measure_option — Option for similarity and dissimilarity measures Description Syntax Options References Also see Syntax command ..., ... measure(measure) ... or command ..., ... measure ... Description measure similarity or dissimilarity measure for continuous data cont_measure similarity measure for binary data binary_measure mixed_measure dissimilarity measure for a mix of binary and continuous data cont_measure Description L2 Euclidean distance (Minkowski with argument 2) alias for L2 Euclidean L(2) alias for L2 squared Euclidean distance L2squared Lpower(2) alias for L2squared absolute-value distance (Minkowski with argument 1) L1 alias for L1 absolute cityblock alias for L1 alias for L1 manhattan alias for L1 L(1)Lpower(1) alias for L1 Linfinity maximum-value distance (Minkowski with infinite argument) maximum alias for Linfinity L(#) Minkowski distance with # arguments Minkowski distance with # arguments raised to # power Lpower(#) Canberra Canberra distance correlation correlation coefficient similarity measure angular angular separation similarity measure alias for angular angle

2 measure_option — Option for similarity and dissimilarity measures

binary_measure	Description			
matching	simple matching similarity coefficient			
Jaccard	Jaccard binary similarity coefficient			
Russell	Russell and Rao similarity coefficient			
Hamann	Hamann similarity coefficient			
Dice	Dice similarity coefficient			
antiDice	anti-Dice similarity coefficient			
Sneath	Sneath and Sokal similarity coefficient			
Rogers	Rogers and Tanimoto similarity coefficient			
Ochiai	Ochiai similarity coefficient			
Yule	Yule similarity coefficient			
Anderberg	Anderberg similarity coefficient			
Kulczynski	Kulczynski similarity coefficient			
Pearson	Pearson's ϕ similarity coefficient			
Gower2	similarity coefficient with same denominator as Pearson			
mixed_measure	Description			

Description

Gower

Several commands have options that allow you to specify a similarity or dissimilarity measure designated as *measure* in the syntax; see [MV] **cluster**, [MV] **mds**, [MV] **discrim knn**, and [MV] **matrix dissimilarity**. These options are documented here. Most analysis commands (for example, cluster and mds) transform similarity measures to dissimilarity measures as needed.

Gower's dissimilarity coefficient

Options

Measures are divided into those for continuous data and binary data. *measure* is not case sensitive. Full definitions are presented in *Similarity and dissimilarity measures for continuous data, Similarity measures for binary data, and Dissimilarity measures for mixed data.*

The similarity or dissimilarity measure is most often used to determine the similarity or dissimilarity between observations. However, sometimes the similarity or dissimilarity between variables is of interest.

Similarity and dissimilarity measures for continuous data

Here are the similarity and dissimilarity measures for continuous data available in Stata. In the following formulas, p represents the number of variables, N is the number of observations, and x_{iv} denotes the value of observation i for variable v.

The formulas are presented in two forms. The first is the formula used when computing the similarity or dissimilarity between observations. The second is the formula used when computing the similarity or dissimilarity between variables.

L2 (aliases Euclidean and L(2))

requests the Minkowski distance metric with argument 2. For comparing observations i and j, the formula is

$$\left\{\sum_{a=1}^{p} (x_{ia} - x_{ja})^2\right\}^{1/2}$$

and for comparing variables u and v the formula is

$$\left\{\sum_{k=1}^{N} (x_{ku} - x_{kv})^2\right\}^{1/2}$$

L2 is best known as Euclidean distance and is the default dissimilarity measure for discrim knn, mds, matrix dissimilarity, and all the cluster subcommands except for centroidlinkage, medianlinkage, and wardslinkage, which default to using L2squared; see [MV] discrim knn, [MV] mds, [MV] matrix dissimilarity, and [MV] cluster.

L2squared (alias Lpower(2))

requests the square of the Minkowski distance metric with argument 2. For comparing observations i and j, the formula is

$$\sum_{a=1}^{p} (x_{ia} - x_{ja})^2$$

and for comparing variables u and v, the formula is

$$\sum_{k=1}^{N} (x_{ku} - x_{kv})^2$$

L2squared is best known as squared Euclidean distance and is the default dissimilarity measure for the centroidlinkage, medianlinkage, and wardslinkage subcommands of cluster; see [MV] cluster.

