xtprobit — Random-effects and population-averaged probit models

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Description

xtprobit fits random-effects and population-averaged probit models for a binary dependent variable. The probability of a positive outcome is assumed to be determined by the standard normal cumulative distribution function.

Quick start

Random-effects probit model of y as a function of x1, x2, and indicators for levels of categorical variable a using xtset data

xtprobit y x1 x2 i.a

Population-averaged model with robust standard errors

xtprobit y x1 x2 i.a, pa vce(robust)

Same as above, but specify an autoregressive correlation structure of order 1

xtprobit y x1 x2 i.a, pa vce(robust) corr(ar 1)

Random-effects model with cluster-robust standard errors for panels nested within cvar

xtprobit y x1 x2 i.a, vce(cluster cvar)

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Syntax

Random-effects (RE) model

xtprobit depvar [indepvars] [if] [in] [weight] [, re RE_options]

Population-averaged (PA) model

xtprobit depvar [indepvars] [if] [in] [weight], pa [PA_options]

Description RE_options

Model

suppress constant term noconstant

use random-effects estimator; the default

include varname in model with coefficient constrained to 1 offset(varname)

apply specified linear constraints constraints(constraints) retain perfect predictor variables asis

SF/Robust

vce(vcetype) vcetype may be oim, robust, cluster clustvar, bootstrap, or

jackknife

Reporting

level(#) set confidence level; default is level (95)

lrmodel perform the likelihood-ratio model test instead of the default Wald test

do not display constraints nocnsreport

control columns and column formats, row spacing, line width, display_options

display of omitted variables and base and empty cells, and

factor-variable labeling

Integration

integration method; intmethod may be mvaghermite (the default) or intmethod(intmethod)

ghermite

use # quadrature points; default is intpoints (12) intpoints(#)

Maximization

maximize_options control the maximization process; seldom used

keep collinear variables collinear

display legend instead of statistics coeflegend

PA_options	Description
Model	
<u>nocons</u> tant	suppress constant term
pa	use population-averaged estimator
<pre>offset(varname)</pre>	include <i>varname</i> in model with coefficient constrained to 1
asis	retain perfect predictor variables
Correlation	
<pre>corr(correlation)</pre>	within-panel correlation structure
force	estimate even if observations unequally spaced in time
SE/Robust	
vce(vcetype)	vcetype may be conventional, robust, bootstrap, or jackknife
nmp	use divisor $N-P$ instead of the default N
<u>s</u> cale(<i>parm</i>)	overrides the default scale parameter; parm may be x2, dev, phi, or #
Reporting	
<u>l</u> evel(#)	set confidence level; default is level(95)
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
optimize_options	control the optimization process; seldom used
<u>coefl</u> egend	display legend instead of statistics
correlation	Description
<u>exc</u> hangeable	exchangeable
<u>ind</u> ependent	independent
<u>un</u> structured	unstructured
<u>fix</u> ed <i>matname</i>	user-specified
ar#	autoregressive of order #
${ t \underline{sta}}$ tionary #	stationary of order #
$\underline{\mathtt{non}}\mathtt{stationary}~\#$	nonstationary of order #

A panel variable must be specified. For xtprobit, pa, correlation structures other than exchangeable and independent require that a time variable also be specified. Use xtset; see [XT] xtset.

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, collect, mi estimate, and statsby are allowed; see [U] 11.1.10 Prefix commands. bayes is allowed for the randomeffects model. For more details, see [BAYES] bayes: xtprobit. fp is allowed for the random-effects model.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

iweights, fweights, and pweights are allowed for the population-averaged model, and iweights are allowed for the random-effects model; see [U] 11.1.6 weight. Weights must be constant within panel.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options for RE model

____ Model

noconstant; see [R] Estimation options.

re requests the random-effects estimator. re is the default if neither re nor pa is specified.

offset(varname), constraints(constraints); see [R] Estimation options.

asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] probit.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from
asymptotic theory (oim), that are robust to some kinds of misspecification (robust), that allow for
intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap,
jackknife); see [XT] vce_options.

Specifying vce(robust) is equivalent to specifying vce(cluster *panelvar*); see *xtprobit*, re and the robust VCE estimator in Methods and formulas.

Reporting

level(#), lrmodel, nocnsreport; see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Integration

intmethod(intmethod), intpoints(#); see [R] Estimation options.

Maximization

maximize_options: difficult, technique(algorithm_spec), iterate(#), [no]log, trace,
 gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#),
 nrtolerance(#), nonrtolerance, and from(init_specs); see [R] Maximize. These options are
 seldom used.

The following options are available with xtprobit but are not shown in the dialog box:

collinear, coeflegend; see [R] Estimation options.

Options for PA model

Model

noconstant; see [R] Estimation options.

pa requests the population-averaged estimator.

offset (varname); see [R] Estimation options.

asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] probit.

Correlation

corr(correlation) specifies the within-panel correlation structure; the default corresponds to the equal-correlation model, corr(exchangeable).

When you specify a correlation structure that requires a lag, you indicate the lag after the structure's name with or without a blank; for example, corr(ar 1) or corr(ar1).

