svsolve() — Solve AX=B for X using singular value decomposition

Syntax

\[
\text{numeric matrix} \quad \text{svsolve}(A, B) \\
\text{numeric matrix} \quad \text{svsolve}(A, B, \text{rank}) \\
\text{numeric matrix} \quad \text{svsolve}(A, B, \text{rank}, \text{tol}) \\
\text{real scalar} \quad \_\text{svsolve}(A, B) \\
\text{real scalar} \quad \_\text{svsolve}(A, B, \text{tol})
\]

where
- \(A\): numeric matrix
- \(B\): numeric matrix
- \(\text{rank}\): irrelevant; real scalar returned
- \(\text{tol}\): real scalar

Description

\text{svsolve}(A, B, \ldots) \text{ uses singular value decomposition to solve } AX = B \text{ and return } X. \text{ When } A \text{ is singular, } \text{svsolve()} \text{ computes the minimum-norm least-squares generalized solution. When } \text{rank} \text{ is specified, it is placed the rank of } A.

\_\text{svsolve}(A, B, \ldots) \text{ does the same thing, except that it destroys the contents of } A \text{ and it overwrites } B \text{ with the solution. Returned is the rank of } A.

In both cases, \(\text{tol}\) specifies the tolerance for determining whether \(A\) is of full rank. \(\text{tol}\) is interpreted in the standard way—as a multiplier for the default if \(\text{tol} > 0\) is specified and as an absolute quantity to use in place of the default if \(\text{tol} \leq 0\) is specified.

Remarks and examples

\text{svsolve}(A, B, \ldots) \text{ is suitable for use with square or nonsquare, full-rank or rank-deficient matrix } A. \text{ When } A \text{ is of full rank, } \text{qrsolve()} \text{ returns the same solution as } \text{lusolve()} \text{ (see [M-5] lusolve()), ignoring roundoff error. When } A \text{ is singular, } \text{svsolve()} \text{ returns the minimum-norm least-squares generalized solution. } \text{qrsolve()} \text{ (see [M-5] qrsolve()), an alternative, returns a generalized least-squares solution that amounts to dropping rows of } A.

Remarks are presented under the following headings:

- Derivation
- Relationship to inversion
- Tolerance
Derivation

We wish to solve for \( X \)

\[ AX = B \]  \hspace{1cm} (1)

Perform singular value decomposition on \( A \) so that we have \( A = USV' \). Then (1) can be rewritten as

\[ USV'X = B \]

Premultiplying by \( U' \) and remembering that \( U'U = I \), we have

\[ SV'X = U'B \]

Matrix \( S \) is diagonal and thus its inverse is easily calculated, and we have

\[ V'X = S^{-1}U'B \]

When we premultiply by \( V \), remembering that \( VV' = I \), the solution is

\[ X = VS^{-1}U'B \]  \hspace{1cm} (2)

See [M-5] \texttt{svd()} for more information on the SVD.

Relationship to inversion

For a general discussion, see \textit{Relationship to inversion} in [M-5] \texttt{lusolve()}.

For an inverse based on the SVD, see [M-5] \texttt{pinv()}. \texttt{pinv(A)} amounts to \texttt{svsolve(A, I(rows(A)))}, although \texttt{pinv()} has separate code that uses less memory.

Tolerance

In (2) above, we are required to calculate the inverse of diagonal matrix \( S \). The generalized solution is obtained by substituting zero for the \( i \)th diagonal element of \( S^{-1} \), where the \( i \)th diagonal element of \( S \) is less than or equal to \( \text{eta} \) in absolute value. The default value of \( \text{eta} \) is

\[ \text{eta} = \text{epsilon}(1) \times \text{rows}(A) \times \text{max}(S) \]

If you specify \( \text{tol} > 0 \), the value you specify is used to multiply \( \text{eta} \). You may instead specify \( \text{tol} \leq 0 \) and then the negative of the value you specify is used in place of \( \text{eta} \); see [M-1] \texttt{tolerance}. 


Conformability

\[ \text{svsolve}(A, B, \text{rank}, \text{tol}): \]

\text{input:}
- \( A \): \( m \times n \)
- \( B \): \( m \times k \)
- \( \text{tol} \): \( 1 \times 1 \) (optional)

\text{output:}
- \( \text{rank} \): \( 1 \times 1 \) (optional)
- \( \text{result} \): \( n \times k \)

\[ \text{svsolve}(A, B, \ldots) \]

\text{input:}
- \( A \): \( m \times n \)
- \( B \): \( m \times k \)
- \( \text{tol} \): \( 1 \times 1 \) (optional)

\text{output:}
- \( A \): \( 0 \times 0 \)
- \( B \): \( m \times k \)
- \( \text{result} \): \( 1 \times 1 \)

Diagnostics

\[ \text{svsolve}(A, B, \ldots) \text{ and } \text{svsolve}(A, B, \ldots) \text{ return missing results if } A \text{ or } B \text{ contain missing.} \]

\[ \text{svsolve}(A, B, \ldots) \text{ aborts with error if } A \text{ (but not } B) \text{ is a view.} \]

Also see

[M-5] \text{solvelower}() — Solve \( AX=B \) for \( X \), \( A \) triangular

[M-5] \text{cholsolve}() — Solve \( AX=B \) for \( X \) using Cholesky decomposition

[M-5] \text{lusolve}() — Solve \( AX=B \) for \( X \) using LU decomposition

[M-5] \text{qrsolve}() — Solve \( AX=B \) for \( X \) using QR decomposition

[M-4] \text{matrix} — Matrix functions

[M-4] \text{solvers} — Functions to solve \( AX=B \) and to obtain \( A \) inverse