Title

varnorm — Test for normally distributed disturbances after var or svar

Syntax

`varnorm [ , options ]`

options	Description
`jbera`	report Jarque–Bera statistic; default is to report all three statistics
`skewness`	report skewness statistic; default is to report all three statistics
`kurtosis`	report kurtosis statistic; default is to report all three statistics
`estimates(`*estname*`)`	use previously stored results *estname*; default is to use active results
`cholesky`	use Cholesky decomposition
`separator(`*#*`)`	draw separator line after every # rows

`varnorm` can be used only after `var` or `svar`; see [TS] **var** and [TS] **var svar**.
You must `tsset` your data before using `varnorm`; see [TS] **tsset**.

Menu

Statistics > Multivariate time series > VAR diagnostics and tests > Test for normally distributed disturbances

Description

`varnorm` computes and reports a series of statistics against the null hypothesis that the disturbances in a VAR are normally distributed. For each equation, and for all equations jointly, up to three statistics may be computed: a skewness statistic, a kurtosis statistic, and the Jarque–Bera statistic. By default, all three statistics are reported.

Options

`jbera` requests that the Jarque–Bera statistic and any other explicitly requested statistic be reported. By default, the Jarque–Bera, skewness, and kurtosis statistics are reported.

`skewness` requests that the skewness statistic and any other explicitly requested statistic be reported. By default, the Jarque–Bera, skewness, and kurtosis statistics are reported.

`kurtosis` requests that the kurtosis statistic and any other explicitly requested statistic be reported. By default, the Jarque–Bera, skewness, and kurtosis statistics are reported.

`estimates(`*estname*`)` specifies that `varnorm` use the previously obtained set of `var` or `svar` estimates stored as *estname*. By default, `varnorm` uses the active results. See [R] **estimates** for information on manipulating estimation results.

`cholesky` specifies that `varnorm` use the Cholesky decomposition of the estimated variance–covariance matrix of the disturbances, $\widehat{\mathbf{\Sigma}}$, to orthogonalize the residuals when `varnorm` is applied to `svar` results. By default, when `varnorm` is applied to `svar` results, it uses the estimated structural decomposition $\widehat{\mathbf{A}}^{-1}\widehat{\mathbf{B}}$ on $\widehat{\mathbf{C}}$ to orthogonalize the residuals. When applied to `var e()` results, `varnorm` always uses the Cholesky decomposition of $\widehat{\mathbf{\Sigma}}$. For this reason, the `cholesky` option may not be specified when using `var` results.

`separator(#)` specifies how often separator lines should be drawn between rows. By default, separator lines do not appear. For example, `separator(1)` would draw a line between each row, `separator(2)` between every other row, and so on.

Remarks and examples

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Some of the postestimation statistics for VAR and SVAR assume that the K disturbances have a K-dimensional multivariate normal distribution. `varnorm` uses the estimation results produced by `var` or `svar` to produce a series of statistics against the null hypothesis that the K disturbances in the VAR are normally distributed.

Per the notation in Lütkepohl (2005), call the skewness statistic $\widehat{\lambda}_1$, the kurtosis statistic $\widehat{\lambda}_2$, and the Jarque–Bera statistic $\widehat{\lambda}_3$. The Jarque–Bera statistic is a combination of the other two statistics. The single-equation results are from tests against the null hypothesis that the disturbance for that particular equation is normally distributed. The results for all the equations are from tests against the null hypothesis that the K disturbances follow a K-dimensional multivariate normal distribution. Failure to reject the null hypothesis indicates a lack of model misspecification.

▷ Example 1: After var

We refit the model with German data described in [TS] **var** and then call `varnorm`.

```
. use http://www.stata-press.com/data/r13/lutkepohl2
(Quarterly SA West German macro data, Bil DM, from Lutkepohl 1993 Table E.1)
. var dln_inv dln_inc dln_consump if qtr<=tq(1978q4), dfk
  (output omitted)
. varnorm
  Jarque-Bera test
```

Equation	chi2	df	Prob > chi2
dln_inv	2.821	2	0.24397
dln_inc	3.450	2	0.17817
dln_consump	1.566	2	0.45702
ALL	7.838	6	0.25025

```
  Skewness test
```

Equation	Skewness	chi2	df	Prob > chi2
dln_inv	.11935	0.173	1	0.67718
dln_inc	-.38316	1.786	1	0.18139
dln_consump	-.31275	1.190	1	0.27532
ALL		3.150	3	0.36913

```
  Kurtosis test
```

Equation	Kurtosis	chi2	df	Prob > chi2
dln_inv	3.9331	2.648	1	0.10367
dln_inc	3.7396	1.664	1	0.19710
dln_consump	2.6484	0.376	1	0.53973
ALL		4.688	3	0.19613

```
  dfk estimator used in computations
```

In this example, neither the single-equation Jarque–Bera statistics nor the joint Jarque–Bera statistic come close to rejecting the null hypothesis.

