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**xthtaylor** — Hausman–Taylor estimator for error-components models

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Also see

# Syntax 5 4 1

xthtaylor depvar indepvars [if] [in] [weight], endog(varlist) [options]

us

<sup>\*</sup>endog(varlist) is required.

A panel variable must be specified. For xthtaylor, amacurdy, a time variable must also be specified. Use xtset; see [XT] xtset.

depvar, indepvars, and all varlists may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, statsby, and xi are allowed; see [U] 11.1.10 Prefix commands.

iweights and fweights are allowed unless the amacurdy option is specified. Weights must be constant within panel; see [U] 11.1.6 weight.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

#### Menu

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# Description

xthtaylor fits panel-data random-effects models in which some of the covariates are correlated with the unobserved individual-level random effect. The estimators, originally proposed by Hausman and Taylor (1981) and by Amemiya and MaCurdy (1986), are based on instrumental variables. By default, xthtaylor uses the Hausman-Taylor estimator. When the amacurdy option is specified, xthtaylor uses the Amemiya-MaCurdy estimator.

Although the estimators implemented in xthtaylor and xtivreg (see [XT] xtivreg) use the method of instrumental variables, each command is designed for different problems. The estimators implemented in xtivreg assume that a subset of the explanatory variables in the model are correlated with the idiosyncratic error  $\epsilon_{it}$ . In contrast, the Hausman-Taylor and Amemiya-MaCurdy estimators that are implemented in xthtaylor assume that some of the explanatory variables are correlated with the individual-level random effects,  $u_i$ , but that none of the explanatory variables are correlated with the idiosyncratic error,  $\epsilon_{it}$ .

# **Options**

Model

noconstant; see [R] estimation options.

endog(varlist) specifies that a subset of explanatory variables in indepvars be treated as endogenous variables, that is, the explanatory variables that are assumed to be correlated with the unobserved random effect. endog() is required.

constant  $(varlist_{ti})$  specifies the subset of variables in *indepvars* that are time invariant, that is, constant within panel. By using this option, you assert not only that the variables specified in varlist<sub>ti</sub> are time invariant but also that all other variables in *indepvars* are time varying. If this assertion is false, xthtaylor does not perform the estimation and will issue an error message. xthtaylor automatically detects which variables are time invariant and which are not. However, users may want to check their understanding of the data and specify which variables are time invariant and which are not.

varying(varlist<sub>tv</sub>) specifies the subset of variables in *indepvars* that are time varying. By using this option, you assert not only that the variables specified in  $varlist_{tv}$  are time varying but also that all other variables in *indepvars* are time invariant. If this assertion is false, xthtaylor does not perform the estimation and issues an error message. xthtaylor automatically detects which variables are time varying and which are not. However, users may want to check their understanding of the data and specify which variables are time varying and which are not.

amacurdy specifies that the Amemiya-MaCurdy estimator be used. This estimator uses extra instruments to gain efficiency at the cost of additional assumptions on the data-generating process. This option may be specified only for samples containing balanced panels, and weights may not be specified. The panels must also have a common initial period.

SE

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for this Hausman-Taylor model.

Reporting

level(#); see [R] estimation options.

small specifies that the p-values from the Wald tests in the output and all subsequent Wald tests obtained via test use t and F distributions instead of the large-sample normal and  $\chi^2$  distributions. By default, the p-values are obtained using the normal and  $\chi^2$  distributions.

# Remarks and examples

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If you have not read [XT] xt, please do so.

Consider a random-effects model of the form

$$y_{it} = \mathbf{X}_{1it}\boldsymbol{\beta}_1 + \mathbf{X}_{2it}\boldsymbol{\beta}_2 + \mathbf{Z}_{1i}\boldsymbol{\delta}_1 + \mathbf{Z}_{2i}\boldsymbol{\delta}_2 + \mu_i + \epsilon_{it}$$

where

 $\mathbf{X}_{1it}$  is a  $1 \times k_1$  vector of observations on exogenous, time-varying variables assumed to be uncorrelated with  $\mu_i$  and  $\epsilon_{it}$ ;

