## xtreg — Linear models for panel data

Description Options Acknowledgments Quick start Remarks and examples References Menu Stored results Also see Syntax Methods and formulas

# Description

xtreg fits linear regression models for panel data. xtreg with the re option fits random-effects models using generalized least squares (GLS); xtreg with the fe option fits fixed-effects models using the within regression estimator; and xtreg with the pa option fits population-averaged models. Alternatively, random-effects models can be fit by using maximum likelihood (mle option) or the between-effects estimator (be option). xtreg with the cre option fits correlated random-effects models using a Mundlak regression. See [XT] xtdata for a faster way to fit fixed- and random-effects models.

# **Quick start**

Random-effects linear regression by GLS of y on x1 and x2 using xtset data

xtreg y x1 x2

Same as above, but estimate by maximum likelihood

xtreg y x1 x2, mle

- Fixed-effects model with cluster-robust standard errors for panels nested within cvar xtreg y x1 x2, fe vce(cluster cvar)
- Fixed-effects model absorbing indicator variables for the levels of cvar xtreg y x1 x2, fe absorb(cvar)
- Correlated random-effects model by Mundlak regression

xtreg y x1 x2, cre

- Population-averaged model with an exchangeable within-panel correlation structure xtreg y x1 x2, pa
- Same as above, but specify an autoregressive correlation structure of order 1 xtreg y x1 x2, pa corr(ar 1)
- Between-effects model

xtreg y x1 x2, be

## Menu

Statistics > Longitudinal/panel data > Linear models > Linear regression (FE, RE, PA, BE, CRE)

# Syntax

GLS random-effects (RE) model xtreg depvar [indepvars] [if] [in] [, re RE\_options]

### Correlated random-effects (CRE) model

xtreg depvar [ indepvars ] [ if ] [ in ], cre [ CRE\_options ]

Between-effects (BE) model

xtreg depvar [indepvars] [if] [in], be [BE\_options]

Fixed-effects (FE) model

```
xtreg depvar [indepvars] [if] [in] [weight], fe [FE_options]
```

#### Random-effects model using maximum likelihood estimation (MLE)

xtreg depvar [indepvars] [if] [in] [weight], mle [MLE\_options]

#### Population-averaged (PA) model

xtreg depvar [indepvars] [if] [in] [weight], pa [PA\_options]

RE_options	Description
Model	
re	use random-effects estimator; the default
sa	use Swamy-Arora estimator of the variance components
SE/Robust	
vce( <i>vcetype</i> )	vcetype may be conventional, <u>r</u> obust, <u>cl</u> uster <i>clustvar</i> , <u>boot</u> strap, or <u>jackknife</u>
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>th</u> eta	report $\theta$
<u>ef</u> orm[( <i>string</i> )]	report exponentiated coefficients and, optionally, label as string
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<u>coefl</u> egend	display legend instead of statistics

CRE_options	Description
Model	
cre	use correlated random-effects estimator
sa	use Swamy-Arora estimator of variance components in Mundlak regression
SE/Robust	
vce( <i>vcetype</i> )	<i>vcetype</i> may be conventional, <u>r</u> obust, <u>cl</u> uster <i>clustvar</i> , <u>boot</u> strap, or <u>jack</u> knife
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
nomeans	do not display coefficients for panel means
<u>ef</u> orm[( <i>string</i> )]	report exponentiated coefficients and, optionally, label as string
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<u>coefl</u> egend	display legend instead of statistics
BE_options	Description
Model	
be	use between-effects estimator
wls	use weighted least squares
SE	
vce( <i>vcetype</i> )	<i>vcetype</i> may be conventional, <u>boot</u> strap, or <u>jack</u> knife
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>ef</u> orm[( <i>string</i> )]	report exponentiated coefficients and, optionally, label as string
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
<u>coefl</u> egend	display legend instead of statistics

FE_options	Description
Model	
fe	use fixed-effects estimator
<pre><u>absorb(varlist[, method])</u></pre>	specify categorical variables to be absorbed
<sup>†</sup> <u>dfabs</u> orb	adjust degrees of freedom for collinearity among absorbed variables
nosigmau	do not estimate $\sigma_u$
nouitest	suppress the $F$ test for $u_i$ and do not estimate $\sigma_u$
SE/Robust	
vce( <i>vcetype</i> )	<pre>vcetype may be conventional, robust, cluster clustvarlist, bootstrap, jackknife, or hc2 [clustvar]</pre>
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>clustert</u> able	display table of multiway cluster combinations
<u>ef</u> orm[( <i>string</i> )]	report exponentiated coefficients and, optionally, label as string
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
<pre>iterate(#)</pre>	maximum number of iterations for alternating projection method (APM); default is iterate(50)
nolog	suppress the APM iteration log
<pre>tolerance(#)</pre>	convergence tolerance for maximum absolute difference; default is tolerance(1e-8)
<u>coefl</u> egend	display legend instead of statistics

 $^{\dagger}$ Ignored if absorb() is not specified.

MLE_options	Description
Model	
<u>nocons</u> tant	suppress constant term
mle	use maximum-likelihood random-effects estimator
SE/Robust	
vce( <i>vcetype</i> )	<pre>vcetype may be oim, robust, cluster clustvar, bootstrap,     or jackknife</pre>
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>ef</u> orm[( <i>string</i> )]	report exponentiated coefficients and, optionally, label as string
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Maximization	
maximize_options	control the maximization process; seldom used
<u>coefl</u> egend	display legend instead of statistics

PA_options	Description
Model	
<u>nocons</u> tant	suppress constant term
pa	use population-averaged estimator
<u>off</u> set( <i>varname</i> )	include varname in model with coefficient constrained to 1
Correlation	
<u>c</u> orr( <i>correlation</i> )	within-panel correlation structure
force	estimate even if observations unequally spaced in time
SE/Robust	
vce( <i>vcetype</i> )	<i>vcetype</i> may be conventional, <u>r</u> obust, <u>boot</u> strap, or jackknife
nmp	use divisor $N - P$ instead of the default $N$
rgf	multiply the robust variance estimate by $(N-1)/(N-P)$
<u>s</u> cale( <i>parm</i> )	overrides the default scale parameter; $parm$ may be x2, dev, phi, or #
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
<u>ef</u> orm[( <i>string</i> )]	report exponentiated coefficients and, optionally, label as string
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
optimize_options	control the optimization process; seldom used
<u>coefl</u> egend	display legend instead of statistics
correlation	Description
<u>exc</u> hangeable	exchangeable
<u>ind</u> ependent	independent
<u>un</u> structured	unstructured
<u>fix</u> ed <i>matname</i>	user-specified
ar #	autoregressive of order #
<u>sta</u> tionary #	stationary of order #
<u>non</u> stationary#	nonstationary of order #

A panel variable must be specified. For xtreg, pa, correlation structures other than exchangeable and independent require that a time variable also be specified. Use xtset; see [XT] xtset.

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, collect, mi estimate, and statsby are allowed; see [U] 11.1.10 Prefix commands. bayes is allowed for the GLS random-effects model. For more details, see [BAYES] bayes: xtreg. fp is allowed for the between-effects, fixed-effects, and maximum-likelihood random-effects models.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

vce() is not allowed with the by prefix.

aweights, fweights, and pweights are allowed for the fixed-effects model. iweights, fweights, and pweights are allowed for the population-averaged model. iweights are allowed for the maximum-likelihood random-effects model. See [U] 11.1.6 weight. Weights must be constant within panel.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

# Options

Options are presented under the following headings:

Options for RE model Options for CRE model Options for BE model Options for MLE model Options for PA model

#### **Options for RE model**

\_ Model

re, the default, requests the GLS random-effects estimator.

sa specifies that the small-sample Swamy–Arora estimator for the individual-level variance component be used instead of the default consistent estimator. See *xtreg*, *re* in *Methods and formulas* for details.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*); see xtreg, re in Methods and formulas.

Reporting

level(#); see [R] Estimation options.

theta specifies that the output include the estimated value of  $\theta$  used in combining the between and fixed estimators. For balanced data, this is a constant, and for unbalanced data, a summary of the values is presented in the header of the output.

eform and eform(*string*) specify that the coefficient table be displayed in exponentiated form and that exp(b) and *string*, respectively, be used to label the exponentiated coefficients in the table. Standard errors and confidence intervals are also transformed.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

## **Options for CRE model**

Model

cre requests the correlated random-effects estimator.

sa specifies that the small-sample Swamy–Arora estimator for the individual-level variance component be used when estimating the Mundlak regression by random effects. See *xtreg*, *re* in *Methods and formulas* for details.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Specifying vce(robust) is equivalent to specifying vce(cluster panelvar).

Reporting

level(#); see [R] Estimation options.

nomeans specifies that the coefficients of the panel means not be displayed in the output table.

- eform and eform(*string*) specify that the coefficient table be displayed in exponentiated form and that exp(b) and *string*, respectively, be used to label the exponentiated coefficients in the table. Standard errors and confidence intervals are also transformed.
- display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

## Options for BE model

Model

be requests the between regression estimator.

w1s specifies that, for unbalanced data, weighted least squares be used rather than the default OLS. Both methods produce consistent estimates. The true variance of the between-effects residual is  $\sigma_{\nu}^{2} + T_{i}\sigma_{\epsilon}^{2}$  (see *xtreg*, be in *Methods and formulas* below). WLS produces a "stabilized" variance of  $\sigma_{\nu}^{2}/T_{i} + \sigma_{\epsilon}^{2}$ , which is also not constant. Thus the choice between OLS and WLS amounts to which is more stable.

Comment: xtreg, be is rarely used anyway, but between estimates are an ingredient in the randomeffects estimate. Our implementation of xtreg, re uses the OLS estimates for this ingredient, based on our judgment that  $\sigma_{\nu}^2$  is large relative to  $\sigma_{\epsilon}^2$  in most models. Formally, only a consistent estimate of the between estimates is required.

SE

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Reporting

level(#); see [R] Estimation options.

eform and eform(*string*) specify that the coefficient table be displayed in exponentiated form and that exp(b) and *string*, respectively, be used to label the exponentiated coefficients in the table. Standard errors and confidence intervals are also transformed.

```
display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt),
sformat(%fmt), and nolstretch; see [R] Estimation options.
```

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

## **Options for FE model**

[ Model ]

fe requests the fixed-effects (within) regression estimator.

absorb(*varlist*[, *method*]) specifies the categorical variables to be absorbed. The results are adjusted as if indicator variables for each level of each variable in *varlist* were included in the regression. The panel variable is included as the first absorbed variable.

An APM iterative algorithm is used to project the *depvar* and *indepvars* to absorb these variables and the panel variable. *method* specifies the APM and is one of halperin or cimmino.

halperin, the default, uses the product of the projection matrices.

cimmino uses the mean of the projection matrices.

The two methods typically perform similarly. See Stammann (2018) for details.

absorb() may not be specified with the vce(hc2) option.

- dfabsorb adjusts the degrees of freedom to account for collinearity among the absorbed variables. The default degrees of freedom assumes all absorbed variables are independent. If absorb() is not also specified, only the panel variable is absorbed, and the dfabsorb option is ignored.
- nosigmau prevents the computation of  $\sigma_u$ . When absorb(*varlist*) is specified, the estimation of  $\sigma_u$  requires an iterative search. If  $\sigma_u$  is not of interest, you may specify nosigmau to save computational time.
- nouitest prevents computation of  $\sigma_u$  and the F test that all  $u_i$  are jointly zero. If  $\sigma_u$  and the F test are of no interest, you may specify nouitest to save computational time.

SE/Robust

- vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), that allow for intragroup correlations (cluster *clustvarlist*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.
  - vce(conventional), the default, uses the conventionally derived variance estimator for ordinary least-squares regression.
  - vce(robust) is equivalent to specifying vce(cluster panelvar).
  - vce(cluster clustvarlist) specifies that standard errors allow for intragroup correlation within groups defined by one or more variables in clustvarlist, relaxing the usual requirement that the observations be independent. For example, vce(cluster clustvar1) produces cluster-robust standard errors that allow for observations that are independent across groups defined by clustvar1 but not necessarily independent within groups. You could also type vce(cluster clustvar1 clustvar2...clusterp) to account for correlation within groups formed by p variables (multiway clustering).

xtreg also allows the following:

vce(hc2[clustvar][, dfadjust]) specifies a cluster-robust variance calculation that uses a bias correction that is an alternative to the one provided by vce(robust) and vce(cluster clustvarlist). Specifying vce(hc2[, dfadjust]) is equivalent to specifying vce(hc2 panelvar[, dfadjust]). dfadjust computes the Bell and McCaffrey (2002) adjusted degrees of freedom based on the clusters of clustvar; see xtreg, fe in Methods and formulas. Note that dfadjust does not affect multiple-imputation results when the command is used with mi estimate.

vce(hc2) may not be specified with the absorb() option.

