Title

glm — Generalized linear models

Syntax

glm *depvar* [*indepvars*] [*if*] [*in*] [*weight*] [, *options*]

options	Description
Model	
family(*familyname*)	distribution of *depvar*; default is family(gaussian)
link(*linkname*)	link function; default is canonical link for family() specified
Model 2	
noconstant	suppress constant term
exposure(*varname*)	include ln(*varname*) in model with coefficient constrained to 1
offset(*varname*)	include *varname* in model with coefficient constrained to 1
constraints(*constraints*)	apply specified linear constraints
collinear	keep collinear variables
asis	retain perfect predictor variables
mu(*varname*)	use *varname* as the initial estimate for the mean of *depvar*
init(*varname*)	synonym for mu(*varname*)
SE/Robust	
vce(*vcetype*)	*vcetype* may be oim, robust, cluster *clustvar*, eim, opg, bootstrap, jackknife, hac *kernel*, jackknife1, or unbiased
vfactor(#)	multiply variance matrix by scalar #
disp(#)	quasilikelihood multiplier
scale(x2 | dev | #)	set the scale parameter
Reporting	
level(#)	set confidence level; default is level(95)
eform	report exponentiated coefficients
nocnsreport	do not display constraints
display_options	control column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
ml	use maximum likelihood optimization; the default
irls	use iterated, reweighted least-squares optimization of the deviance
maximize_options	control the maximization process; seldom used
fisher(#)	use the Fisher scoring Hessian or expected information matrix (EIM)
search	search for good starting values

noheader	suppress header table from above coefficient table
notable	suppress coefficient table
nodisplay	suppress the output; iteration log is still displayed
coeflegend	display legend instead of statistics

familyname	Description
gaussian	Gaussian (normal)
igaussian	inverse Gaussian
binomial $\left[\, varname_N \,\vert\, \#_N \,\right]$	Bernoulli/binomial
poisson	Poisson
nbinomial $\left[\, \#_k \,\vert\, \text{ml} \,\right]$	negative binomial
gamma	gamma

linkname	Description
identity	identity
log	log
logit	logit
probit	probit
cloglog	cloglog
power #	power
opower #	odds power
nbinomial	negative binomial
loglog	log-log
logc	log-complement

indepvars may contain factor variables; see [U] **11.4.3 Factor variables**.

depvar and *indepvars* may contain time-series operators; see [U] **11.4.4 Time-series varlists**.

bootstrap, by, fp, jackknife, mfp, mi estimate, nestreg, rolling, statsby, stepwise, and svy are allowed; see [U] **11.1.10 Prefix commands**.

vce(bootstrap), vce(jackknife), and vce(jackknife1) are not allowed with the mi estimate prefix; see [MI] **mi estimate**.

Weights are not allowed with the bootstrap prefix; see [R] **bootstrap**.

aweights are not allowed with the jackknife prefix; see [R] **jackknife**.

vce(), vfactor(), disp(), scale(), irls, fisher(), noheader, notable, nodisplay, and weights are not allowed with the svy prefix; see [SVY] **svy**.

fweights, aweights, iweights, and pweights are allowed; see [U] **11.1.6 weight**.

noheader, notable, nodisplay, and coeflegend do not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

Menu

Statistics > Generalized linear models > Generalized linear models (GLM)

Description

glm fits generalized linear models. It can fit models by using either IRLS (maximum quasilikelihood) or Newton–Raphson (maximum likelihood) optimization, which is the default.

See [U] **26 Overview of Stata estimation commands** for a description of all of Stata's estimation commands, several of which fit models that can also be fit using `glm`.

Options

Model

family(*familyname*) specifies the distribution of *depvar*; family(gaussian) is the default.

link(*linkname*) specifies the link function; the default is the canonical link for the family() specified (except for family(nbinomial)).

Model 2

noconstant, exposure(*varname*), offset(*varname*), constraints(*constraints*), collinear; see [R] **estimation options**. constraints(*constraints*) and collinear are not allowed with irls.

asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] **probit**. This option is only allowed with option family(binomial) with a denominator of 1.

mu(*varname*) specifies *varname* as the initial estimate for the mean of *depvar*. This option can be useful with models that experience convergence difficulties, such as family(binomial) models with power or odds-power links. init(*varname*) is a synonym.

SE/Robust

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] **vce_option**.

In addition to the standard *vcetype*s, glm allows the following alternatives:

vce(eim) specifies that the EIM estimate of variance be used.

vce(jackknife1) specifies that the one-step jackknife estimate of variance be used.

vce(hac *kernel* [*#*]) specifies that a heteroskedasticity- and autocorrelation-consistent (HAC) variance estimate be used. HAC refers to the general form for combining weighted matrices to form the variance estimate. There are three kernels built into glm. *kernel* is a user-written program or one of

$$\underline{n}\text{west} \mid \text{gallant} \mid \underline{an}\text{derson}$$

specifies the number of lags. If *#* is not specified, $N - 2$ is assumed. If you wish to specify vce(hac ...), you must tsset your data before calling glm.

vce(unbiased) specifies that the unbiased sandwich estimate of variance be used.

vfactor(*#*) specifies a scalar by which to multiply the resulting variance matrix. This option allows you to match output with other packages, which may apply degrees of freedom or other small-sample corrections to estimates of variance.

disp(*#*) multiplies the variance of *depvar* by *#* and divides the deviance by *#*. The resulting distributions are members of the quasilikelihood family.

scale(x2 | dev | *#*) overrides the default scale parameter. This option is allowed only with Hessian (information matrix) variance estimates.

By default, scale(1) is assumed for the discrete distributions (binomial, Poisson, and negative binomial), and scale(x2) is assumed for the continuous distributions (Gaussian, gamma, and inverse Gaussian).

scale(x2) specifies that the scale parameter be set to the Pearson chi-squared (or generalized chi-squared) statistic divided by the residual degrees of freedom, which is recommended by McCullagh and Nelder (1989) as a good general choice for continuous distributions.

scale(dev) sets the scale parameter to the deviance divided by the residual degrees of freedom. This option provides an alternative to scale(x2) for continuous distributions and overdispersed or underdispersed discrete distributions.

scale(#) sets the scale parameter to #. For example, using scale(1) in family(gamma) models results in exponential-errors regression. Additional use of link(log) rather than the default link(power -1) for family(gamma) essentially reproduces Stata's streg, dist(exp) nohr command (see [ST] **streg**) if all the observations are uncensored.

┌─ Reporting ───

level(#); see [R] **estimation options**.

eform displays the exponentiated coefficients and corresponding standard errors and confidence intervals. For family(binomial) link(logit) (that is, logistic regression), exponentiation results are odds ratios; for family(nbinomial) link(log) (that is, negative binomial regression) and for family(poisson) link(log) (that is, Poisson regression), exponentiated coefficients are incidence-rate ratios.

nocnsreport; see [R] **estimation options**.

display_options: noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvla-bel, fvwrap(#), fvwrapon(*style*), cformat(%*fmt*), pformat(%*fmt*), sformat(%*fmt*), and nolstretch; see [R] **estimation options**.

┌─ Maximization ──

ml requests that optimization be carried out using Stata's ml commands and is the default.

irls requests iterated, reweighted least-squares (IRLS) optimization of the deviance instead of Newton–Raphson optimization of the log likelihood. If the irls option is not specified, the optimization is carried out using Stata's ml commands, in which case all options of ml maximize are also available.

maximize_options: difficult, technique(*algorithm_spec*), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(*init_specs*); see [R] **maximize**. These options are seldom used.

Setting the optimization type to technique(bhhh) resets the default *vcetype* to vce(opg).

fisher(#) specifies the number of Newton–Raphson steps that should use the Fisher scoring Hessian or EIM before switching to the observed information matrix (OIM). This option is useful only for Newton–Raphson optimization (and not when using irls).

search specifies that the command search for good starting values. This option is useful only for Newton–Raphson optimization (and not when using irls).

The following options are available with glm but are not shown in the dialog box:

noheader suppresses the header information from the output. The coefficient table is still displayed.

notable suppresses the table of coefficients from the output. The header information is still displayed.

nodisplay suppresses the output. The iteration log is still displayed.

coeflegend; see [R] **estimation options**.

