Title

xtrc — Random-coefficients model

Syntax

`xtrc` *depvar indepvars* [*if*] [*in*] [, *options*]

options	Description
Main	
`noconstant`	suppress constant term
`offset(`*varname*`)`	include *varname* in model with coefficient constrained to 1
SE	
`vce(`*vcetype*`)`	*vcetype* may be `conventional`, `bootstrap`, or `jackknife`
Reporting	
`level(#)`	set confidence level; default is `level(95)`
`betas`	display group-specific best linear predictors
display_options	control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
`coeflegend`	display legend instead of statistics

A panel variable must be specified; use `xtset`; see [XT] **xtset**.

indepvars may contain factor variables; see [U] **11.4.3 Factor variables**.

`by`, `mi estimate`, and `statsby` are allowed; see [U] **11.1.10 Prefix commands**.

`vce(bootstrap)` and `vce(jackknife)` are not allowed with the `mi estimate` prefix; see [MI] **mi estimate**.

`coeflegend` does not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

Menu

Statistics > Longitudinal/panel data > Random-coefficients regression by GLS

Description

`xtrc` fits the Swamy (1970) random-coefficients linear regression model.

Options

Main

`noconstant`, `offset(`*varname*`)`; see [R] **estimation options**

SE

`vce(vcetype)` specifies the type of standard error reported, which includes types that are derived from asymptotic theory (`conventional`) and that use bootstrap or jackknife methods (`bootstrap`, `jackknife`); see [XT] ***vce_options***.

`vce(conventional)`, the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Reporting

`level(#)`; see [R] **estimation options**.

`betas` requests that the group-specific best linear predictors also be displayed.

display_options: `noomitted`, `vsquish`, `noemptycells`, `baselevels`, `allbaselevels`, `nofvlabel`, `fvwrap(#)`, `fvwrapon(style)`, `cformat(%fmt)`, `pformat(%fmt)`, `sformat(%fmt)`, and `nolstretch`; see [R] **estimation options**.

The following option is available with `xtrc` but is not shown in the dialog box:

`coeflegend`; see [R] **estimation options**.

Remarks and examples

stata.com

In random-coefficients models, we wish to treat the parameter vector as a realization (in each panel) of a stochastic process. `xtrc` fits the Swamy (1970) random-coefficients model, which is suitable for linear regression of panel data. See Greene (2012, chap. 11) and Poi (2003) for more information about this and other panel-data models.

▷ Example 1

Greene (2012, 1112) reprints data from a classic study of investment demand by Grunfeld and Griliches (1960). In [XT] **xtgls**, we use this dataset to illustrate many of the possible models that may be fit with the `xtgls` command. Although the models included in the `xtgls` command offer considerable flexibility, they all assume that there is no parameter variation across firms (the cross-sectional units).

To take a first look at the assumption of parameter constancy, we should `reshape` our data so that we may fit a simultaneous-equation model with `sureg`; see [R] **sureg**. Because there are only five panels here, this is not too difficult.

```
. use http://www.stata-press.com/data/r13/invest2

. reshape wide invest market stock, i(time) j(company)
(note: j = 1 2 3 4 5)
Data                               long   ->   wide
-----------------------------------------------------------------------------
Number of obs.                      100   ->      20
Number of variables                   5   ->      16
j variable (5 values)           company   ->   (dropped)
xij variables:
                                 invest   ->   invest1 invest2 ... invest5
                                 market   ->   market1 market2 ... market5
                                  stock   ->   stock1 stock2 ... stock5
-----------------------------------------------------------------------------
```

```
. sureg (invest1 market1 stock1) (invest2 market2 stock2)
> (invest3 market3 stock3) (invest4 market4 stock4) (invest5 market5 stock5)
Seemingly unrelated regression
```

Equation	Obs	Parms	RMSE	"R-sq"	chi2	P
invest1	20	2	84.94729	0.9207	261.32	0.0000
invest2	20	2	12.36322	0.9119	207.21	0.0000
invest3	20	2	26.46612	0.6876	46.88	0.0000
invest4	20	2	9.742303	0.7264	59.15	0.0000
invest5	20	2	95.85484	0.4220	14.97	0.0006