Reporting

level(#); see [R] Estimation options.

- clustertable displays a table reporting cluster combinations and the number of clusters per combination. This option is available only when vce(cluster *clustvarlist*) is specified with more than one variable in *clustvarlist* to compute multiway cluster-robust standard errors.
- eform and eform(*string*) specify that the coefficient table be displayed in exponentiated form and that exp(b) and *string*, respectively, be used to label the exponentiated coefficients in the table. Standard errors and confidence intervals are also transformed.

dfci specifies that parameter degrees of freedom and confidence intervals be reported in the coefficient table.

dfpvalues specifies that parameter degrees of freedom and p-values be reported in the coefficient table.

Optimization

iterate(#) specifies the maximum number of iterations for the APM. The default is iterate(50).

nolog specifies that no APM iterative log be displayed.

tolerance(#) specifies the limit for the maximum absolute difference between iterations for the projected depvar and indepvars. The default is tolerance(1e-8).

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

#### Options for MLE model

Model

noconstant; see [R] Estimation options.

mle requests the maximum-likelihood random-effects estimator.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster *clustvar*), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

Reporting

level(#); see [R] Estimation options.

eform and eform(*string*) specify that the coefficient table be displayed in exponentiated form and that exp(b) and *string*, respectively, be used to label the exponentiated coefficients in the table. Standard errors and confidence intervals are also transformed.

```
display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt),
sformat(%fmt), and nolstretch; see [R] Estimation options.
```

Maximization

maximize\_options: iterate(#), [no]log, trace, tolerance(#), ltolerance(#), and from(init\_specs); see [R] Maximize. These options are seldom used.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

## **Options for PA model**

Model

noconstant; see [R] Estimation options.

pa requests the population-averaged estimator. For linear regression, this is the same as a random-effects estimator (both interpretations hold).

xtreg, pa is equivalent to xtgee, family(gaussian) link(id) corr(exchangeable), which are the defaults for the xtgee command. xtreg, pa allows all the relevant xtgee options such as vce(robust). Whether you use xtreg, pa or xtgee makes no difference. See [XT] xtgee.

offset(varname); see [R] Estimation options.

Correlation

corr(correlation) specifies the within-panel correlation structure; the default corresponds to the equalcorrelation model, corr(exchangeable).

When you specify a correlation structure that requires a lag, you indicate the lag after the structure's name with or without a blank; for example, corr(ar1) or corr(ar1).

If you specify the fixed correlation structure, you specify the name of the matrix containing the assumed correlations following the word fixed, for example, corr(fixed myr).

force specifies that estimation be forced even though the time variable is not equally spaced. This is relevant only for correlation structures that require knowledge of the time variable. These correlation structures require that observations be equally spaced so that calculations based on lags correspond to a constant time change. If you specify a time variable indicating that observations are not equally spaced, the (time dependent) model will not be fit. If you also specify force, the model will be fit, and it will be assumed that the lags based on the data ordered by the time variable are appropriate.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

nmp; see [XT] vce\_options.

rgf specifies that the robust variance estimate is multiplied by (N-1)/(N-P), where N is the total number of observations and P is the number of coefficients estimated. This option can be used with family(gaussian) only when vce(robust) is either specified or implied by the use of pweights. Using this option implies that the robust variance estimate is not invariant to the scale of any weights used.

scale(x2|dev|phi|#); see [XT] vce\_options.

Reporting

level(#); see [R] Estimation options.

eform and eform(*string*) specify that the coefficient table be displayed in exponentiated form and that exp(b) and *string*, respectively, be used to label the exponentiated coefficients in the table. Standard errors and confidence intervals are also transformed.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Optimization

optimize\_options control the iterative optimization process. These options are seldom used.

<u>iter</u>ate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimization stops and presents the current results, even if convergence has not been reached. The default is iterate(100).

<u>tol</u>erance(#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

trace specifies that the current estimates be printed at each iteration.

The following option is available with xtreg but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

## **Remarks and examples**

If you have not read [XT] **xt**, please do so.

See Baltagi (2013, chap. 2) and Wooldridge (2020, chap. 14) for good overviews of fixed-effects and random-effects models. Allison (2009) provides perspective on the use of fixed- versus random-effects estimators and provides many examples using Stata.

Consider fitting models of the form

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it} \tag{1}$$

In this model,  $\nu_i + \epsilon_{it}$  is the error term that we have little interest in; we want estimates of  $\beta$ .  $\nu_i$  is the unit-specific error term; it differs between units, but for any particular unit, its value is constant. In the pulmonary data of [XT] **xt**, a person who exercises less would presumably have a lower forced expiratory volume year after year and so would have a negative  $\nu_i$ .

 $\epsilon_{it}$  is the "usual" error term with the usual properties (mean 0, uncorrelated with itself, uncorrelated with x, uncorrelated with  $\nu$ , and homoskedastic), although in a more thorough development, we could decompose  $\epsilon_{it} = v_t + \omega_{it}$ , assume that  $\omega_{it}$  is a conventional error term, and better describe  $v_t$ .

Before making the assumptions necessary for estimation, let's perform some useful algebra on (1). Whatever the properties of  $\nu_i$  and  $\epsilon_{it}$ , if (1) is true, it must also be true that

$$\overline{y}_{i} = \alpha + \overline{\mathbf{x}}_{i} \boldsymbol{\beta} + \nu_{i} + \overline{\epsilon}_{i} \tag{2}$$

where  $\overline{y}_i = \sum_t y_{it}/T_i$ ,  $\overline{\mathbf{x}}_i = \sum_t \mathbf{x}_{it}/T_i$ , and  $\overline{\epsilon}_i = \sum_t \epsilon_{it}/T_i$ . Subtracting (2) from (1), it must be equally true that

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta} + (\epsilon_{it} - \overline{\epsilon}_i)$$
(3)

These three equations provide the basis for estimating  $\beta$ . In particular, xtreg, fe provides what is known as the fixed-effects estimator—also known as the within estimator—and amounts to using OLS to perform the estimation of (3). xtreg, be provides what is known as the between estimator and amounts to using OLS to perform the estimation of (2). xtreg, re provides the random-effects estimator and is a (matrix) weighted average of the estimates produced by the between and within estimators. In particular, the random-effects estimator turns out to be equivalent to estimation of

$$(y_{it} - \theta_i \bar{y}_i) = (1 - \theta_i)\alpha + (\mathbf{x}_{it} - \theta_i \bar{\mathbf{x}}_i)\beta + \{(1 - \theta_i)\nu_i + (\epsilon_{it} - \theta_i \bar{\epsilon}_i)\}$$
(4)

where  $\theta_i$  is a function of  $\sigma_{\nu}^2$ ,  $\sigma_{\epsilon}^2$  and  $T_i$ . If  $\sigma_{\nu}^2 = 0$ , meaning that  $\nu_i$  is always 0,  $\theta_i = 0$  for all *i* and (1) can be estimated by OLS directly. Alternatively, if  $\sigma_{\epsilon}^2 = 0$ , meaning that  $\epsilon_{it}$  is 0,  $\theta_i = 1$  for all *i* and the within estimator returns all the information available (which will, in fact, be a regression with an  $R^2$  of 1).

Consider an extended version of (4), where we add the panel averages of the covariates as regressors, so that

$$(y_{it} - \theta_i \bar{y}_i) = (1 - \theta_i)\alpha + (\mathbf{x}_{it} - \theta_i \bar{\mathbf{x}}_i)\beta + (1 - \theta_i)\bar{\mathbf{x}}_i\gamma + \{(1 - \theta_i)\nu_i + (\epsilon_{it} - \theta_i\bar{\epsilon}_i)\}$$
(5)

This extended model is known as the Mundlak regression or, more recently, as the correlated randomeffects model. Mundlak (1978) showed that, for balanced panels, the estimation of (5) by OLS produces the same coefficients for time-variant regressors as xtreg, fe. This equivalence result was extended to unbalanced panels in Wooldridge (2019). With unbalanced panels, for the equivalence to hold, the panel averages in (5) should be calculated using only observations in the estimation sample. xtreg, cre fits the correlated random-effects model.

Few assumptions are required to justify the fixed-effects estimator of (3). The estimates are, however, conditional on the sample in that the  $\nu_i$  are not assumed to have a distribution but are instead treated as fixed and estimable. This statistical fine point can lead to difficulty when making out-of-sample predictions, but that aside, the fixed-effects estimator has much to recommend it.

More is required to justify the between estimator of (2), but the conditioning on the sample is not assumed because  $\nu_i + \overline{\epsilon}_i$  is treated as an error term. Newly required is that we assume that  $\nu_i$  and  $\overline{\mathbf{x}}_i$  are uncorrelated. This follows from the assumptions of the OLS estimator but is also transparent: were  $\nu_i$  and  $\overline{\mathbf{x}}_i$  correlated, the estimator could not determine how much of the change in  $\overline{y}_i$ , associated with an increase in  $\overline{\mathbf{x}}_i$ , to assign to  $\beta$  versus how much to attribute to the unknown correlation. (This, of course, suggests the use of an instrumental-variable estimator,  $\overline{\mathbf{z}}_i$ , which is correlated with  $\overline{\mathbf{x}}_i$  but uncorrelated with  $\nu_i$ , though that approach is not implemented here.)

The random-effects estimator of (4) requires the same no-correlation assumption. In comparison with the between estimator, the random-effects estimator produces more efficient results, albeit ones with unknown small-sample properties. The between estimator is less efficient because it discards the over-time information in the data in favor of simple means; the random-effects estimator uses both the within and the between information.

The correlated random-effects estimator of (5) does not require the no-correlation assumption that is required by the random-effects estimator.

All of this would seem to leave the between estimator of (2) with no role (except for a minor, technical part it plays in helping to estimate  $\sigma_{\nu}^2$  and  $\sigma_{\epsilon}^2$ , which are used in the calculation of  $\theta$ , on which the random-effects estimates depend). Let's, however, consider a variation on (1):

$$y_{it} = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta}_1 + (\mathbf{x}_{it} - \overline{\mathbf{x}}_i) \boldsymbol{\beta}_2 + \nu_i + \epsilon_{it} \tag{1'}$$

xtreg — Linear models for panel data 14

In this model, we postulate that changes in the average value of  $\mathbf{x}$  for an individual have a different effect from temporary departures from the average. In an economic situation, y might be purchases of some item and  $\mathbf{x}$  income; a change in average income should have more effect than a transitory change. In a clinical situation, y might be a physical response and  $\mathbf{x}$  the level of a chemical in the brain; the model allows a different response to permanent rather than transitory changes.

The variations of (2) and (3) corresponding to (1') are

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta}_1 + \nu_i + \overline{\epsilon}_i \tag{2'}$$

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\boldsymbol{\beta}_2 + (\epsilon_{it} - \overline{\epsilon}_i) \tag{3'}$$

That is, the between estimator estimates  $\beta_1$  and the within  $\beta_2$ , and neither estimates the other. Thus even when estimating equations like (1), it is worth comparing the within and between estimators. Differences in results can suggest models like (1'), or at the least some other specification error.

Finally, it is worth understanding the role of the between and within estimators with regressors that are constant over time or constant over units. Consider the model

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta}_1 + \mathbf{s}_i\boldsymbol{\beta}_2 + \mathbf{z}_t\boldsymbol{\beta}_3 + \nu_i + \epsilon_{it}$$
(1")

This model is the same as (1), except that we explicitly identify the variables that vary over both time and i ( $\mathbf{x}_{it}$ , such as output or forced expiratory volume; variables that are constant over time ( $\mathbf{s}_i$ , such as race or sex); and variables that vary solely over time ( $\mathbf{z}_t$ , such as the consumer price index or age in a cohort study). The corresponding between and within equations are

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta}_1 + \mathbf{s}_i \boldsymbol{\beta}_2 + \overline{\mathbf{z}} \boldsymbol{\beta}_3 + \nu_i + \overline{\epsilon}_i$$

$$(2'')$$

$$(y_{it} - \overline{y}_i) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_i) \boldsymbol{\beta}_1 + (\mathbf{z}_t - \overline{\mathbf{z}}) \boldsymbol{\beta}_3 + (\epsilon_{it} - \overline{\epsilon}_i)$$

$$(3'')$$

In the between estimator of (2''), no estimate of  $\beta_3$  is possible because  $\overline{z}$  is a constant across the *i* observations; the regression-estimated intercept will be an estimate of  $\alpha + \overline{z}\beta_3$ . On the other hand, it can provide estimates of  $\beta_1$  and  $\beta_2$ . It can estimate effects of factors that are constant over time, such as race and sex, but to do so it must assume that  $\nu_i$  is uncorrelated with those factors.

