

irt grm — Graded response model

[Description](#)
[Options](#)
[References](#)

[Quick start](#)
[Remarks and examples](#)
[Also see](#)

[Menu](#)
[Stored results](#)

[Syntax](#)
[Methods and formulas](#)

Description

`irt grm` fits graded response models (GRMs) to ordinal items. In the GRM, items vary in their difficulty and discrimination. This model is an extension of the 2PL model to ordered categorical items.

Quick start

GRM for ordinal items `o1` to `o5`

```
irt grm o1-o5
```

Report items sorted by discrimination

```
estat report, sort(a)
```

Plot CCCs for `o1`

```
irtgraph icc o1
```

Menu

Statistics > IRT (item response theory)

Syntax

```
irt grm varlist [if] [in] [weight] [, options]
```

| <i>options</i> | Description |
|---|--|
| Model | |
| <code>listwise</code> | drop observations with any missing items |
| SE/Robust | |
| <code>vce(<i>vcetype</i>)</code> | <i>vcetype</i> may be <code>oim</code> , <code>robust</code> , <code>cluster <i>clustvar</i></code> , <code>bootstrap</code> , or <code>jackknife</code> |
| Reporting | |
| <code>level(#)</code> | set confidence level; default is <code>level(95)</code> |
| <code>notable</code> | suppress coefficient table |
| <code>noheader</code> | suppress output header |
| <code>display_<options></code> | control columns and column formats |
| Integration | |
| <code>intmethod(<i>intmethod</i>)</code> | integration method |
| <code>intpoints(#)</code> | set the number of integration points; default is <code>intpoints(7)</code> |
| Maximization | |
| <code>maximize_<options></code> | control the maximization process; seldom used |
| <code>startvalues(<i>svmethod</i>)</code> | method for obtaining starting values |
| <code>noestimate</code> | do not fit the model; show starting values instead |
| <code>dnumerical</code> | use numerical derivative techniques |
| <code>coeflegend</code> | display legend instead of statistics |

| <i>intmethod</i> | Description |
|--------------------------|--|
| <code>mvaghermite</code> | mean–variance adaptive Gauss–Hermite quadrature; the default |
| <code>mcaghermite</code> | mode-curvature adaptive Gauss–Hermite quadrature |
| <code>ghermite</code> | nonadaptive Gauss–Hermite quadrature |

`bootstrap`, `by`, `jackknife`, `statsby`, and `svy` are allowed; see [U] 11.1.10 Prefix commands.

Weights are not allowed with the `bootstrap` prefix; see [R] bootstrap.

`vce()` and weights are not allowed with the `svy` prefix; see [SVY] svy.

`fweights`, `iweights`, and `pweights` are allowed; see [U] 11.1.6 weight.

`startvalues()`, `noestimate`, `dnumerical`, and `coeflegend` do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options

Model

`listwise` handles missing values through listwise deletion, which means that the entire observation is omitted from the estimation sample if any of the items are missing for that observation. By default, all nonmissing items in an observation are included in the likelihood calculation; only missing items are excluded.

SE/Robust

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`oim`), that are robust to some kinds of misspecification (`robust`), that allow for intragroup correlation (`cluster clustvar`), and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [R] [vce_option](#).

Reporting

`level(#)`; see [R] [estimation options](#).

`notable` suppresses the estimation table, either at estimation or upon replay.

`noheader` suppresses the output header, either at estimation or upon replay.

`display_options`: `noci`, `nopvalues`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] [estimation options](#).

Integration

`intmethod(intmethod)` specifies the integration method to be used for computing the log likelihood. `mvaghermite` performs mean and variance adaptive Gauss–Hermite quadrature; `mcaghermite` performs mode and curvature adaptive Gauss–Hermite quadrature; and `ghermite` performs non-adaptive Gauss–Hermite quadrature.

The default integration method is `mvaghermite`.

`intpoints(#)` sets the number of integration points for quadrature. The default is `intpoints(7)`, which means that seven quadrature points are used to compute the log likelihood.

The more integration points, the more accurate the approximation to the log likelihood. However, computation time increases with the number of integration points.

Maximization

`maximize_options`: `difficult`, `technique(algorithm_spec)`, `iterate(#)`, `[no]log`, `trace`, `gradient`, `showstep`, `hessian`, `showtolerance`, `tolerance(#)`, `ltolerance(#)`, `nrtolerance(#)`, `nonrtolerance`, and `from(init_specs)`; see [R] [maximize](#). Those that require special mention for `irt` are listed below.

