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# Title

hetoprobit --- Heteroskedastic ordered probit regression

Description Options References Quick start Remarks and examples Also see Menu Stored results Syntax Methods and formulas

# Description

hetoprobit fits a heteroskedastic ordered probit model for an ordinal dependent variable. hetoprobit is a generalization of oprobit that allows the variance to be modeled as a function of independent variables and to differ between subjects or groups in the population.

# **Quick start**

Heteroskedastic ordinal probit model of y on x1, using x2 to model the variance hetoprobit y x1, het(x2)

With robust standard errors

hetoprobit y x1, het(x2) vce(robust)

Perform a Wald test on the variance instead of a likelihood-ratio (LR) test hetoprobit y x1, het(x2) waldhet

## Menu

Statistics > Ordinal outcomes > Heteroskedastic ordered probit regression

### Syntax

```
hetoprobit depvar [indepvars] [if] [in] [weight],
het(varlist[, offset(varname<sub>o</sub>)]) [options]
```

options	Description			
Model				
* het (varlist $[]$ )	independent variables to model the variance and optional offset variable			
<u>off</u> set( <i>varname</i> ) <u>const</u> raints( <i>constraints</i> )	include <i>varname</i> in model with coefficient constrained to 1 apply specified linear constraints			
SE/Robust				
vce( <i>vcetype</i> )	<pre>vcetype may be oim, robust, cluster clustvar, opg, bootstrap, or jackhnife</pre>			
Reporting				
<u>l</u> evel(#)	set confidence level; default is level(95)			
waldhet	perform Wald test on variance instead of LR test			
<u>nocnsr</u> eport	do not display constraints			
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling			
Maximization				
maximize_options	control the maximization process; seldom used			
noheader	do not display header above coefficient table			
notable	do not display coefficient table			
<u>col</u> linear	keep collinear variables			
<u>coefl</u> egend	display legend instead of statistics			

\*het() is required. The full specification is  $het(varlist [, offset(varname_o)])$ . indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.

depvar, indepvars, and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayes, bootstrap, by, collect, fp, jackknife, rolling, statsby, and svy are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: hetoprobit.

Weights are not allowed with the bootstrap prefix; see [R] bootstrap.

vce() and weights are not allowed with the svy prefix; see [SVY] svy.

fweights, iweights, and pweights are allowed; see [U] 11.1.6 weight.

noheader, notable, collinear, and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

# Options

\_ Model ]

het(varlist [, offset(varname<sub>o</sub>)]) specifies the independent variables and, optionally, the offset
variable in the variance function. het() is required.

offset(varname<sub>o</sub>) specifies that offset varname<sub>o</sub> be included in the variance model with the coefficient constrained to be 1.

offset(varname), constraints(constraints); see [R] Estimation options.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (oim, opg), that are robust to some kinds of misspecification (robust), that allow for intragroup correlation (cluster clustvar), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [R] vce\_option.

Reporting

level(#); see [R] Estimation options.

waldhet specifies that a Wald test of whether lnsigma = 0 be performed instead of the LR test.

nocnsreport; see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(% fmt), pformat(% fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize\_options: difficult, technique(algorithm\_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(init\_specs); see [R] Maximize. These options are seldom used.

The following options are available with hetoprobit but are not shown in the dialog box:

noheader suppresses the header above the coefficient table.

notable suppresses the display of the coefficient table.

collinear, coeflegend; see [R] Estimation options.

### Remarks and examples

hetoprobit fits a maximum-likelihood heteroskedastic ordered probit model, which is a generalization of the ordered probit model (see [R] oprobit).

In ordinal regression models, the outcome is an ordinal variable—a variable that is categorical and ordered, for instance, "poor", "good", and "excellent". The specific values of the ordinal variable are irrelevant. It matters only that larger values are assumed to correspond to "higher" outcomes. To simplify the discussion in this entry, we assume without loss of generality that the dependent variable takes on the integer values  $0, 1, \ldots, H$ , for some value H > 1.