 $\mathbf{X}_{2it}$  is a  $1 \times k_2$  vector of observations on endogenous, time-varying variables assumed to be (possibly) correlated with  $\mu_i$  but orthogonal to  $\epsilon_{it}$ ;

 $\mathbf{Z}_{1i}$  is a  $1 \times q_1$  vector of observations on exogenous, time-invariant variables assumed to be uncorrelated with  $\mu_i$  and  $\epsilon_{it}$ ;

 $\mathbf{Z}_{2i}$  is a  $1 \times g_2$  vector of observations on endogenous, time-invariant variables assumed to be (possibly) correlated  $\mu_i$  but orthogonal to  $\epsilon_{it}$ ;

 $\mu_i$  is the unobserved, panel-level random effect that is assumed to have zero mean and finite variance  $\sigma_{\mu}^2$  and to be independently and identically distributed (i.i.d.) over the panels;

 $\epsilon_{it}$  is the idiosyncratic error that is assumed to have zero mean and finite variance  $\sigma_{\epsilon}^2$  and to be i.i.d. over all the observations in the data;

 $\beta_1, \beta_2, \delta_1$ , and  $\delta_2$  are  $k_1 \times 1$ ,  $k_2 \times 1$ ,  $g_1 \times 1$ , and  $g_2 \times 1$  coefficient vectors, respectively; and  $i=1,\ldots,n$ , where n is the number of panels in the sample and, for each  $i, t=1,\ldots,T_i$ .

Because  $X_{2it}$  and  $Z_{2i}$  may be correlated with  $\mu_i$ , the simple random-effects estimators—xtreg, re and xtreg, mle—are generally not consistent for the parameters in this model. Because the within estimator, xtreg, fe, removes the  $\mu_i$  by mean-differencing the data before estimating  $\beta_1$  and  $\beta_2$ , it is consistent for these parameters. However, in the process of removing the  $\mu_i$ , the within estimator also eliminates the  $\mathbf{Z}_{1i}$  and the  $\mathbf{Z}_{2i}$ . Thus it cannot estimate  $\delta_1$  nor  $\delta_2$ . The Hausman-Taylor and Amemiya—MaCurdy estimators implemented in xthtaylor are designed to resolve this problem.

The within estimator consistently estimates  $\beta_1$  and  $\beta_2$ . Using these estimates, we can obtain the within residuals, called  $\widehat{d}_i$ . Intermediate, albeit consistent, estimates of  $\delta_1$  and  $\delta_2$ —called  $\widehat{\delta}_{1 \text{IV}}$  and  $\delta_{2IV}$ , respectively—are obtained by regressing the within residuals on  $\mathbf{Z}_{1i}$  and  $\mathbf{Z}_{2i}$ , using  $\mathbf{X}_{1it}$  and  $\mathbf{Z}_{1i}$  as instruments. The order condition for identification requires that the number of variables in  $X_{1it}$ ,  $k_1$ , be at least as large as the number of elements in  $Z_{2i}$ ,  $g_2$  and that there be sufficient correlation between the instruments and  $\mathbf{Z}_{2i}$  to avoid a weak-instrument problem.

The within estimates of  $\beta_1$  and  $\beta_2$  and the intermediate estimates  $\delta_{1\text{IV}}$  and  $\delta_{2\text{IV}}$  can be used to obtain sets of within and overall residuals. These two sets of residuals can be used to estimate the variance components (see *Methods and formulas* for details).

The estimated variance components can then be used to perform a GLS transform on each of the variables. For what follows, define the general notation  $\check{w}_{it}$  to represent the GLS transform of the variable  $w_{it}$ ,  $\overline{w}_i$  to represent the within-panel mean of  $w_{it}$ , and  $\widetilde{w}_{it}$  to represent the within transform of  $w_{it}$ . With this notational convention, the Hausman-Taylor (1981) estimator of the coefficients of interest can be obtained by the instrumental-variables regression

$$\check{y}_{it} = \check{\mathbf{X}}_{1it}\boldsymbol{\beta}_1 + \check{\mathbf{X}}_{2it}\boldsymbol{\beta}_2 + \check{\mathbf{Z}}_{1i}\boldsymbol{\delta}_1 + \check{\mathbf{Z}}_{2i}\boldsymbol{\delta}_2 + \check{\mu}_i + \check{\epsilon}_{it} \tag{1}$$

using  $\widetilde{\mathbf{X}}_{1it}$ ,  $\widetilde{\mathbf{X}}_{2it}$ ,  $\overline{\mathbf{X}}_{1i}$ ,  $\overline{\mathbf{X}}_{2i}$ , and  $\mathbf{Z}_{1i}$  as instruments.