The within estimator of (3"), like the between estimator, provides an estimate of  $\beta_1$  but provides no estimate of  $\beta_2$  for time-invariant factors. Instead, it provides an estimate of  $\beta_3$ , the effects of the time-varying factors. The within estimator can also provide estimates  $u_i$  for  $\nu_i$ . More correctly, the estimator  $u_i$  is an estimator of  $\nu_i + \mathbf{s}_i \beta_2$ . Thus  $u_i$  is an estimator of  $\nu_i$  only if there are no time-invariant variables in the model. If there are time-invariant variables,  $u_i$  is an estimate of  $\nu_i$  plus the effects of the time-invariant variables.

The correlated random-effects estimator of (5) can provide estimates of coefficients on all three types of variables—those that vary across time and panels, those that vary only across time, and those that vary only across panels.

Remarks are presented under the following headings:

Assessing goodness of fit xtreg and associated commands

## Assessing goodness of fit

 $R^2$  is a popular measure of goodness of fit in ordinary regression. In our case, given  $\hat{\alpha}$  and  $\hat{\beta}$  estimates of  $\alpha$  and  $\beta$ , we can assess the goodness of fit with respect to (1), (2), or (3). The prediction equations are, respectively,

$$\begin{split} \hat{y}_{it} &= \hat{\alpha} + \mathbf{x}_{it} \widehat{\boldsymbol{\beta}} \\ \hat{y}_{i} &= \hat{\alpha} + \overline{\mathbf{x}}_{i} \widehat{\boldsymbol{\beta}} \\ \hat{\bar{y}}_{i} &= (\hat{y}_{it} - \hat{\bar{y}}_{i}) = (\mathbf{x}_{it} - \overline{\mathbf{x}}_{i}) \widehat{\boldsymbol{\beta}} \end{split} \tag{1'''}$$

xtreg reports "*R*-squares" corresponding to these three equations. *R*-squares is in quotes because the *R*-squares reported do not have all the properties of the OLS  $R^2$ .

The ordinary properties of  $R^2$  include being equal to the squared correlation between  $\hat{y}$  and y and being equal to the fraction of the variation in y explained by  $\hat{y}$ —formally defined as  $Var(\hat{y})/Var(y)$ . The identity of the definitions is from a special property of the OLS estimates; in general, given a prediction  $\hat{y}$  for y, the squared correlation is not equal to the ratio of the variances, and the ratio of the variances is not required to be less than 1.

xtreg reports  $R^2$  values calculated as correlations squared, calling them  $R^2$  overall, corresponding to (1''');  $R^2$  between, corresponding to (2'''); and  $R^2$  within, corresponding to (3'''). In fact, you can think of each of these three numbers as having all the properties of ordinary  $R^2$ 's, if you bear in mind that the prediction being judged is not  $\hat{y}_{it}$ ,  $\hat{y}_i$ , and  $\hat{y}_{it}$ , but  $\gamma_1 \hat{y}_{it}$  from the regression  $y_{it} = \gamma_1 \hat{y}_{it}$ ;  $\gamma_2 \hat{\overline{y}}_i$  from the regression  $\overline{y}_i = \gamma_2 \hat{\overline{y}}_i$ ; and  $\gamma_3 \hat{\overline{y}}_{it}$  from  $\tilde{y}_{it} = \gamma_3 \hat{\overline{y}}_{it}$ .

In particular, xtreg, be obtains its estimates by performing OLS on (2), and therefore its reported  $R^2$  between is an ordinary  $R^2$ . The other two reported  $R^2$ 's are merely correlations squared, or, if you prefer,  $R^2$ 's from the second-round regressions  $y_{it} = \gamma_{11}\hat{y}_{it}$  and  $\tilde{y}_{it} = \gamma_{13}\hat{y}_{it}$ .

xtreg, fe obtains its estimates by performing OLS on (3), so its reported  $R^2$  within is an ordinary  $R^2$ . As with be, the other  $R^2$ 's are correlations squared, or, if you prefer,  $R^2$ 's from the second-round regressions  $\overline{y_i} = \gamma_{22} \hat{\overline{y}}_i$  and, as with be,  $\tilde{y}_{it} = \gamma_{23} \hat{\overline{y}}_{it}$ .

xtreg, re obtains its estimates by performing OLS on (4); none of the  $R^2$ 's corresponding to (1'''), (2'''), or (3''') correspond directly to this estimator [the "relevant"  $R^2$  is the one corresponding to (4)]. All three reported  $R^2$ 's are correlations squared, or, if you prefer, from second-round regressions.

xtreg, cre obtains its estimates by performing OLS on the extended model in (5). Because this is equivalent to performing a random-effects regression with the panel means included as additional controls, the three  $R^2$ 's reported by xtreg, cre correspond to the  $R^2$ 's that xtreg, re reports for the extended model.

#### xtreg and associated commands

#### Example 1: Between-effects model

Using nlswork.dta described in [XT] xt, we will model ln\_wage in terms of completed years of schooling (grade), current age and age squared, current years worked (experience) and experience squared, current years of tenure on the current job and tenure squared, whether black (race = 2), whether residing in an area not designated a standard metropolitan statistical area (SMSA), and whether residing in the South.

. use https://www.stata-press.com/data/r19/nlswork (National Longitudinal Survey of Young Women, 14-24 years old in 1968)

To obtain the between-effects estimates, we use xtreg, be. nlswork.dta has previously been xtset idcode year because that is what is true of the data, but for running xtreg, it would have been sufficient to have xtset idcode by itself.

<pre>. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure &gt; c.tenure#c.tenure 2.race not_smsa south, be</pre>							
Between regression (regression on group means) Group variable: idcode				Number o		28,091	
Group variable	e: 1dcode				of groups =	4,697	
R-squared:				Obs per	0 1		
	= 0.1591				min =	1	
Between =					avg =	6.0	
Overall =	= 0.3695				max =	15	
				F(10,468		450.23	
sd(u_i + avg(e	e_i.)) = .3036	114		Prob > I	? =	0.0000	
ln_wage	Coefficient	Std. err.	t	P> t	[95% conf.	interval]	
grade	.0607602	.0020006	30.37	0.000	.0568382	.0646822	
age	.0323158	.0087251	3.70	0.000	.0152105	.0494211	
c.age#c.age	0005997	.0001429	-4.20	0.000	0008799	0003194	
ttl_exp	.0138853	.0056749	2.45	0.014	.0027598	.0250108	
c.ttl exp#							
c.ttl_exp	.0007342	.0003267	2.25	0.025	.0000936	.0013747	
tenure	.0698419	.0060729	11.50	0.000	.0579361	.0817476	
c.tenure#							
c.tenure	0028756	.0004098	-7.02	0.000	0036789	0020722	
race							
Black	0564167	.0105131	-5.37	0.000	0770272	0358061	
not_smsa	1860406	.0112495	-16.54	0.000	2080949	1639862	
south	0993378	.010136	-9.80	0.000	1192091	0794665	
_cons	.3339113	.1210434	2.76	0.006	.0966093	.5712133	

The between-effects regression is estimated on person-averages, so the "n = 4697" result is relevant. xtreg, be reports the "number of observations" and group-size information: describe in [XT] xt showed that we have 28,534 "observations"—person-years, really—of data. If we take the subsample that has no missing values in ln\_wage, grade, ..., south leaves us with 28,091 observations on person-years, reflecting 4,697 persons, each observed for an average of 6.0 years.

For goodness of fit, the  $R^2$  between is directly relevant; our  $R^2$  is 0.4900. If, however, we use these estimates to predict the within model, we have an  $R^2$  of 0.1591. If we use these estimates to fit the overall data, our  $R^2$  is 0.3695.

The F statistic tests that the coefficients on the regressors grade, age, ..., south are all jointly zero. Our model is significant.

The root mean squared error of the fitted regression, which is an estimate of the standard deviation of  $\nu_i + \overline{\epsilon}_i$ , is 0.3036.

For our coefficients, each year of schooling increases hourly wages by 6.1%; age increases wages up to age 26.9 and thereafter decreases them (because the quadratic  $ax^2 + bx + c$  turns over at x = -b/2a, which for our age and c.age#c.age coefficients is  $0.0323158/(2 \times 0.0005997) \approx 26.9$ ); total experience increases wages at an increasing rate (which is surprising and bothersome); tenure on the current job increases wages up to a tenure of 12.1 years and thereafter decreases them; wages of blacks are, these things held constant, (approximately) 5.6% below that of nonblacks (approximately because 2.race is an indicator variable); residing in a non-SMSA (rural area) reduces wages by 18.6%; and residing in the South reduces wages by 9.9%. Example 2: Fixed-effects model

To fit the same model with the fixed-effects estimator, we specify the fe option.

<pre>&gt; c.tenure#c.t note: grade on note: 2.race of</pre>	tenure 2.race	not_smsa so e of colline	outh, fe earity.	tl_exp#c.	ttl_exp tenur	e
Fixed-effects Group variable	0	ression	-	Number Number	of obs = of groups =	28,091 4,697
R-squared:				Obs per	group:	
Within = Between = Overall =	= 0.3505			-	min = avg = max =	1 6.0 15
corr(u_i, Xb)	= 0.1936			F(8, 23 Prob >		610.12 0.0000
ln_wage	Coefficient	Std. err.	t	P> t	[95% conf.	interval]
grade age	0 .0359987	(omitted) .0033864	10.63	0.000	.0293611	.0426362
c.age#c.age	000723	.0000533	-13.58	0.000	0008274	0006186
ttl_exp	.0334668	.0029653	11.29	0.000	.0276545	.039279
c.ttl_exp# c.ttl_exp	.0002163	.0001277	1.69	0.090	0000341	.0004666
tenure	.0357539	.0018487	19.34	0.000	.0321303	.0393775
c.tenure# c.tenure	0019701	.000125	-15.76	0.000	0022151	0017251
race Black not_smsa south _cons	0 0890108 0606309 1.03732	(omitted) .0095316 .0109319 .0485546	-9.34 -5.55 21.36	0.000 0.000 0.000	1076933 0820582 .9421496	0703282 0392036 1.13249
sigma_u sigma_e rho	.35562203 .29068923 .59946283	(fraction	of varia	nce due t	o u_i)	

The observation summary at the top is the same as for the between-effects model, although this time it is the "Number of obs" that is relevant.

Our three  $R^2$ 's are not too different from those reported previously; the  $R^2$  within is slightly higher (0.1727 versus 0.1591), and the  $R^2$  between is a little lower (0.3505 versus 0.4900), as expected, because the between estimator maximizes  $R^2$  between and the within estimator  $R^2$  within. In terms of overall fit, these estimates are somewhat worse (0.2625 versus 0.3695).

If the unobserved time-invariant component  $\nu$  is not correlated with the regressors, estimates from the fixed-effects model are consistent but inefficient relative to estimates from the random-effects model. In this case, the interpretation of sigma\_u in the coefficient table is the same for the fixed-effects and random-effects models. However, sigma\_u is a nuisance parameter when  $\nu$  is correlated with the covariates.

The postestimation command estat mundlak performs a Mundlak specification test to help decide between random-effects and fixed-effects estimation. A rejection of the null hypothesis in this test indicates that  $\nu$  is probably correlated with the regressors and that the random-effects estimator is inconsistent.

```
. estat mundlak
Mundlak specification test
H0: Covariates are uncorrelated with unobserved panel-level effects
chi2(8) = 151.62
Prob > chi2 = 0.0000
Notes: Fixed effects and correlated random effects are
consistent under H0 and Ha.
Random effects are efficient under H0.
```

In this case, the test favors the implemented fixed-effects model.

In the xtreg output, we see that both grade and 2.race were omitted from the model because they do not vary over time. Our estimate  $u_i$  is an estimate of  $\nu_i$  plus the effects of any time-invariant variables, including grade and 2.race, so our estimate of the standard deviation is based on the variation in  $\nu_i$ , grade, 2.race, and any other time-invariant variables. On the other hand, had 2.race and grade been omitted merely because they were collinear with the other regressors in our model,  $u_i$  would be an estimate of  $\nu_i$ . (xtsum and xttab allow you to determine whether a variable is time invariant; see [XT] xtsum and [XT] xttab.)

Regardless of the status of  $u_i$ , our estimate of the standard deviation of  $\epsilon_{it}$  is valid (and, in fact, is the estimate that would be used by the random-effects estimator to produce its results).