L1 (aliases <u>absolute</u>, <u>cityblock</u>, <u>manhattan</u>, and L(1)) requests the Minkowski distance metric with argument 1. For comparing observations i and j, the formula is

$$\sum_{a=1}^{p} |x_{ia} - x_{ja}|$$

and for comparing variables u and v, the formula is

$$\sum_{k=1}^{N} |x_{ku} - x_{kv}|$$

L1 is best known as absolute-value distance.

Linfinity (alias maximum)

requests the Minkowski distance metric with infinite argument. For comparing observations i and j, the formula is

$$\max_{a=1,\dots,p} |x_{ia} - x_{ja}|$$

and for comparing variables u and v, the formula is

$$\max_{k=1,\dots,N} |x_{ku} - x_{kv}|$$

Linfinity is best known as maximum-value distance.

L(#)

requests the Minkowski distance metric with argument #. For comparing observations i and j, the formula is

$$\left(\sum_{a=1}^{p} |x_{ia} - x_{ja}|^{\#}\right)^{1/\#} \qquad \# \ge 1$$

and for comparing variables u and v, the formula is

$$\left(\sum_{k=1}^{N} |x_{ku} - x_{kv}|^{\#}\right)^{1/\#} \qquad \# \ge 1$$

We discourage using extremely large values for #. Because the absolute value of the difference is being raised to the value of #, depending on the nature of your data, you could experience numeric overflow or underflow. With a large value of #, the L() option will produce results similar to those of the Linfinity option. Use the numerically more stable Linfinity option instead of a large value for # in the L() option.

See Anderberg (1973) for a discussion of the Minkowski metric and its special cases.

Lpower(#)

requests the Minkowski distance metric with argument #, raised to the # power. For comparing observations i and j, the formula is

$$\sum_{a=1}^{p} |x_{ia} - x_{ja}|^{\#} \qquad \# \ge 1$$

and for comparing variables u and v, the formula is

$$\sum_{k=1}^{N} |x_{ku} - x_{kv}|^{\#} \qquad \# \ge 1$$

As with L(#), we discourage using extremely large values for #; see the discussion above.

<u>Canb</u>erra

requests the following distance metric when comparing observations i and j

$$\sum_{a=1}^{p} \frac{|x_{ia} - x_{ja}|}{|x_{ia}| + |x_{ja}|}$$

and the following distance metric when comparing variables u and v

$$\sum_{k=1}^{N} \frac{|x_{ku} - x_{kv}|}{|x_{ku}| + |x_{kv}|}$$

When comparing observations, the Canberra metric takes values between 0 and p, the number of variables. When comparing variables, the Canberra metric takes values between 0 and N, the number of observations; see Gordon (1999) and Gower (1985). Gordon (1999) explains that the Canberra distance is sensitive to small changes near zero.

<u>corr</u>elation

requests the correlation coefficient similarity measure. For comparing observations i and j, the formula is

$$\frac{\sum_{a=1}^{p} (x_{ia} - \overline{x}_{i.})(x_{ja} - \overline{x}_{j.})}{\left\{\sum_{a=1}^{p} (x_{ia} - \overline{x}_{i.})^2 \sum_{b=1}^{p} (x_{jb} - \overline{x}_{j.})^2\right\}^{1/2}}$$

and for comparing variables u and v, the formula is

$$\frac{\sum_{k=1}^{N} (x_{ku} - \overline{x}_{.u})(x_{kv} - \overline{x}_{.v})}{\left\{\sum_{k=1}^{N} (x_{ku} - \overline{x}_{.u})^2 \sum_{l=1}^{N} (x_{lv} - \overline{x}_{.v})^2\right\}^{1/2}}$$

where $\overline{x}_{i.} = (\sum_{a=1}^{p} x_{ia})/p$ and $\overline{x}_{.u} = (\sum_{k=1}^{N} x_{ku})/N$.