	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
invest1						
market1	.120493	.0216291	5.57	0.000	.0781007	.1628853
stock1	.3827462	.032768	11.68	0.000	.318522	.4469703
_cons	-162.3641	89.45922	-1.81	0.070	-337.7009	12.97279
invest2						
market2	.0695456	.0168975	4.12	0.000	.0364271	.1026641
stock2	.3085445	.0258635	11.93	0.000	.2578529	.3592362
_cons	.5043112	11.51283	0.04	0.965	-22.06042	23.06904
invest3						
market3	.0372914	.0122631	3.04	0.002	.0132561	.0613268
stock3	.130783	.0220497	5.93	0.000	.0875663	.1739997
_cons	-22.43892	25.51859	-0.88	0.379	-72.45443	27.57659
invest4						
market4	.0570091	.0113623	5.02	0.000	.0347395	.0792788
stock4	.0415065	.0412016	1.01	0.314	-.0392472	.1222602
_cons	1.088878	6.258805	0.17	0.862	-11.17815	13.35591
invest5						
market5	.1014782	.0547837	1.85	0.064	-.0058958	.2088523
stock5	.3999914	.1277946	3.13	0.002	.1495186	.6504642
_cons	85.42324	111.8774	0.76	0.445	-133.8525	304.6989

Here we instead fit a random-coefficients model:

```
. use http://www.stata-press.com/data/r13/invest2
. xtrc invest market stock
Random-coefficients regression                  Number of obs      =       100
Group variable: company                         Number of groups   =         5
                                                Obs per group: min =        20
                                                               avg =      20.0
                                                               max =        20
                                                Wald chi2(2)       =     17.55
                                                Prob > chi2        =    0.0002
```

invest	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
market	.0807646	.0250829	3.22	0.001	.0316031	.1299261
stock	.2839885	.0677899	4.19	0.000	.1511229	.4168542
_cons	-23.58361	34.55547	-0.68	0.495	-91.31108	44.14386

```
Test of parameter constancy:    chi2(12) =   603.99       Prob > chi2 = 0.0000
```

Just as the results of our simultaneous-equation model do not support the assumption of parameter constancy, the test included with the random-coefficients model also indicates that the assumption is not valid for these data. With large panel datasets, we would not want to take the time to look at a simultaneous-equations model (aside from the fact that our doing so was subjective).

◁

Stored results

`xtrc` stores the following in `e()`:

Scalars	
`e(N)`	number of observations
`e(N_g)`	number of groups
`e(df_m)`	model degrees of freedom
`e(chi2)`	χ^2
`e(chi2_c)`	χ^2 for comparison test
`e(df_chi2c)`	degrees of freedom for comparison χ^2 test
`e(g_min)`	smallest group size
`e(g_avg)`	average group size
`e(g_max)`	largest group size
`e(rank)`	rank of `e(V)`
Macros	
`e(cmd)`	`xtrc`
`e(cmdline)`	command as typed
`e(depvar)`	name of dependent variable
`e(ivar)`	variable denoting groups
`e(tvar)`	variable denoting time within groups
`e(title)`	title in estimation output
`e(offset)`	linear offset variable
`e(chi2type)`	`Wald`; type of model χ^2 test
`e(vce)`	*vcetype* specified in `vce()`
`e(vcetype)`	title used to label Std. Err.
`e(properties)`	`b V`
`e(predict)`	program used to implement `predict`
`e(marginsnotok)`	predictions disallowed by `margins`
`e(asbalanced)`	factor variables `fvset` as `asbalanced`
`e(asobserved)`	factor variables `fvset` as `asobserved`
Matrices	
`e(b)`	coefficient vector
`e(Sigma)`	$\widehat{\boldsymbol{\Sigma}}$ matrix
`e(beta_ps)`	matrix of best linear predictors
`e(V)`	variance–covariance matrix of the estimators
`e(V_ps)`	matrix of variances for the best linear predictors; row i contains vec of variance matrix for group i predictor
Functions	
`e(sample)`	marks estimation sample

Methods and formulas

In a random-coefficients model, the parameter heterogeneity is treated as stochastic variation. Assume that we write

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i$$

where $i = 1, \dots, m$, and $\boldsymbol{\beta}_i$ is the coefficient vector ($k \times 1$) for the ith cross-sectional unit, such that

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\nu}_i \qquad E(\boldsymbol{\nu}_i) = \mathbf{0} \qquad E(\boldsymbol{\nu}_i \boldsymbol{\nu}_i') = \boldsymbol{\Sigma}$$

Our goal is to find $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\Sigma}}$.

