Title

> **matrix symeigen —** Eigenvalues and eigenvectors of symmetric matrices

Syntax

> <u>matrix</u> <u>symeigen</u> $\mathbf{X}$ $\mathbf{v}$ = $\mathbf{A}$

where $\mathbf{A}$ is an $n \times n$ symmetric matrix.

Menu

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Description

matrix symeigen returns the eigenvectors in the columns of $\mathbf{X}$: $n \times n$ and the corresponding eigenvalues in $\mathbf{v}$: $1 \times n$. The eigenvalues are sorted: $\mathbf{v}[1,1]$ contains the largest eigenvalue (and $\mathbf{X}[1\ldots,1]$ its corresponding eigenvector), and $\mathbf{v}[1,n]$ contains the smallest eigenvalue (and $\mathbf{X}[1\ldots,n]$ its corresponding eigenvector).

If you want the eigenvalues for a nonsymmetric matrix, see [P] **matrix eigenvalues**.

Also see [M-5] **eigensystem( )** for other routines for obtaining eigenvalues and eigenvectors.

Remarks and examples

Typing matrix symeigen $\mathbf{X}$ $\mathbf{v}$ = $\mathbf{A}$ for $\mathbf{A}$: $n \times n$ returns

$$\mathbf{v} = (\lambda_1, \lambda_2, \ldots, \lambda_n)$$
$$\mathbf{X} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_n)$$

where $\lambda_1 \geq \lambda_2 \geq \ldots \geq \lambda_n$. Each $\mathbf{x}_i$ and λ_i is a solution to

$$\mathbf{A}\mathbf{x}_i = \lambda_i \mathbf{x}_i$$

or, more compactly,

$$\mathbf{A}\mathbf{X} = \mathbf{X}\,\mathrm{diag}(\mathbf{v})$$

▷ Example 1

Eigenvalues and eigenvectors have many uses. We will demonstrate that symeigen returns matrices meeting the definition:

```
. use http://www.stata-press.com/data/r13/auto
(1978 Automobile Data)
. matrix accum A = weight mpg length, noconstant deviation
(obs=74)
```

```
. matrix list A

symmetric A[3,3]
             weight         mpg       length
weight     44094178
   mpg   -264948.11   2443.4595
length   1195077.3   -7483.5135   36192.662

. matrix symeigen X lambda = A

. matrix list lambda

lambda[1,3]
             e1          e2          e3
r1     44128163   3830.4869   820.73955

. matrix list X

X[3,3]
                  e1          e2          e3
weight   .99961482  -.02756261   .00324179
   mpg  -.00600667   -.1008305   .99488549
length   .02709477   .99452175   .10095722

. matrix AX = A*X

. matrix XLambda = X*diag(lambda)

. matrix list AX

AX[3,3]
                  e1          e2          e3
weight     44111166  -105.57823   2.6606641
   mpg    -265063.5  -386.22991   816.54187
length   1195642.6   3809.5025   82.859585

. matrix list XLambda

XLambda[3,3]
                  e1          e2          e3
weight     44111166  -105.57823   2.6606641
   mpg    -265063.5  -386.22991   816.54187
length   1195642.6   3809.5025   82.859585
```

◁

Methods and formulas

Stata's internal eigenvalue and eigenvector extraction routines are translations of the public domain EISPACK routines, Smith et al. (1976), which are in turn based on Wilkinson and Reinsch (1971). EISPACK was developed under contract for the Office of Scientific and Technical Information, U.S. Department of Energy, by Argonne National Laboratory and supported by funds provided by the Nuclear Regulatory Commission. Stata's use of these routines is by permission of the National Energy Software Center of the Argonne National Laboratory. A brief but excellent introduction to the techniques used by these routines can be found in Press et al. (2007, 563–599).

References

Press, W. H., S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery. 2007. *Numerical Recipes: The Art of Scientific Computing*. 3rd ed. New York: Cambridge University Press.

Smith, B. T., J. M. Boyle, J. J. Dongarra, B. S. Garbow, Y. Ikebe, V. C. Klema, and C. B. Moler. 1976. *Matrix Eigensystem Routines–EISPACK Guide*. 2nd ed. Berlin: Springer.

Wilkinson, J. H., and C. Reinsch. 1971. *Handbook for Automatic Computation, Vol. 2: Linear Algebra*. New York: Springer.

Also see

[P] **matrix** — Introduction to matrix commands

[P] **matrix eigenvalues** — Eigenvalues of nonsymmetric matrices

[M-4] **matrix** — Matrix functions

[U] **14 Matrix expressions**