The correlation similarity measure takes values between -1 and 1. With this measure, the relative direction of the two vectors is important. The correlation similarity measure is related to the angular separation similarity measure (described next). The correlation similarity measure gives the cosine of the angle between the two vectors measured from the mean; see Gordon (1999).

angular (alias angle)

requests the angular separation similarity measure. For comparing observations i and j, the formula is

$$\frac{\sum_{a=1}^{p} x_{ia} x_{ja}}{\left(\sum_{a=1}^{p} x_{ia}^2 \sum_{b=1}^{p} x_{jb}^2\right)^{1/2}}$$

and for comparing variables u and v, the formula is

$$\frac{\sum_{k=1}^{N} x_{ku} x_{kv}}{\left(\sum_{k=1}^{N} x_{ku}^2 \sum_{l=1}^{N} x_{lv}^2\right)^{1/2}}$$

The angular separation similarity measure is the cosine of the angle between the two vectors measured from zero and takes values from -1 to 1; see Gordon (1999).

Similarity measures for binary data

Similarity measures for binary data are based on the four values from the cross-tabulation of observation i and j (when comparing observations) or variables u and v (when comparing variables).

For comparing observation i and j, the cross-tabulation is

		obs. j	
		1	0
$\overline{obs.}_i$	1 0	$a \\ c$	$egin{array}{c} b \ d \end{array}$

ī.

a is the number of variables where observations i and j both had ones, and d is the number of variables where observations i and j both had zeros. The number of variables where observation i is one and observation j is zero is b, and the number of variables where observation i is zero and observation j is one is c.

For comparing variables u and v, the cross-tabulation is

		var 1	$\begin{array}{c} v \\ 0 \end{array}$
var.	1 0	$a \\ c$	$egin{array}{c} b \ d \end{array}$

a is the number of observations where variables u and v both had ones, and d is the number of observations where variables u and v both had zeros. The number of observations where variable u is one and variable v is zero is b, and the number of observations where variable u is zero and variable v is one is c.

Stata treats nonzero values as one when a binary value is expected. Specifying one of the binary similarity measures imposes this behavior unless some other option overrides it (for instance, the allbinary option of matrix dissimilarity; see [MV] matrix dissimilarity).

Hubálek (1982) gives an extensive list of binary similarity measures. Gower (1985) lists 15 binary similarity measures, 14 of which are implemented in Stata. (The excluded measure has many cases where the quantity is undefined, so it was not implemented.) Anderberg (1973) gives an interesting table where many of these measures are compared based on whether the zero-zero matches are included in the numerator, whether these matches are included in the denominator, and how the weighting of matches and mismatches is handled. Hilbe (1992b, 1992a) implemented an early Stata command for computing some of these (as well as other) binary similarity measures.

The formulas for some of these binary similarity measures are undefined when either one or both of the vectors (observations or variables depending on which are being compared) are all zeros (or, sometimes, all ones). Gower (1985) says concerning these cases, "These coefficients are then conventionally assigned some appropriate value, usually zero."

The following binary similarity coefficients are available. Unless stated otherwise, the similarity measures range from 0 to 1.

matching

requests the simple matching (Zubin 1938, Sokal and Michener 1958) binary similarity coefficient

$$\frac{a+d}{a+b+c+d}$$

which is the proportion of matches between the 2 observations or variables.

<u>Jac</u>card

requests the Jaccard (1901, 1908) binary similarity coefficient

$$\frac{a}{a+b+c}$$

which is the proportion of matches when at least one of the vectors had a one. If both vectors are all zeros, this measure is undefined. Stata then declares the answer to be one, meaning perfect agreement. This is a reasonable choice for most applications and will cause an all-zero vector to have similarity of one only with another all-zero vector. In all other cases, an all-zero vector will have Jaccard similarity of zero to the other vector.

The Jaccard coefficient was discovered earlier by Gilbert (1884).