Remarks and examples

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Remarks are presented under the following headings:

General use
Variance estimators
User-defined functions

General use

glm fits generalized linear models of y with covariates $\mathbf{x}$:

$$g\{E(y)\} = \mathbf{x}\boldsymbol{\beta}, \qquad y \sim F$$

$g(\,)$ is called the link function, and F is the distributional family. Substituting various definitions for $g(\,)$ and F results in a surprising array of models. For instance, if y is distributed as Gaussian (normal) and $g(\,)$ is the identity function, we have

$$E(y) = \mathbf{x}\boldsymbol{\beta}, \qquad y \sim \text{Normal}$$

or linear regression. If $g(\,)$ is the logit function and y is distributed as Bernoulli, we have

$$\text{logit}\{E(y)\} = \mathbf{x}\boldsymbol{\beta}, \qquad y \sim \text{Bernoulli}$$

or logistic regression. If $g(\,)$ is the natural log function and y is distributed as Poisson, we have

$$\ln\{E(y)\} = \mathbf{x}\boldsymbol{\beta}, \qquad y \sim \text{Poisson}$$

or Poisson regression, also known as the log-linear model. Other combinations are possible.

Although glm can be used to perform linear regression (and, in fact, does so by default), this regression should be viewed as an instructional feature; regress produces such estimates more quickly, and many postestimation commands are available to explore the adequacy of the fit; see [R] **regress** and [R] **regress postestimation**.

In any case, you specify the link function by using the link() option and specify the distributional family by using family(). The available link functions are

Link function	glm option
identity	link(identity)
log	link(log)
logit	link(logit)
probit	link(probit)
complementary log-log	link(cloglog)
odds power	link(opower #)
power	link(power #)
negative binomial	link(nbinomial)
log-log	link(loglog)
log-complement	link(logc)

Define $\mu = E(y)$ and $\eta = g(\mu)$, meaning that $g(\cdot)$ maps $E(y)$ to $\eta = \mathbf{x}\boldsymbol{\beta} + \text{offset}$.

Link functions are defined as follows:

identity is defined as $\eta = g(\mu) = \mu$.

log is defined as $\eta = \ln(\mu)$.

logit is defined as $\eta = \ln\{\mu/(1-\mu)\}$, the natural log of the odds.

probit is defined as $\eta = \Phi^{-1}(\mu)$, where $\Phi^{-1}(\ )$ is the inverse Gaussian cumulative.

cloglog is defined as $\eta = \ln\{-\ln(1-\mu)\}$.

opower is defined as $\eta = \left[\{\mu/(1-\mu)\}^n - 1\right]/n$, the power of the odds. The function is generalized so that link(opower 0) is equivalent to link(logit), the natural log of the odds.

power is defined as $\eta = \mu^n$. Specifying link(power 1) is equivalent to specifying link(identity). The power function is generalized so that $\mu^0 \equiv \ln(\mu)$. Thus link(power 0) is equivalent to link(log). Negative powers are, of course, allowed.

nbinomial is defined as $\eta = \ln\{\mu/(\mu+k)\}$, where $k = 1$ if family(nbinomial) is specified, $k = \#_k$ if family(nbinomial $\#_k$) is specified, and k is estimated via maximum likelihood if family(nbinomial ml) is specified.

loglog is defined as $\eta = -\ln\{-\ln(\mu)\}$.

logc is defined as $\eta = \ln(1-\mu)$.

The available distributional families are

Family	glm option
Gaussian (normal)	family(gaussian)
inverse Gaussian	family(igaussian)
Bernoulli/binomial	family(binomial)
Poisson	family(poisson)
negative binomial	family(nbinomial)
gamma	family(gamma)

family(normal) is a synonym for family(gaussian).

The binomial distribution can be specified as 1) family(binomial), 2) family(binomial $\#_N$), or 3) family(binomial $varname_N$). In case 2, $\#_N$ is the value of the binomial denominator N, the number of trials. Specifying family(binomial 1) is the same as specifying family(binomial). In case 3, $varname_N$ is the variable containing the binomial denominator, allowing the number of trials to vary across observations.

The negative binomial distribution can be specified as 1) family(nbinomial), 2) family(nbinomial $\#_k$), or 3) family(nbinomial ml). Omitting $\#_k$ is equivalent to specifying family(nbinomial 1). In case 3, the value of $\#_k$ is estimated via maximum likelihood. The value $\#_k$ enters the variance and deviance functions. Typical values range between 0.01 and 2; see the technical note below.

You do not have to specify both family() and link(); the default link() is the canonical link for the specified family() (except for nbinomial):

Family	Default link
family(gaussian)	link(identity)
family(igaussian)	link(power -2)
family(binomial)	link(logit)
family(poisson)	link(log)
family(nbinomial)	link(log)
family(gamma)	link(power -1)

If you specify both family() and link(), not all combinations make sense. You may choose from the following combinations:

	identity	log	logit	probit	cloglog	power	opower	nbinomial	loglog	logc
Gaussian	X	X				X				
inverse Gaussian	X	X				X				
binomial	X	X	X	X	X	X	X		X	X
Poisson	X	X				X				
negative binomial	X	X				X		X		
gamma	X	X				X				

❏ Technical note

Some family() and link() combinations result in models already fit by Stata. These are

family()	link()	Options	Equivalent Stata command
gaussian	identity	*nothing* | irls | irls vce(oim)	regress
gaussian	identity	t(*var*) vce(hac nwest #) vfactor(#$_v$)	newey, t(*var*) lag(#) (see note 1)
binomial	cloglog	*nothing* | irls vce(oim)	cloglog (see note 2)
binomial	probit	*nothing* | irls vce(oim)	probit (see note 2)
binomial	logit	*nothing* | irls | irls vce(oim)	logit or logistic (see note 3)
poisson	log	*nothing* | irls | irls vce(oim)	poisson (see note 3)
nbinomial	log	*nothing* | irls vce(oim)	nbreg (see note 4)
gamma	log	scale(1)	streg, dist(exp) nohr (see note 5)

Notes:

1. The variance factor #$_v$ should be set to $n/(n-k)$, where n is the number of observations and k the number of regressors. If the number of regressors is not specified, the estimated standard errors will, as a result, differ by this factor.

2. Because the link is not the canonical link for the binomial family, you must specify the vce(oim) option if using irls to get equivalent standard errors. If irls is used without vce(oim), the regression coefficients will be the same but the standard errors will be only asymptotically equivalent. If no options are specified (*nothing*), glm will optimize using Newton–Raphson, making it equivalent to the other Stata command.

    See [R] **cloglog** and [R] **probit** for more details about these commands.

3. Because the canonical link is being used, the standard errors will be equivalent whether the EIM or the OIM estimator of variance is used.

4. Family negative binomial, log-link models—also known as negative binomial regression models—are used for data with an overdispersed Poisson distribution. Although `glm` can be used to fit such models, using Stata's maximum likelihood `nbreg` command is probably better. In the GLM approach, you specify `family(nbinomial #`$_k$`)` and then search for a $#_k$ that results in the deviance-based dispersion being 1. You can also specify `family(nbinomial ml)` to estimate $#_k$ via maximum likelihood, which will report the same value returned from `nbreg`. However, `nbreg` also reports a confidence interval for it; see [R] **nbreg** and Rogers (1993). Of course, `glm` allows links other than `log`, and for those links, including the canonical `nbinomial` link, you will need to use `glm`.

5. `glm` can be used to estimate parameters from exponential regressions, but this method requires specifying `scale(1)`. However, censoring is not available. Censored exponential regression may be modeled using `glm` with `family(poisson)`. The log of the original response is entered into a Poisson model as an offset, whereas the new response is the censor variable. The result of such modeling is identical to the log relative hazard parameterization of `streg, dist(exp) nohr`. See [ST] **streg** for details about the `streg` command.

In general, where there is overlap between a capability of `glm` and that of some other Stata command, we recommend using the other Stata command. Our recommendation is not because of some inferiority of the GLM approach. Rather, those other commands, by being specialized, provide options and ancillary commands that are missing in the broader `glm` framework. Nevertheless, `glm` does produce the same answers where it should.