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In ordered probit models, an underlying score is estimated as a linear function of the independent variables and a set of cutpoints. The probability of observing outcome  $y_j = h$ , where h = 0, 1, ..., H, corresponds to the probability that the value of the linear function, plus random error, is within the range of the cutpoints associated with the outcome

$$\Pr(y_j = h) = \Pr(\kappa_h < \beta_1 x_{1j} + \beta_2 x_{2j} + \dots + \beta_k x_{kj} + u_j \le \kappa_{h+1})$$
$$= \Phi(\kappa_{h+1} - \mathbf{x}_j \beta) - \Phi(\kappa_h - \mathbf{x}_j \beta)$$

where  $\mathbf{x}_j = (x_{1j}, x_{2j}, \dots, x_{kj})$  are the k independent variables that model the mean function;  $\boldsymbol{\beta}$  is a column vector of unknown parameters in the mean function;  $u_j$ , where  $j = 1, \dots, N$ , are normally distributed error terms;  $\kappa_h$ , where  $h = 1, \dots, H$ , are the unknown cutpoints that separate the different possible values of h; and  $\Phi(\cdot)$  is the cumulative distribution function of the standard normal distribution. Also, by convention, to complete the intervals for the lowest and highest values of the outcome,  $\kappa_0 = -\infty$  and  $\kappa_{H+1} = \infty$ .

In conventional ordinal probit models, the error term is assumed i.i.d. normal with unit variance for all observations. hetoprobit generalizes the ordered probit model by representing the variance of the error term  $u_j$  as a multiplicative function of explanatory variables  $\mathbf{z}_j = (z_{1j}, z_{2j}, \ldots, z_{mj})$ . This approach was introduced by Harvey (1976), though we depart from Harvey slightly by modeling standard deviation rather than variance. More specifically, we model the natural logarithm of the standard deviation as a linear combination of the explanatory variables,

$$\ln \sigma_j = \mathbf{z}_j oldsymbol{\gamma}$$

where  $\gamma$  is a column vector of unknown parameters in the variance function.

With this generalization, the error variance may differ between subjects or between groups in the population, and

$$\Pr(y_j = h) = \Phi\left\{\frac{\kappa_{h+1} - \mathbf{x}_j\beta}{\exp(\mathbf{z}_j\gamma)}\right\} - \Phi\left\{\frac{\kappa_h - \mathbf{x}_j\beta}{\exp(\mathbf{z}_j\gamma)}\right\}$$

For the model to be identifiable, there can be no constant term in  $z_j \gamma$ . Also, as with [R] **oprobit**, there is no constant term in  $x_j \beta$ . The role of the constant is subsumed by the cutpoints.

We estimate the coefficients  $\beta_1, \beta_2, \ldots, \beta_k$  and  $\gamma_1, \gamma_2, \ldots, \gamma_m$  together with the cutpoints  $\kappa_1, \kappa_2, \ldots, \kappa_H$ . If the model has no independent variables in  $\mathbf{x}_j$ , only the cutpoints and the  $\gamma$  parameters are estimated.

Modeling of heteroskedastic variance has both constructive and defensive uses. It is known that differences in variance between subjects or between groups in the population can cause biased coefficient estimates and can complicate comparison of distinct groups. Thus, incorporating a model for variance can be necessary for proper inference, even if the variance function itself is not a topic of interest to the researcher. For discussion, see Williams (2010) and the references cited therein. There are also cases where modeling the differences between variances of different subjects or different groups in the population is one of the principal purposes of the study. We will discuss such a scenario in the examples below. See Reardon et al. (2017) and Alvarez and Brehm (1995) for additional examples.

#### Example 1: Modeling heteroskedasticity of reported health status

In this example, we will use a slightly modified subset of data from the 2015 Eating & Health Module of the American Time Use Survey (ATUS), conducted by the U.S. Bureau of Labor Statistics. Our analysis will not account for the survey design. The ATUS measures the amount of time people spend doing various activities, such as working, caring for children, volunteering, and socializing. Of interest to us is an ordinal response variable, health, which contains individuals' self-assessments of their overall health status on a five-point scale: 1 for "poor", 2 for "fair", 3 for "good", 4 for "very good", and 5 for "excellent".

We want to examine the role that age and other factors play in an individual's self-assessment of health. Age is a natural variable to include when modeling mean or typical health status. But we also suspect that the variation in health status is greater in an older population, as compared with a youthful population, which consists mainly of healthy individuals. If our suspicion is true, quantifying the relationship between variation in health status and age may have value, for example, in planning a healthcare strategy that is appropriately tailored both for the older Medicare population and for a younger cohort.

Thus, we will use hetoprobit to model heteroskedasticity induced by age. In modeling the variance term, in addition to age, we will include a factor variable, exercise, which indicates whether or not an individual exercised during the previous week. For purposes of illustration, imagine that we are not interested in exercise as a topic in its own right, but we are concerned that health variability among those who exercise may differ from the variability among those who do not. Therefore, we include exercise in the variance term to help insulate our estimation results against a possible hidden bias.

