

gschurd() — Generalized Schur decomposition

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Description

`gschurd(A, B, T, R, U, V, w, b)` computes the generalized Schur decomposition of two square, numeric matrices, A and B , and the [generalized eigenvalues](#). The decomposition is returned in the [Schur-form](#) matrix, T ; the upper-triangular matrix, R ; and the orthogonal (unitary) matrices, U and V . The generalized eigenvalues are returned in the complex vectors w and b .

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m)` computes the generalized Schur decomposition of two square, numeric matrices, A and B , and the [generalized eigenvalues](#), and groups the results according to whether a condition on each generalized eigenvalue is satisfied. f is a pointer to the function that implements the condition on each generalized eigenvalue, as discussed [below](#). The number of generalized eigenvalues for which the condition is true is returned in m .

`_gschurd()` mirrors `gschurd()`, the difference being that it returns T in A and R in B .

`_gschurdgroupby()` mirrors `gschurdgroupby()`, the difference being that it returns T in A and R in B .

`_gschurd_la()` and `_gschurdgroupby_la()` are the interfaces into the LAPACK routines used to implement the above functions; see [\[M-1\] LAPACK](#). Their direct use is not recommended.

Syntax

```
void          gschurd(A, B, T, R, U, V, w, b)
```

```
void          _gschurd(A, B, U, V, w, b)
```

```
void gschurdgroupby(A, B, f, T, R, U, V, w, b, m)
```

```
void _gschurdgroupby(A, B, f, U, V, w, b, m)
```

Remarks and examples

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Remarks are presented under the following headings:

Generalized Schur decomposition
Grouping the results

Generalized Schur decomposition

The generalized Schur decomposition of a pair of square, numeric matrices, A and B , can be written as

$$U' \times A \times V = T$$

$$U' \times B \times V = R$$

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where **T** is in Schur form, **R** is upper triangular, and **U** and **V** are orthogonal if **A** and **B** are real and are unitary if **A** or **B** is complex. The complex vectors **w** and **b** contain the generalized eigenvalues.

If **A** and **B** are real, **T** is in real Schur form and **R** is a real upper-triangular matrix. If **A** or **B** is complex, **T** is in complex Schur form and **R** is a complex upper-triangular matrix.

In the example below, we define **A** and **B**, obtain the generalized Schur decomposition, and list **T** and **R**.

```
:A = (6, 2, 8, -1\ -3, -4, -6, 4\ 0, 8, 4, 1\ -8, -7, -3, 5)
:B = (8, 0, -8, -1\ -6, -2, -6, -1\ -7, -6, 2, -6\ 1, -7, 9, 2)
:gschurd(A, B, T=., R=., U=., V=., w=., b=.)
: T
      1          2          3          4
1  12.99313938   1.746927947   3.931212285  -10.91622337
2      0         .014016016   6.153566902   1.908835695
3      0        -4.362999645   1.849905717  -2.998194791
4      0          0          0          0  -5.527285433

: R
      1          2          3          4
1  4.406836593   6.869534063  -1.840892081   1.740906311
2      0        13.88730687      0         -.6995556735
3      0          0         9.409495218  -4.659386723
4      0          0          0          0   9.453808732

: w
      1          2          3          4
1  12.9931394   .409611804+1.83488354i   .024799819-.111092453i  -5.52728543

: b
      1          2          3          4
1  4.406836593   4.145676341   .2509986829   9.453808732
```

Generalized eigenvalues can be obtained by typing

```
: w:/b
      1          2          3          4
1  2.94840508   .098804579+.442601735i   .098804579-.442601735i  -.584662287
```

Grouping the results

`gschurdgroupby()` reorders the generalized Schur decomposition so that a selected group of generalized eigenvalues appears in the leading block of the pair **w** and **b**. It also reorders the generalized Schur form **T**, **R**, and orthogonal (unitary) matrices, **U** and **V**, correspondingly.

We must pass `gschurdgroupby()` a [pointer](#) to a function that implements our criterion. The function must accept two arguments, a complex scalar and a real scalar, so that it can receive a generalized eigenvalue, and it must return the real value 0 to indicate rejection and a nonzero real value to indicate selection.

In the following example, we use `gschurdgroupby()` to put the finite, real, generalized eigenvalues first. One of the arguments to `gschurdgroupby()` is a pointer to the function `onlyreal()` which accepts two arguments, a complex scalar and a real scalar that define a generalized eigenvalue. `onlyreal()` returns 1 if the generalized eigenvalue is finite and real; it returns zero otherwise.

```

: real scalar onlyreal(complex scalar w, real scalar b)
> {
>     if(b==0) return(0)
>     if(Im(w/b)==0) return(1)
>     return(0)
> }
: gschurdgroupby(A, B, &onlyreal(), T=., R=., U=., V=., w=., b=., m=.)

```

We obtain

```

: T
      1          2          3          4
1  12.99313938   8.19798168   6.285710813   5.563547054
2      0   -5.952366071   -1.473533834   2.750066482
3      0           0   -0.2015830885   3.882051743
4      0           0   6.337230739   1.752690714

: R
      1          2          3          4
1  4.406836593   2.267479575   -6.745927817   1.720793701
2      0   10.18086202   -2.253089622   5.74882307
3      0           0   -12.5704981           0
4      0           0           0   9.652818299

: w
      1          2          3          4
1  12.9931394  -5.95236607  .36499234+1.63500766i  .36499234-1.63500766i

: b
      1          2          3          4
1  4.406836593  10.18086202  3.694083258  3.694083258

: w:/b
      1          2          3          4
1  2.94840508  -.584662287  .098804579+.442601735i  .098804579-.442601735i

```

`m` contains the number of real, generalized eigenvalues

```

: m
2

```

Conformability

`gschurd(A, B, T, R, U, V, w, b):`

input:

A: $n \times n$

B: $n \times n$

output:

T: $n \times n$

R: $n \times n$

U: $n \times n$

V: $n \times n$

w: $1 \times n$

b: $1 \times n$

`_gschurd(A, B, U, V, w, b):`

input:

A: $n \times n$

B: $n \times n$

output:

A: $n \times n$

B: $n \times n$

U: $n \times n$

V: $n \times n$

w: $1 \times n$

b: $1 \times n$

`gschurdgroupby(A, B, f, T, R, U, V, w, b, m):`

input:

A: $n \times n$

B: $n \times n$

f: 1×1

output:

T: $n \times n$

R: $n \times n$

U: $n \times n$

V: $n \times n$

w: $1 \times n$

b: $1 \times n$

m: 1×1

`_gschurdgroupby(A, B, f, U, V, w, b, m)`:

input:

A: $n \times n$
B: $n \times n$
f: 1×1

output:

A: $n \times n$
B: $n \times n$
U: $n \times n$
V: $n \times n$
w: $1 \times n$
b: $1 \times n$
m: 1×1

Diagnostics

`_gschurd()` and `_gschurdgroupby()` abort with error if *A* or *B* is a view.

`gschurd()`, `_gschurd()`, `gschurdgroupby()`, and `_gschurdgroupby()` return missing results if *A* or *B* contains missing values.

Also see

[M-1] [LAPACK](#) — The LAPACK linear-algebra routines

[M-5] [ghessenbergd\(\)](#) — Generalized Hessenberg decomposition

[M-5] [geigensystem\(\)](#) — Generalized eigenvectors and eigenvalues

[M-4] [matrix](#) — Matrix functions