For the instruments to be valid, this estimator requires that  $\overline{\mathbf{X}}_{1i}$  and  $\mathbf{Z}_{1i}$  be uncorrelated with the random-effect  $\mu_i$ . More precisely, the instruments are valid when

$$\operatorname{plim}_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \overline{\mathbf{X}}_{1i.} \mu_i = 0$$

and

$$\operatorname{plim}_{n\to\infty} \frac{1}{n} \sum_{i=1}^{n} \mathbf{Z}_{1i} \mu_i = 0$$

Amemiya and MaCurdy (1986) place stricter requirements on the instruments that vary within panels to obtain a more efficient estimator. Specifically, Amemiya and MaCurdy (1986) assume that  $\mathbf{X}_{1it}$  is orthogonal to  $\mu_i$  in every period; that is,  $\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^n\mathbf{X}_{1it}\mu_i=0$  for  $t=1,\ldots,T$ . With this restriction, they derive the Amemiya–MaCurdy estimator as the instrumental-variables regression of (1) using instruments  $\widetilde{\mathbf{X}}_{1it}$ ,  $\widetilde{\mathbf{X}}_{2it}$ ,  $\mathbf{X}_{1it}^*$ , and  $\mathbf{Z}_{1i}$ . The order condition for the Amemiya–MaCurdy estimator is now  $Tk_1>g_2$ . xthtaylor uses the Amemiya–MaCurdy estimator when the amacurdy option is specified.

### Example 1

This example replicates the results of Baltagi and Khanti-Akom (1990, table II, column HT) using 595 observations on individuals over 1976–1982 that were extracted from the Panel Study of Income Dynamics (PSID). In the model, the log-transformed wage lwage is assumed to be a function of how long the person has worked for a firm, wks; binary variables indicating whether a person lives in a large metropolitan area or in the south, smsa and south; marital status is ms; years of education, ed; a quadratic of work experience, exp and exp2; occupation, occ; a binary variable indicating employment in a manufacture industry, ind; a binary variable indicating that wages are set by a union contract, union; a binary variable indicating gender, fem; and a binary variable indicating whether the individual is African American, blk.

We suspect that the time-varying variables exp, exp2, wks, ms, and union are all correlated with the unobserved individual random effect. We can inspect these variables to see if they exhibit sufficient within-panel variation to serve as their own instruments.

- . use http://www.stata-press.com/data/r13/psidextract
- . xtsum exp exp2 wks ms union

Variable	е	Mean	Std. Dev.	Min	Max	Observ	ations
exp	overall between within	19.85378	10.96637 10.79018 2.00024	1 4 16.85378	51 48 22.85378	N = n = T =	4165 595 7
exp2	overall between within	514.405	496.9962 489.0495 90.44581	1 20 231.405	2601 2308 807.405	N = n = T =	4165 595 7
wks	overall between within	46.81152	5.129098 3.284016 3.941881	5 31.57143 12.2401	52 51.57143 63.66867	N = n = T =	4165 595 7
ms	overall between within	.8144058	.3888256 .3686109 .1245274	0 0 0427371	1 1 1.671549	N = n = T =	4165 595 7
union	overall between within	.3639856	.4812023 .4543848 .1593351	0 0 4931573	1 1 1.221128	N = n = T =	4165 595 7

We are also going to assume that the exogenous variables occ, south, smsa, ind, fem, and blk are instruments for the endogenous, time-invariant variable ed. The output below indicates that although fem appears to be a weak instrument, the remaining instruments are probably sufficiently correlated to identify the coefficient on ed. (See Baltagi and Khanti-Akom [1990] for more discussion.)