Our model will include three explanatory variables for the mean function: age, bmi (body mass index), and exercise.

	/www.stata-pre ting & Health			health15		
	health age bmi			e i.exer	cise)	
(output omitted	)		_			
Fitting order	ed probit mode	1:				
Iteration 0:	Log likelihoo		943			
Iteration 1:	Log likelihood = -2717.2752					
Iteration 2:	0					
Iteration 3:	Log likelihoo	d = -2716.9	679			
Fitting full r	nodel:					
Iteration 0:	Log likelihoo					
Iteration 1: Iteration 2:	Log likelihoo Log likelihoo					
Iteration 3:	Log likelihoo					
Heteroskedast	0				Number of ob	s = 2.009
					LR chi2(3)	= 366.91
Log likelihood	d = -2708.5491				Prob > chi2	= 0.0000
health	Coefficient	Std. err.	Z	P> z	[95% conf.	interval]
health						
age	0083348	.0015969	-5.22	0.000	0114646	005205
bmi	0564072	.0057392	-9.83	0.000	0676558	0451586
exercise						
Yes	.6493794	.0732137	8.87	0.000	.5058833	.7928755
lnsigma						
age	.0041401	.0011611	3.57	0.000	.0018643	.0064159
exercise						
Yes	0773166	.0423038	-1.83	0.068	1602305	.0055973
/cut1	-3.903773	.2913163			-4.474742	-3.332803
/cut2	-2.776111	.2262442			-3.219541	-2.33268
/cut3	-1.576396	.174352			-1.918119	-1.234672
/cut4	4189882	.1524084			7177031	1202733
LR test of lns	sigma=0: chi2(	2) = 16.84			Prob > chi	2 = 0.000

The LR test at the bottom of the output is a test of homogeneity of the variance function. The  $\chi^2(2)$  statistic of 16.84 is significant, indicating that heteroskedasticity is present. If you prefer the Wald test for heteroskedasticity, you can specify the waldhet option. The coefficients for the variance function are reported in the section of the table labeled lnsigma. Our results indicate that age is a significant contributor to the variance function but that exercise is not significant at a 0.05 level.

The LR test for the model that appears above the coefficient table is a joint test for inclusion of age, bmi, and exercise in the mean function. The null model for this test is the model consisting only of cutpoints and the heteroskedastic term. Coefficients for the mean function are reported in the section of the table labeled health. In this example, age, bmi, and exercise are significant components of the linear predictor of the mean.

The signs of the coefficients in the fitted model are directly interpretable. For example, the negative value for the coefficient of bmi implies that higher values of bmi predict lower values of health status. However, because of the probit link and the fact that we estimate variance with a log transformation, the numerical relationships between the coefficients of the model and the outcome variables are nonlinear. Postestimation commands recognize and account for these nonlinearities.

#### Example 2: Predict the probability of a poor health rating

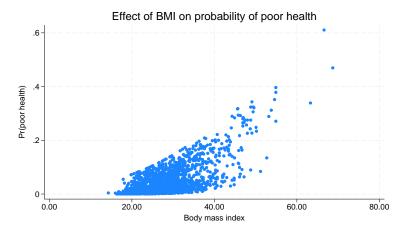
Ordered probit models allow us to look at the probabilities of different outcomes of interest. Suppose we are interested in predictions of a reported health status of "poor" (health = 1) and how it differs across levels of bmi. First, we obtain the predicted probability of poor health.

. predict pr1, pr outcome(1)

We can now visualize how the predicted probability of poor health status differs across the range of bmi values in our sample.

. twoway scatter pr1 bmi, ytitle("Pr(poor health)")

> title("Effect of BMI on probability of poor health")



We see that predicted probabilities of poor health increase as body mass index increases.

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#### Example 3: Predictive margins and average marginal effect

The graph above plots the predicted probability of poor health for each individual in our dataset. We may also want to evaluate how the average predicted probability changes across levels of the covariates in the model. For instance, we can use the margins command to obtain the expected probability of having poor health across a range of ages.

```
. margins, at(age = (30(10)70)) predict(outcome(1))
Predictive margins
                                                            Number of obs = 2,009
Model VCE: OIM
Expression: Pr(health==1), predict(outcome(1))
1._at: age = 30
2. at: age = 40
3._at: age = 50
4._at: age = 60
5._at: age = 70
                           Delta-method
                                                              [95% conf. interval]
                    Margin
                             std. err.
                                              z
                                                   P>|z|
          _at
                                           6.02
                                                                           .0299306
          1
                  .0225765
                              .0037522
                                                   0.000
                                                              .0152224
          2
                  .0299079
                              .0038887
                                           7.69
                                                   0.000
                                                              .0222862
                                                                           .0375297
          З
                  .0388244
                              .0041936
                                           9.26
                                                   0.000
                                                              .0306051
                                                                           .0470438
          4
                  .0494246
                              .0049748
                                           9.93
                                                   0.000
                                                              .0396742
                                                                           .0591751
```

