matrix eigenvalues — Eigenvalues of nonsymmetric matrices

Syntax

`matrix eigenvalues r c = A`

where **A** is an $n \times n$ nonsymmetric, real matrix.

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Data > Matrices, ado language > Eigenvalues of square matrices

Description

`matrix eigenvalues` returns the real part of the eigenvalues in the $1 \times n$ row vector **r** and the imaginary part of the eigenvalues in the $1 \times n$ row vector **c**. Thus the jth eigenvalue is `r[1,`j`]` $+$ i $*$ `c[1,`j`]`.

The eigenvalues are sorted by their moduli; `r[1,1]` $+$ i $*$ `c[1,1]` has the largest modulus, and `r[1,`n`]` $+$ i $*$ `c[1,`n`]` has the smallest modulus.

If you want the eigenvalues for a symmetric matrix, see [P] **matrix symeigen**.

Also see [M-5] **eigensystem()** for alternative routines for obtaining eigenvalues and eigenvectors.

Remarks and examples

Typing `matrix eigenvalues r c = A` for **A** $n \times n$ returns

$$\mathbf{r} = \left(r_1, r_2, \ldots, r_n\right)$$
$$\mathbf{c} = \left(c_1, c_2, \ldots, c_n\right)$$

where $\mathbf{r}_j$ is the real part and $\mathbf{c}_j$ the imaginary part of the jth eigenvalue. The eigenvalues are part of the solution to the problem

$$\mathbf{A}\mathbf{x}_j = \lambda_j \mathbf{x}_j$$

and, in particular,

$$\lambda_j = \mathbf{r}_j + \mathrm{i} * \mathbf{c}_j$$

The corresponding eigenvectors, $\mathbf{x}_j$, are not saved by `matrix eigenvalues`. The returned **r** and **c** are ordered so that $|\lambda_1| \geq |\lambda_2| \geq \cdots \geq |\lambda_n|$, where $|\lambda_j| = \sqrt{\mathbf{r}_j^2 + \mathbf{c}_j^2}$.

Example 1

In time-series analysis, researchers often use eigenvalues to verify the stability of the fitted model.

Suppose that we have fit a univariate time-series model and that the stability condition requires the moduli of all the eigenvalues of a "companion" matrix **A** to be less than 1. (See Hamilton [1994] for a discussion of these models and conditions.)

First, we form the companion matrix.

```
. matrix A = (0.66151492, .2551595, .35603325, -0.15403902, -.12734386)
. matrix A = A \ (I(4), J(4,1,0))
. mat list A
A[5,5]
            c1          c2          c3          c4          c5
r1   .66151492    .2551595   .35603325  -.15403902  -.12734386
r1           1           0           0           0           0
r2           0           1           0           0           0
r3           0           0           1           0           0
r4           0           0           0           1           0
```

Next we use `matrix eigenvalues` to obtain the eigenvalues, which we will then list:

```
. matrix eigenvalues re im = A
. mat list re
re[1,5]
              c1          c2          c3          c4          c5
real   .99121823   .66060006  -.29686008  -.29686008   -.3965832
. mat list im
im[1,5]
                 c1          c2          c3          c4          c5
complex           0           0   .63423776  -.63423776           0
```

Finally, we compute and list the moduli, which are all less than 1, although the first is close:

```
. forvalues i = 1/5 {
  2.          di sqrt(re[1,`i']^2 + im[1,`i']^2)
  3. }
.99121823
.66060006
.70027384
.70027384
.3965832
```

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Methods and formulas

Stata's internal eigenvalue extraction routine for nonsymmetric matrices is based on the public domain LAPACK routine DGEEV. Anderson et al. (1999) provide an excellent introduction to these routines. Stata's internal routine also uses, with permission, **f2c** (©1990–1997 by AT&T, Lucent Technologies, and Bellcore).

References

Anderson, E., Z. Bai, C. Bischof, S. Blackford, J. Demmel, J. J. Dongarra, J. Du Croz, A. Greenbaum, S. Hammarling, A. McKenney, and D. Sorensen. 1999. *LAPACK Users' Guide*. 3rd ed. Philadelphia: Society for Industrial and Applied Mathematics.

Gould, W. W. 2011a. Understanding matrices intuitively, part 1. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2011/03/03/understanding-matrices-intuitively-part-1/.

———. 2011b. Understanding matrices intuitively, part 2, eigenvalues and eigenvectors. The Stata Blog: Not Elsewhere Classified. http://blog.stata.com/2011/03/09/understanding-matrices-intuitively-part-2/.

Hamilton, J. D. 1994. *Time Series Analysis*. Princeton: Princeton University Press.

Also see

[P] **matrix** — Introduction to matrix commands

[P] **matrix symeigen** — Eigenvalues and eigenvectors of symmetric matrices

[M-4] **matrix** — Matrix functions

[U] 14 Matrix expressions