Title

estat aroots — Check the stability condition of ARIMA estimates

Syntax

`estat aroots` [, *options*]

options	Description
`nograph`	suppress graph of eigenvalues for the companion matrices
`dlabel`	label eigenvalues with the distance from the unit circle
`modlabel`	label eigenvalues with the modulus
Grid	
`nogrid`	suppress polar grid circles
`pgrid(`[...]`)`	specify radii and appearance of polar grid circles; see *Options* for details
Plot	
marker_options	change look of markers (color, size, etc.)
Reference unit circle	
`rlopts(`*cline_options*`)`	affect rendition of reference unit circle
Y axis, X axis, Titles, Legend, Overall	
twoway_options	any options other than `by()` documented in [G-3] ***twoway_options***

Menu for estat

Statistics > Postestimation > Reports and statistics

Description

`estat aroots` checks the eigenvalue stability condition after estimating the parameters of an ARIMA model using `arima`. A graph of the eigenvalues of the companion matrices for the AR and MA polynomials is also produced.

`estat aroots` is available only after `arima`; see [TS] **arima**.

Options

`nograph` specifies that no graph of the eigenvalues of the companion matrices be drawn.

`dlabel` labels each eigenvalue with its distance from the unit circle. `dlabel` cannot be specified with `modlabel`.

`modlabel` labels the eigenvalues with their moduli. `modlabel` cannot be specified with `dlabel`.

Grid

`nogrid` suppresses the polar grid circles.

`pgrid(`[*numlist*] [`,` *line_options*]`)` determines the radii and appearance of the polar grid circles. By default, the graph includes nine polar grid circles with radii 0.1, 0.2, ..., 0.9 that have the `grid` line style. The *numlist* specifies the radii for the polar grid circles. The *line_options* determine the appearance of the polar grid circles; see [G-3] ***line_options***. Because the `pgrid()` option can be repeated, circles with different radii can have distinct appearances.

Plot

marker_options specify the look of markers. This look includes the marker symbol, the marker size, and its color and outline; see [G-3] ***marker_options***.

Reference unit circle

`rlopts(`*cline_options*`)` affect the rendition of the reference unit circle; see [G-3] ***cline_options***.

Y axis, X axis, Titles, Legend, Overall

twoway_options are any of the options documented in [G-3] ***twoway_options***, except `by()`. These include options for titling the graph (see [G-3] ***title_options***) and for saving the graph to disk (see [G-3] ***saving_option***).

Remarks and examples

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Inference after `arima` requires that the variable y_t be covariance stationary. The variable y_t is covariance stationary if its first two moments exist and are time invariant. More explicitly, y_t is covariance stationary if

1. $E(y_t)$ is finite and not a function of t;
2. $\text{Var}(y_t)$ is finite and independent of t; and
3. $\text{Cov}(y_t, y_s)$ is a finite function of $|t - s|$ but not of t or s alone.

The stationarity of an ARMA process depends on the autoregressive (AR) parameters. If the inverse roots of the AR polynomial all lie inside the unit circle, the process is stationary, invertible, and has an infinite-order moving-average (MA) representation. Hamilton (1994, chap. 1) shows that if the modulus of each eigenvalue of the matrix $\mathbf{F}(\boldsymbol{\rho})$ is strictly less than 1, the estimated ARMA is stationary; see *Methods and formulas* for the definition of the matrix $\mathbf{F}(\boldsymbol{\rho})$.

The MA part of an ARMA process can be rewritten as an infinite-order AR process provided that the MA process is invertible. Hamilton (1994, chap. 1) shows that if the modulus of each eigenvalue of the matrix $\mathbf{F}(\boldsymbol{\theta})$ is strictly less than 1, the estimated ARMA is invertible; see *Methods and formulas* for the definition of the matrix $\mathbf{F}(\boldsymbol{\theta})$.

▷ Example 1

In this example, we check the stability condition of the SARIMA model that we fit in example 3 of [TS] **arima**. We begin by reestimating the parameters of the model.

```
. use http://www.stata-press.com/data/r13/air2
(TIMESLAB: Airline passengers)
. generate lnair = ln(air)
```