The derivation of the estimator assumes that the cross-sectional specific coefficient vector $\boldsymbol{\beta}_i$ is the outcome of a random process with mean vector $\boldsymbol{\beta}$ and covariance matrix $\boldsymbol{\Sigma}$,

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i = \mathbf{X}_i(\boldsymbol{\beta} + \boldsymbol{\nu}_i) + \boldsymbol{\epsilon}_i = \mathbf{X}_i\boldsymbol{\beta} + (\mathbf{X}_i\boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i) = \mathbf{X}_i\boldsymbol{\beta} + \boldsymbol{\omega}_i$$

where $E(\boldsymbol{\omega}_i) = \mathbf{0}$ and

$$E(\boldsymbol{\omega}_i\boldsymbol{\omega}_i') = E\Big\{(\mathbf{X}_i\boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)(\mathbf{X}_i\boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)'\Big\} = E(\boldsymbol{\epsilon}_i\boldsymbol{\epsilon}_i') + \mathbf{X}_iE(\boldsymbol{\nu}_i\boldsymbol{\nu}_i')\mathbf{X}_i' = \sigma_i^2\mathbf{I} + \mathbf{X}_i\boldsymbol{\Sigma}\,\mathbf{X}_i' = \boldsymbol{\Pi}_i$$

Stacking the m equations, we have

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega}$$

where $\boldsymbol{\Pi} \equiv E(\boldsymbol{\omega}\boldsymbol{\omega}')$ is a block diagonal matrix with $\boldsymbol{\Pi}_i$, $i = 1...m$, along the main diagonal and zeros elsewhere. The GLS estimator of $\widehat{\boldsymbol{\beta}}$ is then

$$\widehat{\boldsymbol{\beta}} = \left(\sum_i \mathbf{X}_i'\boldsymbol{\Pi}_i^{-1}\mathbf{X}_i\right)^{-1} \sum_i \mathbf{X}_i'\boldsymbol{\Pi}_i^{-1}\mathbf{y}_i = \sum_{i=1}^{m}\mathbf{W}_i\mathbf{b}_i$$

where

$$\mathbf{W}_i = \left\{\sum_{i=1}^{m}(\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}\right\}^{-1}(\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}$$

$\mathbf{b}_i = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{y}_i$ and $\mathbf{V}_i = \sigma_i^2\,(\mathbf{X}_i'\mathbf{X}_i)^{-1}$, showing that the resulting GLS estimator is a matrix-weighted average of the panel-specific OLS estimators. The variance of $\widehat{\boldsymbol{\beta}}$ is

$$\mathrm{Var}(\widehat{\boldsymbol{\beta}}) = \sum_{i=1}^{m}(\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}$$

To calculate the above estimator $\widehat{\boldsymbol{\beta}}$ for the unknown $\boldsymbol{\Sigma}$ and $\mathbf{V}_i$ parameters, we use the two-step approach suggested by Swamy (1970):

$$\begin{aligned}
\mathbf{b}_i &= \text{OLS panel-specific estimator}\\
\widehat{\sigma}_i^2 &= \frac{\widehat{\boldsymbol{\epsilon}}_i'\widehat{\boldsymbol{\epsilon}}_i}{n_i - k}\\
\widehat{\mathbf{V}}_i &= \widehat{\sigma}_i^2\,(\mathbf{X}_i'\mathbf{X}_i)^{-1}\\
\overline{\mathbf{b}} &= \frac{1}{m}\sum_{i=1}^{m}\mathbf{b}_i\\
\widehat{\boldsymbol{\Sigma}} &= \frac{1}{m-1}\left(\sum_{i=1}^{m}\mathbf{b}_i\mathbf{b}_i' - m\overline{\mathbf{b}}\,\overline{\mathbf{b}}'\right) - \frac{1}{m}\sum_{i=1}^{m}\widehat{\mathbf{V}}_i
\end{aligned}$$

The two-step procedure begins with the usual OLS estimates of $\boldsymbol{\beta}_i$. With those estimates, we may proceed by obtaining estimates of $\widehat{\mathbf{V}}_i$ and $\widehat{\boldsymbol{\Sigma}}$ (and thus $\widehat{\mathbf{W}}_i$) and then obtain an estimate of $\boldsymbol{\beta}$.