Special note. When equivalence is expected, for some datasets, you may still see very slight differences in the results, most often only in the later digits of the standard errors. When you compare `glm` output to an equivalent Stata command, these tiny discrepancies arise for many reasons:

a. `glm` uses a general methodology for starting values, whereas the equivalent Stata command may be more specialized in its treatment of starting values.

b. When using a canonical link, `glm, irls` should be equivalent to the maximum likelihood method of the equivalent Stata command, yet the convergence criterion is different (one is for deviance, the other for log likelihood). These discrepancies are easily resolved by adjusting one convergence criterion to correspond to the other.

c. When both `glm` and the equivalent Stata command use Newton–Raphson, small differences may still occur if the Stata command has a different default convergence criterion from that of `glm`. Adjusting the convergence criterion will resolve the difference. See [R] **ml** and [R] **maximize** for more details.

❑

▷ Example 1

In example 1 of [R] **logistic**, we fit a model based on data from a study of risk factors associated with low birthweight (Hosmer, Lemeshow, and Sturdivant 2013, 24). We can replicate the estimation by using `glm`:

```
. use http://www.stata-press.com/data/r13/lbw
(Hosmer & Lemeshow data)

. glm low age lwt i.race smoke ptl ht ui, family(binomial) link(logit)

Iteration 0:   log likelihood =  -101.0213
Iteration 1:   log likelihood = -100.72519
Iteration 2:   log likelihood =   -100.724
Iteration 3:   log likelihood =   -100.724
```

```
Generalized linear models                      No. of obs       =         189
Optimization     : ML                          Residual df      =         180
                                               Scale parameter  =           1
Deviance         =   201.4479911               (1/df) Deviance  =    1.119156
Pearson          =   182.0233425               (1/df) Pearson   =    1.011241

Variance function: V(u) = u*(1-u)              [Bernoulli]
Link function    : g(u) = ln(u/(1-u))          [Logit]

                                               AIC              =      1.1611
Log likelihood   = -100.7239956                BIC              =  -742.0665
```

		OIM				
low	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
age	-.0271003	.0364504	-0.74	0.457	-.0985418	.0443412
lwt	-.0151508	.0069259	-2.19	0.029	-.0287253	-.0015763
race						
black	1.262647	.5264101	2.40	0.016	.2309024	2.294392
other	.8620792	.4391532	1.96	0.050	.0013548	1.722804
smoke	.9233448	.4008266	2.30	0.021	.137739	1.708951
ptl	.5418366	.346249	1.56	0.118	-.136799	1.220472
ht	1.832518	.6916292	2.65	0.008	.4769494	3.188086
ui	.7585135	.4593768	1.65	0.099	-.1418484	1.658875
_cons	.4612239	1.20459	0.38	0.702	-1.899729	2.822176

glm, by default, presents coefficient estimates, whereas `logistic` presents the exponentiated coefficients—the odds ratios. glm's `eform` option reports exponentiated coefficients, and glm, like Stata's other estimation commands, replays results.

```
. glm, eform
```

```
Generalized linear models                          No. of obs       =          189
Optimization     : ML                              Residual df      =          180
                                                   Scale parameter =            1
Deviance         =   201.4479911                   (1/df) Deviance =    1.119156
Pearson          =   182.0233425                   (1/df) Pearson  =    1.011241

Variance function: V(u) = u*(1-u)                  [Bernoulli]
Link function    : g(u) = ln(u/(1-u))              [Logit]

                                                   AIC              =      1.1611
Log likelihood   = -100.7239956                    BIC              =  -742.0665
```

low	Odds Ratio	OIM Std. Err.	z	P>|z|	[95% Conf. Interval]	
age	.9732636	.0354759	-0.74	0.457	.9061578	1.045339
lwt	.9849634	.0068217	-2.19	0.029	.9716834	.9984249
race						
black	3.534767	1.860737	2.40	0.016	1.259736	9.918406
other	2.368079	1.039949	1.96	0.050	1.001356	5.600207
smoke	2.517698	1.00916	2.30	0.021	1.147676	5.523162
ptl	1.719161	.5952579	1.56	0.118	.8721455	3.388787
ht	6.249602	4.322408	2.65	0.008	1.611152	24.24199
ui	2.1351	.9808153	1.65	0.099	.8677528	5.2534
_cons	1.586014	1.910496	0.38	0.702	.1496092	16.8134

These results are the same as those reported in example 1 of [R] **logistic**.

Included in the output header are values for the Akaike (1973) information criterion (AIC) and the Bayesian information criterion (BIC) (Raftery 1995). Both are measures of model fit adjusted for the number of parameters that can be compared across models. In both cases, a smaller value generally indicates a better model fit. AIC is based on the log likelihood and thus is available only when Newton–Raphson optimization is used. BIC is based on the deviance and thus is always available. ◁

❑ Technical note

The values for AIC and BIC reported in the output after glm are different from those reported by estat ic:

```
. estat ic
```

Akaike's information criterion and Bayesian information criterion

Model	Obs	ll(null)	ll(model)	df	AIC	BIC
.	189	.	-100.724	9	219.448	248.6237

Note: N=Obs used in calculating BIC; see **[R] BIC note**

There are various definitions of these information criteria (IC) in the literature; glm and estat ic use different definitions. glm bases its computation of the BIC on deviance, whereas estat ic uses the likelihood. Both glm and estat ic use the likelihood to compute the AIC; however, the AIC from estat ic is equal to N, the number of observations, times the AIC from glm. Refer to *Methods and formulas* in this entry and [R] **estat ic** for the references and formulas used by glm and estat ic, respectively, to compute AIC and BIC. Inferences based on comparison of IC values reported by glm

for different GLM models will be equivalent to those based on comparison of IC values reported by
`estat ic` after `glm`.

❑

▷ Example 2

We use data from an early insecticide experiment, given in Pregibon (1980). The variables are
`ldose`, the log dose of insecticide; `n`, the number of flour beetles subjected to each dose; and `r`, the
number killed.

```
. use http://www.stata-press.com/data/r13/ldose
. list, sep(4)
```

	ldose	n	r
1.	1.6907	59	6
2.	1.7242	60	13
3.	1.7552	62	18
4.	1.7842	56	28
5.	1.8113	63	52
6.	1.8369	59	53
7.	1.861	62	61
8.	1.8839	60	60

The aim of the analysis is to estimate a dose–response relationship between p, the proportion
killed, and X, the log dose.

As a first attempt, we will formulate the model as a linear logistic regression of p on `ldose`; that
is, we will take the logit of p and represent the dose–response curve as a straight line in X:

$$\ln\{p/(1-p)\} = \beta_0 + \beta_1 X$$

Because the data are grouped, we cannot use Stata's `logistic` command to fit the model. Stata does,
however, already have a command for performing logistic regression on data organized in this way,
so we could type

```
. blogit r n ldose
```

Instead, we will fit the model by using `glm`:

```
. glm r ldose, family(binomial n) link(logit)

Iteration 0:   log likelihood = -18.824848
Iteration 1:   log likelihood = -18.715271
Iteration 2:   log likelihood = -18.715123
Iteration 3:   log likelihood = -18.715123

Generalized linear models                          No. of obs      =        8
Optimization     : ML                              Residual df     =        6
                                                   Scale parameter =        1
Deviance         =   11.23220702                   (1/df) Deviance =  1.872035
Pearson          =    10.0267936                   (1/df) Pearson  =  1.671132

Variance function: V(u) = u*(1-u/n)                [Binomial]
Link function    : g(u) = ln(u/(n-u))              [Logit]

                                                   AIC             =  5.178781
Log likelihood   = -18.71512262                    BIC             = -1.244442
```

r	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf. Interval]
ldose	34.27034	2.912141	11.77	0.000	28.56265 39.97803
_cons	-60.71747	5.180713	-11.72	0.000	-70.87149 -50.56346

The only difference between `blogit` and `glm` here is how they obtain the answer. `blogit` expands the data to contain 481 observations (the sum of n) so that it can run Stata's standard, individual-level logistic command. `glm`, on the other hand, uses the information on the binomial denominator directly. We specified `family(binomial n)`, meaning that variable n contains the denominator. Parameter estimates and standard errors from the two approaches do not differ.