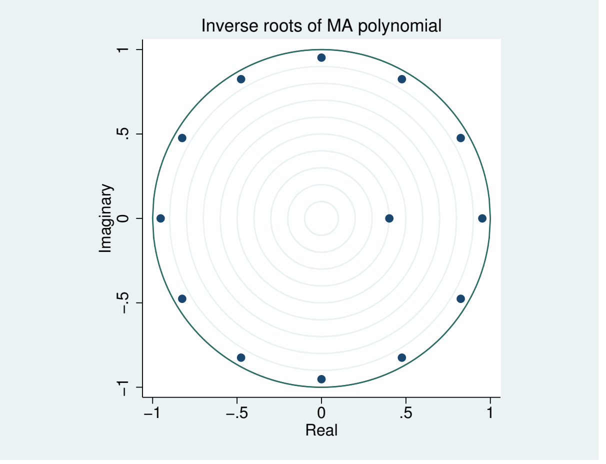
```
. arima lnair, arima(0,1,1) sarima(0,1,1,12) noconstant
(setting optimization to BHHH)
Iteration 0:   log likelihood =   223.8437
Iteration 1:   log likelihood =  239.80405
Iteration 2:   log likelihood =  244.10265
Iteration 3:   log likelihood =  244.65895
Iteration 4:   log likelihood =  244.68945
(switching optimization to BFGS)
Iteration 5:   log likelihood =  244.69431
Iteration 6:   log likelihood =  244.69647
Iteration 7:   log likelihood =  244.69651
Iteration 8:   log likelihood =  244.69651
ARIMA regression
Sample:  14 - 144                               Number of obs     =        131
                                                Wald chi2(2)      =      84.53
Log likelihood =  244.6965                      Prob > chi2       =     0.0000

------------------------------------------------------------------------------
             |                 OPG
  DS12.lnair |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
ARMA         |
          ma |
         L1. |  -.4018324   .0730307    -5.50   0.000    -.5449698   -.2586949
-------------+----------------------------------------------------------------
ARMA12       |
          ma |
         L1. |  -.5569342   .0963129    -5.78   0.000     -.745704   -.3681644
-------------+----------------------------------------------------------------
      /sigma |   .0367167   .0020132    18.24   0.000     .0327708    .0406625
------------------------------------------------------------------------------
Note: The test of the variance against zero is one sided, and the two-sided
    confidence interval is truncated at zero.
```

We can now use `estat aroots` to check the stability condition of the MA part of the model.

```
. estat aroots
   Eigenvalue stability condition
```

Eigenvalue	Modulus
.824798 + .4761974*i*	.952395
.824798 - .4761974*i*	.952395
.9523947	.952395
-.824798 + .4761974*i*	.952395
-.824798 - .4761974*i*	.952395
-.4761974 + .824798*i*	.952395
-.4761974 - .824798*i*	.952395
2.776e-16 + .9523947*i*	.952395
2.776e-16 - .9523947*i*	.952395
.4761974 + .824798*i*	.952395
.4761974 - .824798*i*	.952395
-.9523947	.952395
.4018324	.401832

```
   All the eigenvalues lie inside the unit circle.
   MA parameters satisfy invertibility condition.
```



Because the modulus of each eigenvalue is strictly less than 1, the MA process is invertible and can be represented as an infinite-order AR process.

The graph produced by `estat aroots` displays the eigenvalues with the real components on the x axis and the imaginary components on the y axis. The graph indicates visually that these eigenvalues are just inside the unit circle.

◁

Stored results

`aroots` stores the following in `r()`:

Matrices

`r(Re_ar)`	real part of the eigenvalues of $F(\rho)$
`r(Im_ar)`	imaginary part of the eigenvalues of $F(\rho)$
`r(Modulus_ar)`	modulus of the eigenvalues of $F(\rho)$
`r(ar)`	$F(\rho)$, the AR companion matrix
`r(Re_ma)`	real part of the eigenvalues of $F(\theta)$
`r(Im_ma)`	imaginary part of the eigenvalues of $F(\theta)$
`r(Modulus_ma)`	modulus of the eigenvalues of $F(\theta)$
`r(ma)`	$F(\theta)$, the MA companion matrix

Methods and formulas

Recall the general form of the ARMA model,

$$\boldsymbol{\rho}(L^p)(y_t - \mathbf{x}_t\boldsymbol{\beta}) = \boldsymbol{\theta}(L^q)\epsilon_t$$

where

$$\boldsymbol{\rho}(L^p) = 1 - \rho_1 L - \rho_2 L^2 - \cdots - \rho_p L^p$$
$$\boldsymbol{\theta}(L^q) = 1 + \theta_1 L + \theta_2 L^2 + \cdots + \theta_q L^q$$

and $L^j y_t = y_{t-j}$.

estat aroots forms the companion matrix

$$\mathbf{F}(\gamma) = \begin{pmatrix} \gamma_1 & \gamma_2 & \dots & \gamma_{r-1} & \gamma_r \\ 1 & 0 & \dots & 0 & 0 \\ 0 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \dots & 1 & 0 \end{pmatrix}$$

where $\boldsymbol{\gamma} = \boldsymbol{\rho}$ and $r = p$ for the AR part of ARMA, and $\boldsymbol{\gamma} = -\boldsymbol{\theta}$ and $r = q$ for the MA part of ARMA. aroots obtains the eigenvalues of $\mathbf{F}$ by using matrix eigenvalues. The modulus of the complex eigenvalue $r + c\text{i}$ is $\sqrt{r^2 + c^2}$. As shown by Hamilton (1994, chap. 1), a process is stable and invertible if the modulus of each eigenvalue of $\mathbf{F}$ is strictly less than 1.

Reference

Hamilton, J. D. 1994. *Time Series Analysis*. Princeton: Princeton University Press.

Also see

[TS] **arima** — ARIMA, ARMAX, and other dynamic regression models