCE)
estimates	cataloging estimation results
hausman	Hausman's specification test
lincom	point estimates, standard errors, testing, and inference for linear combinations of coefficients
lrtest	likelihood-ratio test
margins	marginal means, predictive margins, marginal effects, and average marginal effects
marginsplot	graph the results from margins (profile plots, interaction plots, etc.)
nlcom	point estimates, standard errors, testing, and inference for nonlinear combinations of coefficients
predict	predictions, residuals, influence statistics, and other diagnostic measures
predictnl	point estimates, standard errors, testing, and inference for generalized predictions
pwcompare	pairwise comparisons of estimates
test	Wald tests of simple and composite linear hypotheses
testnl	Wald tests of nonlinear hypotheses

predict

Description for predict

predict creates a new variable containing predictions such as mean responses; linear predictions; standard errors; and Pearson, deviance, and Anscombe residuals.

Menu for predict

Statistics > Postestimation

Syntax for predict

Syntax for obtaining estimated random effects and their standard errors

predict [*type*] *newvarsspec* [*if*] [*in*], <u>ref</u>fects [reses(*newvarsspec*)

<u>releve</u>l(*levelvar*)]

Syntax for obtaining other predictions

predict [*type*] *newvar* [*if*] [*in*] [, *statistic* <u>nooff</u>set <u>fixed</u>only]

newvarsspec is *stub** or *newvarlist*.

statistic	Description
Main	
mu	predicted mean; the default
xb	linear predictor for the fixed portion of the model only
stdp	standard error of the fixed-portion linear prediction
<u>pearson</u>	Pearson residuals
<u>deviance</u>	deviance residuals
<u>ans</u>combe	Anscombe residuals

These statistics are available both in and out of sample; type predict ... if e(sample) ... if wanted only for the estimation sample.

Options for predict

Main

reffects calculates posterior modal estimates of the random effects. By default, estimates for all random effects in the model are calculated. However, if the relevel(*levelvar*) option is specified, then estimates for only level *levelvar* in the model are calculated. For example, if classes are nested within schools, then typing

. predict b*, reffects relevel(school)

would yield random-effects estimates at the school level. You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub** and let Stata name the variables *stub*1, *stub*2, ..., *stub*q for you.

reses(*newvarsspec*) calculates standard errors for the random-effects estimates. By default, standard errors for all random effects in the model are calculated. However, if the relevel(*levelvar*) option is specified, then standard errors for only level *levelvar* in the model are calculated; see the reffects option.

You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub** and let Stata name the variables *stub*1, *stub*2, ..., *stubq* for you. The new variables will have the same storage type as the corresponding random-effects variables.

The reffects and reses() options often generate multiple new variables at once. When this occurs, the random effects (or standard errors) contained in the generated variables correspond to the order in which the variance components are listed in the output of meqrpoisson. Still, examining the variable labels of the generated variables (with the describe command, for instance) can be useful in deciphering which variables correspond to which terms in the model.

relevel(*levelvar*) specifies the level in the model at which predictions for random effects and their standard errors are to be obtained. *levelvar* is the name of the model level and is either the name of the variable describing the grouping at that level or is _all, a special designation for a group comprising all the estimation data.

mu, the default, calculates the predicted mean, that is, the predicted count. By default, this is based on a linear predictor that includes both the fixed effects and the random effects, and the predicted mean is conditional on the values of the random effects. Use the fixedonly option (see below) if you want predictions that include only the fixed portion of the model, that is, if you want random effects set to 0.

xb calculates the linear prediction $\mathbf{x}\boldsymbol{\beta}$ based on the estimated fixed effects (coefficients) in the model. This is equivalent to fixing all random effects in the model to their theoretical (prior) mean value of 0.

stdp calculates the standard error of the fixed-effects linear predictor $\mathbf{x}\boldsymbol{\beta}$.

pearson calculates Pearson residuals. Pearson residuals large in absolute value may indicate a lack of fit. By default, residuals include both the fixed portion and the random portion of the model. The fixedonly option modifies the calculation to include the fixed portion only.