An alternative model, which gives asymmetric sigmoid curves for p, involves the complementary log-log, or cloglog, function:

$$\ln\{-\ln(1-p)\} = \beta_0 + \beta_1 X$$

We fit this model by using `glm`:

```
. glm r ldose, family(binomial n) link(cloglog)

Iteration 0:   log likelihood = -14.883594
Iteration 1:   log likelihood = -14.822264
Iteration 2:   log likelihood = -14.822228
Iteration 3:   log likelihood = -14.822228

Generalized linear models                          No. of obs      =        8
Optimization     : ML                              Residual df     =        6
                                                   Scale parameter =        1
Deviance         =   3.446418004                   (1/df) Deviance =  .574403
Pearson          =   3.294675153                   (1/df) Pearson  =  .5491125

Variance function: V(u) = u*(1-u/n)                [Binomial]
Link function    : g(u) = ln(-ln(1-u/n))           [Complementary log-log]

                                                   AIC             =  4.205557
Log likelihood   = -14.82222811                    BIC             = -9.030231
```

r	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf. Interval]
ldose	22.04118	1.793089	12.29	0.000	18.52679 25.55557
_cons	-39.57232	3.229047	-12.26	0.000	-45.90114 -33.24351

The complementary log-log model is preferred; the deviance for the logistic model, 11.23, is much higher than the deviance for the cloglog model, 3.45. This change also is evident by comparing log likelihoods, or equivalently, AIC values.

This example also shows the advantage of the glm command—we can vary assumptions easily. Note the minor difference in what we typed to obtain the logistic and cloglog models:

```
. glm r ldose, family(binomial n) link(logit)
. glm r ldose, family(binomial n) link(cloglog)
```

If we were performing this work for ourselves, we would have typed the commands in a more abbreviated form:

```
. glm r ldose, f(b n) l(l)
. glm r ldose, f(b n) l(cl)
```

◁

❑ Technical note

Factor variables may be used with glm. Say that, in the example above, we had ldose, the log dose of insecticide; n, the number of flour beetles subjected to each dose; and r, the number killed—all as before—except that now we have results for three different kinds of beetles. Our hypothetical data include beetle, which contains the values 1 ("Destructive flour"), 2 ("Red flour"), and 3 ("Mealworm").

```
. use http://www.stata-press.com/data/r13/beetle
. list, sep(0)
```

	beetle	ldose	n	r
1.	1	1.6907	59	6
2.	1	1.7242	60	13
3.	1	1.7552	62	18
4.	1	1.7842	56	28
5.	1	1.8113	63	52
	(output omitted)			
23.	3	1.861	64	23
24.	3	1.8839	58	22

Let's assume that, at first, we wish merely to add a shift factor for the type of beetle. We could type

```
. glm r i.beetle ldose, family(bin n) link(cloglog)
Iteration 0:   log likelihood = -79.012269
Iteration 1:   log likelihood =  -76.94951
Iteration 2:   log likelihood = -76.945645
Iteration 3:   log likelihood = -76.945645

Generalized linear models                         No. of obs      =         24
Optimization     : ML                             Residual df     =         20
                                                  Scale parameter =          1
Deviance         =   73.76505595                  (1/df) Deviance =   3.688253
Pearson          =    71.8901173                  (1/df) Pearson  =   3.594506

Variance function: V(u) = u*(1-u/n)               [Binomial]
Link function    : g(u) = ln(-ln(1-u/n))          [Complementary log-log]
                                                  AIC             =    6.74547
Log likelihood   = -76.94564525                   BIC             =   10.20398
```

```
                             OIM
          r        Coef.   Std. Err.      z     P>|z|     [95% Conf. Interval]

     beetle
  Red flour    -.0910396   .1076132    -0.85    0.398    -.3019576     .1198783
   Mealworm    -1.836058   .1307125   -14.05    0.000     -2.09225    -1.579867

      ldose     19.41558   .9954265    19.50    0.000     17.46458     21.36658
      _cons    -34.84602    1.79333   -19.43    0.000    -38.36089    -31.33116
```

We find strong evidence that the insecticide works differently on the mealworm. We now check whether the curve is merely shifted or also differently sloped:

```
. glm r beetle##c.ldose, family(bin n) link(cloglog)

Iteration 0:   log likelihood = -67.270188
Iteration 1:   log likelihood = -65.149316
Iteration 2:   log likelihood = -65.147978
Iteration 3:   log likelihood = -65.147978

Generalized linear models                       No. of obs      =         24
Optimization     : ML                           Residual df     =         18
                                                Scale parameter =          1
Deviance         =  50.16972096                 (1/df) Deviance = 2.787207
Pearson          =  49.28422567                 (1/df) Pearson  = 2.738013

Variance function: V(u) = u*(1-u/n)             [Binomial]
Link function    : g(u) = ln(-ln(1-u/n))        [Complementary log-log]

                                                AIC             = 5.928998
Log likelihood   = -65.14797776                 BIC             = -7.035248
```

```
                             OIM
          r        Coef.   Std. Err.      z     P>|z|     [95% Conf. Interval]

     beetle
  Red flour     -.79933   4.470882    -0.18    0.858    -9.562098     7.963438
   Mealworm    17.78741   4.586429     3.88    0.000     8.798172     26.77664

      ldose    22.04118   1.793089    12.29    0.000     18.52679     25.55557

    beetle#
    c.ldose
  Red flour    .3838708   2.478477     0.15    0.877    -4.473855     5.241596
   Mealworm     -10.726   2.526412    -4.25    0.000    -15.67768    -5.774321

      _cons   -39.57232   3.229047   -12.26    0.000    -45.90114    -33.24351
```

We find that the (complementary log-log) dose–response curve for the mealworm has roughly half the slope of that for the destructive flour beetle.

See [U] **25 Working with categorical data and factor variables**; what is said there concerning linear regression is applicable to any GLM model.

❑

Variance estimators

glm offers many variance options and gives different types of standard errors when used in various combinations. We highlight some of them here, but for a full explanation, see Hardin and Hilbe (2012).

▷ Example 3

Continuing with our flour beetle data, we rerun the most recently displayed model, this time requesting estimation via IRLS.

```
. use http://www.stata-press.com/data/r13/beetle
. glm r beetle##c.ldose, f(bin n) l(cloglog) ltol(1e-13) irls
Iteration 1:   deviance =  54.41414
Iteration 2:   deviance =  50.19424
Iteration 3:   deviance =  50.16973
  (output omitted )
Generalized linear models                         No. of obs      =        24
Optimization     : MQL Fisher scoring              Residual df     =        18
                   (IRLS EIM)                       Scale parameter =         1
Deviance         =  50.16972096                    (1/df) Deviance =  2.787207
Pearson          =  49.28422567                    (1/df) Pearson  =  2.738013
Variance function: V(u) = u*(1-u/n)                [Binomial]
Link function    : g(u) = ln(-ln(1-u/n))           [Complementary log-log]
                                                   BIC             = -7.035248
```

r	Coef.	EIM Std. Err.	z	P>|z|	[95% Conf. Interval]	
beetle						
Red flour	-.79933	4.586649	-0.17	0.862	-9.788997	8.190337
Mealworm	17.78741	4.624834	3.85	0.000	8.7229	26.85192
ldose	22.04118	1.799356	12.25	0.000	18.5145	25.56785
beetle# c.ldose						
Red flour	.3838708	2.544068	0.15	0.880	-4.602411	5.370152
Mealworm	-10.726	2.548176	-4.21	0.000	-15.72033	-5.731665
_cons	-39.57232	3.240274	-12.21	0.000	-45.92314	-33.2215

Note our use of the ltol() option, which, although unrelated to our discussion on variance estimation, was used so that the regression coefficients would match those of the previous Newton–Raphson (NR) fit.

Because IRLS uses the EIM for optimization, the variance estimate is also based on EIM. If we want optimization via IRLS but the variance estimate based on OIM, we specify glm, irls vce(oim):

```
. glm r beetle##c.ldose, f(b n) l(cl) ltol(1e-15) irls vce(oim) noheader nolog
```

r	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf. Interval]	
beetle						
Red flour	-.79933	4.470882	-0.18	0.858	-9.562098	7.963438
Mealworm	17.78741	4.586429	3.88	0.000	8.798172	26.77664
ldose	22.04118	1.793089	12.29	0.000	18.52679	25.55557
beetle# c.ldose						
Red flour	.3838708	2.478477	0.15	0.877	-4.473855	5.241596
Mealworm	-10.726	2.526412	-4.25	0.000	-15.67768	-5.774321
_cons	-39.57232	3.229047	-12.26	0.000	-45.90114	-33.24351

This approach is identical to NR except for the convergence path. Because the cloglog link is not the canonical link for the binomial family, EIM and OIM produce different results. Both estimators, however, are asymptotically equivalent.