If you specify the fixed correlation structure, you specify the name of the matrix containing the assumed correlations following the word fixed, for example, corr(fixed myr).

force specifies that estimation be forced even though the time variable is not equally spaced. This is relevant only for correlation structures that require knowledge of the time variable. These correlation structures require that observations be equally spaced so that calculations based on lags correspond to a constant time change. If you specify a time variable indicating that observations are not equally spaced, the (time dependent) model will not be fit. If you also specify force, the model will be fit, and it will be assumed that the lags based on the data ordered by the time variable are appropriate.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

nmp, scale(x2 | dev | phi | #); see [XT] vce_options.

Reporting

level(#); see [R] Estimation options.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Optimization

optimize_options control the iterative optimization process. These options are seldom used.

<u>iter</u>ate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimization stops and presents the current results, even if convergence has not been reached. The default is iterate(100).

<u>tolerance</u>(#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

trace specifies that the current estimates be printed at each iteration.

The following option is available with xtprobit but is not shown in the dialog box:

coeflegend; see [R] Estimation options.

Remarks and examples

xtprobit may be used to fit a population-averaged model or a random-effects probit model. There is no command for a conditional fixed-effects model, as there does not exist a sufficient statistic allowing the fixed effects to be conditioned out of the likelihood. Unconditional fixed-effects probit models may be fit with the probit command with indicator variables for the panels. However, unconditional fixedeffects estimates are biased. We do not discuss fixed-effects further in this entry.

By default, the population-averaged model is an equal-correlation model; that is, xtprobit, pa assumes corr (exchangeable). Thus, xtprobit is a convenience command for obtaining the populationaveraged model using xtgee; see [XT] xtgee. Typing

```
. xtprobit ..., pa ...
```

is equivalent to typing

```
. xtgee ..., ... family(binomial) link(probit) corr(exchangeable)
```

See also [XT] **xtgee** for information about xtprobit.

By default or when re is specified, xtprobit fits via maximum likelihood the random-effects model

$$Pr(y_{it} \neq 0 | \mathbf{x}_{it}) = \Phi(\mathbf{x}_{it}\boldsymbol{\beta} + \nu_i)$$

for $i=1,\ldots,n$ panels, where $t=1,\ldots,n_i, \nu_i$ are i.i.d., $N(0,\sigma_{\nu}^2)$, and Φ is the standard normal cumulative distribution function.

Underlying this model is the variance components model

$$y_{it} \neq 0 \iff \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i + \epsilon_{it} > 0$$

where ϵ_{it} are i.i.d. Gaussian distributed with mean zero and variance $\sigma_{\epsilon}^2 = 1$, independently of ν_i .

1

Prob >= chibar2 = 0.000

Example 1: Random-effects model

We are studying unionization of women in the United States and are using the union dataset; see [XT] xt. We wish to fit a random-effects model of union membership:

```
. use https://www.stata-press.com/data/r19/union
(NLS Women 14-24 in 1968)
. xtprobit union age grade i.not_smsa south##c.year
Fitting comparison model:
Iteration 0: Log likelihood = -13864.23
Iteration 1: Log likelihood = -13545.541
Iteration 2: Log likelihood = -13544.385
Iteration 3: Log likelihood = -13544.385
Fitting full model:
rho = 0.0
              Log likelihood = -13544.385
rho = 0.1
              Log likelihood = -12237.655
rho = 0.2
              Log likelihood = -11590.282
rho = 0.3
            Log likelihood = -11211.185
rho = 0.4
           Log likelihood = -10981.319
rho = 0.5
           Log likelihood = -10852.793
rho = 0.6
              Log likelihood = -10808.759
rho = 0.7
             Log likelihood = -10865.57
Iteration 0: Log likelihood = -10807.712
Iteration 1: Log likelihood = -10599.332
Iteration 2: Log likelihood = -10552.287
Iteration 3:
              Log likelihood = -10552.225
Iteration 4: Log likelihood = -10552.225
Random-effects probit regression
                                                     Number of obs
                                                                   = 26,200
Group variable: idcode
                                                     Number of groups = 4,434
Random effects u i ~ Gaussian
                                                     Obs per group:
                                                                  min =
                                                                           5.9
                                                                  avg =
                                                                  max =
                                                                            12
Integration method: mvaghermite
                                                     Integration pts. =
                                                                            12
                                                     Wald chi2(6)
                                                                      = 220.91
Log likelihood = -10552.225
                                                     Prob > chi2
                                                                      = 0.0000
       union
               Coefficient
                            Std. err.
                                                P>|z|
                                                          [95% conf. interval]
                                           z
                                         0.98
         age
                 .0082967
                            .0084599
                                                0.327
                                                         -.0082843
                                                                     .0248778
      grade
                 .0482731
                            .0099469
                                        4.85
                                                0.000
                                                          .0287776
                                                                      .0677686
  1.not smsa
                 -.139657
                            .0460548
                                        -3.03
                                                0.002
                                                         -.2299227
                                                                     -.0493913
     1.south
                -1.584394
                            .358473
                                        -4.42
                                                0.000
                                                         -2.286989
                                                                     -.8818002
                -.0039854
                                        -0.45
                                                0.652
                                                         -.0213113
                                                                      .0133406
        year
                            .0088399
south#c.year
          1
                 .0134017
                            .0044622
                                         3.00
                                                0.003
                                                          .0046559
                                                                      .0221475
                -1.668202
                                        -3.51
                                                0.000
       cons
                            .4751819
                                                         -2.599542
                                                                     -.7368628
                 .6103616
                            .0458783
                                                          .5204418
                                                                      .7002814
    /lnsig2u
     sigma u
                  1.35687
                            .0311255
                                                          1.297217
                                                                      1.419267
                 .6480233
                            .0104643
                                                          .6272511
                                                                      .6682502
         rho
```

LR test of rho=0: chibar2(01) = 5984.32

The output includes the additional panel-level variance component, which is parameterized as the log of the variance $\ln(\sigma_{\nu}^2)$ (labeled lnsig2u in the output). The standard deviation σ_{ν} is also included in the output (labeled sigma_u) together with ρ (labeled rho), where

$$\rho = \frac{\sigma_{\nu}^2}{\sigma_{\nu}^2 + 1}$$

which is the proportion of the total variance contributed by the panel-level variance component.