. correlate fem blk occ south smsa ind ed (obs=4165)

	fem	blk	occ	south	smsa	ind	ed
fem	1.0000						
blk	0.2086	1.0000					
осс	-0.0847	0.0837	1.0000				
south	0.0516	0.1218	0.0413	1.0000			
smsa	0.1044	0.1154	-0.2018	-0.1350	1.0000		
ind	-0.1778	-0.0475	0.2260	-0.0769	-0.0689	1.0000	
ed	-0.0012	-0.1196	-0.6194	-0.1216	0.1843	-0.2365	1.0000

We will assume that the correlations are strong enough and proceed with the estimation. The output below gives the Hausman-Taylor estimates for this model.

```
. xthtaylor lwage occ south smsa ind exp exp2 wks ms union fem blk ed,
> endog(exp exp2 wks ms union ed)
```

Hausman-Taylor estimation	Number of obs =	4165
Group variable: id	Number of groups =	595
	Obs per group: min =	- 7
	avg =	- 7
	max =	= 7
Random effects u_i ~ i.i.d.	Wald chi2(12) =	0001.01
	Prob > chi2 =	0.0000

lwage	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]		
TVexogenous								
осс	0207047	.0137809	-1.50	0.133	0477149	.0063055		
south	.0074398	.031955	0.23	0.816	0551908	.0700705		
smsa	0418334	.0189581	-2.21	0.027	0789906	0046761		
ind	.0136039	.0152374	0.89	0.372	0162608	.0434686		
TVendogenous								
exp	.1131328	.002471	45.79	0.000	.1082898	.1179758		
exp2	0004189	.0000546	-7.67	0.000	0005259	0003119		
wks	.0008374	.0005997	1.40	0.163	0003381	.0020129		
ms	0298508	.01898	-1.57	0.116	0670508	.0073493		
union	.0327714	.0149084	2.20	0.028	.0035514	.0619914		
TIexogenous								
fem	1309236	.126659	-1.03	0.301	3791707	.1173234		
blk	2857479	.1557019	-1.84	0.066	5909179	.0194221		
TIendogenous								
ed	.137944	.0212485	6.49	0.000	.0962977	.1795902		
_cons	2.912726	. 2836522	10.27	0.000	2.356778	3.468674		
sigma_u	.94180304							
sigma_e	.15180273							
rho	.97467788	(fraction of variance due to u_i)						

Note: TV refers to time varying; TI refers to time invariant.

The estimated  $\sigma_{\mu}$  and  $\sigma_{\epsilon}$  are 0.9418 and 0.1518, respectively, indicating that a large fraction of the total error variance is attributed to  $\mu_i$ . The z statistics indicate that several the coefficients may not be significantly different from zero. Whereas the coefficients on the time-invariant variables fem and blk have relatively large standard errors, the standard error for the coefficient on ed is relatively small.

Baltagi and Khanti-Akom (1990) also present evidence that the efficiency gains of the Amemiya-MaCurdy estimator over the Hausman-Taylor estimator are small for these data. This point is especially important given the additional restrictions that the estimator places on the data-generating process. The output below replicates the Baltagi and Khanti-Akom (1990) results from column AM of table II.

max =

7

4

. xthtaylor lwage occ south smsa ind exp exp2 wks ms union fem blk ed, > endog(exp exp2 wks ms union ed) amacurdy

Amemiya-MaCurdy estimation Number of obs 4165 Group variable: id Number of groups 595 Time variable: t Obs per group: min = 7 7 avg =

