**Syntax**

- **numeric matrix** `sin(numeric matrix Z)`
- **numeric matrix** `cos(numeric matrix Z)`
- **numeric matrix** `tan(numeric matrix Z)`

- **numeric matrix** `asin(numeric matrix Z)`
- **numeric matrix** `acos(numeric matrix Z)`
- **numeric matrix** `atan(numeric matrix Z)`

- **real matrix** `atan2(real matrix X, real matrix Y)`

- **real matrix** `arg(complex matrix Z)`

- **numeric matrix** `sinh(numeric matrix Z)`
- **numeric matrix** `cosh(numeric matrix Z)`
- **numeric matrix** `tanh(numeric matrix Z)`

- **numeric matrix** `asinh(numeric matrix Z)`
- **numeric matrix** `acosh(numeric matrix Z)`
- **numeric matrix** `atanh(numeric matrix Z)`

- **real scalar** `pi()`

**Description**

`sin(Z)`, `cos(Z)`, and `tan(Z)` return the appropriate trigonometric functions. Angles are measured in radians. All return real if the argument is real and complex if the argument is complex.

- `sin(x)`, `x` real, returns the sine of `x`. `sin()` returns a value between $-1$ and $1$.

- `sin(z)`, `z` complex, returns the complex sine of `z`, mathematically defined as $\frac{\exp(i \cdot z) - \exp(-i \cdot z)}{2i}$.

- `cos(x)`, `x` real, returns the cosine of `x`. `cos()` returns a value between $-1$ and $1$.

- `cos(z)`, `z` complex, returns the complex cosine of `z`, mathematically defined as $\frac{\exp(i \cdot z) + \exp(-i \cdot z)}{2}$. 


\[ \tan(x), \text{ } x \text{ real, returns the tangent of } x. \]

\[ \tan(z), \text{ } z \text{ complex, returns the complex tangent of } z, \text{ mathematically defined as } \frac{\sin(z)}{\cos(z)}. \]

\[ \text{asin}(Z), \text{ acos}(Z), \text{ and atan}(Z) \text{ return the appropriate inverse trigonometric functions. Returned results are in radians. All return real if the argument is real and complex if the argument is complex.} \]

\[ \text{asin}(x), \text{ } x \text{ real, returns arcsine in the range } [-\pi/2, \pi/2]. \text{ If } x < -1 \text{ or } x > 1, \text{ missing (.) is returned.} \]

\[ \text{asin}(z), \text{ } z \text{ complex, returns the complex arcsine, mathematically defined as } -i \ln\{i * z + \sqrt{1 - z^2}\}. \text{ Re(asin(z)) is chosen to be in the interval } [-\pi/2, \pi/2]. \]

\[ \text{acos}(x), \text{ } x \text{ real, returns arccosine in the range } [0, \pi]. \text{ If } x < -1 \text{ or } x > 1, \text{ missing (.) is returned.} \]

\[ \text{acos}(z), \text{ } z \text{ complex, returns the complex arccosine, mathematically defined as } -i \ln\{z + \sqrt{z^2 - 1}\}. \text{ Re(acos(z)) is chosen to be in the interval } [0, \pi]. \]

\[ \text{atan}(x), \text{ } x \text{ real, returns arctangent in the range } (-\pi/2, \pi/2). \]

\[ \text{atan}(z), \text{ } z \text{ complex, returns the complex arctangent, mathematically defined as } \ln\{(1 + iz)/(1 - iz)\}/(2i). \text{ Re(atan(z)) is chosen to be in the interval } [0, \pi]. \]

\[ \text{atan2}(X, Y) \text{ returns the radian value in the range } (-\pi, \pi] \text{ of the angle of the vector determined by } (X,Y), \text{ the result being in the range } [0, \pi] \text{ for quadrants 1 and 2 and } [0, -\pi) \text{ for quadrants 4 and 3.} \text{ X and Y must be real. atan2(X, Y) is equivalent to arg(C(X, Y)).} \]