When rho is zero, the panel-level variance component is unimportant, and the panel estimator is not different from the pooled estimator. A likelihood-ratio test of this is included at the bottom of the output. This test formally compares the pooled estimator (probit) with the panel estimator.

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□ Technical note

The random-effects model is calculated using quadrature, which is an approximation whose accuracy depends partially on the number of integration points used. We can use the quadchk command to see if changing the number of integration points affects the results. If the results change, the quadrature approximation is not accurate given the number of integration points. Try increasing the number of integration points using the intpoints() option and run quadchk again. Do not attempt to interpret the results of estimates when the coefficients reported by quadchk differ substantially.

. quadchk, nooutput Refitting model intpoints() = 8 Refitting model intpoints() = 16

adratı		

	quadrature check					
	Fitted quadrature 12 points	Comparison quadrature 8 points	Comparison quadrature 16 points			
Log likelihood	-10552.225	-10554.496 -2.2712569 .00021524	-10552.399 17396615 .00001649	Difference Relative difference		
union: age	.00829671	.00828745 -9.265e-06 0011167	.00831488 .00001817 .00218987	Difference Relative difference		
union: grade	.0482731	.04860277 .00032967 .00682917	.04826287 00001023 00021188	Difference Relative difference		
union: 1.not_smsa	13965702	14057441 00091739 .00656891	13953521 .00012181 00087218	Difference Relative difference		
union: 1.south	-1.5843944	-1.5909857 00659135 .00416017	-1.5843375 .00005689 00003591	Difference Relative difference		
union: year	00398535	00397811 7.237e-06 00181578	00400181 00001646 .00412982	Difference Relative difference		
union: 1.south#c.~r	.01340169	.01344457 .00004288 .00319946	.01340388 2.193e-06 .0001636	Difference Relative difference		
union: _cons	-1.6682022	-1.6757524 00755024 .00452597	-1.6665327 .00166948 00100077	Difference Relative difference		
/: lnsig2u	.61036163	.61780789 .00744626 .01219976	.60974814 00061349 00100513	Difference Relative difference		

The results obtained for 12 quadrature points were closer to the results for 16 points than to the results for eight points. Although the relative and absolute differences are a bit larger than we would like, they are not large. We can increase the number of quadrature points with the intpoints() option; if we choose intpoints (20) and do another quadchk we will get acceptable results, with relative differences around 0.01%.

This is not the case if we use nonadaptive quadrature. Then the results we obtain are

```
. xtprobit union age grade i.not_smsa south##c.year, intmethod(ghermite)
Fitting comparison model:
Iteration 0: Log likelihood = -13864.23
Iteration 1: Log likelihood = -13545.541
Iteration 2: Log likelihood = -13544.385
Iteration 3: Log likelihood = -13544.385
Fitting full model:
rho = 0.0
             Log likelihood = -13544.385
rho = 0.1
             Log likelihood = -12237.655
rho = 0.2
             Log likelihood = -11590.282
rho = 0.3
             Log likelihood = -11211.185
             Log likelihood = -10981.319
rho = 0.4
rho = 0.5
           Log likelihood = -10852.793
rho = 0.6
             Log likelihood = -10808.759
rho = 0.7
             Log likelihood = -10865.57
Iteration 0: Log likelihood = -10808.759
Iteration 1: Log likelihood = -10594.349
Iteration 2: Log likelihood = -10560.913
Iteration 3: Log likelihood = -10560.876
Iteration 4: Log likelihood = -10560.876
Random-effects probit regression
                                                     Number of obs
                                                                   = 26,200
Group variable: idcode
                                                     Number of groups = 4,434
Random effects u_i ~ Gaussian
                                                     Obs per group:
                                                                  min =
                                                                             1
                                                                  avg =
                                                                           5.9
                                                                  max =
                                                                            12
Integration method: ghermite
                                                     Integration pts. =
                                                     Wald chi2(6)
                                                                      = 218.99
Log likelihood = -10560.876
                                                     Prob > chi2
                                                                      = 0.0000
                                                          [95% conf. interval]
               Coefficient Std. err.
                                                P>|z|
       union
                                           z
                 .0093488
                            .0083385
                                         1.12
                                                0.262
                                                         -.0069945
                                                                       .025692
         age
      grade
                 .0488014
                            .0101168
                                        4.82
                                                0.000
                                                          .0289728
                                                                        .06863
                -.1364862
                                                                      -.045773
  1.not smsa
                            .0462831
                                       -2.95
                                                0.003
                                                         -.2271995
     1.south
                -1.592711
                            .3576715
                                       -4.45
                                                0.000
                                                         -2.293734
                                                                     -.8916877
       year
                -.0053723
                            .0087219
                                        -0.62
                                                0.538
                                                         -.0224668
                                                                      .0117223
south#c.year
          1
                 .0136764
                            .0044532
                                         3.07
                                                0.002
                                                          .0049482
                                                                      .0224046
       _cons
               -1.575539
                            .4639881
                                        -3.40
                                                0.001
                                                         -2.484939
                                                                     -.6661388
    /lnsig2u
                 .5615976
                            .0432021
                                                           .476923
                                                                      .6462722
                 1.324187
                            .0286038
                                                          1.269295
                                                                      1.381453
     sigma_u
                 .6368221
                            .0099918
                                                           .617021
                                                                      .6561699
LR test of rho=0: chibar2(01) = 5967.02
                                                       Prob >= chibar2 = 0.000
```

