Title

xtrc — Random-coefficients model

Description

`xtrc` fits the Swamy (1970) random-coefficients linear regression model, which does not impose the assumption of constant parameters across panels. Average coefficient estimates are reported by default, but panel-specific coefficients may be requested.

Quick start

Random-coefficients regression of `y` on `x1` and `x2` using `xtset` data

```
xtrc y x1 x2
```

As above, but report panel-specific best linear predictors

```
xtrc y x1 x2, betas
```

Multiple-imputation estimates of random-coefficients regression using `mi xtset` data

```
mi estimate: xtrc y x
```

Menu

Statistics > Longitudinal/panel data > Random-coefficients regression by GLS

Syntax

xtrc *depvar indepvars* [*if*] [*in*] [, *options*]

options	Description
Main	
noconstant	suppress constant term
offset(*varname*)	include *varname* in model with coefficient constrained to 1
SE	
vce(*vcetype*)	*vcetype* may be conventional, bootstrap, or jackknife
Reporting	
level(#)	set confidence level; default is level(95)
betas	display group-specific best linear predictors
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
coeflegend	display legend instead of statistics

A panel variable must be specified; use xtset; see [XT] **xtset**.

indepvars may contain factor variables; see [U] **11.4.3 Factor variables**.

by, mi estimate, and statsby are allowed; see [U] **11.1.10 Prefix commands**.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] **mi estimate**.

coeflegend does not appear in the dialog box.

See [U] **20 Estimation and postestimation commands** for more capabilities of estimation commands.

Options

Main

noconstant, offset(*varname*); see [R] **estimation options**

SE

vce(*vcetype*) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional) and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] ***vce_options***.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

Reporting

level(#); see [R] **estimation options**.

betas requests that the group-specific best linear predictors also be displayed.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(*style*), cformat(%*fmt*), pformat(%*fmt*), sformat(%*fmt*), and nolstretch; see [R] **estimation options**.

The following option is available with `xtrc` but is not shown in the dialog box:

`coeflegend`; see [R] **estimation options**.

Remarks and examples

stata.com

In random-coefficients models, we wish to treat the parameter vector as a realization (in each panel) of a stochastic process. `xtrc` fits the Swamy (1970) random-coefficients model, which is suitable for linear regression of panel data. See Greene (2012, chap. 11) and Poi (2003) for more information about this and other panel-data models.

▷ Example 1

Greene (2012, 1112) reprints data from a classic study of investment demand by Grunfeld and Griliches (1960). In [XT] **xtgls**, we use this dataset to illustrate many of the possible models that may be fit with the `xtgls` command. Although the models included in the `xtgls` command offer considerable flexibility, they all assume that there is no parameter variation across firms (the cross-sectional units).

To take a first look at the assumption of parameter constancy, we should `reshape` our data so that we may fit a simultaneous-equation model with `sureg`; see [R] **sureg**. Because there are only five panels here, this is not too difficult.

```
. use http://www.stata-press.com/data/r15/invest2

. reshape wide invest market stock, i(time) j(company)
(note: j = 1 2 3 4 5)

Data                                long   ->   wide
-----------------------------------------------------------------------------
Number of obs.                       100   ->      20
Number of variables                    5   ->      16
j variable (5 values)            company   ->   (dropped)
xij variables:
                                  invest   ->   invest1 invest2 ... invest5
                                  market   ->   market1 market2 ... market5
                                   stock   ->   stock1 stock2 ... stock5
-----------------------------------------------------------------------------
```

```
. sureg (invest1 market1 stock1) (invest2 market2 stock2)
> (invest3 market3 stock3) (invest4 market4 stock4) (invest5 market5 stock5)
Seemingly unrelated regression
--------------------------------------------------------------------------
Equation             Obs   Parms        RMSE    "R-sq"       chi2        P
--------------------------------------------------------------------------
invest1               20       2    84.94729    0.9207     261.32   0.0000
invest2               20       2    12.36322    0.9119     207.21   0.0000
invest3               20       2    26.46612    0.6876      46.88   0.0000
invest4               20       2    9.742303    0.7264      59.15   0.0000
invest5               20       2    95.85484    0.4220      14.97   0.0006
--------------------------------------------------------------------------

------------------------------------------------------------------------------
             |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
invest1      |
     market1 |    .120493   .0216291     5.57   0.000     .0781007    .1628853
      stock1 |   .3827462    .032768    11.68   0.000      .318522    .4469703
       _cons |  -162.3641   89.45922    -1.81   0.070    -337.7009    12.97279
-------------+----------------------------------------------------------------
invest2      |
     market2 |   .0695456   .0168975     4.12   0.000     .0364271    .1026641
      stock2 |   .3085445   .0258635    11.93   0.000     .2578529    .3592362
       _cons |   .5043112   11.51283     0.04   0.965    -22.06042    23.06904
-------------+----------------------------------------------------------------
invest3      |
     market3 |   .0372914   .0122631     3.04   0.002     .0132561    .0613268
      stock3 |    .130783   .0220497     5.93   0.000     .0875663    .1739997
       _cons |  -22.43892   25.51859    -0.88   0.379    -72.45443    27.57659
-------------+----------------------------------------------------------------
invest4      |
     market4 |   .0570091   .0113623     5.02   0.000     .0347395    .0792788
      stock4 |   .0415065   .0412016     1.01   0.314    -.0392472    .1222602
       _cons |   1.088878   6.258805     0.17   0.862    -11.17815    13.35591
-------------+----------------------------------------------------------------
invest5      |
     market5 |   .1014782   .0547837     1.85   0.064    -.0058958    .2088523
      stock5 |   .3999914   .1277946     3.13   0.002     .1495186    .6504642
       _cons |   85.42324   111.8774     0.76   0.445    -133.8525    304.6989
------------------------------------------------------------------------------
```

