Title

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mkspline — Linear and restricted cubic spline construction

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Syntax

Linear spline with knots at specified points
mkspline newvar1 #1 [newvar2 #2 [...]] newvark = oldvar [if] [in] [, marginal
displayknots]

Linear spline with knots equally spaced or at percentiles of data

```
mkspline stubname # = oldvar [if] [in] [weight] [, marginal pctile
displayknots]
```

Restricted cubic spline

mkspline stubname = oldvar [if] [in] [weight], cubic [<u>nknots(#) knots(numlist</u>) <u>di</u>splayknots]

fweights are allowed with the second and third syntax; see [U] 11.1.6 weight.

Menu

Data > Create or change data > Other variable-creation commands > Linear and cubic spline construction

Description

mkspline creates variables containing a linear spline or a restricted cubic spline of *oldvar*.

In the first syntax, mkspline creates $newvar_1, \ldots, newvar_k$ containing a linear spline of *oldvar* with knots at the specified $\#_1, \ldots, \#_{k-1}$.

In the second syntax, mkspline creates # variables named *stubname*1, ..., *stubname*# containing a linear spline of *oldvar*. The knots are equally spaced over the range of *oldvar* or are placed at the percentiles of *oldvar*.

In the third syntax, mkspline creates variables containing a restricted cubic spline of *oldvar*. This is also known as a natural spline. The location and spacing of the knots is determined by the specification of the nknots() and knots() options.

Options

∫ Options]

- marginal is allowed with the first or second syntax. It specifies that the new variables be constructed so that, when used in estimation, the coefficients represent the change in the slope from the preceding interval. The default is to construct the variables so that, when used in estimation, the coefficients measure the slopes for the interval.
- displayknots displays the values of the knots that were used in creating the linear or restricted cubic spline.
- pctile is allowed only with the second syntax. It specifies that the knots be placed at percentiles of the data rather than being equally spaced over the range.
- nknots(#) is allowed only with the third syntax. It specifies the number of knots that are to be used for a restricted cubic spline. This number must be between 3 and 7 unless the knot locations are specified using knots(). The default number of knots is 5.
- knots(numlist) is allowed only with the third syntax. It specifies the exact location of the knots to be used for a restricted cubic spline. The values of these knots must be given in increasing order. When this option is omitted, the default knot values are based on Harrell's recommended percentiles with the additional restriction that the smallest knot may not be less than the fifth-smallest value of *oldvar* and the largest knot may not be greater than the fifth-largest value of *oldvar*. If both nknots() and knots() are given, they must specify the same number of knots.

Remarks and examples

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Remarks are presented under the following headings:

Linear splines Restricted cubic splines

Linear splines

Linear splines allow estimating the relationship between y and x as a piecewise linear function, which is a function composed of linear segments—straight lines. One linear segment represents the function for values of x below x_0 , another linear segment handles values between x_0 and x_1 , and so on. The linear segments are arranged so that they join at x_0, x_1, \ldots , which are called the knots. An example of a piecewise linear function is shown below.



Example 1

We wish to fit a model of log income on education and age by using a piecewise linear function for age:

$$lninc = b_0 + b_1 educ + f(age) + u$$

The knots are to be placed at 10-year intervals: 20, 30, 40, 50, and 60.

- . use http://www.stata-press.com/data/r13/mksp1
- . mkspline age1 20 age2 30 age3 40 age4 50 age5 60 age6 = age, marginal
- . regress lninc educ age1-age6
- (output omitted)

Because we specified the marginal option, we could test whether the age effect is the same in the 30-40 and 40-50 intervals by asking whether the age4 coefficient is zero. With the marginal option, coefficients measure the change in slope from the preceding group. Specifying marginal changes only the interpretation of the coefficients; the same model is fit in either case. Without the marginal option, the interpretation of the coefficients would have been

$$\frac{dy}{d \texttt{age}} = \begin{cases} a_1 & \text{if age} < 20 \\ a_2 & \text{if } 20 \leq \texttt{age} < 30 \\ a_3 & \text{if } 30 \leq \texttt{age} < 40 \\ a_4 & \text{if } 40 \leq \texttt{age} < 50 \\ a_5 & \text{if } 50 \leq \texttt{age} < 60 \\ a_6 & \text{otherwise} \end{cases}$$

With the marginal option, the interpretation is

$$\frac{dy}{d \mathsf{age}} = \begin{cases} a_1 & \text{if } \mathsf{age} < 20 \\ a_1 + a_2 & \text{if } 20 \leq \mathsf{age} < 30 \\ a_1 + a_2 + a_3 & \text{if } 30 \leq \mathsf{age} < 40 \\ a_1 + a_2 + a_3 + a_4 & \text{if } 40 \leq \mathsf{age} < 50 \\ a_1 + a_2 + a_3 + a_4 + a_5 & \text{if } 50 \leq \mathsf{age} < 60 \\ a_1 + a_2 + a_3 + a_4 + a_5 + a_6 & \text{otherwise} \end{cases}$$

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Example 2

Say that we have a binary outcome variable called outcome. We are beginning an analysis and wish to parameterize the effect of dosage on outcome. We wish to divide the data into five equal-width groups of dosage for the piecewise linear function.