We now check the stability of the quadrature technique for this nonadaptive quadrature model. We expect it to be less stable.

```
. quadchk, nooutput
Refitting model intpoints() = 8
Refitting model intpoints() = 16
```

Ü	•	Quadrature chec	τ.	
	Fitted quadrature 12 points	Comparison quadrature 8 points	Comparison quadrature 16 points	
Log likelihood	-10560.876	-10574.239 -13.362535 .00126529	-10555.792 5.0839579 0004814	Difference Relative difference
union:	.00934876	.01264615 .0032974 .35270966	.00731888 00202987 21712744	Difference Relative difference
union: grade	.04880139	.05710089 .00829951 .17006703	.04432417 00447722 09174372	Difference Relative difference
union: 1.not_smsa	13648624	13327724 .003209 0235115	14094541 00445917 .03267123	Difference Relative difference
union: 1.south	-1.592711	-1.5275627 .06514823 04090399	-1.6059143 01320331 .00828983	Difference Relative difference
union: year	00537226	00867673 00330447 .61509968	00307042 .00230184 4284678	Difference Relative difference
union: 1.south#c.~r	.01367641	.01278071 0008957 06549266	.01369009 .00001368 .00100054	Difference Relative difference
union: _cons	-1.5755388	-1.4888646 .08667418 0550124	-1.6505526 0750138 .04761152	Difference Relative difference
/: lnsig2u	.56159763	.49290978 06868786 12230795	.58068904 .0190914 .03399481	Difference Relative difference

Once again, the results obtained for 12 quadrature points were closer to the results for 16 points than to the results for eight points. However, here the convergence point seems to be sensitive to the number of quadrature points, so we should not trust these results. We should increase the number of quadrature points with the intpoints() option and then use quadchk again. We should not use the results of a random-effects specification when there is evidence that the numeric technique for calculating the model is not stable (as shown by quadchk).

Generally, the relative differences in the coefficients should not change by more than 1% if the quadrature technique is stable. See [XT] quadchk for details. Increasing the number of quadrature points can often improve the stability, and for models with high rho we may need many. We can also switch between adaptive and nonadaptive quadrature. As a rule, adaptive quadrature, which is the default integration method, is much more flexible and robust.

Because the xtprobit, re likelihood function is calculated by Gauss-Hermite quadrature, on large problems the computations can be slow. Computation time is roughly proportional to the number of points used for the quadrature.

Example 2: Equal-correlation model

south#c.year

_cons

.0086294

-.8670997

As an alternative to the random-effects specification, we can fit an equal-correlation probit model:

. xtprobit union age grade i.not_smsa south##c.year, pa

```
Iteration 1: Tolerance = .12544249
Iteration 2: Tolerance = .0034686
Iteration 3: Tolerance = .00017448
Iteration 4: Tolerance = 8.382e-06
Iteration 5: Tolerance = 3.997e-07
GEE population-averaged model
                                                     Number of obs
                                                                       = 26,200
Group variable: idcode
                                                     Number of groups = 4,434
Family: Binomial
                                                     Obs per group:
Link:
       Probit
                                                                   min =
                                                                              1
Correlation: exchangeable
                                                                   avg =
                                                                            5.9
                                                                             12
                                                                   max =
                                                     Wald chi2(6)
                                                                       = 242.57
Scale parameter = 1
                                                     Prob > chi2
                                                                       = 0.0000
      union
               Coefficient
                            Std. err.
                                                P>|z|
                                                           [95% conf. interval]
                                           7.
                            .0053208
                                         1.69
                                                0.092
                                                          -.0014586
                                                                       .0193985
         age
                 .0089699
      grade
                 .0333174
                            .0062352
                                         5.34
                                                0.000
                                                          .0210966
                                                                      .0455382
                -.0715717
                             .027543
                                        -2.60
                                                0.009
                                                          -.1255551
                                                                      -.0175884
  1.not_smsa
    1.south
                -1.017368
                             .207931
                                        -4.89
                                                0.000
                                                         -1.424905
                                                                     -.6098308
       year
                -.0062708
                            .0055314
                                        -1.13
                                                0.257
                                                          -.0171122
                                                                       .0045706
```