Swamy (1970) further points out that the matrix $\widehat{\boldsymbol{\Sigma}}$ may not be positive definite and that because the second term is of order $1/(mT)$, it is negligible in large samples. A simple and asymptotically expedient solution is simply to drop this second term and instead use

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{m-1}\left(\sum_{i=1}^{m} \mathbf{b}_i \mathbf{b}_i' - m\overline{\mathbf{b}}\,\overline{\mathbf{b}}'\right)$$

As discussed by Judge et al. (1985, 541), the feasible best linear predictor of $\boldsymbol{\beta}_i$ is given by

$$\begin{aligned}
\widehat{\boldsymbol{\beta}}_i &= \widehat{\boldsymbol{\beta}} + \widehat{\boldsymbol{\Sigma}}\mathbf{X}_i' \left(\mathbf{X}_i\widehat{\boldsymbol{\Sigma}}\mathbf{X}_i' + \widehat{\sigma}_i^2\mathbf{I}\right)^{-1} \left(\mathbf{y}_i - \mathbf{X}_i\widehat{\boldsymbol{\beta}}\right) \\
&= \left(\widehat{\boldsymbol{\Sigma}}^{-1} + \widehat{\mathbf{V}}_i^{-1}\right)^{-1} \left(\widehat{\boldsymbol{\Sigma}}^{-1}\widehat{\boldsymbol{\beta}} + \widehat{\mathbf{V}}_i^{-1}\mathbf{b}_i\right)
\end{aligned}$$

The conventional variance of $\widehat{\boldsymbol{\beta}}_i$ is given by

$$\mathrm{Var}(\widehat{\boldsymbol{\beta}}_i) = \mathrm{Var}(\widehat{\boldsymbol{\beta}}) + (\mathbf{I} - \mathbf{A}_i)\left\{\widehat{\mathbf{V}}_i - \mathrm{Var}(\widehat{\boldsymbol{\beta}})\right\}(\mathbf{I} - \mathbf{A}_i)'$$

where

$$\mathbf{A}_i = \left(\widehat{\boldsymbol{\Sigma}}^{-1} + \widehat{\mathbf{V}}_i^{-1}\right)^{-1} \widehat{\boldsymbol{\Sigma}}^{-1}$$

To test the model, we may look at the difference between the OLS estimate of $\boldsymbol{\beta}$, ignoring the panel structure of the data and the matrix-weighted average of the panel-specific OLS estimators. The test statistic suggested by Swamy (1970) is given by

$$\chi^2_{k(m-1)} = \sum_{i=1}^{m} (\mathbf{b}_i - \overline{\boldsymbol{\beta}}^*)' \widehat{\mathbf{V}}_i^{-1} (\mathbf{b}_i - \overline{\boldsymbol{\beta}}^*) \qquad \text{where} \qquad \overline{\boldsymbol{\beta}}^* = \left(\sum_{i=1}^{m} \widehat{\mathbf{V}}_i^{-1}\right)^{-1} \sum_{i=1}^{m} \widehat{\mathbf{V}}_i^{-1}\mathbf{b}_i$$

Johnston and DiNardo (1997) have shown that the test is algebraically equivalent to testing

$$H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_m$$

in the generalized (groupwise heteroskedastic) `xtgls` model, where $\mathbf{V}$ is block diagonal with ith diagonal element $\boldsymbol{\Pi}_i$.

References

Eberhardt, M. 2012. Estimating panel time-series models with heterogeneous slopes. *Stata Journal* 12: 61–71.

Greene, W. H. 2012. *Econometric Analysis*. 7th ed. Upper Saddle River, NJ: Prentice Hall.

Grunfeld, Y., and Z. Griliches. 1960. Is aggregation necessarily bad? *Review of Economics and Statistics* 42: 1–13.

Johnston, J., and J. DiNardo. 1997. *Econometric Methods*. 4th ed. New York: McGraw–Hill.

Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lütkepohl, and T.-C. Lee. 1985. *The Theory and Practice of Econometrics*. 2nd ed. New York: Wiley.

Nichols, A. 2007. Causal inference with observational data. *Stata Journal* 7: 507–541.

Poi, B. P. 2003. From the help desk: Swamy's random-coefficients model. *Stata Journal* 3: 302–308.

Swamy, P. A. V. B. 1970. Efficient inference in a random coefficient regression model. *Econometrica* 38: 311–323.

——. 1971. *Statistical Inference in Random Coefficient Regression Models*. New York: Springer.

Also see

[XT] **xtrc postestimation** — Postestimation tools for xtrc

[XT] **xtreg** — Fixed-, between-, and random-effects and population-averaged linear models

[XT] **xtset** — Declare data to be panel data

[ME] **mixed** — Multilevel mixed-effects linear regression

[MI] **estimation** — Estimation commands for use with mi estimate

[U] **20 Estimation and postestimation commands**