.0064443

Based on our model, what would we expect if everyone was 30 years old but had the same distributions of bmi and exercise that we observed in our data? The first line in this table reports that the average predicted probability of poor health is 0.0226 in this case. The second line shows the average predicted probability of poor health if we set age = 40, and so on. We find that for age = 70, the average probability of reporting a poor health status has increased to 0.0618. We can visualize this by typing marginsplot after margins.

9.58

0.000

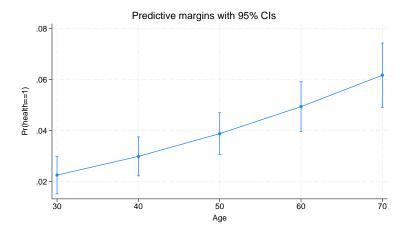
.0491226

.0743839

```
. marginsplot
Variables that uniquely identify margins: age
```

.0617532

5



We have focused on the prediction of poor health. We could instead simultaneously obtain average predicted probabilities of poor, fair, good, very good, and excellent health status and plot them across our requested age range. In that case, we would type

. margins, at(age = (30(10)70))

. marginsplot

We might also be interested in characterizing the relationship between bmi and the probability of reporting poor health. The coefficients and cutpoints reported in hetoprobit are not easily interpreted. We can, however, use margins to estimate the average marginal effect of bmi on the probability of reporting poor health. Because the average marginal effect depends on the value of bmi, we estimate it across a range of bmi values by typing

```
. margins, dydx(bmi) at(bmi = (20(5)35)) predict(outcome(1))
Average marginal effects
                                                            Number of obs = 2,009
Model VCE: OIM
Expression: Pr(health==1), predict(outcome(1))
dy/dx wrt: bmi
1._at: bmi = 20
2._at: bmi = 25
3._at: bmi = 30
4._at: bmi = 35
                           Delta-method
                             std. err.
                                                   P>|z|
                                                             [95% conf. interval]
                     dy/dx
                                             z
bmi
         _at
                              .0001721
                  .0015875
                                           9.23
                                                   0.000
                                                              .0012503
                                                                          .0019248
          1
          2
                  .0024512
                              .0002665
                                           9.20
                                                   0.000
                                                              .0019288
                                                                          .0029736
          3
                  .0036447
                              .0004377
                                           8.33
                                                   0.000
                                                              .0027868
                                                                          .0045027
          4
                  .0052124
                              .0007013
                                                   0.000
                                                                           .006587
                                           7.43
                                                              .0038379
```

The average marginal effect of bmi on the probability of reporting poor health increases as bmi itself increases.

#### Example 4: Interpreting the variance function

2

1.355853

From the output of our hetoprobit command, we determined that variance of health status is affected by age. Let's consider to what extent. In this example, we assess the effect of age on the variance by using the margins command. We use the predict(sigma) option to obtain the average predicted standard deviation of the errors. We will look at ages 15 and 85, which are the youngest and oldest ages, respectively, in our dataset.

```
. margins, predict(sigma) at(age = (15,85)) noatlegend
Predictive margins
                                                           Number of obs = 2,009
Model VCE: OIM
Expression: Heteroskedastic standard deviation, predict(sigma)
                           Delta-method
                             std. err.
                                                  P>|z|
                                                             [95% conf. interval]
                   Margin
                                             7.
         _at
          1
                  1.014732
                             .0320905
                                         31.62
                                                  0.000
                                                             .9518354
                                                                         1.077628
```

.1398144

Variation increases with age. The expected standard deviation of the error term changes from 1.015 at age 15 to 1.356 at age 85.