.00258

.294771

0.001

0.003

.0035727

-1.44484

.013686

-.2893592

3.34

-2.94

Example 3: Population-averaged model

In example 3 of [R] probit, we showed the above results and compared them with probit, vce(cluster id). xtprobit with the pa option allows a vce(robust) option, so we can obtain the population-averaged probit estimator with the robust variance calculation by typing

```
. xtprobit union age grade i.not_smsa south##c.year, pa vce(robust) nolog
GEE population-averaged model
                                                    Number of obs
Group variable: idcode
                                                    Number of groups = 4,434
Family: Binomial
                                                    Obs per group:
Link: Probit
                                                                min =
Correlation: exchangeable
                                                                avg =
                                                                         5.9
                                                                max =
                                                                         12
                                                    Wald chi2(6) = 156.33
                                                    Prob > chi2
Scale parameter = 1
                                                                    = 0.0000
                                (Std err adjusted for clustering on ideode)
```

(blu.	ell.	aujusteu	101	Clusteling	OII	iucoue)	
_							

union	Coefficient	Semirobust std. err.	z	P> z	[95% conf.	interval]
age grade 1.not_smsa 1.south year	.0089699 .0333174 0715717 -1.017368 0062708	.0051169 .0076425 .0348659 .3026981 .0055745	1.75 4.36 -2.05 -3.36 -1.12	0.080 0.000 0.040 0.001 0.261	001059 .0183383 1399076 -1.610645 0171965	.0189988 .0482965 0032359 4240906 .0046549
south#c.year 1	.0086294	.0037866	2.28	0.023	.0012078	.0160509
_cons	8670997	.3243959	-2.67	0.008	-1.502904	2312955

These standard errors are similar to those shown for probit, vce(cluster id) in [R] probit.

4

Example 4: Random-effects model with stable quadrature

In a previous example, we showed how quadchk indicated that the quadrature technique was numerically unstable. Here we present an example in which the quadrature is stable.

In this example, we have (synthetic) data on whether workers complain to managers at fast-food restaurants. The covariates are age (in years of the worker), grade (years of schooling completed by the worker), south (equal to 1 if the restaurant is located in the South), tenure (the number of years spent on the job by the worker), gender (of the worker), race (of the worker), income (in thousands of dollars by the restaurant), genderm (gender of the manager), burger (equal to 1 if the restaurant specializes in hamburgers), and chicken (equal to 1 if the restaurant specializes in chicken). The model is given by

```
. use https://www.stata-press.com/data/r19/chicken
. xtprobit complain age grade south tenure gender race income genderm burger
> chicken, nolog
Random-effects probit regression
                                                        Number of obs
                                                                             2,763
Group variable: restaurant
                                                        Number of groups =
                                                                               500
Random effects u_i ~ Gaussian
                                                        Obs per group:
                                                                                 3
                                                                      min =
                                                                      avg =
                                                                               5.5
                                                                      max =
                                                                                 8
Integration method: mvaghermite
                                                        Integration pts. =
                                                                                12
                                                        Wald chi2(10)
                                                                          = 126.59
Log likelihood = -1318.2088
                                                        Prob > chi2
                                                                          = 0.0000
    complain
               Coefficient
                             Std. err.
                                                  P>|z|
                                                             [95% conf. interval]
                                             7.
                -.0430409
                              .0130211
                                          -3.31
                                                  0.001
                                                            -.0685617
                                                                           -.01752
         age
                  .0330934
                             .0264572
                                           1.25
                                                  0.211
                                                            -.0187618
                                                                          .0849486
       grade
       south
                     .1012
                             .0707196
                                           1.43
                                                  0.152
                                                             -.037408
                                                                          .2398079
      tenure
                -.0440079
                              .0987099
                                          -0.45
                                                  0.656
                                                            -.2374758
                                                                            .14946
      gender
                  .3318499
                              .0601382
                                           5.52
                                                  0.000
                                                             .2139812
                                                                          .4497185
                                                             .2668703
        race
                  .3417901
                              .0382251
                                           8.94
                                                  0.000
                                                                          .4167098
                -.0022702
                              .0008885
                                          -2.56
                                                  0.011
                                                            -.0040117
                                                                         -.0005288
      income
                  .0524577
                              .0706585
                                           0.74
                                                  0.458
                                                            -.0860305
                                                                          .1909459
     genderm
                  .0448931
                                           0.47
                                                  0.639
                                                            -.1425091
                                                                          .2322953
      burger
                              .0956151
                                           2.00
                                                  0.046
     chicken
                  .1904714
                              .0953067
                                                             .0036737
                                                                          .3772691
       _cons
                -.2145311
                              .6240549
                                          -0.34
                                                  0.731
                                                            -1.437656
                                                                          1.008594
                -1.704494
    /lnsig2u
                              .2502057
                                                            -2.194888
                                                                         -1.214099
                  .4264557
                              .0533508
                                                              .333723
                                                                          .5449563
     sigma_u
         rho
                  .1538793
                              .0325769
                                                             .1002105
                                                                          .2289765
LR test of rho=0: chibar2(01) = 29.91
                                                          Prob >= chibar2 = 0.000
```

Again we would like to check the stability of the quadrature technique of the model before interpreting the results. Given the estimate of ρ and the small size of the panels (between 3 and 8), we should find that the quadrature technique is numerically stable.