Going back to NR, we can also specify vce(robust) to get the Huber/White/sandwich estimator of variance:

```
. glm r beetle##c.ldose, f(b n) l(cl) vce(robust) noheader nolog
```

r	Coef.	Robust Std. Err.	z	P>|z|	[95% Conf. Interval]	
beetle						
Red flour	-.79933	5.733049	-0.14	0.889	-12.0359	10.43724
Mealworm	17.78741	5.158477	3.45	0.001	7.676977	27.89784
ldose	22.04118	.8998551	24.49	0.000	20.27749	23.80486
beetle# c.ldose						
Red flour	.3838708	3.174427	0.12	0.904	-5.837892	6.605633
Mealworm	-10.726	2.800606	-3.83	0.000	-16.21508	-5.236912
_cons	-39.57232	1.621306	-24.41	0.000	-42.75003	-36.39462

The sandwich estimator gets its name from the form of the calculation—it is the multiplication of three matrices, with the outer two matrices (the "bread") set to the OIM variance matrix. When irls is used along with vce(robust), the EIM variance matrix is instead used as the bread. Using a result from McCullagh and Nelder (1989), Newson (1999) points out that the EIM and OIM variance matrices are equivalent under the canonical link. Thus if irls is specified with the canonical link, the resulting variance is labeled "Robust". When the noncanonical link for the family is used, which is the case in the example below, the EIM and OIM variance matrices differ, so the resulting variance is labeled "Semirobust".

```
. glm r beetle##c.ldose, f(b n) l(cl) irls ltol(1e-15) vce(robust) noheader
> nolog
```

r	Coef.	Semirobust Std. Err.	z	P>|z|	[95% Conf. Interval]	
beetle						
Red flour	-.79933	6.288963	-0.13	0.899	-13.12547	11.52681
Mealworm	17.78741	5.255307	3.38	0.001	7.487194	28.08762
ldose	22.04118	.9061566	24.32	0.000	20.26514	23.81721
beetle#						
c.ldose						
Red flour	.3838708	3.489723	0.11	0.912	-6.455861	7.223603
Mealworm	-10.726	2.855897	-3.76	0.000	-16.32345	-5.128542
_cons	-39.57232	1.632544	-24.24	0.000	-42.77205	-36.3726

The outer product of the gradient (OPG) estimate of variance is one that avoids the calculation of second derivatives. It is equivalent to the "middle" part of the sandwich estimate of variance and can be specified by using glm, vce(opg), regardless of whether NR or IRLS optimization is used.

```
. glm r beetle##c.ldose, f(b n) l(cl) vce(opg) noheader nolog
```

r	Coef.	OPG Std. Err.	z	P>|z|	[95% Conf. Interval]	
beetle						
Red flour	-.79933	6.664045	-0.12	0.905	-13.86062	12.26196
Mealworm	17.78741	6.838505	2.60	0.009	4.384183	31.19063
ldose	22.04118	3.572983	6.17	0.000	15.03826	29.0441
beetle#						
c.ldose						
Red flour	.3838708	3.700192	0.10	0.917	-6.868372	7.636114
Mealworm	-10.726	3.796448	-2.83	0.005	-18.1669	-3.285097
_cons	-39.57232	6.433101	-6.15	0.000	-52.18097	-26.96368

The OPG estimate of variance is a component of the BHHH (Berndt et al. 1974) optimization technique. This method of optimization is also available with glm with the technique() option; however, the technique() option is not allowed with the irls option.

◁

▷ Example 4

The Newey–West (1987) estimator of variance is a sandwich estimator with the "middle" of the sandwich modified to take into account possible autocorrelation between the observations. These estimators are a generalization of those given by the Stata command newey for linear regression. See [TS] **newey** for more details.

For example, consider the dataset given in [TS] **newey**, which has time-series measurements on usr and idle. We want to perform a linear regression with Newey–West standard errors.

```
. use http://www.stata-press.com/data/r13/idle2

. list usr idle time
```

	usr	idle	time
1.	0	100	1
2.	0	100	2
3.	0	97	3
4.	1	98	4
5.	2	94	5
	(output omitted)		
29.	1	98	29
30.	1	98	30

Examining *Methods and formulas* of [TS] **newey**, we see that the variance estimate is multiplied by a correction factor of $n/(n-k)$, where k is the number of regressors. `glm, vce(hac ...)` does not make this correction, so to get the same standard errors, we must use the `vfactor()` option within `glm` to make the correction manually.

```
. display 30/28
1.0714286

. tsset time
        time variable:  time, 1 to 30
                delta:  1 unit

. glm usr idle, vce(hac nwest 3) vfactor(1.0714286)

Iteration 0:   log likelihood = -71.743396
```

```
Generalized linear models                          No. of obs      =         30
Optimization     : ML                              Residual df     =         28
                                                   Scale parameter =   7.493297
Deviance         =  209.8123165                    (1/df) Deviance =   7.493297
Pearson          =  209.8123165                    (1/df) Pearson  =   7.493297

Variance function: V(u) = 1                        [Gaussian]
Link function    : g(u) = u                        [Identity]

HAC kernel (lags): Newey-West (3)
                                                   AIC             =   4.916226
Log likelihood   = -71.74339627                    BIC             =   114.5788
```

		HAC				
usr	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
idle	-.2281501	.0690928	-3.30	0.001	-.3635694	-.0927307
_cons	23.13483	6.327033	3.66	0.000	10.73407	35.53558

The `glm` command above reproduces the results given in [TS] **newey**. We may now generalize this output to models other than simple linear regression and to different kernel weights.

```
. glm usr idle, fam(gamma) link(log) vce(hac gallant 3)

Iteration 0:   log likelihood = -61.76593
Iteration 1:   log likelihood = -60.963233
Iteration 2:   log likelihood =  -60.95097
Iteration 3:   log likelihood = -60.950965

Generalized linear models                      No. of obs       =         30
Optimization     : ML                          Residual df      =         28
                                               Scale parameter =   .431296
Deviance        =  9.908506707                 (1/df) Deviance =  .3538752
Pearson         =  12.07628677                 (1/df) Pearson  =   .431296

Variance function: V(u) = u^2                  [Gamma]
Link function    : g(u) = ln(u)                [Log]

HAC kernel (lags): Gallant (3)
                                               AIC              =   4.196731
Log likelihood   = -60.95096484                BIC              = -85.32502
```

		HAC				
usr	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
idle	-.0796609	.0184647	-4.31	0.000	-.115851	-.0434708
_cons	7.771011	1.510198	5.15	0.000	4.811078	10.73094

glm also offers variance estimators based on the bootstrap (resampling your data with replacement) and the jackknife (refitting the model with each observation left out in succession). Also included is the one-step jackknife estimate, which, instead of performing full reestimation when each observation is omitted, calculates a one-step NR estimate, with the full data regression coefficients as starting values.

```
. set seed 1
. glm usr idle, fam(gamma) link(log) vce(bootstrap, reps(100) nodots)

Generalized linear models                      No. of obs       =         30
Optimization     : ML                          Residual df      =         28
                                               Scale parameter =   .431296
Deviance        =  9.908506707                 (1/df) Deviance =  .3538752
Pearson         =  12.07628677                 (1/df) Pearson  =   .431296

Variance function: V(u) = u^2                  [Gamma]
Link function    : g(u) = ln(u)                [Log]
                                               AIC              =   4.196731
Log likelihood   = -60.95096484                BIC              = -85.32502
```

	Observed	Bootstrap			Normal-based	
usr	Coef.	Std. Err.	z	P>|z|	[95% Conf. Interval]	
idle	-.0796609	.0216591	-3.68	0.000	-.1221119	-.0372099
_cons	7.771011	1.80278	4.31	0.000	4.237627	11.3044

◁

See Hardin and Hilbe (2012) for a full discussion of the variance options that go with glm and, in particular, of how the different variance estimators are modified when vce(cluster *clustvar*) is specified. Finally, not all variance options are supported with all types of weights. See help glm for a current table of the variance options that are supported with the different weights.

User-defined functions

glm may be called with a user-written link function, variance (family) function, Newey–West kernel-weight function, or any combination of the three.