Here we instead fit a random-coefficients model:

```
. use http://www.stata-press.com/data/r15/invest2
. xtrc invest market stock
Random-coefficients regression                  Number of obs     =        100
Group variable: company                         Number of groups  =          5
                                                Obs per group:
                                                              min =         20
                                                              avg =       20.0
                                                              max =         20
                                                Wald chi2(2)      =      17.55
                                                Prob > chi2       =     0.0002
------------------------------------------------------------------------------
      invest |      Coef.   Std. Err.      z    P>|z|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      market |   .0807646   .0250829     3.22   0.001     .0316031    .1299261
       stock |   .2839885   .0677899     4.19   0.000     .1511229    .4168542
       _cons |  -23.58361   34.55547    -0.68   0.495    -91.31108    44.14386
------------------------------------------------------------------------------
Test of parameter constancy:    chi2(12) =   603.99         Prob > chi2 = 0.0000
```

Just as the results of our simultaneous-equation model do not support the assumption of parameter constancy, the test included with the random-coefficients model also indicates that the assumption is not valid for these data. With large panel datasets, we would not want to take the time to look at a simultaneous-equations model (aside from the fact that our doing so was subjective).

◁

Stored results

`xtrc` stores the following in `e()`:

Scalars

`e(N)`	number of observations
`e(N_g)`	number of groups
`e(df_m)`	model degrees of freedom
`e(chi2)`	χ^2
`e(chi2_c)`	χ^2 for comparison test
`e(df_chi2c)`	degrees of freedom for comparison χ^2 test
`e(g_min)`	smallest group size
`e(g_avg)`	average group size
`e(g_max)`	largest group size
`e(rank)`	rank of `e(V)`

Macros

`e(cmd)`	`xtrc`
`e(cmdline)`	command as typed
`e(depvar)`	name of dependent variable
`e(ivar)`	variable denoting groups
`e(tvar)`	variable denoting time within groups
`e(title)`	title in estimation output
`e(offset)`	linear offset variable
`e(chi2type)`	`Wald`; type of model χ^2 test
`e(vce)`	*vcetype* specified in `vce()`
`e(vcetype)`	title used to label Std. Err.
`e(properties)`	`b V`
`e(predict)`	program used to implement `predict`
`e(marginsnotok)`	predictions disallowed by `margins`
`e(asbalanced)`	factor variables `fvset` as `asbalanced`
`e(asobserved)`	factor variables `fvset` as `asobserved`

Matrices

`e(b)`	coefficient vector
`e(Sigma)`	$\widehat{\boldsymbol{\Sigma}}$ matrix
`e(beta_ps)`	matrix of best linear predictors
`e(V)`	variance–covariance matrix of the estimators
`e(V_ps)`	matrix of variances for the best linear predictors; row i contains vec of variance matrix for group i predictor

Functions

`e(sample)`	marks estimation sample

Methods and formulas

In a random-coefficients model, the parameter heterogeneity is treated as stochastic variation. Assume that we write

$$\mathbf{y}_i = \mathbf{X}_i \boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i$$

where $i = 1, \dots, m$, and $\boldsymbol{\beta}_i$ is the coefficient vector ($k \times 1$) for the ith cross-sectional unit, such that

$$\boldsymbol{\beta}_i = \boldsymbol{\beta} + \boldsymbol{\nu}_i \qquad E(\boldsymbol{\nu}_i) = \mathbf{0} \qquad E(\boldsymbol{\nu}_i \boldsymbol{\nu}_i') = \boldsymbol{\Sigma}$$

Our goal is to find $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\Sigma}}$.