Our estimate of the correlation of  $u_i$  with  $\mathbf{x}_{it}$  suffers from the problem of what  $u_i$  measures. We find correlation but cannot say whether this is correlation of  $\nu_i$  with  $\mathbf{x}_{it}$  or merely correlation of grade and 2.race with  $\mathbf{x}_{it}$ . In any case, the fixed-effects estimator is robust to such a correlation, and the other estimates it produces are unbiased.

So, although this estimator produces no estimates of the effects of grade and 2.race, it does predict that age has a positive effect on wages up to age 24.9 years (compared with 26.9 years estimated by the between estimator); that total experience still increases wages at an increasing rate (which is still bothersome); that tenure increases wages up to 9.1 years (compared with 12.1); that living in a non-SMSA reduces wages by 8.9% (compared with a more drastic 18.6%); and that living in the South reduces wages by 6.1% (as compared with 9.9%).

#### Example 3: Fixed-effects models with robust standard errors

If we suspect that there is heteroskedasticity or within-panel serial correlation in the idiosyncratic error term  $\epsilon_{it}$ , we could specify the vce(robust) or the vce(hc2) option:

	- ·				•	
<pre>. xtreg ln_w g &gt; c.tenure#c.t note: grade on note: 2.race of</pre>	tenure 2.race nitted because	not_smsa so e of colline	outh, fe v earity.			re
Fixed-effects Group variable	0	Number Number	of obs = of groups =	28,091 4,697		
R-squared:				Obs per	group:	
	= 0.1727			F	min =	1
Between =	= 0.3505				avg =	6.0
Overall =	= 0.2625				max =	15
				F(8, 46		273.86
corr(u i, Xb)	= 0 1936			Prob >		0.0000
coll(u_1, ND)	0.1500	(2.)				
		(Std. er	c. adjuste	ed for 4,	697 clusters	in idcode)
		Robust				
ln_wage	Coefficient	std. err.	t	P> t	[95% conf	. interval]
wage	coefficient	stu. eii.		12101	[35% COIII	. Incervarj
grade	0	(omitted)				
age	.0359987	.0052407	6.87	0.000	.0257243	.046273
-0-						
c.age#c.age	000723	.0000845	-8.56	0.000	0008887	0005573
0 0						
ttl_exp	.0334668	.004069	8.22	0.000	.0254896	.0414439
c.ttl_exp#						
c.ttl_exp	.0002163	.0001763	1.23	0.220	0001294	.0005619
tenure	.0357539	.0024683	14.49	0.000	.0309148	.040593
c.tenure#						
c.tenure	0019701	.0001696	-11.62	0.000	0023026	0016376
race						
Black	0	(omitted)				
not_smsa	0890108	.0137629	-6.47	0.000	1159926	062029
south	0606309	.0163366	-3.71	0.000	0926583	0286035
_cons	1.03732	.0739644	14.02	0.000	.8923149	1.182325
	255,600,00					
sigma_u	.35562203 .29068923					
sigma_e rho	. 29068923	(fraction	of worder	nco duc +	o 11 i)	
1110	.03340203	(fraction	UI VAIIA		0 u_1)	

Although the estimated coefficients are the same with and without the vce(robust) option, the robust estimator produced larger standard errors and a *p*-value for c.ttl\_exp#c.ttl\_exp above the conventional 10%. The *F* test of  $\nu_i = 0$  is suppressed because it is too difficult to compute the robust form of the statistic when there are more than a few panels.

## Technical note

The robust standard errors reported above are identical to those obtained by clustering on the panel variable idcode. Clustering on the panel variable produces an estimator of the VCE that is robust to cross-sectional heteroskedasticity and within-panel (serial) correlation that is asymptotically equivalent to that proposed by Arellano (1987). Although the example above applies the fixed-effects estimator, the robust and cluster–robust VCE estimators are also available for the random-effects estimator. Wooldridge (2020) and Arellano (2003) discuss these robust and cluster–robust VCE estimators. More details are available in *Methods and formulas*.

## Example 4: Random-effects model

Refitting our log-wage model with the random-effects estimator, we obtain

. xtreg ln_w g > c.tenure#c.t					ttl_exp tenur	e		
Random-effects GLS regression Group variable: idcode				Number Number	of obs = of groups =	28,091 4,697		
R-squared:					Obs per group:			
-	= 0.1715			F	min =	1		
Between =	= 0.4784				avg =	6.0		
Overall =	Overall = 0.3708					15		
				Wald ch	= i2(10) =	9244.74		
<pre>corr(u_i, X) =</pre>				Prob >	chi2 =	0.0000		
min 5% 0.2520 0.252	theta - median 20 0.5499	95% 0.7016	max 0.7206					
ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]		
grade	.0646499	.0017812	36.30	0.000	.0611589	.0681409		
age	.0368059	.0031195	11.80	0.000	.0306918	.0429201		
c.age#c.age	0007133	.00005	-14.27	0.000	0008113	0006153		
ttl_exp	.0290208	.002422	11.98	0.000	.0242739	.0337678		
c.ttl_exp#								
c.ttl_exp	.0003049	.0001162	2.62	0.009	.000077	.0005327		
tenure	.0392519	.0017554	22.36	0.000	.0358113	.0426925		
c.tenure# c.tenure	0020035	.0001193	-16.80	0.000	0022373	0017697		
race								
Black	053053	.0099926	-5.31	0.000	0726381	0334679		
not smsa	1308252	.0071751	-18.23	0.000	1448881	1167622		
south	0868922	.0073032	-11.90	0.000	1012062	0725781		
_cons	.2387207	.049469	4.83	0.000	.1417633	.3356781		
sigma u	.25790526							
sigma_e	.29068923							
rho	.44045273	(fraction	of varian	ce due t	o u_i)			
cons sigma_u sigma_e	.2387207 .25790526 .29068923	.049469	4.83	0.000	.1417633			

According to the  $R^2$ 's, this estimator performs worse within than the within fixed-effects estimator and worse between than the between estimator, as it must, and slightly better overall.

We estimate that  $\sigma_{\nu}$  is 0.2579 and  $\sigma_{\epsilon}$  is 0.2907 and, by assertion, assume that the correlation of  $\nu$  and **x** is zero.

All that is known about the random-effects estimator is its asymptotic properties, so rather than reporting an F statistic for overall significance, xtreg, re reports a  $\chi^2$ . Taken jointly, our coefficients are significant.

xtreg, re also reports a summary of the distribution of  $\theta_i$ , an ingredient in the estimation of (4).  $\theta$  is not a constant here because we observe women for unequal periods.

We estimate that schooling has a rate of return of 6.5% (compared with 6.1% between and no estimate within); that the increase of wages with age turns around at 25.8 years (compared with 26.9 between and 24.9 within); that total experience yet again increases wages increasingly; that the effect of job tenure turns around at 9.8 years (compared with 12.1 between and 9.1 within); that being black reduces wages by 5.3% (compared with 5.6% between and no estimate within); that living in a non-SMSA reduces wages 13.1% (compared with 18.6% between and 8.9% within); and that living in the South reduces wages 8.7% (compared with 9.9% between and 6.1% within).

1

= 0.0000

### Example 5: Random-effects model fit using MLE

We could also have fit this random-effects model with MLE:

. xtreg ln\_w grade age c.age#c.age ttl\_exp c.ttl\_exp#c.ttl\_exp tenure > c.tenure#c.tenure 2.race not smsa south, mle Fitting constant-only model: Iteration 0: Log likelihood = -12663.954 Iteration 1: Log likelihood = -12649.756 Iteration 2: Log likelihood = -12649.614 Iteration 3: Log likelihood = -12649.614 Fitting full model: Iteration 0: Log likelihood = -8922.145 Iteration 1: Log likelihood = -8853.6409 Iteration 2: Log likelihood = -8853.4255 Iteration 3: Log likelihood = -8853.4254 Random-effects ML regression Number of obs = 28,091 Group variable: idcode Number of groups = 4,697 Random effects u i ~ Gaussian Obs per group: min = 6.0 avg = max = 15 = 7592.38 LR chi2(10)

Log likelihood = -8853.4254

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
grade	.0646093	.0017372	37.19	0.000	.0612044	.0680142
age	.0368531	.0031226	11.80	0.000	.030733	.0429732
c.age#c.age	0007132	.0000501	-14.24	0.000	0008113	000615
ttl_exp	.0288196	.0024143	11.94	0.000	.0240877	.0335515
c.ttl exp#						
c.ttl_exp	.000309	.0001163	2.66	0.008	.0000811	.0005369
tenure	.0394371	.0017604	22.40	0.000	.0359868	.0428875
c.tenure#						
c.tenure	0020052	.0001195	-16.77	0.000	0022395	0017709
race						
Black	0533394	.0097338	-5.48	0.000	0724172	0342615
not_smsa	1323433	.0071322	-18.56	0.000	1463221	1183644
south	0875599	.0072143	-12.14	0.000	1016998	0734201
_cons	.2390837	.0491902	4.86	0.000	.1426727	.3354947
/sigma_u	.2485556	.0035017			.2417863	.2555144
/sigma_e	.2918458	.001352			.289208	.2945076
rho	.4204033	.0074828			.4057959	.4351212

LR test of sigma\_u=0: chibar2(01) = 7339.84

 $Prob \ge chibar2 = 0.000$ 

Prob > chi2

The estimates are nearly the same as those produced by xtreg, re—the GLS estimator. For instance, xtreg, re estimated the coefficient on grade to be 0.0646499, xtreg, mle estimated 0.0646093, and the ratio is 0.0646499/0.0646093 = 1.001 to three decimal places. Similarly, the standard errors are nearly equal: 0.0017811/0.0017372 = 1.025. Below we compare all 11 coefficients:

	Coefficient ratio			S	SE ratio		
Estimator	mean	min.	max.	mean	min.	max.	
xtreg, mle (MLE)	1.	1.	1.	1.	1.	1.	
xtreg, re (GLS)	.997	.987	1.007	1.006	.997	1.027	

#### Example 6: Population-averaged model

We could also have fit this model with the population-averaged estimator:

. xtreg ln\_w grade age c.age#c.age ttl\_exp c.ttl\_exp#c.ttl\_exp tenure > c.tenure#c.tenure 2.race not\_smsa south, pa Iteration 1: Tolerance = .0310561 Iteration 2: Tolerance = .00074898 Iteration 3: Tolerance = .0000147 Iteration 4: Tolerance = 2.880e-07 GEE population-averaged model Number of obs = 28.091 Group variable: idcode Number of groups = 4,697 Family: Gaussian Obs per group: Link: Identity min = Correlation: exchangeable 6.0 avg = max = 15 Wald chi2(10) = 9598.89 Prob > chi2 = 0.0000

Scale parameter = .1436709

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf	. interval]
grade	.0645427	.0016829	38.35	0.000	.0612442	.0678412
age	.036932	.0031509	11.72	0.000	.0307564	.0431076
c.age#c.age	0007129	.0000506	-14.10	0.000	0008121	0006138
ttl_exp	.0284878	.0024169	11.79	0.000	.0237508	.0332248
c.ttl_exp# c.ttl_exp	.0003158	.0001172	2.69	0.007	.000086	.0005456
tenure	.0397468	.0017779	22.36	0.000	.0362621	.0432315
c.tenure# c.tenure	002008	.0001209	-16.61	0.000	0022449	0017711
race						
Black	0538314	.0094086	-5.72	0.000	072272	0353909
not_smsa	1347788	.0070543	-19.11	0.000	1486049	1209526
south	0885969	.0071132	-12.46	0.000	1025386	0746552
_cons	.2396286	.0491465	4.88	0.000	.1433034	.3359539

4

1

These results differ from those produced by xtreg, re and xtreg, mle. Coefficients are larger and standard errors smaller. xtreg, pa is simply another way to run the xtgee command. That is, we would have obtained the same output had we typed

. xtgee ln\_w grade age c.age#c.age ttl\_exp c.ttl\_exp#c.ttl\_exp > tenure c.tenure#c.tenure 2.race not\_smsa south (output omitted because it is the same as above)

See [XT] **xtgee**. In the language of xtgee, the random-effects model corresponds to an exchangeable correlation structure and identity link, and xtgee also allows other correlation structures. Let's stay with the random-effects model, however. xtgee will also produce robust estimates of variance, and we refit this model that way by typing

. xtgee ln\_w grade age c.age#c.age ttl\_exp c.ttl\_exp#c.ttl\_exp > tenure c.tenure#c.tenure 2.race not\_smsa south, vce(robust) (output omitted, coefficients the same, standard errors different)

In the previous example, we presented a table comparing xtreg, re with xtreg, mle. Below we add the results from the estimates shown and the ones we did with xtgee, vce(robust):

		С	oefficie	nt ratio		SE ratio	
Estimator		mean	min.	max.	mear	n min.	max.
xtreg, mle	(MLE)	1.	1.	1.	1.	1.	1.
xtreg, re	(GLS)	.997	.987	1.007	1.006	5 .997	1.027
xtreg, pa	(PA)	1.060	.847	1.317	.853	.626	.986
<pre>xtgee, vce(robust)</pre>	(PA)	1.060	.847	1.317	1.306	5 .957	1.545

So, which are right? This is a real dataset, and we do not know. However, in example 2 in [XT] **xtreg postestimation**, we will present evidence that the assumptions underlying the xtreg, re and xtreg, mle results are not met.