Refitting model intpoints() = 8
Refitting model intpoints() = 16

Quadrature check

		quadrature check	2	
	Fitted	Comparison	Comparison	
	quadrature	quadrature	quadrature	
	12 points	8 points	16 points	
Log	-1318.2088	-1318.2088	-1318.2088	
likelihood		-2.002e-06	-1.194e-09	Difference
		1.519e-09	9.061e-13	Relative difference
complain:	04304086	04304086	04304086	
age		-3.896e-10	-2.625e-12	Difference
		9.051e-09	6.100e-11	Relative difference
complain:	.0330934	.0330934	.0330934	
grade		2.208e-11	1.867e-12	Difference
		6.673e-10	5.642e-11	Relative difference
complain:	.10119998	.10119999	.10119998	
south		2.369e-09	3.957e-11	Difference
		2.341e-08	3.910e-10	Relative difference
complain:	04400789	0440079	04400789	
tenure		-3.362e-09	-2.250e-11	Difference
		7.640e-08	5.114e-10	Relative difference
complain:	.33184986	.33184986	.33184986	
gender		3.190e-09	2.546e-11	Difference
Ü		9.612e-09	7.673e-11	Relative difference
complain:	.34179006	.34179007	.34179006	
race		3.801e-09	2.990e-11	Difference
		1.112e-08	8.749e-11	Relative difference
complain:	00227021	00227021	00227021	
income		-4.468e-11	-9.252e-13	Difference
		1.968e-08	4.075e-10	Relative difference
complain:	.05245769	.05245769	.05245769	
genderm		1.963e-09	4.481e-11	Difference
		3.742e-08	8.542e-10	Relative difference
complain:	.04489311	.04489311	.04489311	
burger		4.173e-10	6.628e-12	Difference
· ·		9.296e-09	1.476e-10	Relative difference
complain:	.19047138	.19047139	.19047138	
chicken		3.096e-09	4.916e-11	Difference
		1.625e-08	2.581e-10	Relative difference
complain:	21453112	21453111	21453112	
_cons		1.281e-08	2.682e-10	Difference
		-5.972e-08	-1.250e-09	Relative difference
/:	-1.7044935	-1.7044934	-1.7044935	
lnsig2u		1.255e-07	-4.135e-10	Difference
-		-7.365e-08	2.426e-10	Relative difference

The relative and absolute differences are all small between the default 12 quadrature points and the result with 16 points. We do not have any coefficients that have a large difference between the default 12 quadrature points and eight quadrature points.

We conclude that the quadrature technique is stable. Because the differences here are so small, we would plan on using and interpreting these results rather than trying to rerun with more quadrature points.

1

Stored results

xtprobit, re stores the following in e():

```
e(N)
                                number of observations
                                number of groups
    e(N_g)
                                number of parameters
    e(k)
    e(k_aux)
                                number of auxiliary parameters
                                number of equations in e(b)
    e(k_eq)
    e(k_eq_model)
                                number of equations in overall model test
                                number of dependent variables
    e(k_dv)
    e(df_m)
                                model degrees of freedom
                                log likelihood
    e(11)
                                log likelihood, constant-only model
    e(11_0)
    e(11_c)
                                log likelihood, comparison model
    e(chi2)
                                \chi^2
                                \chi^2 for comparison test
    e(chi2_c)
                                number of clusters
    e(N_clust)
    e(rho)
    e(sigma_u)
                                panel-level standard deviation
                                number of quadrature points
    e(n_quad)
    e(g_min)
                                smallest group size
                                average group size
    e(g_avg)
    e(g_max)
                                largest group size
                                p-value for model test
    e(p)
                                rank of e(V)
    e(rank)
    e(rank0)
                                rank of e(V) for constant-only model
                                number of iterations
    e(ic)
    e(rc)
                                return code
                                1 if converged, 0 otherwise
    e(converged)
Macros
    e(cmd)
                                xtprobit
    e(cmdline)
                                command as typed
                                name of dependent variable
    e(depvar)
    e(ivar)
                                variable denoting groups
    e(model)
                                re
    e(wtype)
                                weight type
    e(wexp)
                                weight expression
    e(title)
                                title in estimation output
                                name of cluster variable
    e(clustvar)
                                linear offset variable
    e(offset)
                                Wald or LR; type of model \chi^2 test
    e(chi2type)
                                Wald or LR; type of model \chi^2 test corresponding to e(chi2_c)
    e(chi2_ct)
                                vcetype specified in vce()
    e(vce)
    e(vcetype)
                                title used to label Std. err.
    e(intmethod)
                                integration method
    e(distrib)
                                Gaussian; the distribution of the random effect
```

```
e(opt)
                              type of optimization
    e(which)
                              max or min; whether optimizer is to perform maximization or minimization
    e(ml_method)
                              type of ml method
    e(user)
                              name of likelihood-evaluator program
    e(technique)
                              maximization technique
    e(properties)
    e(predict)
                              program used to implement predict
    e(marginsdefault)
                              default predict() specification for margins
                              factor variables fyset as asbalanced
    e(asbalanced)
    e(asobserved)
                              factor variables fyset as asobserved
Matrices
    e(b)
                              coefficient vector
                              constraints matrix
    e(Cns)
    e(ilog)
                              iteration log
                              gradient vector
    e(gradient)
                              variance-covariance matrix of the estimators
    e(V)
    e(V_modelbased)
                              model-based variance
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
```