`from()` accepts a properly labeled vector of initial values or a list of coefficient names with values. A list of values is not allowed.

The following options are available with `irt` but are not shown in the dialog box:

`startvalues()` specifies how starting values are to be computed. Starting values specified in `from()` override the computed starting values.

`startvalues(zero)` specifies that all starting values be set to 0. This option is typically useful only when specified with the `from()` option.

`startvalues(constantly)` builds on `startvalues(zero)` by fitting a constant-only model for each response to obtain estimates of intercept and cutpoint parameters.

`startvalues(fixedonly)` builds on `startvalues(constantly)` by fitting a full fixed-effects model for each response variable to obtain estimates of coefficients along with intercept and cutpoint parameters. You can also add suboption `iterate(#)` to limit the number of iterations `irt` allows for fitting the fixed-effects model.

`startvalues(ivloadings)` builds on `startvalues(fixedonly)` by using instrumental-variable methods with the generalized residuals from the fixed-effects models to compute starting values for latent-variable loadings. This is the default behavior.

`noestimate` specifies that the model is not to be fit. Instead, starting values are to be shown (as modified by the above options if modifications were made), and they are to be shown using the `coeflegend` style of output. An important use of this option is before you have modified starting values at all; you can type the following:

```
. irt ..., ... noestimate
. matrix b = e(b)
. ... (modify elements of b) ...
. irt ..., ... from(b)
```

`dnnumerical` specifies that during optimization, the gradient vector and Hessian matrix be computed using numerical techniques instead of analytical formulas. By default, `irt` uses analytical formulas for computing the gradient and Hessian for all integration methods.

`coeflegend`; see [R] [estimation options](#).

Remarks and examples

stata.com

The following discussion is about how to use `irt` to fit GRMs to ordinal items. If you are new to the IRT features in Stata, we encourage you to read [IRT] [irt](#) first.

In the GRM, item responses are categorical and ordered, for example, “poor”, “good”, and “excellent” or “strongly disagree”, “disagree”, “agree”, and “strongly agree”. If there are only two outcomes, the GRM is equivalent to the 2PL model; see [IRT] [irt 2pl](#). If the item responses are not ordered, see [IRT] [irt nrm](#).

The GRM allows the ordered categories to vary between items; however, to keep the following discussion from being overly complicated, we will assume the outcome levels for all items are given by $k = 0, 1, \dots, K$.

In the GRM, each item is modeled with its own discrimination parameter and cutpoints that identify boundaries between the ordered outcomes. The probability of observing outcome k or higher for item i and person j is given by

$$\Pr(Y_{ij} \geq k | \theta_j) = \frac{\exp\{a_i(\theta_j - b_{ik})\}}{1 + \exp\{a_i(\theta_j - b_{ik})\}} \quad \theta_j \sim N(0, 1)$$

where a_i represents the discrimination of item i , b_{ik} is the k th cutpoint for item i , and θ_j is the latent trait of person j . The cutpoint b_{ik} can be considered the difficulty of responding with category k or higher for item i .

The GRM is defined in terms of cumulative probabilities, but we can calculate the probability of observing outcome k as

$$\Pr(Y_{ij} = k | \theta_j) = \Pr(Y_{ij} \geq k | \theta_j) - \Pr(Y_{ij} \geq k + 1 | \theta_j)$$

where we take $\Pr(Y_{ij} \geq 0) = 1$ and $\Pr(Y_{ij} > K) = 0$. Because of the additional calculation step required to obtain the probability of observing a particular outcome, the GRM is an indirect IRT model, also known as a difference model; see [Thissen and Steinberg \(1986\)](#).

The GRM was proposed by [Samejima \(1969\)](#). In the multilevel literature, the GRM is known as the cumulative logit model; see [\[ME\] meologit](#). When no latent variable is present, the model for a single item is known as the proportional odds model; see [\[R\] ologit](#).