4

#### Example 7: Correlated random-effects models

Below we use the same specification as in the fixed-effects estimation in example 3. Fixed-effects models cannot estimate coefficients for time-invariant regressors such as grade and race. If the effects of these variables are of interest, we can fit a correlated random-effects model by specifying the cre option.

```
. xtreg ln_w grade age c.age#c.age ttl_exp c.ttl_exp#c.ttl_exp tenure
> c.tenure#c.tenure 2.race not_smsa south, cre
note: grade omitted from xt_means because of collinearity.
note: 2.race omitted from xt_means because of collinearity.
```

						28,091 4,697
R-squared: Within = Between = Overall =	= 0.4899			Obs per	group: min = avg = max =	1 6.0 15
corr(xit_vars*		r) = 0.3941		Wald ch Prob >	i2(10) =	5984.86 0.0000
ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
xit vars						
grade age	.0610924 .0359987	.0019795 .0033749	30.86 10.67	0.000	.0572127 .029384	.0649722 .0426134
c.age#c.age	000723	.0000531	-13.62	0.000	000827	000619
ttl_exp	.0334668	.0029553	11.32	0.000	.0276745	.039259
c.ttl_exp#						
c.ttl_exp	.0002163	.0001273	1.70	0.089	0000332	.0004658
tenure	.0357539	.0018424	19.41	0.000	.0321428	.039365
c.tenure#						
c.tenure	0019701	.0001246	-15.82	0.000	0022142	001726
race						
Black	0603623	.0102465	-5.89	0.000	0804451	0402795
not_smsa	0890108	.0094992	-9.37	0.000	1076289	0703926
south _cons	0606309 .311389	.0108948 .1282604	-5.57 2.43	0.000 0.015	0819844 .0600033	0392774 .5627747
xt_means grade	0	(omitted)				
age	0021612	.009836	-0.22	0.826	0214394	.017117
c.age#c.age	.0001041	.0001603	0.65	0.516	0002102	.0004184
ttl_exp	0194687	.0065442	-2.97	0.003	032295	0066424
c.ttl_exp#						
c.ttl_exp	.0004933	.0003588	1.38	0.169	0002098	.0011965
tenure	.0317887	.0064167	4.95	0.000	.0192122	.0443651
c.tenure# c.tenure	0007977	.0004349	-1.83	0.067	00165	.0000547
race Black	0	(omitted)				
not_smsa	0982075	.0144899	-6.78	0.000	1266072	0698077
south	0405623	.0147223	-2.76	0.006	0694174	0117072
sigma_u	.25790526					
sigma_e	.29068923					
rho	.44045273	(fraction	of varia	nce due t	o u_i)	
	( )					

Mundlak test (xt\_means = 0): chi2(8) = 151.6234

The first section of the coefficient table, labeled xit\_vars, reports coefficients and related statistics for both the time-variant and time-invariant variables. All the coefficients for time-variant regressors are numerically equivalent to those from xtreg, fe. This result is known as the Mundlak equivalence. As for the time-invariant regressors, we estimate that schooling has a rate of return of 6.1% (compared with 6.5% in the random-effects model) and being black reduces wages by 6.0% (compared with 5.3% in the random-effects model). As a result of the Mundlak equivalence, the correlated random-effects model has the same  $R^2$  within, 0.1727, as the fixed-effects model. It also has a higher  $R^2$  overall, 0.3800, because it includes as additional regressors the time-invariant independent variables and the panel means.

The second section of the coefficient table, labeled xt\_means, reports coefficients and related statistics for the panel means of the time-varying variables. The panel means for time-invariant variables such as grade are omitted because the means are collinear with the original variables.

xtreg, cre also performs a Mundlak specification test to help decide between estimation using the random-effects model and estimation using the correlated random-effects model or fixed-effects model. This test consists of testing whether all the coefficients of the panel means are equal to zero. The results of the test can be found at the bottom of the output table. In this example, with a Wald test statistic of 151.6234 and a *p*-value of 0.0000, we reject the null hypothesis and favor this correlated random-effects model.

4

## Example 8: Controlling for high-dimensional categorical variables

Let's assume that in the labor market model we have been fitting, we would like to account for county effects. The United States, as of 2024, had approximately 3,200 counties. To fit a fixed-effects model, we could type

```
. xtreg ... i.county, fe
```

However, this would add the 3,200 county indicators as regressors in our model. If we are not interested in the coefficients on the county indicators, this creates an unnecessary computational burden. We can fit the model in a more efficient way by absorbing the county effects. To do this, we type

. xtreg ..., fe absorb(county)

Now, instead of estimating 3,200 coefficients, we absorb the county effects as well as the panel effects.

For illustrative purposes, we generate a county variable that mimics the description above.

```
. set seed 123
. generate county = runiformint(1,3200)
```

Then we fit the model.

```
. set seed 123
. generate county = runiformint(1,3200)
```

Below the command, we see iteration logs showing the maximum absolute difference (MAD) for the APM, which is the algorithm used to absorb the effects of panel and county variables. The smaller the MAD, the closer the APM is to the projection of the model fit by including indicator variables for all levels of county. Thus, the MAD should be as close to 0 as possible to absorb county and panel effects and obtain regression coefficients for the other predictors.

Following the iteration logs, we obtain output similar to the output from xtreg, fe when the absorb() option was not specified. However, we see an additional table that reports which variables were absorbed and the number of categories in each of the variables. No coefficients are reported for the counties. The county effects are absorbed.

Note that this output includes an estimate of  $\sigma_u$  and an F test that all  $u_i$ s are 0; each of these requires additional computation time. If you are not interested in the estimate of  $\sigma_u$ , you can specify the nosigmau option. If you are not interested in  $\sigma_u$  or the F test that the  $u_i$ s are 0, you can specify the nouitest option.

. xtreg ln\_w age c.age#c.age ttl\_exp c.ttl\_exp#c.ttl\_exp tenure > c.tenure#c.tenure not smsa south, fe > absorb(county) nouitest (output omitted) Fixed-effects (within) regression Number of obs 28,093 Group variable: idcode Number of groups = 4,699 R-squared: Obs per group: Within = 0.2881 min = 1 Between = 0.3479avg = 6.0 Overall = 0.2604max = 15 F(8, 20187) = 533.12 Absorbed variable Levels 3,200 county Coefficient Std. err. P>|t| [95% conf. interval] ln wage t .0359171 .0036425 9.86 0.000 .0287774 .0430567 age -.0007245 -12.65 0.000 -.0008368 -.0006122 c.age#c.age .0000573 ttl exp .0343766 .0031888 10.78 0.000 .0281262 .040627 c.ttl exp# c.ttl exp .0001783 .0001374 1.30 0.194 -.0000911 .0004477 0.000 tenure .0356387 .0019824 17.98 .0317531 .0395244 c.tenure# -.0019394 .0001339 -14.48 0.000 -.0022019-.0016769 c.tenure -.0909885 .0102319 -8.89 0.000 -.1110438 -.0709331 not\_smsa .0117182 south -.0514181 -4.39 0.000 -.0743867 -.0284494 1.034046 .0522015 0.000 1.136366 \_cons 19.81 .9317273 sigma e .29022084

Specifying nouitest can be particularly helpful in improving speed when absorbing categorical variables with many levels.

# Stored results

xtreg, re stores the following in e(): Scalars number of observations e(N) number of groups e(N\_g) e(df\_m) model degrees of freedom e(g\_min) smallest group size e(g\_avg) average group size e(g\_max) largest group size e(Tcon) 1 if T is constant square root of the sum of e(sigma\_e)<sup>2</sup> and e(sigma\_u)<sup>2</sup> e(sigma) e(sigma\_u) panel-level standard deviation standard deviation of  $\epsilon_{it}$ e(sigma\_e)  $R^2$  for within model e(r2\_w)  $R^2$  for overall model e(r2\_o)  $R^2$  for between model e(r2\_b) number of clusters e(N\_clust)  $\chi^2$ e(chi2) e(p) p-value for model test e(rho) ρ e(thta\_min) minimum  $\theta$ e(thta\_5)  $\theta$ , 5th percentile e(thta\_50)  $\theta$ , 50th percentile e(thta\_95)  $\theta$ , 95th percentile e(thta\_max) maximum  $\theta$ e(rmse) root mean squared error of GLS regression e(Tbar) harmonic mean of group sizes e(rank) rank of e(V)Macros e(cmd) xtreg command as typed e(cmdline) e(depvar) name of dependent variable e(ivar) variable denoting groups e(model) re e(clustvar) name of cluster variable Wald; type of model  $\chi^2$  test e(chi2type) e(vce) vcetype specified in vce() title used to label Std. err. e(vcetype) sa, if specified e(sa) ъV e(properties) program used to implement predict e(predict) e(marginsnotok) predictions disallowed by margins factor variables fyset as asbalanced e(asbalanced) factor variables fvset as asobserved e(asobserved) Matrices e(b) coefficient vector coefficient vector for fixed-effects model e(bf) e(theta) θ e(V) variance-covariance matrix of the estimators e(VCEf) VCE for fixed-effects model Functions e(sample) marks estimation sample

Matrices

r(table)

matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtreg, cre stores the following in e():

Sc	alars	
	e(N)	number of observations
	e(N_g)	number of groups
	e(df_m)	model degrees of freedom
	e(g_min)	smallest group size
	e(g_avg)	average group size
	e(g_max)	largest group size
	e(Tcon)	1 if $T_i$ is constant
	e(sigma)	square root of the sum of $e(sigma_e)^2$ and $e(sigma_u)^2$
	e(sigma_u)	panel-level standard deviation
	e(sigma_e)	standard deviation of $\epsilon_{it}$
	e(r2_w)	$R^2$ for within model
	e(r2_o)	$R^2$ for overall model
	e(r2_b)	$R^2$ for between model
	e(N_clust)	number of clusters
	e(chi2)	$\chi^2$
	e(p)	<i>p</i> -value for model test
	e(rho)	ρ
	e(rmse)	root mean squared error of Mundlak GLS regression
	e(Tbar)	harmonic mean of group sizes
	e(rank)	rank of e(V)
	e(corr)	$\operatorname{corr}(\mathbf{x}_{it}\boldsymbol{\beta}, \overline{\mathbf{x}}_i \boldsymbol{\gamma})$
	e(chi2_mundlak)	Mundlak test $\chi^2$ statistic
	e(p_mundlak)	Mundlak test p-value
	e(df_mundlak)	Mundlak test degrees of freedom
Ma	acros	
	e(cmd)	xtreg
	e(cmdline)	command as typed
	e(depvar)	name of dependent variable
	e(covariates)	list of covariates
	e(ivar)	variable denoting groups
	e(model)	cre
	e(clustvar)	name of cluster variable
	e(chi2type)	Wald; type of model $\chi^2$ test
	e(vce)	vcetype specified in vce()
	e(vcetype)	title used to label Std. err.
	e(sa)	sa, if specified
	e(properties)	b V
	e(predict)	program used to implement predict
	e(estat_cmd)	program used to implement estat
	e(marginsok)	predictions allowed by margins
	e(marginsnotok)	predictions disallowed by margins
	e(asbalanced)	factor variables fvset as asbalanced
	e(asobserved)	factor variables fvset as asobserved
Ma	atrices	
	e(b)	coefficient vector
	e(bf)	coefficient vector for fixed-effects model

variance–covariance matrix of the estimators
VCE for fixed-effects model
marks estimation sample

Matrices

r(table)	matrix containing the coefficients with their standard errors, test statistics, p-values, and
	confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtreg, be stores the following in e():

Scalars	
e(N)	number of observations
e(N_g)	number of groups
e(mss)	model sum of squares
e(df_m)	model degrees of freedom
e(rss)	residual sum of squares
e(df_r)	residual degrees of freedom
e(11)	log likelihood
e(11_0)	log likelihood, constant-only model
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(Tcon)	1 if T is constant
e(r2)	$R^2$
e(r2_a)	adjusted $R^2$
e(r2_w)	$R^2$ for within model
e(r2_o)	$R^2$ for overall model
e(r2_b)	$R^2$ for between model
e(F)	F statistic
e(p)	<i>p</i> -value for model test
e(rmse)	root mean squared error
e(Tbar)	harmonic mean of group sizes
e(rank)	rank of e(V)
Macros	
e(cmd)	xtreg
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups
e(model)	be
e(typ)	WLS, if wls specified
e(title)	title in estimation output
e(vce)	vcetype specified in vce()
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(V)	variance-covariance matrix of the estimators