Syntax of link functions

```
program progname
        version 13
        args todo eta mu return
        if `todo' == -1 {
                /* Set global macros for output */
                global SGLM_lt "title for link function"
                global SGLM_lf "subtitle showing link definition"
                exit
        }
        if `todo' == 0 {
                /* set η=g(μ) */
                /* Intermediate calculations go here */
                generate double `eta' = ...
                exit
        }
        if `todo' == 1 {
                /* set μ=g⁻¹(η) */
                /* Intermediate calculations go here */
                generate double `mu' = ...
                exit
        }
        if `todo' == 2 {
                /* set return = ∂μ/∂η */
                /* Intermediate calculations go here */
                generate double `return' = ...
                exit
        }
        if `todo' == 3 {
                /* set return = ∂²μ/∂η² */
                /* Intermediate calculations go here */
                generate double `return' = ...
                exit
        }
        display as error "Unknown call to glm link function"
        exit 198
end
```

Syntax of variance functions

```
program progname
        version 13
        args todo eta mu return
        if 'todo' == -1 {
                /* Set global macros for output */
                /* Also check that depvar is in proper range */
                /* Note: For this call, eta contains indicator for whether each obs. is in est. sample */
                global SGLM_vt "title for variance function"
                global SGLM_vf "subtitle showing function definition"
                global SGLM_mu "program to call to enforce boundary conditions on μ"
                exit
        }
        if 'todo' == 0 {
                /* set η to initial value. */
                /* Intermediate calculations go here */
                generate double 'eta' = ...
                exit
        }
        if 'todo' == 1 {
                /* set return = V(μ) */
                /* Intermediate calculations go here */
                generate double 'return' = ...
                exit
        }
        if 'todo' == 2 {
                /* set return = ∂V(μ)/∂μ */
                /* Intermediate calculations go here */
                generate double 'return' = ...
                exit
        }
        if 'todo' == 3 {
                /* set return = squared deviance (per observation) */
                /* Intermediate calculations go here */
                generate double 'return' = ...
                exit
        }
        if 'todo' == 4 {
                /* set return = Anscombe residual */
                /* Intermediate calculations go here */
                generate double 'return' = ...
                exit
        }
        if 'todo' == 5 {
                /* set return = log likelihood */
                /* Intermediate calculations go here */
                generate double 'return' = ...
                exit
        }
        if 'todo' == 6 {
                /* set return = adjustment for deviance residuals */
                /* Intermediate calculations go here */
                generate double 'return' = ...
                exit
        }
        display as error "Unknown call to glm variance function"
        exit 198
end
```

Syntax of Newey–West kernel-weight functions

```
program progname, rclass
        version 13
        args G j
        /* G  is the maximum lag */
        /* j  is the current lag */
        /*  Intermediate calculations go here */
        return scalar wt = computed weight
        return local setype "Newey-West"
        return local sewtype "name of kernel"
end
```

Global macros available for user-written programs

Global macro	Description
SGLM_V	program name of variance (family) evaluator
SGLM_L	program name of link evaluator
SGLM_y	dependent variable name
SGLM_m	binomial denominator
SGLM_a	negative binomial k
SGLM_p	power if power() or opower() is used, or
	an argument from a user-specified link function
SGLM_s1	indicator; set to one if scale is equal to one
SGLM_ph	value of scale parameter

▷ Example 5

Suppose that we wish to perform Poisson regression with a log-link function. Although this regression is already possible with standard glm, we will write our own version for illustrative purposes.

Because we want a log link, $\eta = g(\mu) = \ln(\mu)$, and for a Poisson family the variance function is $V(\mu) = \mu$.

The Poisson density is given by

$$f(y_i) = \frac{e^{-\exp(\mu_i)} e^{\mu_i y_i}}{y_i!}$$

resulting in a log likelihood of

$$L = \sum_{i=1}^{n} \{-e^{\mu_i} + \mu_i y_i - \ln(y_i!)\}$$

The squared deviance of the ith observation for the Poisson family is given by

$$d_i^2 = \begin{cases} 2\widehat{\mu}_i & \text{if } y_i = 0 \\ 2\{y_i \ln(y_i/\widehat{\mu}_i) - (y_i - \widehat{\mu}_i)\} & \text{otherwise} \end{cases}$$

We now have enough information to write our own Poisson-log `glm` module. We create the file `mylog.ado`, which contains

```
program mylog
        version 13
        args todo eta mu return
        if `todo' == -1 {
                global SGLM_lt "My Log"             //  Titles for output
                global SGLM_lf "ln(u)"
                exit
        }
        if `todo' == 0  {
                gen double `eta' = ln(`mu')        //  η = ln(μ)
                exit
        }
        if `todo' == 1  {
                gen double `mu' = exp(`eta')       //  μ = exp(η)
                exit
        }
        if `todo' == 2 {
                gen double `return' = `mu'         //  ∂μ/∂η = exp(η) = μ
                exit
        }
        if `todo' == 3 {
                gen double `return' = `mu'         //  ∂²μ/∂η² = exp(η) = μ
                exit
        }
        di as error "Unknown call to glm link function"
        exit 198
end
```

and we create the file `mypois.ado`, which contains

```
program mypois
        version 13
        args todo eta mu return
        if `todo' == -1 {
                local y      "$SGLM_y"
                local touse "`eta'"                       // `eta' marks estimation sample here
                capture assert `y'>=0 if `touse'    // check range of y
                if _rc {
                        di as error `"dependent variable `y' has negative values"'
                        exit 499
                }
                global SGLM_vt "My Poisson"         // Titles for output
                global SGLM_vf "u"
                global SGLM_mu "glim_mu 0 ."        // see note 1
                exit
        }
        if `todo' == 0 {                             // Initialization of η; see note 2
                gen double `eta' = ln(`mu')
                exit
        }
```

```
                if 'todo' == 1 {
                        gen double 'return' =  'mu'          //   V(μ) = μ
                        exit
                }
                if 'todo' == 2 {                             //   ∂ V(μ)/∂μ
                        gen byte 'return' = 1
                        exit
                }
                if 'todo' == 3 {                             //   squared deviance, defined above
                        local y "$SGLM_y"
                        if "'y'" == "" {
                                local y "'e(depvar)'"
                        }
                        gen double 'return' = cond('y'==0, 2*'mu', /*
                                */ 2*('y'*ln('y'/'mu')-('y'-'mu')))
                        exit
                }
                if 'todo' == 4 {                             //   Anscombe residual; see note 3
                        local y "$SGLM_y"
                        if "'y'" == "" {
                                local y "'e(depvar)'"
                        }
                        gen double 'return' = 1.5*('y'^(2/3)-'mu'^(2/3)) / 'mu'^(1/6)
                        exit
                }
                if 'todo' == 5 {                             //   log likelihood; see note 4
                        local y "$SGLM_y"
                        if "'y'" == "" {
                                local y "'e(depvar)'"
                        }
                        gen double 'return' = -'mu'+'y'*ln('mu')-lngamma('y'+1)
                        exit
                }
                if 'todo' == 6 {                             //   adjustment to residual; see note 5
                        gen double 'return' = 1/(6*sqrt('mu'))
                        exit
                }
                di as error "Unknown call to glm variance function"
                error 198
        end
```

Notes:

1. `glim_mu` is a Stata program that will, at each iteration, bring $\widehat{\mu}$ back into its plausible range, should it stray out of it. Here `glim_mu` is called with the arguments zero and missing, meaning that zero is the lower bound of $\widehat{\mu}$ and there exists no upper bound—such is the case for Poisson models.

2. Here the initial value of η is easy because we intend to fit this model with our user-defined log link. In general, however, the initialization may need to vary according to the link to obtain convergence. If so, the global macro SGLM_L is used to determine which link is being utilized.

3. The Anscombe formula is given here because we know it. If we were not interested in Anscombe residuals, we could merely set 'return' to missing. Also, the local macro y is set either to SGLM_y if it is in current estimation or to e(depvar) if this function is being accessed by `predict`.

4. If we were not interested in ML estimation, we could omit this code entirely and just leave an exit statement in its place. Similarly, if we were not interested in deviance or IRLS optimization, we could set 'return' in the deviance portion of the code ('todo'==3) to missing.

5. This code defines the term to be added to the predicted residuals if the `adjusted` option is specified. Again, if we were not interested, we could set 'return' to missing.