Random effects u\_i ~ i.i.d. Wald chi2(12) 6879.20 Prob > chi2 0.0000

	Coef.	Std. Err.	z	P> z	[OE% Conf	 Intervall
lwage	Coel.	sta. EII.	Z	P/ Z	[95% COIII.	Interval]
TVexogenous						
осс	0208498	.0137653	-1.51	0.130	0478292	.0061297
south	.0072818	.0319365	0.23	0.820	0553126	.0698761
smsa	0419507	.0189471	-2.21	0.027	0790864	0048149
ind	.0136289	.015229	0.89	0.371	0162194	.0434771
TVendogenous						
exp	.1129704	.0024688	45.76	0.000	.1081316	.1178093
exp2	0004214	.0000546	-7.72	0.000	0005283	0003145
wks	.0008381	.0005995	1.40	0.162	0003368	.002013
ms	0300894	.0189674	-1.59	0.113	0672649	.0070861
union	.0324752	.0148939	2.18	0.029	.0032837	.0616667
TIexogenous						
fem	132008	.1266039	-1.04	0.297	380147	.1161311
blk	2859004	.1554857	-1.84	0.066	5906468	.0188459
TIendogenous						
ed	.1372049	.0205695	6.67	0.000	.0968894	.1775205
_cons	2.927338	.2751274	10.64	0.000	2.388098	3.466578
sigma_u sigma_e rho	.94180304 .15180273 .97467788	(fraction of variance due to u_i)				

Note: TV refers to time varying; TI refers to time invariant.

#### ☐ Technical note

We mentioned earlier that insufficient correlation between an endogenous variable and the instruments can give rise to a weak-instrument problem. Suppose that we simulate data for a model of the form

$$y = 3 + 3x_{1a} + 3x_{1b} + 3x_2 + 3z_1 + 3z_2 + u_i + e_{it}$$

and purposely construct the instruments so that they exhibit little correlation with the endogenous variable  $z_2$ .

- . use http://www.stata-press.com/data/r13/xthtaylor1
- . correlate ui z1 z2 x1a x1b x2 eit
  (obs=10000)

	ui	z1	z2	x1a	x1b	x2	eit
ui	1.0000						
z1	0.0268	1.0000					
z2	0.8777	0.0286	1.0000				
x1a	-0.0145	0.0065	-0.0034	1.0000			
x1b	0.0026	0.0079	0.0038	-0.0030	1.0000		
x2	0.8765	0.0191	0.7671	-0.0192	0.0037	1.0000	
eit	0.0060	-0.0198	0.0123	-0.0100	-0.0138	0.0092	1.0000

In the output below, weak instruments have serious consequences on the estimates produced by xthtaylor. The estimate of the coefficient on z2 is three times larger than its true value, and its standard error is rather large. Without sufficient correlation between the endogenous variable and its instruments in a given sample, there is insufficient information for identifying the parameter. Also, given the results of Stock, Wright, and Yogo (2002), weak instruments will cause serious size distortions in any tests performed.

Number of obs

Number of groups

10000

26.32868

3.675269

1000

. xthtaylor yit x1a x1b x2 z1 z2, endog(x2 z2)

9.525973

2.837072

8.729479

3.1657492

.88377062

Hausman-Taylor estimation

Group variable: id

\_cons

sigma\_e

rho

-						
				Obs per	group: min = avg = max =	= 10
Random effects	s u_i ~ i.i.d	Wald ch: Prob > 0		= 24172.91 = 0.0000		
yit	Coef.	Std. Err.	z	P> z	[95% Conf	. Interval]
TVexogenous						
x1a	2.959736	.0330233	89.63	0.000	2.895011	3.02446
x1b	2.953891	.0333051	88.69	0.000	2.888614	3.019168
TVendogenous						
x2	3.022685	.033085	91.36	0.000	2.957839	3.08753
TIexogenous						
z1	2.709179	.587031	4.62	0.000	1.55862	3.859739
TIendogenous						

1.11

6.63

(fraction of variance due to u\_i)

0.266

0.000

-7.276732

1.998875

Note: TV refers to time varying; TI refers to time invariant.

8.572966

.4276595

## Example 2

Now let's consider why we might want to specify the  $constant(varlist_{ti})$  option. For this example, we will use simulated data. In the output below, we fit a model over the full sample. Note the placement in the output of the coefficient on the exogenous variable x1c.