<u>Russ</u>ell

requests the Russell and Rao (1940) binary similarity coefficient

$$\frac{a}{a+b+c+d}$$

Hamann

requests the Hamann (1961) binary similarity coefficient

$$\frac{(a+d) - (b+c)}{a+b+c+d}$$

which is the number of agreements minus disagreements divided by the total. The Hamann coefficient ranges from -1, perfect disagreement, to 1, perfect agreement. The Hamann coefficient is equal to twice the simple matching coefficient minus 1.

Dice

requests the Dice binary similarity coefficient

$$\frac{2a}{2a+b+c}$$

suggested by Czekanowski (1932), Dice (1945), and Sørensen (1948). The Dice coefficient is similar to the Jaccard similarity coefficient but gives twice the weight to agreements. Like the Jaccard coefficient, the Dice coefficient is declared by Stata to be one if both vectors are all zero, thus avoiding the case where the formula is undefined.

antiDice

requests the binary similarity coefficient

$$\frac{a}{a+2(b+c)}$$

which is credited to Anderberg (1973) but was shown earlier by Sokal and Sneath (1963, 129). We did not call this the Anderberg coefficient because there is another coefficient better known by that name; see the Anderberg option. The name anti-Dice is our creation. This coefficient takes the opposite view from the Dice coefficient and gives double weight to disagreements. As with the Jaccard and Dice coefficients, the anti-Dice coefficient is declared to be one if both vectors are all zeros.

Sneath

requests the Sneath and Sokal (1962) binary similarity coefficient

$$\frac{2(a+d)}{2(a+d)+(b+c)}$$

which is similar to the simple matching coefficient but gives double weight to matches. Also compare the Sneath and Sokal coefficient with the Dice coefficient, which differs only in whether it includes d.

Rogers

requests the Rogers and Tanimoto (1960) binary similarity coefficient

$$\frac{a+d}{(a+d)+2(b+c)}$$

which takes the opposite approach from the Sneath and Sokal coefficient and gives double weight to disagreements. Also compare the Rogers and Tanimoto coefficient with the anti-Dice coefficient, which differs only in whether it includes d.

Ochiai

requests the Ochiai (1957) binary similarity coefficient

$$\frac{a}{\left\{(a+b)(a+c)\right\}^{1/2}}$$

The formula for the Ochiai coefficient is undefined when one or both of the vectors being compared are all zeros. If both are all zeros, Stata declares the measure to be one, and if only one of the two vectors is all zeros, the measure is declared to be zero.

The Ochiai coefficient was presented earlier by Driver and Kroeber (1932).

Yule

requests the Yule (see Yule [1900] and Yule and Kendall [1950]) binary similarity coefficient

$$\frac{ad - bc}{ad + bc}$$

which ranges from -1 to 1. The formula for the Yule coefficient is undefined when one or both of the vectors are either all zeros or all ones. Stata declares the measure to be 1 when b + c = 0, meaning that there is complete agreement. Stata declares the measure to be -1 when a + d = 0, meaning that there is complete disagreement. Otherwise, if ad - bc = 0, Stata declares the measure to be 0. These rules, applied before using the Yule formula, avoid the cases where the formula would produce an undefined result.

<u>Ander</u>berg

requests the Anderberg binary similarity coefficient

$$\left(\frac{a}{a+b} + \frac{a}{a+c} + \frac{d}{c+d} + \frac{d}{b+d}\right) \middle/ 4$$

The Anderberg coefficient is undefined when one or both vectors are either all zeros or all ones. This difficulty is overcome by first applying the rule that if both vectors are all ones (or both vectors are all zeros), the similarity measure is declared to be one. Otherwise, if any of the marginal totals (a + b, a + c, c + d, b + d) are zero, then the similarity measure is declared to be zero.

Though this similarity coefficient is best known as the Anderberg coefficient, it appeared earlier in Sokal and Sneath (1963, 130).