Functions e(sample)	marks estimation sample	
In addition to the above, the following is stored in r():		
Matrices		

r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtreg, fe stores the following in e():

Scal	lars	
	e(N)	number of observations
	e(N_g)	number of groups
	e(mss)	model sum of squares
	e(df_m)	model degrees of freedom
	e(rss)	residual sum of squares
	e(df_r)	residual degrees of freedom
	e(tss)	total sum of squares
	e(g_min)	smallest group size
	e(g_avg)	average group size
	e(g_max)	largest group size
	e(Tcon)	1 if T is constant
	e(corr)	$\operatorname{corr}(u_i, \mathbf{x}_{it}\boldsymbol{\beta})$
	e(sigma)	square root of the sum of $e(sigma_e)^2$ and $e(sigma_u)^2$
	e(sigma_u)	panel-level standard deviation
	e(sigma_e)	standard deviation of $\epsilon_{it}$
	e(r2)	$R^2$
	e(r2_a)	adjusted $R^2$
	e(r2_w)	$R^2$ for within model
	e(r2_o)	$R^2$ for overall model
	e(r2_b)	$R^2$ for between model
	e(11)	log likelihood
	e(11_0)	log likelihood, constant-only model
	e(N_clust)	number of clusters
	e(k_absorb)	total number of absorbed categories, including those of the panel variable
	e(rho)	ho
	e(F)	F statistic
	e(F_f)	F statistic for test of $u_i = 0$
	e(p)	<i>p</i> -value for model test
	e(p_f)	$p$ -value for test of $u_i = 0$
	e(df_a)	degrees of freedom for absorbed panel effect
	e(df_b)	numerator degrees of freedom for F statistic
	e(rmse)	root mean squared error
	e(Tbar)	mean of group sizes
	e(rank)	rank of e(V)
	e(converged)	1 if APM converged, 0 otherwise
Mac	cros	
	e(cmd)	xtreg
	e(cmdline)	command as typed
	e(depvar)	name of dependent variable
	e(ivar)	variable denoting groups
	e(absvar)	names of absorbed variables, including the panel variable
	e(apm)	alternating projection method
	• ·	

e(model)	fe
e(wtype)	weight type
e(wexp)	weight expression
e(clustvar)	names of cluster variables
e(cluster#)	cluster combination #
e(vce)	vcetype specified in vce()
e(vcetype)	title used to label Std. err.
e(properties)	b V
e(predict)	program used to implement predict
e(marginsnotok)	predictions disallowed by margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(V)	variance-covariance matrix of the estimators
e(V_modelbased)	model-based variance
e(adj_df)	adjusted degrees of freedom when vce(hc2, dfadjust) is specified
e(kcluster)	cluster sizes, multiway clustering
e(kabsorb)	number of levels for each absorbed variable, including the panel variable
e(dfabsorb)	adjusted degrees of freedom for each absorbed variable
e(ksingle)	number of singletons for each absorbed variable
Functions	
e(sample)	marks estimation sample

```
Matrices
```

ices	
r(table)	matrix containing the coefficients with their standard errors, test statistics, p-values, and
	confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtreg, mle stores the following in e():

Scalars	
e(N)	number of observations
e(N_g)	number of groups
e(df_m)	model degrees of freedom
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(sigma_u)	panel-level standard deviation
e(sigma_e)	standard deviation of $\epsilon_{it}$
e(11)	log likelihood
e(11_0)	log likelihood, constant-only model
e(ll_c)	log likelihood, comparison model
e(N_clust)	number of clusters
e(chi2)	$\chi^2$
e(chi2_c)	$\chi^2$ for comparison test
e(p)	<i>p</i> -value for model test
e(rho)	ho
e(rank)	rank of e(V)
Macros	
e(cmd)	xtreg
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups

<pre>e(model) e(wtype) e(wexp) e(title) e(clustvar) e(vce) e(vcetype) e(chi2type) e(chi2_ct) e(distrib) e(properties) e(predict)</pre>	ml weight type weight expression title in estimation output name of cluster variable <i>vcetype</i> specified in vce() title used to label Std. err. Wald or LR; type of model $\chi^2$ test Wald or LR; type of model $\chi^2$ test Ogaussian; the distribution of the random effects b V program used to implement predict
e(marginsnotok) e(asbalanced)	predictions disallowed by margins factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices e(b) e(V)	coefficient vector variance-covariance matrix of the estimators
Functions e(sample)	marks estimation sample

```
Matrices
```

```
r(table)
```

matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtreg, pa stores the following in e():

Scalars	
e(N)	number of observations
e(N_g)	number of groups
e(df_m)	model degrees of freedom
e(chi2)	$\chi^2$
e(p)	<i>p</i> -value for model test
e(df_pear)	degrees of freedom for Pearson $\chi^2$
e(chi2_dev)	$\chi^2$ test of deviance
e(chi2_dis)	$\chi^2$ test of deviance dispersion
e(deviance)	deviance
e(dispers)	deviance dispersion
e(phi)	scale parameter
e(g_min)	smallest group size
e(g_avg)	average group size
e(g_max)	largest group size
e(rank)	rank of e(V)
e(tol)	target tolerance
e(dif)	achieved tolerance
e(rc)	return code
Macros	
e(cmd)	xtgee
e(cmd2)	xtreg
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(ivar)	variable denoting groups
e(tvar)	variable denoting time within groups

<pre>e(model) e(family) e(link) e(corr) e(scale) e(wtype) e(wexp) e(offset) e(chi2type) e(vce) e(vce) e(vceype) e(rgf) e(nmp) e(properties) e(predict) e(marginsnotok) e(asbalanced)</pre>	pa Gaussian identity; link function correlation structure x2, dev, phi, or #; scale parameter weight type weight expression linear offset variable Wald; type of model $\chi^2$ test <i>vcetype</i> specified in vce() title used to label Std. err. rgf, if rgf specified nmp, if specified b V program used to implement predict predictions disallowed by margins factor variables fvset as asbalanced
e(asbalanced) e(asobserved)	factor variables fvset as asbalanced factor variables fvset as asobserved
Matrices e(b) e(R) e(V) e(V_modelbased) Functions	coefficient vector estimated working correlation matrix variance–covariance matrix of the estimators model-based variance
e(sample)	marks estimation sample

Matrices

r(table) matrix containing the coefficients with their standard errors, test statistics, *p*-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

# Methods and formulas

The model to be fit is

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it}$$

for i = 1, ..., n and, for each i, t = 1, ..., T, of which  $T_i$  periods are actually observed.

Methods and formulas are presented under the following headings:

```
xtreg, fe
Absorbed variables with xtreg, fe
xtreg, be
xtreg, re
xtreg, cre
xtreg, mle
xtreg, pa
```

#### xtreg, fe

xtreg, fe produces estimates by running OLS on

$$(y_{it} - \overline{y}_i + \overline{\overline{y}}) = \alpha + (\mathbf{x}_{it} - \overline{\mathbf{x}}_i + \overline{\overline{\mathbf{x}}})\boldsymbol{\beta} + (\epsilon_{it} - \overline{\epsilon}_i + \overline{\nu}) + \overline{\overline{\epsilon}}$$

where  $\overline{y}_i = \sum_{t=1}^{T_i} y_{it}/T_i$ , and similarly,  $\overline{\overline{y}} = \sum_i \sum_t y_{it}/(nT_i)$ . The conventional covariance matrix of the estimators is adjusted for the extra n-1 estimated means, so results are the same as using OLS on (1) to estimate  $\nu_i$  directly. Specifying vce (robust) or vce (cluster *clustvar*) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] **\_robust**, particularly *Introduction* and *Methods and formulas*. Wooldridge (2020) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2020), Stock and Watson (2008), and Arellano (2003), specifying vce (robust) is equivalent to specifying vce (cluster *panelvar*), where *panelvar* is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in  $\epsilon_{it}$ .

The cluster–robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation induced by the within transform. The panel-nesting restriction is also enforced for multiway clustering, vce(cluster *clustvarlist*), which includes all combinations of the cluster variables. See *Methods and formulas* in [R] **regress** for computational details of multiway clustering.

xtreg, fe with vce(hc2 [*clustvar*][, dfadjust]) specifies an alternative bias correction for the cluster-robust variance calculation. If *clustvar* is not specified, *panelvar* is used in the VCE computation. See *Robust calculation for regress* in *Methods and formulas* in [R] **regress** for a description of the VCE produced by vce(hc2) and the adjusted degrees-of-freedom computations.

From the estimates  $\hat{\alpha}$  and  $\hat{\beta}$ , estimates  $u_i$  of  $\nu_i$  are obtained as  $u_i = \overline{y}_i - \hat{\alpha} - \overline{\mathbf{x}}_i \hat{\beta}$ . Reported from the calculated  $u_i$  are its standard deviation and its correlation with  $\overline{\mathbf{x}}_i \hat{\beta}$ . Reported as the standard deviation of  $e_{it}$  is the regression's estimated root mean squared error, s, which is adjusted (as previously stated) for the n-1 estimated means.

Reported as  $R^2$  within is the  $R^2$  from the mean-deviated regression.

Reported as  $R^2$  between is  $\operatorname{corr}(\overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}, \overline{y}_i)^2$ .

Reported as  $R^2$  overall is corr $(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$ .

#### Absorbed variables with xtreg, fe

Suppose we have h categorical variables,  $C_1, \ldots, C_h$ , that we would like to include as controls in our fixed-effects regression. Let  $m_k$  be the number of categories for variable  $C_k$ , and let  $\mathbf{d}_{k(it)}$  be the  $1 \times m_k$  indicator vector for panel i at time t. Thus, vector  $\mathbf{d}_{k(it)}$  is a unit vector with a 1 in the category of variable  $C_k$  in panel i at time t, and 0 everywhere else. Let  $\mathbf{D}_{k(i)}$  denote the  $T_i \times m_k$  matrix of indicators for variable  $C_k$  and panel i:

$$\mathbf{D}_{k(i)} = \begin{pmatrix} \mathbf{d}_{k(i1)} \\ \mathbf{d}_{k(i2)} \\ \vdots \\ \mathbf{d}_{k(iT_i)} \end{pmatrix}$$

We would like to fit the model

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \mathbf{d}_{1(it)}\boldsymbol{\gamma}_1 + \dots + \mathbf{d}_{h(it)}\boldsymbol{\gamma}_h + \nu_i + \epsilon_{it}$$

for  $t = 1, ..., T_i$ , and i = 1, ..., n.

Estimating the  $\gamma$  coefficients in the same manner as the  $\beta$  coefficients can be computationally expensive if the categorical variables have many levels. Instead, we absorb the effects of these categorical variables along with the panel effects.

When absorbing categorical variables, regression coefficients are computed by first running an APM and then applying Frisch–Waugh–Lovell's theorem (see Hansen [2022, 82]). The APM is used to obtain the residuals of  $y_{it}$  and  $\mathbf{x}_{it}$  after regressing them (separately) on all the categorical variables. The speed of the APM is further improved by using a conjugate gradient acceleration technique (Hernández-Ramos, Escalante, and Raydan 2011).

Two projection methods are available: Halperin (the default) and Cimmino. You can specify which to use with the halperin or cimmino suboption within the absorb() option. We describe both methods below.

Let  $\mathbf{D}_k$  be the  $N \times m_k$  matrix of indicators for the kth absorbed variable (where  $N = \sum_{i=1}^{n} T_i$ ):

$$\mathbf{D}_k = \begin{pmatrix} \mathbf{D}_{k(1)} \\ \mathbf{D}_{k(2)} \\ \vdots \\ \mathbf{D}_{k(n)} \end{pmatrix}$$

The orthonormal projection matrix for  $C_k$  is therefore given by  $\mathbf{P}_k = \mathbf{D}_k (\mathbf{D}'_k \mathbf{D}_k)^{-1} \mathbf{D}'_k$ . By convention, we define the first absorbed variable (k = 1) as the panel variable. Let  $\mathbf{y}$  be the  $N \times 1$  vector with the values of the dependent variable in the sample. Similarly, let  $\mathbf{X}$  be the matrix with the values of the covariates. Thus, the product  $\overline{\mathbf{y}}_k = \mathbf{P}_k \mathbf{y}$  is the projection of the dependent variable onto the column space of  $\mathbf{D}_k$ . That is,  $\overline{\mathbf{y}}_k$  is the  $N \times 1$  vector containing the (repeated) means of  $y_{it}$  for each level of  $C_k$ , in the order that these levels appear in the sample. The same projection can be applied to the columns of covariate matrix  $\mathbf{X}$ , that is,  $\overline{\mathbf{X}}_k = \mathbf{P}_k \mathbf{X}$ .