The derivation of the estimator assumes that the cross-sectional specific coefficient vector $\boldsymbol{\beta}_i$ is the outcome of a random process with mean vector $\boldsymbol{\beta}$ and covariance matrix $\boldsymbol{\Sigma}$,

$$\mathbf{y}_i = \mathbf{X}_i\boldsymbol{\beta}_i + \boldsymbol{\epsilon}_i = \mathbf{X}_i(\boldsymbol{\beta} + \boldsymbol{\nu}_i) + \boldsymbol{\epsilon}_i = \mathbf{X}_i\boldsymbol{\beta} + (\mathbf{X}_i\boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i) = \mathbf{X}_i\boldsymbol{\beta} + \boldsymbol{\omega}_i$$

where $E(\boldsymbol{\omega}_i) = \mathbf{0}$ and

$$E(\boldsymbol{\omega}_i\boldsymbol{\omega}_i') = E\Big\{(\mathbf{X}_i\boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)(\mathbf{X}_i\boldsymbol{\nu}_i + \boldsymbol{\epsilon}_i)'\Big\} = E(\boldsymbol{\epsilon}_i\boldsymbol{\epsilon}_i') + \mathbf{X}_i E(\boldsymbol{\nu}_i\boldsymbol{\nu}_i')\mathbf{X}_i' = \sigma_i^2\mathbf{I} + \mathbf{X}_i\boldsymbol{\Sigma}\,\mathbf{X}_i' = \boldsymbol{\Pi}_i$$

Stacking the m equations, we have

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\omega}$$

where $\boldsymbol{\Pi} \equiv E(\boldsymbol{\omega}\boldsymbol{\omega}')$ is a block diagonal matrix with $\boldsymbol{\Pi}_i$, $i = 1...m$, along the main diagonal and zeros elsewhere. The GLS estimator of $\widehat{\boldsymbol{\beta}}$ is then

$$\widehat{\boldsymbol{\beta}} = \left(\sum_i \mathbf{X}_i'\boldsymbol{\Pi}_i^{-1}\mathbf{X}_i\right)^{-1} \sum_i \mathbf{X}_i'\boldsymbol{\Pi}_i^{-1}\mathbf{y}_i = \sum_{i=1}^m \mathbf{W}_i\mathbf{b}_i$$

where

$$\mathbf{W}_i = \left\{\sum_{i=1}^m (\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}\right\}^{-1} (\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}$$

$\mathbf{b}_i = (\mathbf{X}_i'\mathbf{X}_i)^{-1}\mathbf{X}_i'\mathbf{y}_i$ and $\mathbf{V}_i = \sigma_i^2\,(\mathbf{X}_i'\mathbf{X}_i)^{-1}$, showing that the resulting GLS estimator is a matrix-weighted average of the panel-specific OLS estimators. The variance of $\widehat{\boldsymbol{\beta}}$ is

$$\mathrm{Var}(\widehat{\boldsymbol{\beta}}) = \sum_{i=1}^m (\boldsymbol{\Sigma} + \mathbf{V}_i)^{-1}$$

To calculate the above estimator $\widehat{\boldsymbol{\beta}}$ for the unknown $\boldsymbol{\Sigma}$ and $\mathbf{V}_i$ parameters, we use the two-step approach suggested by Swamy (1970):

$$\begin{aligned}
\mathbf{b}_i &= \text{OLS panel-specific estimator}\\
\widehat{\sigma}_i^2 &= \frac{\widehat{\boldsymbol{\epsilon}}_i'\widehat{\boldsymbol{\epsilon}}_i}{n_i - k}\\
\widehat{\mathbf{V}}_i &= \widehat{\sigma}_i^2\,(\mathbf{X}_i'\mathbf{X}_i)^{-1}\\
\overline{\mathbf{b}} &= \frac{1}{m}\sum_{i=1}^m \mathbf{b}_i\\
\widehat{\boldsymbol{\Sigma}} &= \frac{1}{m-1}\left(\sum_{i=1}^m \mathbf{b}_i\mathbf{b}_i' - m\overline{\mathbf{b}}\,\overline{\mathbf{b}}'\right) - \frac{1}{m}\sum_{i=1}^m \widehat{\mathbf{V}}_i
\end{aligned}$$

The two-step procedure begins with the usual OLS estimates of $\boldsymbol{\beta}_i$. With those estimates, we may proceed by obtaining estimates of $\widehat{\mathbf{V}}_i$ and $\widehat{\boldsymbol{\Sigma}}$ (and thus $\widehat{\mathbf{W}}_i$) and then obtain an estimate of $\boldsymbol{\beta}$.