The skewness and kurtosis results have similar structures.

The Jarque–Bera results use the sum of the skewness and kurtosis statistics. The skewness and kurtosis results are based on the skewness and kurtosis coefficients, respectively. See *Methods and formulas*.

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▷ Example 2: After svar

The test statistics are computed on the orthogonalized VAR residuals; see *Methods and formulas*. When `varnorm` is applied to `var` results, `varnorm` uses a Cholesky decomposition of the estimated variance–covariance matrix of the disturbances, $\widehat{\boldsymbol{\Sigma}}$, to orthogonalize the residuals.

By default, when `varnorm` is applied to `svar` estimation results, it uses the estimated structural decomposition $\widehat{\mathbf{A}}^{-1}\widehat{\mathbf{B}}$ on $\widehat{\mathbf{C}}$ to orthogonalize the residuals of the underlying VAR. Alternatively, when `varnorm` is applied to `svar` results and the `cholesky` option is specified, `varnorm` uses the Cholesky decomposition of $\widehat{\boldsymbol{\Sigma}}$ to orthogonalize the residuals of the underlying VAR.

We fit an SVAR that is based on an underlying VAR with two lags that is the same as the one fit in the previous example. We impose a structural decomposition that is the same as the Cholesky decomposition, as illustrated in [TS] **var svar**.

```
. matrix a = (.,0,0\.,.,0\.,.,.)
. matrix b = I(3)
. svar dln_inv dln_inc dln_consump if qtr<=tq(1978q4), dfk aeq(a) beq(b)
  (output omitted )
. varnorm
```

Jarque-Bera test

Equation	chi2	df	Prob > chi2
dln_inv	2.821	2	0.24397
dln_inc	3.450	2	0.17817
dln_consump	1.566	2	0.45702
ALL	7.838	6	0.25025

Skewness test

Equation	Skewness	chi2	df	Prob > chi2
dln_inv	.11935	0.173	1	0.67718
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Kurtosis test

Equation	Kurtosis	chi2	df	Prob > chi2
dln_inv	3.9331	2.648	1	0.10367
dln_inc	3.7396	1.664	1	0.19710
dln_consump	2.6484	0.376	1	0.53973
ALL		4.688	3	0.19613

```
dfk estimator used in computations
```

Because the estimated structural decomposition is the same as the Cholesky decomposition, the `varnorm` results are the same as those from the previous example.

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❑ Technical note

The statistics computed by `varnorm` depend on $\widehat{\Sigma}$, the estimated variance–covariance matrix of the disturbances. `var` uses the maximum likelihood estimator of this matrix by default, but the `dfk` option produces an estimator that uses a small-sample correction. Thus specifying `dfk` in the call to `var` or `svar` will affect the test results produced by `varnorm`.

❑

Stored results

varnorm stores the following in r():

Macros	
r(dfk)	dfk, if specified
Matrices	
r(kurtosis)	kurtosis test, df, and p-values
r(skewness)	skewness test, df, and p-values
r(jb)	Jarque–Bera test, df, and p-values

Methods and formulas

varnorm is based on the derivations found in Lütkepohl (2005, 174–181). Let $\widehat{\mathbf{u}}_t$ be the $K \times 1$ vector of residuals from the K equations in a previously fitted VAR or the residuals from the K equations of the VAR underlying a previously fitted SVAR. Similarly, let $\widehat{\boldsymbol{\Sigma}}$ be the estimated covariance matrix of the disturbances. (Note that $\widehat{\boldsymbol{\Sigma}}$ depends on whether the dfk option was specified.) The skewness, kurtosis, and Jarque–Bera statistics must be computed using the orthogonalized residuals.

