**Cumulative spectral distribution**

### Syntax

```stata
cumsp varname [if] [in] [ , options ]
```

### Options

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<th>Description</th>
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<tr>
<td><code>generate(newvar)</code></td>
<td>create <code>newvar</code> holding distribution values</td>
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<td><strong>Plot</strong></td>
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<tr>
<td><code>cline_options</code></td>
<td>affect rendition of the plotted points connected by lines</td>
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<tr>
<td><code>marker_options</code></td>
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<tr>
<td><code>twoway_options</code></td>
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You must `tsset` your data before using `cumsp`; see [TS] `tsset`. Also, the time series must be dense (nonmissing with no gaps in the time variable) in the sample specified.

`varname` may contain time-series operators; see [U] 11.4.4 Time-series varlists.

### Menu

Statistics > Time series > Graphs > Cumulative spectral distribution

### Description

`cumsp` plots the cumulative sample spectral-distribution function evaluated at the natural frequencies for a (dense) time series.

### Options

- **Main**
  - `generate(newvar)` specifies a new variable to contain the estimated cumulative spectral-distribution values.

- **Plot**
  - `cline_options` affect the rendition of the plotted points connected by lines; see [G-3] `cline_options`. 
marker_options specify the look of markers. This look includes the marker symbol, the marker size, and its color and outline; see [G-3] marker_options.

marker_label_options specify if and how the markers are to be labeled; see [G-3] marker_label_options.

addplot(plot) provides a way to add other plots to the generated graph; see [G-3] addplot_option.

twoway_options are any of the options documented in [G-3] twoway_options, excluding by(). These include options for titling the graph (see [G-3] title_options) and for saving the graph to disk (see [G-3] saving_option).

Remarks and examples

Example 1

Here we use the international airline passengers dataset (Box, Jenkins, and Reinsel 2008, Series G). This dataset has 144 observations on the monthly number of international airline passengers from 1949 through 1960. In the cumulative sample spectral distribution function for these data, we also request a vertical line at frequency 1/12. Because the data are monthly, there will be a pronounced jump in the cumulative sample spectral-distribution plot at the 1/12 value if there is an annual cycle in the data.

. use http://www.stata-press.com/data/r13/air2
   (TIMESLAB: Airline passengers)
. cumsp air, xline(.083333333)

The cumulative sample spectral-distribution function clearly illustrates the annual cycle.
Methods and formulas

A time series of interest is decomposed into a unique set of sinusoids of various frequencies and amplitudes.

A plot of the sinusoidal amplitudes versus the frequencies for the sinusoidal decomposition of a time series gives us the spectral density of the time series. If we calculate the sinusoidal amplitudes for a discrete set of “natural” frequencies \((1/n, 2/n, \ldots, q/n)\), we obtain the periodogram.

Let \(x(1), \ldots, x(n)\) be a time series, and let \(\omega_k = (k - 1)/n\) denote the natural frequencies for \(k = 1, \ldots, \lfloor n/2 \rfloor + 1\) where \(\lfloor \rfloor\) indicates the greatest integer function. Define

\[
C_k^2 = \frac{1}{n^2} \left| \sum_{t=1}^{n} x(t)e^{2\pi i(t-1)\omega_k} \right|^2
\]

A plot of \(nC_k^2\) versus \(\omega_k\) is then called the periodogram.

The sample spectral density may then be defined as \(\hat{f}(\omega_k) = nC_k^2\).

If we let \(\hat{f}(\omega_1), \ldots, \hat{f}(\omega_Q)\) be the sample spectral density function of the time series evaluated at the frequencies \(\omega_j = (j - 1)/Q\) for \(j = 1, \ldots, Q\) and we let \(q = \lfloor Q/2 \rfloor + 1\), then

\[
\hat{F}(\omega_k) = \frac{\sum_{i=1}^{q} \hat{f}(\omega_j)}{q} \sum_{i=1}^{k} \hat{f}(\omega_j)
\]

is the sample spectral-distribution function of the time series.

References


Also see

[TS] *tsset* — Declare data to be time-series data

[TS] *corrgram* — Tabulate and graph autocorrelations

[TS] *pergram* — Periodogram