deviance calculates deviance residuals. Deviance residuals are recommended by McCullagh and Nelder (1989) as having the best properties for examining the goodness of fit of a GLM. They are approximately normally distributed if the model is correctly specified. They may be plotted against the fitted values or against a covariate to inspect the model's fit. By default, residuals include both the fixed portion and the random portion of the model. The fixedonly option modifies the calculation to include the fixed portion only.

anscombe calculates Anscombe residuals, which are designed to closely follow a normal distribution. By default, residuals include both the fixed portion and the random portion of the model. The fixedonly option modifies the calculation to include the fixed portion only.

nooffset is relevant only if you specified offset(*varname$_o$*) or exposure(*varname$_e$*) for meqrpoisson. It modifies the calculations made by predict so that they ignore the offset/exposure variable; the linear prediction is treated as $\mathbf{X}\beta + \mathbf{Z}\mathbf{u}$ rather than $\mathbf{X}\beta + \mathbf{Z}\mathbf{u} +$ offset, or $\mathbf{X}\beta + \mathbf{Z}\mathbf{u} + \ln(\text{exposure})$, whichever is relevant.

fixedonly modifies predictions to include only the fixed portion of the model, equivalent to setting all random effects equal to 0; see the mu option.

margins

Description for margins

margins estimates margins of response for linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

margins [*marginlist*] [, *options*]

margins [*marginlist*] , <u>pred</u>ict(*statistic* ...) [*options*]

statistic	Description
xb	linear predictor for the fixed portion of the model only; the default
<u>reff</u>ects	not allowed with margins
mu	not allowed with margins
stdp	not allowed with margins
pearson	not allowed with margins
<u>dev</u>iance	not allowed with margins
<u>ans</u>combe	not allowed with margins

Statistics not allowed with margins are functions of stochastic quantities other than e(b).

For the full syntax, see [R] **margins**.

estat

Description for estat

estat group reports the number of groups and minimum, average, and maximum group sizes for each level of the model. Model levels are identified by the corresponding group variable in the data. Because groups are treated as nested, the information in this summary may differ from what you would get if you used the tabulate command on each group variable individually.

estat recovariance displays the estimated variance–covariance matrix of the random effects for each level in the model. Random effects can be either random intercepts, in which case the corresponding rows and columns of the matrix are labeled as _cons, or random coefficients, in which case the label is the name of the associated variable in the data.

Menu for estat

Statistics > Postestimation

Syntax for estat

Summarize the composition of the nested groups

    estat <u>group</u>

Display the estimated random-effects covariance matrix (or matrices)

    estat <u>recov</u>ariance [, <u>relevel</u>(*levelvar*) <u>correl</u>ation *matlist_options*]

Options for estat recovariance

relevel(*levelvar*) specifies the level in the model for which the random-effects covariance matrix
    is to be displayed and returned in r(cov). By default, the covariance matrices for all levels in the
    model are displayed. *levelvar* is the name of the model level and is either the name of the variable
    describing the grouping at that level or is _all, a special designation for a group comprising all
    the estimation data.

correlation displays the covariance matrix as a correlation matrix and returns the correlation matrix
    in r(corr).

matlist_options are style and formatting options that control how the matrix (or matrices) is displayed;
    see [P] **matlist** for a list of options that are available.

Remarks and examples

<div align="right">stata.com</div>

    Various predictions, statistics, and diagnostic measures are available after fitting a Poisson mixed-
effects model with meqrpoisson. For the most part, calculation centers around obtaining estimates
of the subject/group-specific random effects. Random effects are not estimated when the model is fit
but instead need to be predicted after estimation.