Swamy (1970) further points out that the matrix $\widehat{\boldsymbol{\Sigma}}$ may not be positive definite and that because the second term is of order $1/(mT)$, it is negligible in large samples. A simple and asymptotically expedient solution is simply to drop this second term and instead use

$$\widehat{\boldsymbol{\Sigma}} = \frac{1}{m-1}\left(\sum_{i=1}^{m} \mathbf{b}_i\mathbf{b}_i' - m\overline{\mathbf{b}}\,\overline{\mathbf{b}}'\right)$$

As discussed by Judge et al. (1985, 541), the feasible best linear predictor of $\boldsymbol{\beta}_i$ is given by

$$\begin{aligned}\widehat{\boldsymbol{\beta}}_i &= \widehat{\boldsymbol{\beta}} + \widehat{\boldsymbol{\Sigma}}\mathbf{X}_i'\left(\mathbf{X}_i\widehat{\boldsymbol{\Sigma}}\mathbf{X}_i' + \widehat{\sigma}_i^2\mathbf{I}\right)^{-1}\left(\mathbf{y}_i - \mathbf{X}_i\widehat{\boldsymbol{\beta}}\right)\\ &= \left(\widehat{\boldsymbol{\Sigma}}^{-1} + \widehat{\mathbf{V}}_i^{-1}\right)^{-1}\left(\widehat{\boldsymbol{\Sigma}}^{-1}\widehat{\boldsymbol{\beta}} + \widehat{\mathbf{V}}_i^{-1}\mathbf{b}_i\right)\end{aligned}$$

The conventional variance of $\widehat{\boldsymbol{\beta}}_i$ is given by

$$\mathrm{Var}(\widehat{\boldsymbol{\beta}}_i) = \mathrm{Var}(\widehat{\boldsymbol{\beta}}) + (\mathbf{I} - \mathbf{A}_i)\left\{\widehat{\mathbf{V}}_i - \mathrm{Var}(\widehat{\boldsymbol{\beta}})\right\}(\mathbf{I} - \mathbf{A}_i)'$$

where

$$\mathbf{A}_i = \left(\widehat{\boldsymbol{\Sigma}}^{-1} + \widehat{\mathbf{V}}_i^{-1}\right)^{-1}\widehat{\boldsymbol{\Sigma}}^{-1}$$

To test the model, we may look at the difference between the OLS estimate of $\boldsymbol{\beta}$, ignoring the panel structure of the data and the matrix-weighted average of the panel-specific OLS estimators. The test statistic suggested by Swamy (1970) is given by

$$\chi^2_{k(m-1)} = \sum_{i=1}^{m}(\mathbf{b}_i - \overline{\boldsymbol{\beta}}^*)'\widehat{\mathbf{V}}_i^{-1}(\mathbf{b}_i - \overline{\boldsymbol{\beta}}^*) \qquad \text{where} \qquad \overline{\boldsymbol{\beta}}^* = \left(\sum_{i=1}^{m}\widehat{\mathbf{V}}_i^{-1}\right)^{-1}\sum_{i=1}^{m}\widehat{\mathbf{V}}_i^{-1}\mathbf{b}_i$$

Johnston and DiNardo (1997) have shown that the test is algebraically equivalent to testing

$$H_0 : \boldsymbol{\beta}_1 = \boldsymbol{\beta}_2 = \cdots = \boldsymbol{\beta}_m$$

in the generalized (groupwise heteroskedastic) `xtgls` model, where $\mathbf{V}$ is block diagonal with ith diagonal element $\boldsymbol{\Pi}_i$.

References

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Poi, B. P. 2003. From the help desk: Swamy's random-coefficients model. *Stata Journal* 3: 302–308.

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Also see

[XT] **xtrc postestimation** — Postestimation tools for xtrc

[XT] **xtreg** — Fixed-, between-, and random-effects and population-averaged linear models

[XT] **xtset** — Declare data to be panel data

[ME] **mixed** — Multilevel mixed-effects linear regression

[MI] **estimation** — Estimation commands for use with mi estimate

[U] **20 Estimation and postestimation commands**