<u>Kulc</u>zynski

requests the Kulczynski (1927) binary similarity coefficient

$$\left(\frac{a}{a+b} + \frac{a}{a+c}\right) \middle/ 2$$

The formula for this measure is undefined when one or both of the vectors are all zeros. If both vectors are all zeros, Stata declares the similarity measure to be one. If only one of the vectors is all zeros, the similarity measure is declared to be zero.

Pearson

requests Pearson's (1900) ϕ binary similarity coefficient

$$\frac{ad - bc}{\left\{(a+b)(a+c)(d+b)(d+c)\right\}^{1/2}}$$

which ranges from -1 to 1. The formula for this coefficient is undefined when one or both of the vectors are either all zeros or all ones. Stata declares the measure to be 1 when b+c = 0, meaning that there is complete agreement. Stata declares the measure to be -1 when a + d = 0, meaning that there is complete disagreement. Otherwise, if ad - bc = 0, Stata declares the measure to be 0. These rules, applied before using Pearson's ϕ coefficient formula, avoid the cases where the formula would produce an undefined result.

Gower2

requests the binary similarity coefficient

$$\frac{ad}{\{(a+b)(a+c)(d+b)(d+c)\}^{1/2}}$$

which is presented by Gower (1985) but appeared earlier in Sokal and Sneath (1963, 130). Stata uses the name Gower2 to avoid confusion with the better-known Gower coefficient, which is used with a mix of binary and continuous data.

The formula for this similarity measure is undefined when one or both of the vectors are all zeros or all ones. This is overcome by first applying the rule that if both vectors are all ones (or both vectors are all zeros) then the similarity measure is declared to be one. Otherwise, if ad = 0, the similarity measure is declared to be zero.

Dissimilarity measures for mixed data

Here is one measure that works with a mix of binary and continuous data. Binary variables are those containing only zeros, ones, and missing values; all other variables are treated as continuous.

Gower

requests the Gower (1971) dissimilarity coefficient for a mix of binary and continuous variables. For comparing observations i and j, the formula is

$$\frac{\sum_{v} \delta_{ijv} d_{ijv}}{\sum_{v} \delta_{ijv}}$$

where δ_{ijv} is a binary indicator equal to 1 whenever both observations *i* and *j* are nonmissing for variable *v*, and zero otherwise. Observations with missing values are not included when using cluster or mds, and so if an observation is included, $\delta_{ijv} = 1$ and $\sum_{v} \delta_{ijv}$ is the number of variables. However, using matrix dissimilarity with the Gower option does not exclude observations with missing values. See [MV] cluster, [MV] mds, and [MV] matrix dissimilarity.

For binary variables v,

$$d_{ijv} = \begin{cases} 0 & \text{if } x_{iv} = x_{jv} \\ 1 & \text{otherwise} \end{cases}$$

This is the same as the matching measure.

For continuous variables v,

$$d_{ijv} = \frac{|x_{iv} - x_{jv}|}{\left\{\max_k(x_{kv}) - \min_k(x_{kv})\right\}}$$

 d_{ijv} is set to 0 if $\max_k(x_{kv}) - \min_k(x_{kv}) = 0$, that is, if the range of the variable is zero. This is the L1 measure divided by the range of the variable.

For comparing variables u and v, the formula is

$$\frac{\sum_{i} \delta_{iuv} d_{iuv}}{\sum_{i} \delta_{iuv}}$$

where δ_{iuv} is a binary indicator equal to 1 whenever both variables u and v are nonmissing for observation i, and zero otherwise. If there are no missing values, $\sum_i \delta_{iuv}$ is the number of observations; otherwise, it is the number of observations for which neither variable u nor v has a missing value.

If all the variables are binary,

$$d_{iuv} = \begin{cases} 0 & \text{if } x_{iu} = x_{iv} \\ 1 & \text{otherwise} \end{cases}$$

If at least one variable is continuous,

$$d_{iuv} = \frac{|x_{iu} - x_{iv}|}{\left\{\max_{v}(x_{iv}) - \min_{v}(x_{iv})\right\}}$$

 d_{iuv} is set to 0 if $\max_v(x_{iv}) - \min_v(x_{iv}) = 0$, that is, if the range of the observation is zero.