We can now test our Poisson-log module by running it on the airline data presented in [R] **poisson**.

```
. use http://www.stata-press.com/data/r13/airline
. list airline injuries n XYZowned
```

	airline	injuries	n	XYZowned
1.	1	11	0.0950	1
2.	2	7	0.1920	0
3.	3	7	0.0750	0
4.	4	19	0.2078	0
5.	5	9	0.1382	0
6.	6	4	0.0540	1
7.	7	3	0.1292	0
8.	8	1	0.0503	0
9.	9	3	0.0629	1

```
. generate lnN=ln(n)
. glm injuries XYZowned lnN, f(mypois) l(mylog) scale(1)

Iteration 0:   log likelihood = -22.557572
Iteration 1:   log likelihood = -22.332861
Iteration 2:   log likelihood = -22.332276
Iteration 3:   log likelihood = -22.332276

Generalized linear models                      No. of obs       =         9
Optimization     : ML                          Residual df      =         6
                                               Scale parameter  =         1
Deviance         =   12.70432823               (1/df) Deviance  =  2.117388
Pearson          =    12.7695081               (1/df) Pearson   =  2.128251

Variance function: V(u) = u                    [My Poisson]
Link function    : g(u) = ln(u)                [My Log]
                                               AIC              =  5.629395
Log likelihood   =  -22.33227605               BIC              = -.4790192
```

		OIM				
injuries	Coef.	Std. Err.	z	P>|z|	[95% Conf.	Interval]
XYZowned	.6840668	.3895877	1.76	0.079	-.0795111	1.447645
lnN	1.424169	.3725155	3.82	0.000	.6940517	2.154286
_cons	4.863891	.7090501	6.86	0.000	3.474178	6.253603

(Standard errors scaled using dispersion equal to square root of 1.)

These are precisely the results given in [R] **poisson** and are those that would have been given had we run `glm, family(poisson) link(log)`. The only minor adjustment we needed to make was to specify the `scale(1)` option. If `scale()` is left unspecified, `glm` assumes `scale(1)` for discrete distributions and `scale(x2)` for continuous ones. By default, `glm` assumes that any user-defined family is continuous because it has no way of checking. Thus we needed to specify `scale(1)` because our model is discrete.

Because we were careful in defining the squared deviance, we could have fit this model with IRLS. Because log is the canonical link for the Poisson family, we would not only get the same regression coefficients but also the same standard errors.

◁

▷ Example 6

Suppose now that we wish to use our log link (mylog.ado) with glm's binomial family. This task requires some modification because our current function is not equipped to deal with the binomial denominator, which we are allowed to specify. This denominator is accessible to our link function through the global macro SGLM_m. We now make the modifications and store them in mylog2.ado.

```
program mylog2                                              // <-- changed
        version 13
        args todo eta mu return
        if 'todo' == -1 {
                global SGLM_lt "My Log, Version 2"          // <-- changed
                if "$SGLM_m" == "1" {                       // <-- changed
                        global SGLM_lf "ln(u)"              // <-- changed
                }                                           // <-- changed
                else    global SGLM_lf "ln(u/$SGLM_m)"      // <-- changed
                exit
        }
        if 'todo' == 0  {
                gen double 'eta' = ln('mu'/$SGLM_m)         // <-- changed
                exit
        }
        if 'todo' == 1  {
                gen double 'mu' = $SGLM_m*exp('eta')        // <-- changed
                exit
        }
        if 'todo' == 2 {
                gen double 'return' = 'mu'
                exit
        }
        if 'todo' == 3 {
                gen double 'return' = 'mu'
                exit
        }
        di as error "Unknown call to glm link function"
        exit 198
end
```

We can now run our new log link with glm's binomial family. Using the flour beetle data from earlier, we have

```
. use http://www.stata-press.com/data/r13/beetle, clear

. glm r ldose, f(bin n) l(mylog2) irls

Iteration 1:   deviance =  2212.108
Iteration 2:   deviance =  452.9352
Iteration 3:   deviance =    429.95
Iteration 4:   deviance =  429.2745
Iteration 5:   deviance =  429.2192
Iteration 6:   deviance =  429.2082
Iteration 7:   deviance =  429.2061
Iteration 8:   deviance =  429.2057
Iteration 9:   deviance =  429.2056
Iteration 10:  deviance =  429.2056
Iteration 11:  deviance =  429.2056
Iteration 12:  deviance =  429.2056
```

```
Generalized linear models                   No. of obs      =       24
Optimization      : MQL Fisher scoring      Residual df     =       22
                    (IRLS EIM)              Scale parameter =        1
Deviance          =   429.205599            (1/df) Deviance =  19.50935
Pearson           =   413.088142            (1/df) Pearson  =  18.77673

Variance function: V(u) = u*(1-u/n)         [Binomial]
Link function     : g(u) = ln(u/n)          [My Log, Version 2]
                                            BIC             = 359.2884
```

r	Coef.	EIM Std. Err.	z	P>|z|	[95% Conf. Interval]	
ldose	8.478908	.4702808	18.03	0.000	7.557175	9.400642
_cons	-16.11006	.8723167	-18.47	0.000	-17.81977	-14.40035

◁

For a more detailed discussion on user-defined functions, and for an example of a user-defined Newey–West kernel weight, see Hardin and Hilbe (2012).

John Ashworth Nelder (1924–2010) was born in Somerset, England. He studied mathematics and statistics at Cambridge and worked as a statistician at the National Vegetable Research Station and then Rothamsted Experimental Station. In retirement, he was actively affiliated with Imperial College London. Nelder was especially well known for his contributions to the theory of linear models and to statistical computing. He was the principal architect of generalized and hierarchical generalized linear models and of the programs GenStat and GLIM.

Robert William Maclagan Wedderburn (1947–1975) was born in Edinburgh and studied mathematics and statistics at Cambridge. At Rothamsted Experimental Station, he developed the theory of generalized linear models with Nelder and originated the concept of quasilikelihood. He died of anaphylactic shock from an insect bite on a canal holiday.

Stored results

`glm, ml` stores the following in `e()`:

Scalars

e(N)	number of observations
e(k)	number of parameters
e(k_eq)	number of equations in e(b)
e(k_eq_model)	number of equations in overall model test
e(k_dv)	number of dependent variables
e(df_m)	model degrees of freedom
e(df)	residual degrees of freedom
e(phi)	scale parameter
e(aic)	model AIC
e(bic)	model BIC
e(ll)	log likelihood, if NR
e(N_clust)	number of clusters
e(chi2)	χ^2
e(p)	significance
e(deviance)	deviance
e(deviance_s)	scaled deviance
e(deviance_p)	Pearson deviance
e(deviance_ps)	scaled Pearson deviance
e(dispers)	dispersion
e(dispers_s)	scaled dispersion
e(dispers_p)	Pearson dispersion
e(dispers_ps)	scaled Pearson dispersion
e(nbml)	1 if negative binomial parameter estimated via ML, 0 otherwise
e(vf)	factor set by vfactor(), 1 if not set
e(power)	power set by power(), opower()
e(rank)	rank of e(V)
e(ic)	number of iterations
e(rc)	return code
e(converged)	1 if converged, 0 otherwise

Macros
e(cmd)	glm
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(varfunc)	program to calculate variance function
e(varfunct)	variance title
e(varfuncf)	variance function
e(link)	program to calculate link function
e(linkt)	link title
e(linkf)	link function
e(m)	number of binomial trials
e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(clustvar)	name of cluster variable
e(offset)	linear offset variable
e(chi2type)	Wald; type of model χ^2 test
e(cons)	set if noconstant specified
e(hac_kernel)	HAC kernel
e(hac_lag)	HAC lag
e(vce)	*vcetype* specified in vce()
e(vcetype)	title used to label Std. Err.
e(opt)	ml or irls
e(opt1)	optimization title, line 1
e(which)	max or min; whether optimizer is to perform maximization or minimization
e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices
e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions
e(sample)	marks estimation sample

`glm, irls` stores the following in `e()`:

Scalars
e(N)	number of observations
e(k)	number of parameters
e(k_eq_model)	number of equations in overall model test
e(df_m)	model degrees of freedom
e(df)	residual degrees of freedom
e(phi)	scale parameter
e(disp)	dispersion parameter
e(bic)	model BIC
e(N_clust)	number of clusters
e(deviance)	deviance
e(deviance_s)	scaled deviance
e(deviance_p)	Pearson deviance
e(deviance_ps)	scaled Pearson deviance
e(dispers)	dispersion
e(dispers_s)	scaled dispersion
e(dispers_p)	Pearson dispersion
e(dispers_ps)	scaled Pearson dispersion
e(nbml)	1 if negative binomial parameter estimated via ML, 0 otherwise
e(vf)	factor set by vfactor(), 1 if not set
e(power)	power set by power(), opower()
e(rank)	rank of e(V)
e(rc)	return code