The Halperin algorithm first sets  $\tilde{\mathbf{y}}_{0}^{(1)} = \mathbf{y}$  and loops over the *h* absorbed variables computing projection residuals  $\tilde{\mathbf{y}}_{k}^{(1)} = (\mathbf{I} - \mathbf{P}_{k})\tilde{\mathbf{y}}_{k-1}^{(1)}$ , for k = 1, ..., h. Then, it repeats the loop with  $\tilde{\mathbf{y}}_{0}^{(j+1)} = \tilde{\mathbf{y}}_{h}^{(j)}$  until convergence at j = j. Convergence is declared when  $|\tilde{\mathbf{y}}_{h}^{(j)} - \tilde{\mathbf{y}}_{0}^{(j)}|$  is less than the specified tolerance, where the matrix norm  $|\cdot|$  is defined as the largest entry in absolute value. The same computations are applied to the columns of covariate matrix **X**. On convergence, the overall mean  $\overline{\overline{y}}$  is added to  $\tilde{\mathbf{y}}_{h}^{(j)}$ , and the vector of means  $\overline{\overline{\mathbf{x}}}$  is added to the columns of  $\widetilde{\mathbf{X}}_{h}^{(j)}$ .

The Cimmino algorithm first sets  $\tilde{\mathbf{y}}^{(0)} = \mathbf{y}$  and then iteratively computes

$$\tilde{\mathbf{y}}^{(j+1)} = \left(\mathbf{I} - \frac{1}{h}\sum_{k=1}^{h}\mathbf{P}_{k}\right)\tilde{\mathbf{y}}^{(j)}$$

until convergence at j + 1 = j. It then repeats the process for covariance matrix **X** and adds the overall means,  $\overline{\overline{y}}$  and  $\overline{\overline{x}}$ , to  $\widetilde{y}^{(j)}$  and to the columns of  $\widetilde{\mathbf{X}}^{(j)}$ , respectively.

Efficient computation of the projection Py can be done in Mata without generating the  $N \times N$  matrix **P** directly (see [M-5] **panelsum()**, for instance).

An estimate of  $\sigma_u$ , the standard deviation of the panel effects, is obtained by applying the APM algorithm to  $\mathbf{y} - \mathbf{X}\hat{\boldsymbol{\beta}}$ . A discussion of estimating the panel effects can be found in *Remarks and examples* of [XT] **xtreg postestimation**.

See Methods and formulas in [R] areg for a description of the method used to adjust the degrees of freedom to account for collinearity among absorbed variables when the dfabsorb option is specified.

For a discussion of controlling for high-dimensional categorical variables in linear and nonlinear models, see Correia (2016) and Correia, Guimarães, and Zylkin (2020).

#### xtreg, be

xtreg, be fits the following model:

$$\overline{y}_i = \alpha + \overline{\mathbf{x}}_i \boldsymbol{\beta} + \nu_i + \overline{\epsilon}_i$$

Estimation is via OLS unless  $T_i$  is not constant and the wls option is specified. Otherwise, the estimation is performed via WLS. The estimates and conventional VCE are obtained from regress for both cases, but for WLS, [aweight= $T_i$ ] is specified.

Reported as  $R^2$  between is the  $R^2$  from the fitted regression.

Reported as  $R^2$  within is corr $\{(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}, y_{it} - \overline{y}_i\}^2$ .

Reported as  $R^2$  overall is corr $(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$ .

#### xtreg, re

The key to the random-effects estimator is the GLS transform. Given estimates of the idiosyncratic component,  $\hat{\sigma}_e^2$ , and the individual component,  $\hat{\sigma}_u^2$ , the GLS transform of a variable z for the random-effects model is

$$z_{it}^* = z_{it} - \overline{\theta}_i \overline{z}_i$$

where  $\overline{z}_i = 1/T_i \sum_{t=1}^{T_i} z_{it}$  and

$$\hat{\theta}_i = 1 - \sqrt{\frac{\hat{\sigma}_e^2}{T_i \hat{\sigma}_u^2 + \hat{\sigma}_e^2}}$$

Given an estimate of  $\hat{\theta}_i$ , one transforms the dependent and independent variables, and then the coefficient estimates and the conventional variance-covariance matrix come from an OLS regression of  $y_{it}^*$  on  $\mathbf{x}_{it}^*$  and the transformed constant  $1 - \hat{\theta}_i$ . Specifying vce(robust) or vce(cluster *clustvar*) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] \_**robust**; in particular, see *Introduction* and *Methods and formulas*. Wooldridge (2020) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2020), Stock and Watson (2008), and Arellano (2003), specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*), where *panelvar* is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in  $\epsilon_{it}$ .

The cluster–robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation that is generally induced by the random-effects transform when there is heteroskedasticity or within-panel serial correlation in the idiosyncratic errors. Stata has two implementations of the Swamy–Arora method for estimating the variance components. They produce the same results in balanced panels and share the same estimator of  $\sigma_e^2$ . However, the two methods differ in their estimator of  $\sigma_u^2$  in unbalanced panels. We call the first  $\hat{\sigma}_{u\overline{T}}^2$  and the second  $\hat{\sigma}_{uSA}^2$ . Both estimators are consistent; however,  $\hat{\sigma}_{uSA}^2$  has a more elaborate adjustment for small samples than  $\hat{\sigma}_{u\overline{T}}^2$ . (See Baltagi [2013], Baltagi and Chang [1994], and Swamy and Arora [1972] for derivations of these methods.)

Both methods use the same function of within residuals to estimate the idiosyncratic error component  $\sigma_e$ . Specifically,

$$\hat{\sigma}_{e}^{2} = \frac{\sum_{i=1}^{n} \sum_{t=1}^{T_{i}} e_{it}^{2}}{N - n - K + 1}$$

where

$$e_{it} = (y_{it} - \overline{y}_i + \overline{\overline{y}}) - \hat{\alpha}_w - (\mathbf{x}_{it} - \overline{\mathbf{x}}_i + \overline{\overline{\mathbf{x}}}) \widehat{\boldsymbol{\beta}}_w$$

and  $\hat{\alpha}_w$  and  $\hat{\beta}_w$  are the within estimates of the coefficients and  $N = \sum_{i=1}^n T_i$ . After passing the within residuals through the within transform, only the idiosyncratic errors are left.

The default method for estimating  $\sigma_u^2$  is

$$\hat{\sigma}_{u\overline{T}}^2 = \max\left\{0, \frac{\mathrm{SSR}_b}{n-K} - \frac{\hat{\sigma}_e^2}{\overline{T}}\right\}$$

where

$$\mathrm{SSR}_{b} = \sum_{i=1}^{n} \left( \overline{y}_{i} - \hat{\alpha}_{b} - \overline{\mathbf{x}}_{i} \widehat{\boldsymbol{\beta}}_{b} \right)^{2}$$

 $\hat{\alpha}_b$  and  $\hat{\beta}_b$  are coefficient estimates from the between regression and  $\overline{T}$  is the harmonic mean of  $T_i$ :

$$\overline{T} = \frac{n}{\sum_{i=1}^{n} \frac{1}{T_i}}$$

This estimator is consistent for  $\sigma_u^2$  and is computationally less expensive than the second method. The sum of squared residuals from the between model estimate a function of both the idiosyncratic component and the individual component. Using our estimator of  $\sigma_e^2$ , we can remove the idiosyncratic component, leaving only the desired individual component.

The second method is the Swamy–Arora method for unbalanced panels derived by Baltagi and Chang (1994), which has a more precise small-sample adjustment. Using this method,

$$\hat{\sigma}_{\mathrm{uSA}}^2 = \max\left\{0, \frac{\mathrm{SSR}_b^* - (n-K)\hat{\sigma}_e^2}{N-c_{\mathrm{tr}}}\right\}$$

where

$$\begin{split} & \mathrm{SSR}_b^* = \sum_{i=1}^n T_i \left( \overline{y}_i - \hat{\alpha}_b - \overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}_b \right)^2 \\ & c_{\mathrm{tr}} = \mathrm{trace} \left\{ (\mathbf{X}' \mathbf{P} \mathbf{X})^{-1} \mathbf{X}' \mathbf{Z} \mathbf{Z}' \mathbf{X} \right\} \\ & \mathbf{P} = \mathrm{diag} \left\{ \left( \frac{1}{T_i} \right) \boldsymbol{\iota}_{T_i} \boldsymbol{\iota}_{T_i}' \right\} \\ & \mathbf{Z} = \mathrm{diag} \left[ \boldsymbol{\iota}_{T_i} \right] \end{split}$$

X is the  $N \times K$  matrix of covariates, including the constant, and  $\iota_{T_i}$  is a  $T_i \times 1$  vector of ones.

The estimated coefficients  $(\hat{\alpha}, \hat{\beta})$  and their estimated covariance matrix  $\hat{\mathbf{V}}$  are reported together with the previously calculated quantities  $\hat{\sigma}_e$  and  $\hat{\sigma}_u$ . The standard deviation of  $\nu_i + e_{it}$  is calculated as  $\sqrt{\hat{\sigma}_e^2 + \hat{\sigma}_u^2}$ .

Reported as  $R^2$  between is corr $(\overline{\mathbf{x}}_i \widehat{\boldsymbol{\beta}}, \overline{y}_i)^2$ .

Reported as  $R^2$  within is corr $\{(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}, y_{it} - \overline{y}_i\}^2$ .

Reported as  $R^2$  overall is  $\operatorname{corr}(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}}, y_{it})^2$ .

#### xtreg, cre

Let's modify notation slightly to better explain estimation using correlated random effects. Denote by  $\mathbf{z}_i$  the time-invariant regressors in the model, by  $\mathbf{x}_{it}$  the regressors that vary across time, and by  $\overline{\mathbf{x}}_i$  the average of  $\mathbf{x}_{it}$  in panel *i* (using only observations in the estimation sample).

xtreg, cre fits a correlated random-effects model by fitting a random-effects regression of  $y_{it}$  on  $\mathbf{x}_{it}$ ,  $\overline{\mathbf{x}}_i$ ,  $\mathbf{z}_i$ , and a constant term. This is equivalent to fitting an OLS regression of  $(y_{it} - \hat{\theta}_i \overline{y}_i)$  on  $(\mathbf{x}_{it} - \hat{\theta}_i \overline{\mathbf{x}}_i)$ ,  $(1 - \hat{\theta}_i)\overline{\mathbf{x}}_i$ ,  $(1 - \hat{\theta}_i)\overline{\mathbf{x}}_i$ ,  $(1 - \hat{\theta}_i)\mathbf{z}_i$ , and the transformed constant term  $(1 - \hat{\theta}_i)$ . The GLS transformation term,  $\hat{\theta}_i$ , is a function of  $T_i$  and the variance estimators  $\hat{\sigma}_u^2$  and  $\hat{\sigma}_e^2$ . Option sa requests that the small-sample Swamy–Arora estimator be used for  $\sigma_u^2$  instead of the default. The formulas for  $\hat{\theta}_i$ ,  $\hat{\sigma}_e^2$ , and the available estimators for  $\sigma_u^2$  can be found in xtreg, re.

Specifying vce (cluster *clustvar*) requests that the cluster-robust VCE estimator be used in the underlying OLS regression with the transformed variables. The cluster-robust VCE estimator requires that i) the panel variable be nested in the clusters defined by *clustvar*, ii) there be many clusters, and iii) the idiosyncratic errors,  $\epsilon_{it}$ , be uncorrelated between clusters. Specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*). See [P] **\_robust** for more on the computation of standard errors in Stata and Wooldridge (2020) and Arellano (2003) for a discussion on the application of the Huber/White VCE estimator for panel data.

For balanced panels, Mundlak (1978) showed that the coefficients for  $\mathbf{x}_{it}$  obtained from xtreg, cre are numerically equivalent to those obtained from a fixed-effects regression of  $y_{it}$  on  $\mathbf{x}_{it}$ . Wooldridge (2019) extended this equivalence result to unbalanced panels. Note that, unlike fixed-effects estimation, xtreg, cre returns coefficients for time-invariant regressors.

xtreg, cre also performs a Mundlak specification test to choose between random-effects estimation and correlated random-effects or fixed-effects estimation. This is an alternative to the Hausman test for comparing fixed-effects and random-effects models, but unlike the Hausman test, the Mundlak test can be used even with a cluster-robust VCE. The Mundlak test is obtained as a test of  $H_0: \gamma = 0$  in the correlated random-effects model,

$$y_{it} = \alpha + \mathbf{x}_{it}\boldsymbol{\beta} + \overline{\mathbf{x}}_i\boldsymbol{\gamma} + \mathbf{z}_i\boldsymbol{\xi} + \nu_i + \epsilon_{it}$$

The overall model Wald test reported in the header of the xtreg, cre output is a test of  $H_0: \beta = \xi = 0$ .