► Example 1: Fitting a GRM

To illustrate the GRM, we use the data from [Zheng and Rabe-Hesketh \(2007\)](#). `charity.dta` contains five survey questions, `ta1` through `ta5`, measuring faith and trust in charity organizations. Responses are strongly agree (0), agree (1), disagree (2), and strongly disagree (3). Higher scores indicate higher levels of distrust. Here we list the first five observations.

```
. use http://www.stata-press.com/data/r14/charity
(Data from Zheng & Rabe-Hesketh (2007))
. list in 1/5, nlabel
```

| | ta1 | ta2 | ta3 | ta4 | ta5 |
|----|-----|-----|-----|-----|-----|
| 1. | . | 2 | 1 | 1 | . |
| 2. | 0 | 0 | 0 | 0 | 0 |
| 3. | 1 | 1 | 2 | 0 | 2 |
| 4. | 1 | 2 | 2 | 0 | 1 |
| 5. | . | 1 | 1 | 1 | 1 |

Looking across the first row, we see that the first respondent did not provide an answer to items `ta1` and `ta5`, answered 2 on item `ta2`, and answered 1 on items `ta3` and `ta4`. All `irt` commands exclude missing items for a given observation from the likelihood calculation but keep the nonmissing items for that observation. If you wish to remove the entire observation from the model, add the `listwise` option at estimation time.

We fit a GRM as follows:

```
. irt grm ta1-ta5
Fitting fixed-effects model:
Iteration 0:   log likelihood = -5467.3926
Iteration 1:   log likelihood = -5467.3926
Fitting full model:
Iteration 0:   log likelihood = -5271.0634
Iteration 1:   log likelihood = -5162.5917
Iteration 2:   log likelihood = -5159.2947
Iteration 3:   log likelihood = -5159.2791
Iteration 4:   log likelihood = -5159.2791
Graded response model                Number of obs    =          945
Log likelihood = -5159.2791
```

| | | Coef. | Std. Err. | z | P> z | [95% Conf. Interval] | |
|---------|--|-----------|-----------|--------|-------|----------------------|-----------|
| ta1 | | | | | | | |
| Discrim | | .907542 | .0955772 | 9.50 | 0.000 | .7202142 | 1.09487 |
| Diff | | | | | | | |
| >=1 | | -1.540098 | .1639425 | -9.39 | 0.000 | -1.861419 | -1.218776 |
| >=2 | | 1.296135 | .1427535 | 9.08 | 0.000 | 1.016343 | 1.575927 |
| =3 | | 3.305059 | .3248468 | 10.17 | 0.000 | 2.668371 | 3.941747 |
| ta2 | | | | | | | |
| Discrim | | .9434675 | .0967483 | 9.75 | 0.000 | .7538444 | 1.133091 |
| Diff | | | | | | | |
| >=1 | | -1.661331 | .167878 | -9.90 | 0.000 | -1.990366 | -1.332296 |
| >=2 | | .0068314 | .0822222 | 0.08 | 0.934 | -.1543208 | .1679836 |
| =3 | | 2.531091 | .2412513 | 10.49 | 0.000 | 2.058247 | 3.003935 |
| ta3 | | | | | | | |
| Discrim | | 1.734201 | .1554383 | 11.16 | 0.000 | 1.429548 | 2.038855 |
| Diff | | | | | | | |
| >=1 | | -1.080079 | .0835119 | -12.93 | 0.000 | -1.243759 | -.9163983 |
| >=2 | | 1.016567 | .0796635 | 12.76 | 0.000 | .8604297 | 1.172705 |
| =3 | | 2.232606 | .1497814 | 14.91 | 0.000 | 1.93904 | 2.526172 |
| ta4 | | | | | | | |
| Discrim | | 1.93344 | .1857629 | 10.41 | 0.000 | 1.569351 | 2.297528 |
| Diff | | | | | | | |
| >=1 | | -.3445057 | .0578468 | -5.96 | 0.000 | -.4578833 | -.2311282 |
| >=2 | | 1.466254 | .0983823 | 14.90 | 0.000 | 1.273428 | 1.65908 |
| =3 | | 2.418954 | .162392 | 14.90 | 0.000 | 2.100672 | 2.737237 |
| ta5 | | | | | | | |
| Discrim | | 1.42753 | .1263962 | 11.29 | 0.000 | 1.179798 | 1.675262 |
| Diff | | | | | | | |
| >=1 | | -.8552358 | .0833158 | -10.26 | 0.000 | -1.018532 | -.6919399 |
| >=2 | | .6805315 | .07469 | 9.11 | 0.000 | .5341418 | .8269211 |
| =3 | | 2.074243 | .1538858 | 13.48 | 0.000 | 1.772632 | 2.375853 |