- . use http://www.stata-press.com/data/r13/xthtaylor2
- . xthtaylor yit x1a x1b x1c x2 z1 z2, endog(x2 z2)

. Autology of year and and all all all, chaos (all all)								
Hausman-Taylor Group variable		Number Number	of obs = of groups =	10000				
				Obs per	group: min = avg = max =	10		
Random effects	s u_i ~ i.i.d.	•		Wald ch Prob >		10011.00		
yit	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]		
TVexogenous								
x1a	3.023647	.0570274	53.02	0.000	2.911875	3.135418		
x1b	2.966666	.0572659	51.81	0.000	2.854427	3.078905		
x1c	.2355318	.123502	1.91	0.057	0065276	.4775912		
TVendogenous								
x2	14.17476	3.128385	4.53	0.000	8.043234	20.30628		
TIexogenous								
z1	1.741709	.4280022	4.07	0.000	.9028398	2.580578		
TIendogenous								
z2	7.983849	.6970903	11.45	0.000	6.617577	9.350121		
_cons	2.146038	.3794179	5.66	0.000	1.402393	2.889684		
sigma_u sigma_e	5.6787791 3.1806188							

Note: TV refers to time varying; TI refers to time invariant.

rho

.76120931

Now suppose that we want to fit the model using only the first eight periods. Below, x1c now appears under the TIexogenous heading rather than the TVexogenous heading because x1c is time invariant in the subsample defined by t<9.

(fraction of variance due to u\_i)

```
. xthtaylor yit x1a x1b x1c x2 z1 z2 if t<9, endog(x2 z2)
Hausman-Taylor estimation
                                                Number of obs
                                                                           8000
Group variable: id
                                                Number of groups =
                                                                           1000
                                                Obs per group: min =
                                                                              8
                                                               avg =
                                                               max =
                                                                              8
Random effects u_i ~ i.i.d.
                                                Wald chi2(6)
                                                                   = 15354.87
                                                Prob > chi2
                                                                        0.0000
```

yit	Coef.	Std. Err.	z	P> z	[95% Conf.	Interval]
TVexogenous						
x1a	3.051966	.0367026	83.15	0.000	2.98003	3.123901
x1b	2.967822	.0368144	80.62	0.000	2.895667	3.039977
TVendogenous						
x2	.7361217	3.199764	0.23	0.818	-5.5353	7.007543
TIexogenous						
x1c	3.215907	.5657191	5.68	0.000	2.107118	4.324696
z1	3.347644	.5819756	5.75	0.000	2.206992	4.488295
TIendogenous						
<b>z</b> 2	2.010578	1.143982	1.76	0.079	231586	4.252742
_cons	3.257004	.5295828	6.15	0.000	2.219041	4.294967
sigma_u	15.445594					
sigma_e	3.175083					
rho	.95945606	(fraction	of varia	nce due t	o u_i)	

Note: TV refers to time varying; TI refers to time invariant.

To prevent a variable from becoming time invariant, you can use either  $constant(varlist_{ti})$  or  $varying(varlist_{tv})$ .  $constant(varlist_{ti})$  specifies the subset of variables in varlist that are time invariant and requires the remaining variables in varlist to be time varying. If you specify  $constant(varlist_{ti})$  and any of the variables contained in  $varlist_{ti}$  are time varying, or if any of the variables not contained in  $varlist_{ti}$  are time invariant,  $varlist_{ti}$  are time varying and will not perform the estimation and will issue an error message.

```
. xthtaylor yit x1a x1b x1c x2 z1 z2 if t<9, endog(x2 z2) constant(z1 z2)
x1c not included in -constant()-.
r(198);</pre>
```

The same thing happens when you use the varying ( $varlist_{tv}$ ) option.