▷ Example 1

    In example 2 of [ME] **meqrpoisson**, we modeled the number of observed epileptic seizures as a
function of treatment with the drug progabide and other covariates,

$$\log(\mu_{ij}) = \beta_0 + \beta_1\texttt{treat}_{ij} + \beta_2\texttt{lbas}_{ij} + \beta_3\texttt{lbas\_trt}_{ij} +$$
$$\beta_4\texttt{lage}_{ij} + \beta_5\texttt{visit}_{ij} + u_j + v_j\texttt{visit}_{ij}$$

where (u_j, v_j) are bivariate normal with 0 mean and variance–covariance matrix

$$\boldsymbol{\Sigma} = \text{Var}\begin{bmatrix} u_j \\ v_j \end{bmatrix} = \begin{bmatrix} \sigma_u^2 & \sigma_{uv} \\ \sigma_{uv} & \sigma_v^2 \end{bmatrix}$$

```
. use http://www.stata-press.com/data/r14/epilepsy
(Epilepsy data; progabide drug treatment)
. meqrpoisson seizures treat lbas lbas_trt lage visit || subject: visit,
> cov(unstructured) intpoints(9)

Refining starting values:

Iteration 0:   log likelihood = -672.17188  (not concave)
Iteration 1:   log likelihood = -660.46056
Iteration 2:   log likelihood =  -655.8713

Performing gradient-based optimization:

Iteration 0:   log likelihood =  -655.8713
Iteration 1:   log likelihood = -655.68217
Iteration 2:   log likelihood = -655.68103
Iteration 3:   log likelihood = -655.68103
```

```
Mixed-effects Poisson regression              Number of obs      =        236
Group variable: subject                       Number of groups   =         59

                                              Obs per group:
                                                            min =          4
                                                            avg =        4.0
                                                            max =          4

Integration points =    9                     Wald chi2(5)       =     115.56
Log likelihood = -655.68103                   Prob > chi2        =     0.0000
```

seizures	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
treat	-.9286589	.4021642	-2.31	0.021	-1.716886	-.1404315
lbas	.8849766	.131252	6.74	0.000	.6277275	1.142226
lbas_trt	.3379758	.2044444	1.65	0.098	-.0627279	.7386794
lage	.4767192	.3536221	1.35	0.178	-.2163673	1.169806
visit	-.2664098	.1647096	-1.62	0.106	-.5892347	.0564151
_cons	2.099555	.2203711	9.53	0.000	1.667635	2.531474

Random-effects Parameters	Estimate	Std. Err.	[95% Conf. Interval]	
subject: Unstructured				
var(visit)	.5314808	.2293851	.2280931	1.238406
var(_cons)	.2514928	.0587892	.1590552	.3976522
cov(visit,_cons)	.0028715	.0887018	-.1709808	.1767238

```
LR test vs. Poisson model: chi2(3) = 324.54          Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.
```

The purpose of this model was to allow subject-specific linear log trends over each subject's four doctor visits, after adjusting for the other covariates. The intercepts of these lines are distributed $N(\beta_0, \sigma_u^2)$, and the slopes are distributed $N(\beta_5, \sigma_v^2)$, based on the fixed effects and assumed distribution of the random effects.

We can use predict to obtain estimates of the random effects u_j and v_j and combine these with our estimates of β_0 and β_5 to obtain the intercepts and slopes of the linear log trends.

```
. predict re_visit re_cons, reffects
. generate b1 = _b[visit] + re_visit
. generate b0 = _b[_cons] + re_cons
. by subject, sort: generate tolist = _n==1
```

```
. list subject treat b1 b0 if tolist & (subject <=5 | subject >=55)
```

	subject	treat	b1	b0
1.	1	0	-.4284563	2.164691
5.	2	0	-.2727145	2.179111
9.	3	0	.0026486	2.450811
13.	4	0	-.3194157	2.268827
17.	5	0	.6063656	2.123723
217.	55	1	-.2304782	2.311494
221.	56	1	.2904741	3.211369
225.	57	1	-.4831492	1.457486
229.	58	1	-.252236	1.168154
233.	59	1	-.1266651	2.204869

We list these slopes (b1) and intercepts (b0) for five control subjects and five subjects on the treatment.

```
. count if tolist & treat
  31
. count if tolist & treat & b1 < 0
  25
. count if tolist & !treat
  28
. count if tolist & !treat & b1 < 0
  20
```

We also find that 25 of the 31 subjects taking progabide were estimated to have a downward trend in seizures over their four doctor visits, compared with 20 of the 28 control subjects.