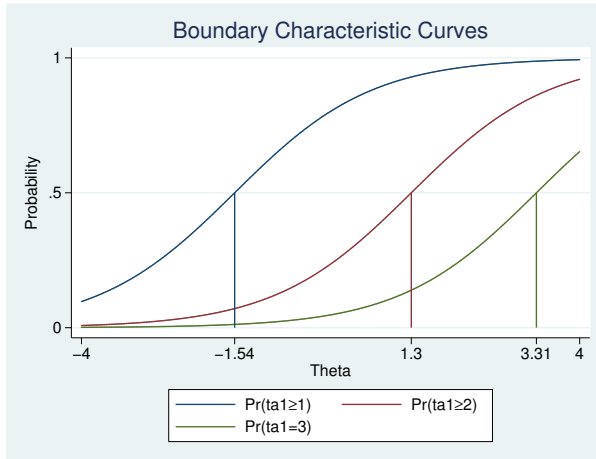
Because the GRM is basically an ordered logistic model, each item's difficulty parameters are naturally estimated in an increasing order. The difficulties represent a point at which a person with trait level $\theta_j = b_{ik}$ has a 50% chance of responding in category k or higher. We make cumulative comparisons because the model is defined in terms of cumulative probabilities.

For example, looking at the estimated parameters of item `ta1`, we see that a person with $\theta = -1.54$ has a 50% chance of answering 0 versus greater than or equal to 1, a person with $\theta = 1.30$ has a

50% chance of answering 0 or 1 versus greater than or equal to 2, and a person with $\theta = 3.31$ has a 50% chance of answering 0, 1, or 2 versus 3.

To illustrate this, we plot the BCCs as a function of θ for `ta1` using the estimated GRM parameters. The `blocation` option adds a vertical line at the estimated difficulties; see [IRT] `irtgraph icc`.

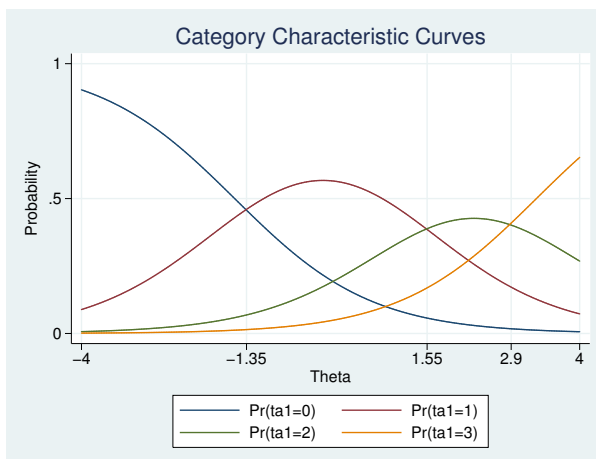
```
. irtgraph icc ta1, blocation
```



We see that the estimated difficulty parameters correspond to the point on the latent trait scale at which $\Pr(Y \geq k|\theta) = 0.5$. You can think of these curves as item characteristic curves where each curve dichotomizes the ordered responses into successive $\Pr(Y \geq k)$ and $\Pr(Y < k)$ categories. The estimated discrimination parameter for `ta1` is 0.91; thus, the curves have relatively flat slopes.

We can also plot category probabilities, $\Pr(Y = k)$, as a function of θ , which in fact is the default behavior of `irtgraph icc`. For categorical responses, such plots are called category characteristic curves (CCCs). Here we plot the CCCs for item `ta1`.

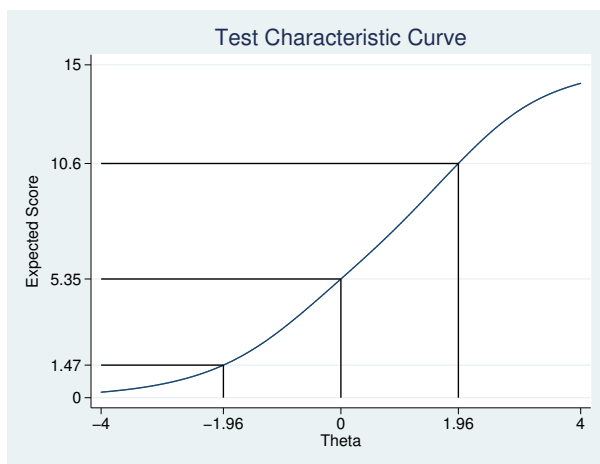
```
. irtgraph icc ta1, xlabel(-4 -1.35 1.55 2.9 4), grid
```



The graph shows that respondents with the latent trait level below approximately -1.35 are most likely to respond in the first category, respondents with the latent trait level between approximately -1.35 and 1.55 are most likely to respond in the second category, and so on.