r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtprobit, pa stores the following in e():

```
Scalars
                               number of observations
    e(N)
    e(N_g)
                               number of groups
                               model degrees of freedom
    e(df_m)
    e(chi2)
                               p-value for model test
    e(p)
                               degrees of freedom for Pearson \chi^2
    e(df_pear)
    e(chi2_dev)
                               \chi^2 test of deviance
    e(chi2_dis)
                               \chi^2 test of deviance dispersion
                               deviance
    e(deviance)
    e(dispers)
                               deviance dispersion
    e(phi)
                               scale parameter
    e(g_min)
                               smallest group size
                               average group size
    e(g_avg)
    e(g_max)
                               largest group size
    e(rank)
                               rank of e(V)
                               target tolerance
    e(tol)
                               achieved tolerance
    e(dif)
    e(rc)
                               return code
Macros
    e(cmd)
                               xtgee
    e(cmd2)
                               xtprobit
                               command as typed
    e(cmdline)
    e(depvar)
                               name of dependent variable
                               variable denoting groups
    e(ivar)
    e(tvar)
                               variable denoting time within groups
    e(model)
```

```
e(family)
                               binomial
    e(link)
                               probit; link function
                               correlation structure
    e(corr)
    e(scale)
                               x2, dev, phi, or #; scale parameter
    e(wtype)
                               weight type
                               weight expression
    e(wexp)
    e(offset)
                               linear offset variable
                               Wald; type of model \chi^2 test
    e(chi2type)
                               vcetvpe specified in vce()
    e(vce)
                               title used to label Std. err.
    e(vcetype)
    e(nmp)
                               nmp, if specified
                               b V
    e(properties)
    e(predict)
                               program used to implement predict
                               predictions disallowed by margins
    e(marginsnotok)
    e(asbalanced)
                               factor variables fyset as asbalanced
    e(asobserved)
                               factor variables fyset as asobserved
Matrices
    e(b)
                               coefficient vector
    e(R.)
                               estimated working correlation matrix
                               variance-covariance matrix of the estimators
    e(V)
    e(V_modelbased)
                               model-based variance
Functions
    e(sample)
                               marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
```

matrix containing the coefficients with their standard errors, test statistics, p-values, and r(table) confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

xtprobit reports the population-averaged results obtained by using xtgee, family(binomial) link(probit) to obtain estimates.

Assuming a normal distribution, $N(0,\sigma_{\nu}^2)$, for the random effects ν_i

$$\Pr(y_{i1},\dots,y_{in_i}|\mathbf{x}_{i1},\dots,\mathbf{x}_{in_i}) = \int_{-\infty}^{\infty} \frac{e^{-\nu_i^2/2\sigma_{\nu}^2}}{\sqrt{2\pi}\sigma_{\nu}} \left\{ \prod_{t=1}^{n_i} F(y_{it},\mathbf{x}_{it}\boldsymbol{\beta} + \nu_i) \right\} d\nu_i$$

where

$$F(y,z) = \begin{cases} \Phi(z) & \text{if } y \neq 0 \\ 1 - \Phi(z) & \text{otherwise} \end{cases}$$

where Φ is the cumulative normal distribution.

The panel-level likelihood l_i is given by

$$\begin{split} l_i &= \int_{-\infty}^{\infty} \frac{e^{-\nu_i^2/2\sigma_{\nu}^2}}{\sqrt{2\pi}\sigma_{\nu}} \left\{ \prod_{t=1}^{n_i} F(y_{it}, \mathbf{x}_{it} \boldsymbol{\beta} + \nu_i) \right\} d\nu_i \\ &\equiv \int_{-\infty}^{\infty} g(y_{it}, x_{it}, \nu_i) d\nu_i \end{split}$$

This integral can be approximated with M-point Gauss-Hermite quadrature

$$\int_{-\infty}^{\infty}e^{-x^2}h(x)dx\approx \sum_{m=1}^{M}w_m^*h(a_m^*)$$

This is equivalent to

$$\int_{-\infty}^{\infty} f(x) dx \approx \sum_{m=1}^{M} w_m^* \exp\left\{(a_m^*)^2\right\} f(a_m^*)$$

where the w_m^* denote the quadrature weights and the a_m^* denote the quadrature abscissas. The log likelihood, L, is the sum of the logs of the panel-level likelihoods l_i .

The default approximation of the log likelihood is by adaptive Gauss-Hermite quadrature, which approximates the panel-level likelihood with

$$l_i \approx \sqrt{2} \hat{\sigma}_i \sum_{m=1}^M w_m^* \exp\left\{(a_m^*)^2\right\} g(y_{it}, x_{it}, \sqrt{2} \hat{\sigma}_i a_m^* + \hat{\mu}_i)$$

where $\hat{\sigma}_i$ and $\hat{\mu}_i$ are the adaptive parameters for panel i. Therefore, with the definition of $g(y_{it}, x_{it}, \nu_i)$, the total log likelihood is approximated by

$$\begin{split} L \approx \sum_{i=1}^n w_i \log \biggl[\sqrt{2} \hat{\sigma}_i \sum_{m=1}^M w_m^* \exp\{(a_m^*)^2\} \frac{\exp\{-(\sqrt{2} \hat{\sigma}_i a_m^* + \hat{\mu}_i)^2/2\sigma_\nu^2\}}{\sqrt{2\pi}\sigma_\nu} \\ & \prod_i F(y_{it}, x_{it} \pmb{\beta} + \sqrt{2} \hat{\sigma}_i a_m^* + \hat{\mu}_i) \biggr] \end{split}$$

where w_i is the user-specified weight for panel i; if no weights are specified, $w_i = 1$.