The Gower measure interprets binary variables as those with only 0, 1, or missing values. All other variables are treated as continuous.

In [MV] **matrix dissimilarity**, missing observations are included only in the calculation of the Gower dissimilarity, but the formula for this dissimilarity measure is undefined when all the values of δ_{ijv} or δ_{iuv} are zero. The dissimilarity is then set to missing.

Technical note: Matrix dissimilarity and the Gower measure

Normally the commands

```
. matrix dissimilarity gm = x1 x2 y1, Gower
```

. clustermat waverage gm, add

and

```
. cluster waverage x1 x2 y1, measure(Gower)
```

will yield the same results, and likewise with mdsmat and mds. However, if any of the variables contain missing observations, this will not be the case. cluster and mds exclude all observations that have missing values for any of the variables of interest, whereas matrix dissimilarity with the Gower option does not. See [MV] cluster, [MV] mds, and [MV] matrix dissimilarity for more information.

Note: matrix dissimilarity without the Gower option does exclude all observations that have missing values for any of the variables of interest.

Technical note: Binary similarity measures applied to averages

Some cluster-analysis methods (such as Stata's kmeans and kmedians clustering) need to compute the similarity or dissimilarity between observations and group averages or group medians; see [MV] **cluster**. With binary data, a group average is interpreted as a proportion.

A group median for binary data will be zero or one, except when there are an equal number of zeros and ones. Here Stata calls the median 0.5, which can also be interpreted as a proportion.

In Stata's cluster kmeans and cluster kmedians commands for comparing a binary observation to a group proportion (see *Partition cluster-analysis methods* in [MV] **cluster**), the values of a, b, c, and d are obtained by assigning the appropriate fraction of the count to these values. In our earlier table showing the relationship of a, b, c, and d in the cross-tabulation of observation i and observation j, we replace observation j by the group-proportions vector. Then when observation i is 1, we add the corresponding proportion to a and add one minus that proportion to b. When observation i is 0, we add the corresponding proportion to c and add one minus that proportion to d. After the values of a, b, c, and d are computed in this way, the binary similarity measures are computed using the formulas as already described. Paul Jaccard (1868–1944) was a Swiss botanist who was born in Sainte-Croix (Vaud) and died in Zürich. He studied at Lausanne, Zürich, and Paris before being appointed to posts at Lausanne in 1894, where he specialized in plant geography, undertaking fieldwork in Egypt, Sweden, and Turkestan. In 1903, Jaccard returned to Zürich to a chair in general botany and plant physiology at ETH. His interests there centered on the microscopic analysis of wood, and anatomical and physiological studies of the growth of trees.

Robert Reuven Sokal (1926–2012) was born in Vienna to a Jewish family. He gained degrees from St. John's University in Shanghai and the University of Chicago. Sokal worked at the University of Kansas–Lawrence and (from 1969) the State University of New York–Stony Brook. He was one of the leaders in the development of numerical taxonomy (Sokal and Sneath 1963; Sneath and Sokal 1973) and was prominent in the application of statistical methods within biological systematics. With F. J. Rohlf, he authored one of the leading biometrics texts (Sokal and Rohlf 2011). In the latter stages of his career, his interests centered on genetic variation in human populations, European ethnohistory, and spatial statistics. Sokal was a member of the U.S. National Academy of Sciences.

Peter Henry Andrews Sneath (1923–2011) was born in Ceylon (now Sri Lanka) and studied medicine in Cambridge and London. After military service, he specialized in microbial systematics and the application of computers to biomedical science, working for the Medical Research Council in the UK and the University of Leicester. With Robert Sokal, Sneath wrote the two initial texts on numerical taxonomy. He is a Fellow of the Royal Society. A bacterial taxon, the genus *Sneathia*, was named after him in 2002.

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Also see

- [MV] **matrix dissimilarity** Compute similarity or dissimilarity measures
- [MV] cluster Introduction to cluster-analysis commands
- [MV] **clustermat** Introduction to clustermat commands