Because

$$\widehat{\boldsymbol{\Sigma}} = \widehat{\mathbf{P}}\widehat{\mathbf{P}}'$$

implies that

$$\widehat{\mathbf{P}}^{-1}\widehat{\boldsymbol{\Sigma}}\widehat{\mathbf{P}}^{-1\prime} = \mathbf{I}_K$$

premultiplying $\widehat{\mathbf{u}}_t$ by $\widehat{\mathbf{P}}$ is one way of performing the orthogonalization. When varnorm is applied to var results, $\widehat{\mathbf{P}}$ is defined to be the Cholesky decomposition of $\widehat{\boldsymbol{\Sigma}}$. When varnorm is applied to svar results, $\widehat{\mathbf{P}}$ is set, by default, to the estimated structural decomposition; that is, $\widehat{\mathbf{P}} = \widehat{\mathbf{A}}^{-1}\widehat{\mathbf{B}}$, where $\widehat{\mathbf{A}}$ and $\widehat{\mathbf{B}}$ are the svar estimates of the $\mathbf{A}$ and $\mathbf{B}$ matrices, or $\widehat{\mathbf{C}}$, where $\widehat{\mathbf{C}}$ is the long-run SVAR estimation of $\mathbf{C}$. (See [TS] var svar for more on the origin and estimation of the $\mathbf{A}$ and $\mathbf{B}$ matrices.) When varnorm is applied to svar results and the cholesky option is specified, $\widehat{\mathbf{P}}$ is set to the Cholesky decomposition of $\widehat{\boldsymbol{\Sigma}}$.

Define $\widehat{\mathbf{w}}_t$ to be the orthogonalized VAR residuals given by

$$\widehat{\mathbf{w}}_t = (\widehat{w}_{1t}, \ldots, \widehat{w}_{Kt})' = \widehat{\mathbf{P}}^{-1}\widehat{\mathbf{u}}_t$$

The $K \times 1$ vectors of skewness and kurtosis coefficients are then computed using the orthogonalized residuals by

$$\widehat{\mathbf{b}}_1 = (\widehat{b}_{11}, \ldots, \widehat{b}_{K1})'; \qquad \widehat{b}_{k1} = \frac{1}{T}\sum_{i=1}^{T} \widehat{w}_{kt}^3$$

$$\widehat{\mathbf{b}}_2 = (\widehat{b}_{12}, \ldots, \widehat{b}_{K2})'; \qquad \widehat{b}_{k2} = \frac{1}{T}\sum_{i=1}^{T} \widehat{w}_{kt}^4$$

Under the null hypothesis of multivariate Gaussian disturbances,

$$\widehat{\lambda}_1 = \frac{T\widehat{\mathbf{b}}_1'\widehat{\mathbf{b}}_1}{6} \quad \xrightarrow{d} \quad \chi^2(K)$$

$$\widehat{\lambda}_2 = \frac{T(\widehat{\mathbf{b}}_2 - 3)'(\widehat{\mathbf{b}}_2 - 3)}{24} \quad \xrightarrow{d} \quad \chi^2(K)$$

and

$$\widehat{\lambda}_3 = \widehat{\lambda}_1 + \widehat{\lambda}_2 \quad \xrightarrow{d} \quad \chi^2(2K)$$

$\widehat{\lambda}_1$ is the skewness statistic, $\widehat{\lambda}_2$ is the kurtosis statistic, and $\widehat{\lambda}_3$ is the Jarque–Bera statistic.

$\widehat{\lambda}_1$, $\widehat{\lambda}_2$, and $\widehat{\lambda}_3$ are for tests of the null hypothesis that the $K \times 1$ vector of disturbances follows a multivariate normal distribution. The corresponding statistics against the null hypothesis that the disturbances from the kth equation come from a univariate normal distribution are

$$\widehat{\lambda}_{1k} = \frac{T\,\widehat{b}_{k1}^{\,2}}{6} \quad \xrightarrow{d} \quad \chi^2(1)$$

$$\widehat{\lambda}_{2k} = \frac{T\,(\widehat{b}_{k2}^{\,2} - 3)^2}{24} \quad \xrightarrow{d} \quad \chi^2(1)$$

and

$$\widehat{\lambda}_{3k} = \widehat{\lambda}_1 + \widehat{\lambda}_2 \quad \xrightarrow{d} \quad \chi^2(2)$$

References

Hamilton, J. D. 1994. *Time Series Analysis*. Princeton: Princeton University Press.

Jarque, C. M., and A. K. Bera. 1987. A test for normality of observations and regression residuals. *International Statistical Review* 2: 163–172.

Lütkepohl, H. 1993. *Introduction to Multiple Time Series Analysis*. 2nd ed. New York: Springer.

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Also see

[TS] **var** — Vector autoregressive models

[TS] **var svar** — Structural vector autoregressive models

[TS] **varbasic** — Fit a simple VAR and graph IRFs or FEVDs

[TS] **var intro** — Introduction to vector autoregressive models