The default method of adaptive Gauss–Hermite quadrature is to calculate the posterior mean and variance and use those parameters for $\hat{\mu}_i$ and $\hat{\sigma}_i$ by following the method of Naylor and Smith (1982), further discussed in Skrondal and Rabe-Hesketh (2004). We start with $\hat{\sigma}_{i,0}=1$ and $\hat{\mu}_{i,0}=0$, and the posterior means and variances are updated in the kth iteration. That is, at the kth iteration of the optimization for l_i , we use

$$l_{i,k} \approx \sum_{m=1}^{M} \sqrt{2} \hat{\sigma}_{i,k-1} w_m^* \exp\{a_m^*)^2\} g(y_{it}, x_{it}, \sqrt{2} \hat{\sigma}_{i,k-1} a_m^* + \hat{\mu}_{i,k-1})$$

Letting

$$\tau_{i.m.k-1} = \sqrt{2}\hat{\sigma}_{i.k-1}a_m^* + \hat{\mu}_{i.k-1}$$

$$\hat{\mu}_{i,k} = \sum_{m=1}^{M} (\tau_{i,m,k-1}) \frac{\sqrt{2} \hat{\sigma}_{i,k-1} w_m^* \exp\{(a_m^*)^2\} g(y_{it}, x_{it}, \tau_{i,m,k-1})}{l_{i,k}}$$

and

$$\hat{\sigma}_{i,k} = \sum_{m=1}^{M} (\tau_{i,m,k-1})^2 \frac{\sqrt{2} \hat{\sigma}_{i,k-1} w_m^* \exp\{(a_m^*)^2\} g(y_{it}, x_{it}, \tau_{i,m,k-1})}{l_{i,k}} - \left(\hat{\mu}_{i,k}\right)^2$$

and this is repeated until $\hat{\mu}_{i,k}$ and $\hat{\sigma}_{i,k}$ have converged for this iteration of the maximization algorithm. This adaptation is applied on every iteration until the log-likelihood change from the preceding iteration is less than a relative difference of 1e–6; after this, the quadrature parameters are fixed.

The log likelihood can also be calculated by nonadaptive Gauss-Hermite quadrature, the intmethod(ghermite) option, where $\rho=\sigma_{\nu}^2/(\sigma_{\nu}^2+1)$:

$$\begin{split} L &= \sum_{i=1}^n w_i \log \Big\{ \Pr(y_{i1}, \dots, y_{in_i} | \mathbf{x}_{i1}, \dots, \mathbf{x}_{in_i}) \Big\} \\ &\approx \sum_{i=1}^n w_i \log \left[\frac{1}{\sqrt{\pi}} \sum_{m=1}^M w_m^* \prod_{t=1}^{n_i} F \bigg\{ y_{it}, \mathbf{x}_{it} \boldsymbol{\beta} + a_m^* \left(\frac{2\rho}{1-\rho} \right)^{1/2} \bigg\} \right] \end{split}$$

Both quadrature formulas require that the integrated function be well approximated by a polynomial of degree equal to the number of quadrature points. The number of periods (panel size) can affect whether

$$\prod_{t=1}^{n_i} F(y_{it}, \mathbf{x}_{it}\boldsymbol{\beta} + \nu_i)$$

is well approximated by a polynomial. As panel size and ρ increase, the quadrature approximation can become less accurate. For large ρ , the random-effects model can also become unidentified. Adaptive quadrature gives better results for correlated data and large panels than nonadaptive quadrature; however, we recommend that you use the quadchk command (see [XT] quadchk) to verify the quadrature approximation used in this command, whichever approximation you choose.

xtprobit, re and the robust VCE estimator

Specifying vce(robust) or vce(cluster clustvar) causes the Huber/White/sandwich VCE estimator to be calculated for the coefficients estimated in this regression. See [P] _robust, particularly Introduction and Methods and formulas. Wooldridge (2020) and Arellano (2003) discuss this application of the Huber/White/sandwich VCE estimator. As discussed by Wooldridge (2020), Stock and Watson (2008), and Arellano (2003), specifying vce(robust) is equivalent to specifying vce(cluster panelvar), where panelvar is the variable that identifies the panels.

Clustering on the panel variable produces a consistent VCE estimator when the disturbances are not identically distributed over the panels or there is serial correlation in ϵ_{it} .

The cluster-robust VCE estimator requires that there are many clusters and the disturbances are uncorrelated across the clusters. The panel variable must be nested within the cluster variable because of the within-panel correlation that is generally induced by the random-effects transform when there is heteroskedasticity or within-panel serial correlation in the idiosyncratic errors.

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Also see

- [XT] **xtprobit postestimation** Postestimation tools for xtprobit
- [XT] **quadchk** Check sensitivity of quadrature approximation
- [XT] **xtcloglog** Random-effects and population-averaged cloglog models
- [XT] **xteprobit** Extended random-effects probit regression
- [XT] **xtgee** GEE population-averaged panel-data models
- [XT] **xtlogit** Fixed-effects, random-effects, and population-averaged logit models
- [XT] **xtset** Declare data to be panel data
- [BAYES] bayes: xtprobit Bayesian random-effects probit model
- [ME] meprobit Multilevel mixed-effects probit regression
- [MI] Estimation Estimation commands for use with mi estimate
- [R] **probit** Probit regression
- [U] 20 Estimation and postestimation commands

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