We use `irtgraph tcc` to plot the TCC using the estimated GRM parameters; see [IRT] `irtgraph tcc`. Because we have 5 items, each coded 0 to 3, the total score ranges from 0 to 15. The `thetalines()` option plots the expected scores at the specified values for θ .

```
. irtgraph tcc, thetalines(-1.96 0 1.96)
```



This plot tells us what kind of scores we can expect from individuals with different levels of the latent trait (trust in charities).

For example, we can expect above-average individuals to score 5.35 or above. Actually, no one is expected to score exactly 5.35 on this survey, so a more realistic statement is that we expect above-average individuals to score above 5 out of a possible score of 15.

Using the 95% critical values from the standard normal distribution (-1.96 and 1.96), this plot also tells us that we can expect 95% of randomly selected people to score between 1.47 and 10.6. Again, a more realistic statement is that we expect about 95% of randomly selected people to score from 2 to 10, which can be interpreted that most people either trust or slightly distrust charities.

◀

Stored results

`irt grm` stores the following in `e()`:

Scalars

| | |
|--------------------------|--|
| <code>e(N)</code> | number of observations |
| <code>e(k)</code> | number of parameters |
| <code>e(k_eq)</code> | number of equations in <code>e(b)</code> |
| <code>e(k_dv)</code> | number of dependent variables |
| <code>e(k_rc)</code> | number of covariances |
| <code>e(k_rs)</code> | number of variances |
| <code>e(irt_k_eq)</code> | number of IRT model groups |
| <code>e(k_items1)</code> | number of items in first IRT model group |
| <code>e(k_cat#)</code> | number of categories for the #th item, ordinal |

| | |
|--------------|--|
| e(ll) | log likelihood |
| e(N_clust) | number of clusters |
| e(n_quad) | number of integration points |
| e(rank) | rank of $\mathbf{e}(V)$ |
| e(ic) | number of iterations |
| e(rc) | return code |
| e(converged) | 1 if target model converged, 0 otherwise |

Macros

| | |
|----------------------|--|
| e(cmd) | gsem |
| e(cmd2) | irt |
| e(cmdline) | command as typed |
| e(model1) | grm |
| e(items1) | names of items in first IRT model group |
| e(n_cuts1) | numlist of cuts in first IRT model group |
| e(depvar) | names of all item variables |
| e(wtype) | weight type |
| e(wexp) | weight expression |
| e(title) | title in estimation output |
| e(clustvar) | name of cluster variable |
| e(family#) | family for the #th <i>item</i> |
| e(link#) | link for the #th <i>item</i> |
| e(intmethod) | integration method |
| e(vce) | <i>vcetype</i> specified in <code>vce()</code> |
| e(vcetype) | title used to label Std. Err. |
| e(opt) | type of optimization |
| e(which) | max or min; whether optimizer is to perform maximization or minimization |
| e(method) | estimation method: ml |
| e(ml_method) | type of ml method |
| e(user) | name of likelihood-evaluator program |
| e(technique) | maximization technique |
| e(datasignature) | the checksum |
| e(datasignaturevars) | variables used in calculation of checksum |
| e(properties) | b V |
| e(estat_cmd) | program used to implement <code>estat</code> |
| e(predict) | program used to implement <code>predict</code> |
| e(covariates) | list of covariates |
| e(footnote) | program used to implement the footnote display |

Matrices

| | |
|-----------------|--|
| e(_N) | sample size for each item |
| e(b) | coefficient vector, slope-intercept parameterization |
| e(b_pclass) | parameter class |
| e(cat#) | categories for the #th item, ordinal |
| e(Cns) | constraints matrix |
| e(i log) | iteration log (up to 20 iterations) |
| e(gradient) | gradient vector |
| e(V) | variance-covariance matrix of the estimators |
| e(V_modelbased) | model-based variance |

Functions

| | |
|-----------|-------------------------|
| e(sample) | marks estimation sample |
|-----------|-------------------------|

Methods and formulas

Let Y_{ij} represent the (yet to be observed) outcome for item i from person j . Without loss of generality, we will assume all items take on the ordered categories, $k = 0, 1, \dots, K$.