#### Stored results

xthtaylor stores the following in e():

```
Scalars
    e(N)
                           number of observations
    e(N_g)
                           number of groups
    e(df_m)
                           model degrees of freedom
    e(df_r)
                           residual degrees of freedom (small only)
                           smallest group size
    e(g_min)
    e(g_avg)
                           average group size
    e(g_max)
                           largest group size
    e(Tcon)
                           1 if panels balanced; 0 otherwise
    e(sigma_u)
                           panel-level standard deviation
    e(sigma_e)
                           standard deviation of \epsilon_{it}
    e(chi2)
                           \chi^2
    e(rho)
                           model F (small only)
    e(F)
    e(Tbar)
                           harmonic mean of group sizes
    e(rank)
                           rank of e(V)
Macros
    e(cmd)
                           xthtaylor
    e(cmdline)
                           command as typed
    e(depvar)
                           name of dependent variable
                           variable denoting groups
    e(ivar)
    e(tvar)
                           variable denoting time within groups, amacurdy only
    e(TVexogenous)
                           exogenous time-varying variables
    e(TIexogenous)
                           exogenous time-invariant variables
    e(TVendogenous)
                           endogenous time-varying variables
    e(TIendogenous)
                           endogenous time-invariant variables
    e(wtype)
                           weight type
    e(wexp)
                           weight expression
                           Hausman-Taylor or Amemiya-MaCurdy
    e(title)
    e(chi2type)
                           Wald; type of model \chi^2 test
                           vcetype specified in vce()
    e(vce)
    e(vcetype)
                           title used to label Std. Err.
    e(properties)
    e(predict)
                           program used to implement predict
Matrices
    e(b)
                           coefficient vector
    e(V)
                           variance-covariance matrix of the estimators
Functions
    e(sample)
                           marks estimation sample
```

## Methods and formulas

Consider an error-components model of the form

$$y_{it} = \mathbf{X}_{1it}\boldsymbol{\beta}_1 + \mathbf{X}_{2it}\boldsymbol{\beta}_2 + \mathbf{Z}_{1i}\delta_1 + \mathbf{Z}_{2i}\delta_2 + \mu_i + \epsilon_{it}$$
 (2)

for  $i=1,\ldots,n$  and, for each  $i,t=1,\ldots,T_i$ , of which  $T_i$  periods are observed; n is the number of panels in the sample. The covariates in  $\mathbf{X}$  are time varying, and the covariates in  $\mathbf{Z}$  are time invariant. Both  $\mathbf{X}$  and  $\mathbf{Z}$  are decomposed into two parts. The covariates in  $\mathbf{X}_1$  and  $\mathbf{Z}_1$  are assumed to be uncorrelated with  $\mu_i$  and  $e_{it}$ , whereas the covariates in  $\mathbf{X}_2$  and  $\mathbf{Z}_2$  are allowed to be correlated with  $\mu_i$  but not with  $\epsilon_{it}$ . Hausman and Taylor (1981) suggest an instrumental-variable estimator for this model.

For some variable w, the within transformation of w is defined as

$$\widetilde{w}_{it} = w_{it} - \overline{w}_{i.}$$
  $\overline{w}_{i.} = \frac{1}{n} \sum_{t=1}^{T_i} w_{it}$ 

Because the within estimator removes Z, the within transformation reduces the model to

$$\widetilde{y}_{it} = \widetilde{\mathbf{X}}_{1it}\boldsymbol{\beta}_1 + \widetilde{\mathbf{X}}_{2it}\boldsymbol{\beta}_2 + \widetilde{\epsilon}_{it}$$

The within estimators  $\widehat{\beta}_{1w}$  and  $\widehat{\beta}_{2w}$  are consistent for  $\beta_1$  and  $\beta_2$ , but they may not be efficient. Also, note that the within estimator cannot estimate  $\delta_1$  and  $\delta_2$ .

From the within estimator, we can be obtain an estimate of the idiosyncratic error component,  $\sigma_{\epsilon}^2$ , as

$$\widehat{\sigma}_{\epsilon}^2 = \frac{\text{RSS}}{N - n}$$

where RSS is the residual sum of squares from the within regression and N is the total number of observations in the sample.

Using the results of the within estimation, we can define

$$\overline{d}_{it} = \overline{y}_{it} - \overline{X}_{1it}\widehat{\beta}_{1w} - \overline{X}_{2it}\widehat{\beta}_{2w}$$

where  $\overline{y}_{it}$ ,  $\overline{X}_{1it}$ , and  $\overline{X}_{2it}$  contain the panel level means of these variables in all observations.

Regressing  $\overline{d}_{it}$  on  $\mathbf{Z}_1$  and  $\mathbf{Z}_2$ , using  $\mathbf{X}_1$  and  $\mathbf{Z}_1$  as instruments, provides intermediate, consistent estimates of  $\delta_1$  and  $\delta_2$ , which we will call  $\widehat{\delta}_{1\mathrm{IV}}$  and  $\widehat{\delta}_{2\mathrm{IV}}$ .