\[ \text{arg}(Z) \text{ returns the arctangent of Im(Z)/Re(Z) in the correct quadrant, the result being in the range } (-\pi, \pi]; [0, \pi] \text{ in quadrants 1 and 2 and } [0, -\pi) \text{ in quadrants 4 and 3. arg(Z) is equivalent to atan2(Re(Z), Im(Z)).} \]

\[ \text{sinh}(Z), \text{ cosh}(Z), \text{ and tanh}(Z) \text{ return the hyperbolic sine, cosine, and tangent, respectively. The returned value is real if the argument is real and complex if the argument is complex.} \]

\[ \text{sinh}(x), \text{ } x \text{ real, returns the inverse hyperbolic sine of } x, \text{ mathematically defined as } \frac{\exp(x) - \exp(-x)}{2}. \]

\[ \text{sinh}(z), \text{ } z \text{ complex, returns the complex hyperbolic sine of } z, \text{ mathematically defined as } \frac{\exp(z) - \exp(-z)}{2}. \]

\[ \text{cosh}(x), \text{ } x \text{ real, returns the inverse hyperbolic cosine of } x, \text{ mathematically defined as } \frac{\exp(x) + \exp(-x)}{2}. \]

\[ \text{cosh}(z), \text{ } z \text{ complex, returns the complex hyperbolic cosine of } z, \text{ mathematically defined as } \frac{\exp(z) + \exp(-z)}{2}. \]

\[ \text{tanh}(x), \text{ } x \text{ real, returns the inverse hyperbolic tangent of } x, \text{ mathematically defined as } \text{sinh}(x)/\text{cosh}(x). \]

\[ \text{tanh}(z), \text{ } z \text{ complex, returns the complex hyperbolic tangent of } z, \text{ mathematically defined as } \text{sinh}(z)/\text{cosh}(z). \]

\[ \text{asinh}(Z), \text{ acosh}(Z), \text{ and atanh}(Z) \text{ return the inverse hyperbolic sine, cosine, and tangent, respectively. The returned value is real if the argument is real and complex if the argument is complex.} \]

\[ \text{asinh}(x), \text{ } x \text{ real, returns the inverse hyperbolic sine.} \]
\( \text{asinh}(z) \), \( z \) complex, returns the complex inverse hyperbolic sine, mathematically defined as \( \ln\{z + \sqrt{z^2 + 1}\} \). \( \text{Im}(\text{asinh}()) \) is chosen to be in the interval \([−\pi/2, \pi/2]\).

\( \text{acosh}(x) \), \( x \) real, returns the inverse hyperbolic cosine. If \( x < 1 \), missing (.) is returned.

\( \text{acosh}(z) \), \( z \) complex, returns the complex inverse hyperbolic cosine, mathematically defined as \( \ln\{z + \sqrt{z^2 - 1}\} \). \( \text{Im}(\text{acosh}()) \) is chosen to be in the interval \([−\pi, \pi]\); \( \text{Re}(\text{acosh}()) \) is chosen to be nonnegative.

\( \text{atanh}(x) \), \( x \) real, returns the inverse hyperbolic tangent. If \( |x| > 1 \), missing (.) is returned.

\( \text{atanh}(z) \), \( z \) complex, returns the complex inverse hyperbolic tangent, mathematically defined as \( \ln\{\frac{1+z}{1-z}\}/2 \). \( \text{Im}(\text{atanh}()) \) is chosen to be in the interval \([−\pi/2, \pi/2]\).

\( \pi() \) returns the value of \( \pi \).

**Conformability**

\( \text{atan2}(X, Y) \):

\[
\begin{align*}
X & : \quad r_1 \times c_1 \\
Y & : \quad r_2 \times c_2, \quad X \text{ and } Y \text{ r-conformable} \\
\text{result} & : \quad \max(r_1, r_2) \times \max(c_1, c_2)
\end{align*}
\]

\( \pi() \) returns a \( 1 \times 1 \) scalar.

All other functions return a matrix of the same dimension as input containing element-by-element calculated results.

**Diagnostics**

All functions return missing for real arguments when the result would be complex. For instance, \( \text{acos}(2) = . \), whereas \( \text{acos}(2+0i) = -1.317i \).

**Also see**

[M-4] scalar — Scalar mathematical functions