We also obtain predictions for number of seizures, and unless we specify the fixedonly option, these predictions will incorporate the estimated subject-specific random effects.

```
. predict n
(option mu assumed; predicted means)
. list subject treat visit seizures n if subject <= 2 | subject >= 58, sep(0)
```

	subject	treat	visit	seizures	n
1.	1	0	-.3	5	3.887582
2.	1	0	-.1	3	3.568324
3.	1	0	.1	3	3.275285
4.	1	0	.3	3	3.00631
5.	2	0	-.3	3	3.705628
6.	2	0	-.1	5	3.508926
7.	2	0	.1	3	3.322664
8.	2	0	.3	3	3.14629
229.	58	1	-.3	0	.9972093
230.	58	1	-.1	0	.9481507
231.	58	1	.1	0	.9015056
232.	58	1	.3	0	.8571552
233.	59	1	-.3	1	2.487858
234.	59	1	-.1	4	2.425625
235.	59	1	.1	3	2.364948
236.	59	1	.3	2	2.305789

◁

❑ Technical note

Out-of-sample predictions are permitted after `meqrpoisson`, but if these predictions involve estimated random effects, the integrity of the estimation data must be preserved. If the estimation data have changed since the model was fit, `predict` will be unable to obtain predicted random effects that are appropriate for the fitted model and will give an error. Thus to obtain out-of-sample predictions that contain random-effects terms, be sure that the data for these predictions are in observations that augment the estimation data.

❑

Stored results

`estat recovariance` stores the following in `r()`:

Scalars
  `r(relevels)`   number of levels

Matrices
  `r(Cov#)`   level-# random-effects covariance matrix
  `r(Corr#)`   level-# random-effects correlation matrix (if option `correlation` was specified)

For a G-level nested model, # can be any integer between 2 and G.

Methods and formulas

Continuing the discussion in *Methods and formulas* of [ME] **meqrpoisson** and using the definitions and formulas defined there, we begin by considering the prediction of the random effects $\mathbf{u}_j$ for the jth cluster in a two-level model.

Given a set of estimated `meqrpoisson` parameters, $(\widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{\Sigma}})$, a profile likelihood in $\mathbf{u}_j$ is derived from the joint distribution $f(\mathbf{y}_j, \mathbf{u}_j)$ as

$$\mathcal{L}_j(\mathbf{u}_j) = \exp\left\{-c\left(\mathbf{y}_j\right)\right\} (2\pi)^{-q/2} |\widehat{\boldsymbol{\Sigma}}|^{-1/2} \exp\left\{g\left(\widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{\Sigma}}, \mathbf{u}_j\right)\right\} \tag{1}$$

The conditional maximum likelihood estimator of $\mathbf{u}_j$—conditional on fixed $(\widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{\Sigma}})$—is the maximizer of $\mathcal{L}_j(\mathbf{u}_j)$ or, equivalently, the value of $\widehat{\mathbf{u}}_j$ that solves

$$\mathbf{0} = g'\left(\widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{\Sigma}}, \widehat{\mathbf{u}}_j\right) = \mathbf{Z}_j' \left\{\mathbf{y}_j - \mathbf{m}(\widehat{\boldsymbol{\beta}}, \widehat{\mathbf{u}}_j)\right\} - \widehat{\boldsymbol{\Sigma}}^{-1} \widehat{\mathbf{u}}_j$$

Because (1) is proportional to the conditional density $f(\mathbf{u}_j|\mathbf{y}_j)$, you can also refer to $\widehat{\mathbf{u}}_j$ as the conditional mode (or posterior mode if you lean toward Bayesian terminology). Regardless, you are referring to the same estimator.