9.70

0.000

1.081822

1.629884

4

# **Stored results**

hetoprobit stores the following in e():

Scalars	
e(N)	number of observations
e(k_cat)	number of categories
e(k)	number of parameters
e(k_eq)	number of equations in e(b)
e(k_eq_model)	number of equations in overall model test
e(k_aux)	number of auxiliary parameters
e(k_dv)	number of dependent variables
e(df_m)	model degrees of freedom ( $x\beta$ term)
e(11)	log likelihood
e(11_0)	log likelihood, cutpoint-only (heteroskedastic) model
e(11_c)	log likelihood, comparison (homoskedastic) model
e(N_clust)	number of clusters
e(chi2)	$\chi^2_{2}$
e(chi2_c)	$\chi^2$ for heteroskedasticity test
e(p)	<i>p</i> -value for model test
e(p_c)	<i>p</i> -value for heteroskedasticity test
e(df_m_c)	degrees of freedom for heteroskedasticity test
e(rank)	rank of e(V)
e(ic)	number of iterations
e(rc)	return code
e(converged)	1 if converged, 0 otherwise
Macros	
e(cmd)	hetoprobit
e(cmdline)	command as typed
e(depvar)	name of dependent variable
e(wtype)	weight type
e(wexp)	weight expression
e(title)	title in estimation output
e(clustvar)	name of cluster variable
e(offset1)	offset for ordered probit equation
e(offset2)	offset for variance equation
e(chi2type)	LR; type of model $\chi^2$ test
e(chi2_ct)	LR or Wald; type of heteroskedasticity test corresponding to e(chi2_c)
e(vce)	vcetype specified in vce()
e(vcetype)	title used to label Std. err.
e(opt)	type of optimization
e(which)	max or min; whether optimizer is to perform maximization or minimization
e(ml_method)	type of ml method
e(user)	name of likelihood-evaluator program
e(technique)	maximization technique
e(properties)	b V
e(predict)	program used to implement predict
e(marginsok)	predictions allowed by margins
e(marginsnotok)	predictions disallowed by margins
e(marginsdefault)	default predict() specification for margins
e(asbalanced)	factor variables fvset as asbalanced
e(asobserved)	factor variables fvset as asobserved
Matrices	
e(b)	coefficient vector
e(Cns)	constraints matrix
e(ilog)	iteration log (up to 20 iterations)
e(gradient)	gradient vector
e(cat)	category values
e(V)	variance-covariance matrix of the estimators
e(V_modelbased)	model-based variance
Functions	
e(sample)	marks estimation sample
-	*

In addition to the above, the following is stored in r():

Matrices r(table)

matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

### Methods and formulas

hetoprobit fits a cumulative probit model with heteroskedastic variance using maximum likelihood estimation. Namely, the model is that, for a subject with explanatory variables x and z,

$$\Pr(Y \le h) = \Phi\left\{\frac{\kappa_{h+1} - \mathbf{x}\beta}{\exp(\mathbf{z}\gamma)}\right\}$$

where Y is an ordinal outcome taking on values h = 0, 1, ..., H, and  $\Phi(\cdot)$  is the cdf of the standard normal distribution. The value  $\kappa_{h+1}$  is a cutpoint that separates the region corresponding to Y = hfrom regions for higher-valued categories. The effects  $\beta$  and the effects  $\gamma$  are the same for each cumulative probability.

The log-likelihood function is

$$\ln L = \sum_{j=1}^{N} w_j \sum_{h=0}^{H} I_h(y_j) \ln \left[ \Phi \left\{ \frac{\kappa_{h+1} - \mathbf{x}_j \boldsymbol{\beta}}{\exp(\mathbf{z}_j \boldsymbol{\gamma})} \right\} - \Phi \left\{ \frac{\kappa_h - \mathbf{x}_j \boldsymbol{\beta}}{\exp(\mathbf{z}_j \boldsymbol{\gamma})} \right\} \right]$$

where

$$I_h(y_j) = \begin{cases} 1 & \text{if } y_j = h \\ 0 & \text{otherwise} \end{cases}$$

and  $y_j$ , where j = 1, ..., N, is an observed value of Y;  $w_j$  are optional weights;  $\kappa_0 = -\infty$  and  $\kappa_{H+1} = \infty$ ; and all other terminology is defined in *Remarks and examples* above.

The log-likelihood function is maximized as described in [R] Maximize.

This command supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster *clustvar*), respectively. See [P] <u>robust</u>, particularly *Maximum likelihood estimators* and *Methods and formulas*.

hetoprobit also supports estimation with survey data. For details on VCEs with survey data, see [SVY] Variance estimation.

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### Also see

- [R] hetoprobit postestimation Postestimation tools for hetoprobit
- [R] **hetprobit** Heteroskedastic probit model
- [R] **oprobit** Ordered probit regression
- [BAYES] **bayes: hetoprobit** Bayesian heteroskedastic ordered probit regression
- [SVY] svy estimation Estimation commands for survey data
- [U] 20 Estimation and postestimation commands

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