Macros
e(cmd)	glm
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(varfunc)	program to calculate variance function
e(varfunct)	variance title
e(varfuncf)	variance function
e(link)	program to calculate link function
e(linkt)	link title
e(linkf)	link function
e(m)	number of binomial trials
e(wtype)	weight type
e(wexp)	weight expression
e(clustvar)	name of cluster variable
e(offset)	linear offset variable
e(cons)	set if noconstant specified
e(hac_kernel)	HAC kernel
e(hac_lag)	HAC lag
e(vce)	*vcetype* specified in vce()
e(vcetype)	title used to label Std. Err.
e(opt)	ml or irls
e(opt1)	optimization title, line 1
e(opt2)	optimization title, line 2
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved

Matrices
e(b)	coefficient vector
e(Cns)	constraints matrix
e(V)	variance–covariance matrix of the estimators
e(V_modelbased)	model-based variance

Functions
e(sample)	marks estimation sample

Methods and formulas

The canonical reference on GLM is McCullagh and Nelder (1989). The term "generalized linear model" is from Nelder and Wedderburn (1972). Many people use the acronym GLIM for GLM models because of the classic GLM software tool GLIM, by Baker and Nelder (1985). See Dobson and Barnett (2008) for a concise introduction and overview. See Rabe-Hesketh and Everitt (2007) for more examples of GLM using Stata. Hoffmann (2004) focuses on applying generalized linear models, using real-world datasets, along with interpreting computer output, which for the most part is obtained using Stata.

This discussion highlights the details of parameter estimation and predicted statistics. For a more detailed treatment, and for information on variance estimation, see Hardin and Hilbe (2012). `glm` supports estimation with survey data. For details on VCEs with survey data, see [SVY] **variance estimation**.

`glm` obtains results by IRLS, as described in McCullagh and Nelder (1989), or by maximum likelihood using Newton–Raphson. The implementation here, however, allows user-specified weights, which we denote as v_j for the jth observation. Let M be the number of "observations" ignoring weights. Define

$$
w_j = \begin{cases} 1 & \text{if no weights are specified} \\ v_j & \text{if \texttt{fweights} or \texttt{iweights} are specified} \\ Mv_j/(\sum_k v_k) & \text{if \texttt{aweights} or \texttt{pweights} are specified} \end{cases}
$$

The number of observations is then $N = \sum_j w_j$ if `fweights` are specified and $N = M$ otherwise. Each IRLS step is performed by `regress` using w_j as the weights.

Let d_j^2 denote the squared deviance residual for the jth observation:

For the Gaussian family, $d_j^2 = (y_j - \widehat{\mu}_j)^2$.

For the Bernoulli family (binomial with denominator 1),

$$
d_j^2 = \begin{cases} -2\ln(1 - \widehat{\mu}_j) & \text{if } y_j = 0 \\ -2\ln(\widehat{\mu}_j) & \text{otherwise} \end{cases}
$$

For the binomial family with denominator m_j,

$$
d_j^2 = \begin{cases} 2y_j\ln(y_j/\widehat{\mu}_j) + 2(m_j - y_j)\ln\{(m_j - y_j)/(m_j - \widehat{\mu}_j)\} & \text{if } 0 < y_j < m_j \\ 2m_j\ln\{m_j/(m_j - \widehat{\mu}_j)\} & \text{if } y_j = 0 \\ 2y_j\ln(y_j/\widehat{\mu}_j) & \text{if } y_j = m_j \end{cases}
$$

For the Poisson family,

$$
d_j^2 = \begin{cases} 2\widehat{\mu}_j & \text{if } y_j = 0 \\ 2\{y_j\ln(y_j/\widehat{\mu}_j) - (y_j - \widehat{\mu}_j)\} & \text{otherwise} \end{cases}
$$

For the gamma family, $d_j^2 = -2\{\ln(y_j/\widehat{\mu}_j) - (y_j - \widehat{\mu}_j)/\widehat{\mu}_j\}$.

For the inverse Gaussian, $d_j^2 = (y_j - \widehat{\mu}_j)^2/(\widehat{\mu}_j^2 y_j)$.

For the negative binomial,

$$
d_j^2 = \begin{cases} 2\ln(1 + k\widehat{\mu}_j)/k & \text{if } y_j = 0 \\ 2y_j\ln(y_j/\widehat{\mu}_j) - 2\{(1 + ky_j)/k\}\ln\{(1 + ky_j)/(1 + k\widehat{\mu}_j)\} & \text{otherwise} \end{cases}
$$

Let $\phi = 1$ if the scale parameter is set to one; otherwise, define $\phi = \widehat{\phi}_0(n - k)/n$, where $\widehat{\phi}_0$ is the estimated scale parameter and k is the number of covariates in the model (including intercept).

Let $\ln L_j$ denote the log likelihood for the jth observation:

For the Gaussian family,

$$
\ln L_j = -\frac{1}{2}\left[\left\{\frac{(y_j - \widehat{\mu}_j)^2}{\phi}\right\} + \ln(2\pi\phi)\right]
$$

For the binomial family with denominator m_j (Bernoulli if all $m_j = 1$),

$$
\ln L_j = \phi \times \begin{cases} \ln\{\Gamma(m_j + 1)\} - \ln\{\Gamma(y_j + 1)\} + \ln\{\Gamma(m_j - y_j + 1)\} & \text{if } 0 < y_j < m_j \\ \quad + (m_j - y_j)\ln(1 - \widehat{\mu}_j/m_j) + y_j\ln(\widehat{\mu}_j/m_j) & \\ m_j\ln(1 - \widehat{\mu}_j/m_j) & \text{if } y_j = 0 \\ m_j\ln(\widehat{\mu}_j/m_j) & \text{if } y_j = m_j \end{cases}
$$

For the Poisson family,

$$
\ln L_j = \phi\left[y_j\ln(\widehat{\mu}_j) - \widehat{\mu}_j - \ln\{\Gamma(y_j + 1)\}\right]
$$

For the gamma family, $\ln L_j = -y_j/\widehat{\mu}_j + \ln(1/\widehat{\mu}_j)$.

For the inverse Gaussian,

$$
\ln L_j = -\frac{1}{2}\left\{\frac{(y_j - \widehat{\mu}_j)^2}{y_j\widehat{\mu}_j^2} + 3\ln(y_j) + \ln(2\pi)\right\}
$$

For the negative binomial (let $m = 1/k$),

$$
\ln L_j = \phi\left[\ln\{\Gamma(m + y_j)\} - \ln\{\Gamma(y_j + 1)\} - \ln\{\Gamma(m)\} \right.
$$
$$
\left. -m\ln(1 + \widehat{\mu}_j/m) + y_j\ln\{\widehat{\mu}_j/(\widehat{\mu}_j + m)\}\right]
$$

The overall deviance reported by `glm` is $D^2 = \sum_j w_j d_j^2$. The dispersion of the deviance is D^2 divided by the residual degrees of freedom.

The Akaike information criterion (AIC) and Bayesian information criterion (BIC) are given by

$$
\text{AIC} = \frac{-2\ln L + 2k}{N}
$$
$$
\text{BIC} = D^2 - (N - k)\ln(N)
$$

where $\ln L = \sum_j w_j \ln L_j$ is the overall log likelihood.

The Pearson deviance reported by `glm` is $\sum_j w_j r_j^2$. The corresponding Pearson dispersion is the Pearson deviance divided by the residual degrees of freedom. `glm` also calculates the scaled versions of all these quantities by dividing by the estimated scale parameter.

Acknowledgments

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Also see

[R] **glm postestimation** — Postestimation tools for glm

[R] **cloglog** — Complementary log-log regression

[R] **logistic** — Logistic regression, reporting odds ratios

[R] **nbreg** — Negative binomial regression

[R] **poisson** — Poisson regression

[R] **regress** — Linear regression

[ME] **meglm** — Multilevel mixed-effects generalized linear model

[MI] **estimation** — Estimation commands for use with mi estimate

[SVY] **svy estimation** — Estimation commands for survey data

[XT] **xtgee** — Fit population-averaged panel-data models by using GEE

[U] **20 Estimation and postestimation commands**