Conditional standard errors for the estimated random effects are derived from standard theory of maximum likelihood, which dictates that the asymptotic variance matrix of $\widehat{\mathbf{u}}_j$ is the negative inverse of the Hessian, which is estimated as

$$g''\left(\widehat{\boldsymbol{\beta}}, \widehat{\boldsymbol{\Sigma}}, \widehat{\mathbf{u}}_j\right) = -\left\{\mathbf{Z}_j' \mathbf{V}(\widehat{\boldsymbol{\beta}}, \widehat{\mathbf{u}}_j)\mathbf{Z}_j + \widehat{\boldsymbol{\Sigma}}^{-1}\right\}$$

Similar calculations extend to models with more than one level of random effects; see Pinheiro and Chao (2006).

For any observation i in the jth cluster in a two-level model, define the linear predictor as

$$\widehat{\eta}_{ij} = \mathbf{x}_{ij}\widehat{\boldsymbol{\beta}} + \mathbf{z}_{ij}\widehat{\mathbf{u}}_j$$

In a three-level model, for the ith observation within the jth level-two cluster within the kth level-three cluster,

$$\widehat{\eta}_{ijk} = \mathbf{x}_{ijk}\widehat{\boldsymbol{\beta}} + \mathbf{z}_{ijk}^{(3)}\widehat{\mathbf{u}}_k^{(3)} + \mathbf{z}_{ijk}^{(2)}\widehat{\mathbf{u}}_{jk}^{(2)}$$

where $\mathbf{z}^{(p)}$ and $\mathbf{u}^{(p)}$ refer to the level p design variables and random effects, respectively. For models with more than three levels, the definition of $\widehat{\eta}$ extends in the natural way, with only the notation becoming more complicated.

If the `fixedonly` option is specified, $\widehat{\eta}$ contains the linear predictor for only the fixed portion of the model, for example, in a two-level model $\widehat{\eta}_{ij} = \mathbf{x}_{ij}\widehat{\boldsymbol{\beta}}$. In what follows, we assume a two-level model, with the only necessary modification for multilevel models being the indexing.

The predicted mean conditional on the random effects $\widehat{\mathbf{u}}_j$ is

$$\widehat{\mu}_{ij} = \exp(\widehat{\eta}_{ij})$$

Pearson residuals are calculated as

$$\nu_{ij}^P = \frac{y_{ij} - \widehat{\mu}_{ij}}{\{V(\widehat{\mu}_{ij})\}^{1/2}}$$

for $V(\widehat{\mu}_{ij}) = \widehat{\mu}_{ij}$.

Deviance residuals are calculated as

$$\nu_{ij}^D = \text{sign}(y_{ij} - \widehat{\mu}_{ij})\sqrt{\widehat{d}_{ij}^2}$$

where

$$\widehat{d}_{ij}^2 = \begin{cases} 2\widehat{\mu}_{ij} & \text{if } y_{ij} = 0 \\ 2\left\{ y_{ij} \log\left(\frac{y_{ij}}{\widehat{\mu}_{ij}}\right) - (y_{ij} - \widehat{\mu}_{ij})\right\} & \text{otherwise} \end{cases}$$

Anscombe residuals are calculated as

$$\nu_{ij}^A = \frac{3\left(y_{ij}^{2/3} - \widehat{\mu}_{ij}^{2/3}\right)}{2\widehat{\mu}_{ij}^{1/6}}$$

For a discussion of the general properties of the above residuals, see Hardin and Hilbe (2012, chap. 4).

References

Hardin, J. W., and J. M. Hilbe. 2012. *Generalized Linear Models and Extensions*. 3rd ed. College Station, TX: Stata Press.

McCullagh, P., and J. A. Nelder. 1989. *Generalized Linear Models*. 2nd ed. London: Chapman & Hall/CRC.

Pinheiro, J. C., and E. C. Chao. 2006. Efficient Laplacian and adaptive Gaussian quadrature algorithms for multilevel generalized linear mixed models. *Journal of Computational and Graphical Statistics* 15: 58–81.

Rabe-Hesketh, S., and A. Skrondal. 2012. *Multilevel and Longitudinal Modeling Using Stata*. 3rd ed. College Station, TX: Stata Press.

Also see

[ME] **meqrpoisson** — Multilevel mixed-effects Poisson regression (QR decomposition)

[U] **20 Estimation and postestimation commands**