Using the IRT parameterization, we see that the probability of person j with latent trait level θ_j (the latent trait) providing response k or above for item i is given by

$$\Pr(Y_{ij} \geq k | a_i, \mathbf{b}_i, \theta_j) = \frac{\exp\{a_i(\theta_j - b_{ik})\}}{1 + \exp\{a_i(\theta_j - b_{ik})\}}$$

where a_i represents the discrimination of item i , $\mathbf{b}_i = (b_{i1}, \dots, b_{iK})$ represent the difficulties that distinguish the ordered categories of item i , and it is understood that $\Pr(Y_{ij} \geq 0 | a_i, \mathbf{b}_i, \theta_j) = 1$ and $\Pr(Y_{ij} > K | a_i, \mathbf{b}_i, \theta_j) = 0$. The probability of observing outcome k is then

$$\Pr(Y_{ij} = k | a_i, \mathbf{b}_i, \theta_j) = \Pr(Y_{ij} \geq k | a_i, \mathbf{b}_i, \theta_j) - \Pr(Y_{ij} \geq k + 1 | a_i, \mathbf{b}_i, \theta_j)$$

`irt grm` fits the model using the slope-intercept form, so the probability for providing response k or above is parameterized as

$$\Pr(Y_{ij} \geq k | \alpha_i, \beta_i, \theta_j) = \frac{\exp(\alpha_i \theta_j - \beta_{ik})}{1 + \exp(\alpha_i \theta_j - \beta_{ik})}$$

The transformation between these two parameterizations is

$$a_i = \alpha_i \quad b_{ik} = \frac{\beta_{ik}}{\alpha_i}$$

Let y_{ij} be the observed response for Y_{ij} and $p_{ij} = \Pr(Y_{ij} = y_{ij} | \alpha_i, \beta_i, \theta_j)$. Conditional on θ_j , the item responses are assumed to be independent, so the conditional density for person j is given by

$$f(\mathbf{y}_j | \mathbf{B}, \theta_j) = \prod_{i=1}^I p_{ij}$$

where $\mathbf{y}_j = (y_{1j}, \dots, y_{Ij})$, $\mathbf{B} = (\alpha_1, \dots, \alpha_I, \beta_1, \dots, \beta_I)$, and I is the number of items.

Missing items are skipped over in the above product by default. When the `listwise` option is specified, persons with any missing items are dropped from the estimation sample.

The likelihood for person j is computed by integrating out the latent variable from the joint density

$$L_j(\mathbf{B}) = \int_{-\infty}^{\infty} f(\mathbf{y}_j | \mathbf{B}, \theta_j) \phi(\theta_j) d\theta_j$$

where $\phi(\cdot)$ is the density function for the standard normal distribution. The log likelihood for the estimation sample is simply the sum of the log likelihoods from the N persons in the estimation sample.

$$\log L(\mathbf{B}) = \sum_{j=1}^N \log L_j(\mathbf{B})$$

The integral in the formula for $L_j(\mathbf{B})$ is generally not tractable, so we must use numerical methods.

Gauss–Hermite quadrature and adaptive quadrature are documented in [Methods and formulas of \[IRT\] irt hybrid](#).

References

- Samejima, F. 1969. Estimation of latent ability using a response pattern of graded scores. *Psychometrika Monograph Supplement*, no. 17.
- Thissen, D., and L. Steinberg. 1986. A taxonomy of item response models. *Psychometrika* 51: 567–577.
- Zheng, X., and S. Rabe-Hesketh. 2007. Estimating parameters of dichotomous and ordinal item response models with `gllamm`. *Stata Journal* 7: 313–333.

Also see

- [IRT] [irt grm postestimation](#) — Postestimation tools for irt grm
- [IRT] [irt](#) — Introduction to IRT models
- [IRT] [irt rsm](#) — Rating scale model
- [SEM] [gsem](#) — Generalized structural equation model estimation command
- [U] [20 Estimation and postestimation commands](#)