. use http://www.stata-press.com/data/r13/mksp2, clear . mkspline dose 5 = dosage, displayknots

| 1 | | 1 5 | | |
|---------------------------------|---------------|-------|-------|-------|
| | knot1 | knot2 | knot3 | knot4 |
| dosage | 20 | 40 | 60 | 80 |
| . logistic out (output omitted) | come dose1-do | se5 | | |

mkspline dose 5 = dosage creates five variables—dose1, dose2, ..., dose5—equally spacing the knots over the range of dosage. Because dosage varied between 0 and 100, the mkspline command above has the same effect as typing

. mkspline dose1 20 dose2 40 dose3 60 dose4 80 dose5 = dosage

The pctile option sets the knots to divide the data into five equal sample-size groups rather than five equal-width ranges. Typing

| . mkspline pc | tdose 5 = dosa | | | |
|---------------|----------------|-------|-------|-------|
| | knot1 | knot2 | knot3 | knot4 |
| dosage | 16 | 36.4 | 55.6 | 82 |

places the knots at the 20th, 40th, 60th, and 80th percentiles of the data.

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Restricted cubic splines

A linear spline can be used to fit many functions well. However, a restricted cubic spline may be a better choice than a linear spline when working with a very curved function. When using a restricted cubic spline, one obtains a continuous smooth function that is linear before the first knot, a piecewise cubic polynomial between adjacent knots, and linear again after the last knot.

Example 3

Returning to the data from example 1, we may feel that a curved function is a better fit. First, we will use the knots() option to specify the five knots that we used previously.

- . use http://www.stata-press.com/data/r13/mksp1, clear
- . mkspline agesp = age, cubic knots(20 30 40 50 60)
- . regress lninc educ agesp* (output omitted)

Harrell (2001, 23) recommends placing knots at equally spaced percentiles of the original variable's marginal distribution. If we do not specify the knots() option, variables will be created containing a restricted cubic spline with five knots determined by Harrell's default percentiles.

- . use http://www.stata-press.com/data/r13/mksp1, clear
- . mkspline agesp = age, cubic displayknots
- . regress lninc educ agesp* (output omitted)

4

Methods and formulas

Methods and formulas are presented under the following headings:

Linear splines Restricted cubic splines

Linear splines

Let V_i , i = 1, ..., n, be the variables to be created; k_i , i = 1, ..., n - 1, be the corresponding knots; and \mathcal{V} be the original variable (the command is mkspline V_1 k_1 V_2 k_2 ... $V_n = \mathcal{V}$). Then

$$V_{1} = \min(\mathcal{V}, k_{1})$$
$$V_{i} = \max\left\{\min(\mathcal{V}, k_{i}), k_{i-1}\right\} - k_{i-1} \quad i = 2, \dots, n-1$$
$$V_{n} = \max(\mathcal{V}, k_{n-1}) - k_{n-1}$$

If the marginal option is specified, the definitions are

$$V_1 = \mathcal{V}$$
$$V_i = \max(0, \mathcal{V} - k_{i-1}) \quad i = 2, \dots, n$$

In the second syntax, mkspline *stubname* $\# = \mathcal{V}$, so let m and M be the minimum and maximum of \mathcal{V} . Without the pctile option, knots are set at m + (M - m)(i/n) for $i = 1, \ldots, n - 1$. If pctile is specified, knots are set at the 100(i/n) percentiles, for $i = 1, \ldots, n - 1$. Percentiles are calculated by centile; see [R] centile.

Restricted cubic splines

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Let k_i , i = 1, ..., n, be the knot values; V_i , i = 1, ..., n-1, be the variables to be created; and \mathcal{V} be the original variable. Then

$$V_{1} = \mathcal{V}$$

$$V_{i+1} = \frac{(\mathcal{V} - k_{i})_{+}^{3} - (k_{n} - k_{n-1})_{-}^{-1} \{ (\mathcal{V} - k_{n-1})_{+}^{3} (k_{n} - k_{i}) - (\mathcal{V} - k_{n})_{+}^{3} (k_{n-1} - k_{i}) \}}{(k_{n} - k_{1})^{2}}$$

$$i = 1, \dots, n-2$$

where

$$(u)_{+} = \begin{cases} u, \text{ if } \quad u > 0\\ 0, \text{ if } \quad u \le 0 \end{cases}$$

Without the knots() option, the locations of the knots are determined by the percentiles recommended in Harrell (2001, 23). These percentiles are based on the chosen number of knots as follows:

| 110. | | | | | | | | |
|----------|-------------|-------|-------|------|-------|-------|------|--|
| of knots | Percentiles | | | | | | | |
| 3 | 10 | 50 | 90 | | | | | |
| 4 | 5 | 35 | 65 | 95 | | | | |
| 5 | 5 | 27.5 | 50 | 72.5 | 95 | | | |
| 6 | 5 | 23 | 41 | 59 | 77 | 95 | | |
| 7 | 2.5 | 18.33 | 34.17 | 50 | 65.83 | 81.67 | 97.5 | |

Harrell provides default percentiles when the number of knots is between 3 and 7. When using a number of knots outside this range, the location of the knots must be specified in knots().

Acknowledgment

The restricted cubic spline portion of mkspline is based on the rc_spline command by William Dupont of the Department of Biostatistics at Vanderbilt University.

References

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Also see

[R] **fp** — Fractional polynomial regression