Reported as  $R^2$  between is  $\operatorname{corr}(\overline{\mathbf{x}}_i(\widehat{\boldsymbol{\beta}} + \widehat{\boldsymbol{\gamma}}) + z_i \widehat{\boldsymbol{\xi}}, \overline{y}_i)^2$ .

Reported as  $R^2$  within is corr $\{(\mathbf{x}_{it} - \overline{\mathbf{x}}_i)\widehat{\boldsymbol{\beta}}, y_{it} - \overline{y}_i\}^2$ .

Reported as  $R^2$  overall is  $\operatorname{corr}(\mathbf{x}_{it}\widehat{\boldsymbol{\beta}} + \overline{\mathbf{x}}_i\widehat{\boldsymbol{\gamma}} + z_i\widehat{\boldsymbol{\xi}}, y_{it})^2$ .

margins, when used after xtreg, cre, recalculates the panel means if working outside the original estimation sample. Once these panel means have been recalculated, however, margins does not recompute the panel means when it adjusts covariate values in the computations of adjusted means, adjusted predictions, and average marginal or partial effects. See Abrevaya and Hsu (2021) for more on estimating partial effects in models with correlated random-effects.

#### xtreg, mle

The log likelihood for the *i*th unit is

$$\begin{split} l_i &= -\frac{1}{2} \left( \frac{1}{\sigma_e^2} \bigg[ \sum_{t=1}^{T_i} (y_{it} - \mathbf{x}_{it} \boldsymbol{\beta})^2 - \frac{{\sigma_u}^2}{T_i \sigma_u^2 + \sigma_e^2} \Big\{ \sum_{t=1}^{T_i} (y_{it} - \mathbf{x}_{it} \boldsymbol{\beta}) \Big\}^2 \bigg] \\ &+ \ln \Big( T_i \frac{\sigma_u^2}{\sigma_e^2} + 1 \Big) + T_i \ln(2\pi\sigma_e^2) \Big) \end{split}$$

The mle and re options yield essentially the same results, except when total  $N = \sum_{i} T_{i}$  is small (200 or less) and the data are unbalanced.

Similarly to xtreg, fe and xtreg, re, specifying vce(robust) or vce(cluster *clustvar*) causes the Huber/White/sandwich VCE estimator to be calculated for the estimated parameters in this regression.

Specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*), where *panelvar* is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in it.

The cluster–robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation that is generally induced by the random-effects transform when there is heteroskedasticity or within-panel serial correlation in the idiosyncratic errors.

#### xtreg, pa

See [XT] **xtgee** for details on the methods and formulas used to calculate the population-averaged model using a generalized estimating equations approach.

# Acknowledgments

We thank Richard Goldstein, who wrote the first draft of the routine that fits random-effects regressions, Badi Baltagi of the Department of Economics at Syracuse University, and Manuelita Ureta of the Department of Economics at Texas A&M University, who assisted us in working our way through the literature. We also thank Sergio Correia at the Federal Reserve Board Paulo Guimarães at the Bank of Portugal, and Fernando Rios-Avila at the Levy Economics Institute at Bard College for their work on controlling for high-dimensional categorical variables.

# References

- Abrevaya, J., and Y.-C. Hsu. 2021. Partial effects in non-linear panel data models with correlated random effects. Econometrics Journal 24: 519–535. https://doi.org/10.1093/ectj/utab004.
- Alejo, J., A. F. Galvao, G. Montes-Rojas, and W. Sosa-Escudero. 2015. Tests for normality in linear panel-data models. Stata Journal 15: 822–832.
- Allison, P. D. 2009. Fixed Effects Regression Models. Newbury Park, CA: Sage.
- Andrews, M. J., T. Schank, and R. Upward. 2006. Practical fixed-effects estimation methods for the three-way errorcomponents model. Stata Journal 6: 461–481.
- Arellano, M. 1987. Computing robust standard errors for within-groups estimators. Oxford Bulletin of Economics and Statistics 49: 431–434. https://doi.org/10.1111/j.1468-0084.1987.mp49004006.x.
- 2003. Panel Data Econometrics. Oxford: Oxford University Press. https://doi.org/10.1093/0199245282.001. 0001.
- Baltagi, B. H. 1985. Pooling cross-sections with unequal time-series lengths. *Economics Letters* 18: 133–136. https://doi.org/10.1016/0165-1765(85)90167-3.
- ———. 2009. A Companion to Econometric Analysis of Panel Data. Chichester, UK: Wiley.
- ——. 2013. Econometric Analysis of Panel Data. 5th ed. Chichester, UK: Wiley.
- Baltagi, B. H., and Y.-J. Chang. 1994. Incomplete panels: A comparative study of alternative estimators for the unbalanced one-way error component regression model. *Journal of Econometrics* 62: 67–89. https://doi.org/10.1016/0304-4076(94)90017-5.
- Baum, C. F. 2001. Residual diagnostics for cross-section time series regression models. Stata Journal 1: 101–104.
- Bell, R. M., and D. F. McCaffrey. 2002. Bias reduction in standard errors for linear regression with multi-stage samples. Survey Methodology 28: 169–181.
- Blackwell, J. L., III. 2005. Estimation and testing of fixed-effect panel-data systems. Stata Journal 5: 202-207.
- Bottai, M., and N. Orsini. 2004. Confidence intervals for the variance component of random-effects linear models. *Stata Journal* 4: 429–435.
- Bruno, G. S. F. 2005. Estimation and inference in dynamic unbalanced panel-data models with a small number of individuals. Stata Journal 5: 473–500.
- Cabanillas, O. B., J. D. Michler, A. Michuda, and E. Tjernström. 2018. Fitting and interpreting correlated randomcoefficient models using Stata. Stata Journal 18: 159–173.
- Christodoulou, D., and V. Sarafidis. 2017. Regression clustering for panel-data models with fixed effects. *Stata Journal* 17: 314–329.
- Correia, S. 2016. A feasible estimator for linear models with multi-way fixed effects. Unpublished manuscript, Duke University, https://scorreia.com/research/hdfe.pdf.
- Correia, S., P. Guimarães, and T. Zylkin. 2020. Fast Poisson estimation with high-dimensional fixed effects. *Stata Journal* 20: 95–115.
- De Hoyos, R. E., and V. Sarafidis. 2006. Testing for cross-sectional dependence in panel-data models. *Stata Journal* 6: 482–496.
- Dwyer, J. H., and M. Feinleib. 1992. "Introduction to statistical models for longitudinal observation". In Statistical Models for Longitudinal Studies of Health, edited by J. H. Dwyer, M. Feinleib, P. Lippert, and H. Hoffmeister, 3–48. New York: Oxford University Press.
- Gardner, J. 2023. Stata tip 149: Weighted estimation of fixed-effects and first-differences models. Stata Journal 23: 276–280.
- Guimarães, P., and P. Portugal. 2010. A simple feasible procedure to fit models with high-dimensional fixed effects. Stata Journal 10: 628–649.
- Hansen, B. E. 2022. Econometrics. Princeton, NJ: Princeton University Press.
- Hernández-Ramos, L. M., R. Escalante, and M. Raydan. 2011. Unconstrained optimization techniques for the acceleration of alternating projection methods. Numerical Functional Analysis and Optimization 32: 1041–1066. https://doi.org/10. 1080/01630563.2011.591954.

- Hoechle, D. 2007. Robust standard errors for panel regressions with cross-sectional dependence. Stata Journal 7: 281-312.
- Hughes, R. A., M. G. Kenward, J. A. C. Sterne, and K. Tilling. 2017. Analyzing repeated measurements while accounting for derivative tracking, varying within-subject variance, and autocorrelation: The xtmixediou command. *Stata Journal* 17: 573–599.
- Judge, G. G., W. E. Griffiths, R. C. Hill, H. Lütkepohl, and T.-C. Lee. 1985. The Theory and Practice of Econometrics. 2nd ed. New York: Wiley.
- Lee, L.-F., and W. E. Griffiths. 1979. The prior likelihood and best linear unbiased prediction in stochastic coefficient linear models. Working Paper 1, Department of Econometrics, University of New England, Armidale, Australia.
- Libois, F., and V. Verardi. 2013. Semiparametric fixed-effects estimator. Stata Journal 13: 329-336.
- Magazzini, L., R. L. Bruno, and M. Stampini. 2020. Using information from singletons in fixed-effects estimation: xtfesing. Stata Journal 20: 965–975.
- McCaffrey, D. F., K. Mihaly, J. R. Lockwood, and T. R. Sass. 2012. A review of Stata commands for fixed-effects estimation in normal linear models. *Stata Journal* 12: 406–432.
- Mundlak, Y. 1978. On the pooling of time series and cross section data. *Econometrica* 46: 69–85. https://doi.org/10.2307/1913646.
- Nichols, A. 2007. Causal inference with observational data. Stata Journal 7: 507-541.
- Pinzon, E. 2015. Fixed effects or random effects: The Mundlak approach. The Stata Blog: Not Elsewhere Classified. https://blog.stata.com/2015/10/29/fixed-effects-or-random-effects-the-mundlak-approach/.
- Rios-Avila, F. 2015. Feasible fitting of linear models with N fixed effects. Stata Journal 15: 881–898.
- Schunck, R. 2013. Within and between estimates in random-effects models: Advantages and drawbacks of correlated random effects and hybrid models. *Stata Journal* 13: 65–76.
- Stammann, A. 2018. Fast and feasible estimation of generalized linear models with high-dimensional *k*-way fixed effects. arXiv:1707.01815 [stat.AP], https://doi.org/10.48550/arXiv.1707.01815.
- Stock, J. H., and M. W. Watson. 2008. Heteroskedasticity-robust standard errors for fixed effects panel data regression. Econometrica 76: 155–174. https://doi.org/10.1111/j.0012-9682.2008.00821.x.
- Swamy, P. A. V. B., and S. S. Arora. 1972. The exact finite sample properties of the estimators of coefficients in the error components regression models. *Econometrica* 40: 261–275. https://doi.org/10.2307/1909405.
- Taub, A. J. 1979. Prediction in the context of the variance-components model. *Journal of Econometrics* 10: 103–107. https://doi.org/10.1016/0304-4076(79)90068-X.
- Twisk, J. W. R. 2013. Applied Longitudinal Data Analysis for Epidemiology: A Practical Guide. 2nd ed. Cambridge: Cambridge University Press.
- Wooldridge, J. M. 2019. Correlated random effects models with unbalanced panels. Journal of Econometrics 211: 137–150. https://doi.org/10.1016/j.jeconom.2018.12.010.
  - —. 2020. Introductory Econometrics: A Modern Approach. 7th ed. Boston: Cengage.
- Wursten, J. 2018. Testing for serial correlation in fixed-effects panel models. Stata Journal 18: 76-100.

# Also see

- [XT] **xtreg postestimation** Postestimation tools for xtreg
- [XT] **xteregress** Extended random-effects linear regression
- [XT] **xtgee** GEE population-averaged panel-data models
- [XT] xtgls GLS linear model with heteroskedastic and correlated errors
- [XT] xtheckman Random-effects regression with sample selection
- [XT] xtivreg Instrumental variables and two-stage least squares for panel-data models
- [XT] xtregar Fixed- and random-effects linear models with an AR(1) disturbance
- [XT] **xtset** Declare data to be panel data
- [BAYES] bayes: xtreg Bayesian random-effects linear model
- [ME] mixed Multilevel mixed-effects linear regression
- [MI] Estimation Estimation commands for use with mi estimate
- [R] areg Linear regression with many indicator variables
- [R] regress Linear regression
- [R] wildbootstrap Wild cluster bootstrap inference
- [SP] **spxtregress** Spatial autoregressive models for panel data
- [TS] forecast Econometric model forecasting
- [TS] prais Prais-Winsten and Cochrane-Orcutt regression
- [U] 20 Estimation and postestimation commands

Stata, Stata Press, Mata, NetCourse, and NetCourseNow are registered trademarks of StataCorp LLC. Stata and Stata Press are registered trademarks with the World Intellectual Property Organization of the United Nations. StataNow is a trademark of StataCorp LLC. Other brand and product names are registered trademarks or trademarks of their respective companies. Copyright © 1985–2025 StataCorp LLC, College Station, TX, USA. All rights reserved.



For suggested citations, see the FAQ on citing Stata documentation.