Using the within estimates,  $\hat{\delta}_{1IV}$ , and  $\hat{\delta}_{2IV}$ , we can obtain an estimate of the variance of the random effect,  $\sigma_{\mu}^2$ . First, let

$$\widehat{e}_{it} = \left(y_{it} - \mathbf{X}_{1it}\widehat{\boldsymbol{\beta}}_{1w} - \mathbf{X}_{2it}\widehat{\boldsymbol{\beta}}_{2w} - \mathbf{Z}_{1it}\widehat{\boldsymbol{\delta}}_{1IV} - \mathbf{Z}_{2it}\widehat{\boldsymbol{\delta}}_{2IV}\right)$$

Then define

$$s^{2} = \frac{1}{N} \sum_{i=1}^{n} \sum_{t=1}^{T_{i}} \left( \frac{1}{T_{i}} \sum_{t=1}^{T_{i}} \widehat{e}_{it} \right)^{2}$$

Hausman and Taylor (1981) showed that, for balanced panels,

$$\mathrm{plim}_{n\to\infty}s^2=T\sigma_\mu^2+\sigma_\epsilon^2$$

For unbalanced panels,

$$\mathrm{plim}_{n\to\infty}s^2=\overline{T}\sigma_u^2+\sigma_\epsilon^2$$

where

$$\overline{T} = \frac{n}{\sum_{i=1}^{n} \frac{1}{T_i}}$$

After we plug in  $\hat{\sigma}_{\epsilon}^2$ , our consistent estimate for  $\sigma_{\epsilon}^2$ , a little algebra suggests the estimate

$$\widehat{\sigma}_{\mu}^2 = (s^2 - \widehat{\sigma}_{\epsilon}^2)(\overline{T})^{-1}$$

Define  $\widehat{\theta}_i$  as

$$\widehat{\theta}_i = 1 - \left(\frac{\widehat{\sigma}_{\epsilon}^2}{\widehat{\sigma}_{\epsilon}^2 + T_i \widehat{\sigma}_{\mu}^2}\right)^{\frac{1}{2}}$$

With  $\widehat{\theta}_i$  in hand, we can perform the standard random-effects GLS transform on each of the variables. The transform is given by

$$w_{it}^* = w_{it} - \widehat{\theta}_i \overline{w}_{i.}$$

where  $\overline{w}_i$  is the within-panel mean.

We can then obtain the Hausman-Taylor estimates of the coefficients in (2) and the conventional VCE by fitting an instrumental-variables regression of the GLS-transformed  $y_{it}^*$  on  $\mathbf{X}_{it}^*$  and  $\mathbf{Z}_{it}^*$ , with instruments  $\widetilde{\mathbf{X}}_{it}$ ,  $\overline{\mathbf{X}}_{1i}$ , and  $\mathbf{Z}_{1i}$ .

We can obtain Amemiya-MaCurdy estimates of the coefficients in (2) and the conventional VCE by fitting an instrumental-variables regression of the GLS-transformed  $y_{it}^*$  on  $\mathbf{X}_{it}^*$  and  $\mathbf{Z}_{it}^*$ , using  $\widetilde{\mathbf{X}}_{it}$ ,  $\check{\mathbf{X}}_{1it}$ , and  $\mathbf{Z}_{1i}$  as instruments, where  $\check{\mathbf{X}}_{1it} = \mathbf{X}_{1i1}, \mathbf{X}_{1i2}, \ldots, \mathbf{X}_{1iT_i}$ . The order condition for the Amemiya-MaCurdy estimator is  $Tk_1 > g_2$ , and this estimator is available only for balanced panels.

#### References

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## Also see

[XT] xthtaylor postestimation — Postestimation tools for xthtaylor

[XT] **xtset** — Declare data to be panel data

[XT] **xtivreg** — Instrumental variables and two-stage least squares for panel-data models

[XT] **xtreg** — Fixed-, between-, and random-effects and population-averaged linear models

[U] 20 Estimation and postestimation commands