Title

power onemean — Power analysis for a one-sample mean test

Description

power onemean computes sample size, power, or target mean for a one-sample mean test. By default, it computes sample size for given power and the values of the mean parameters under the null and alternative hypotheses. Alternatively, it can compute power for given sample size and values of the null and alternative means or the target mean for given sample size, power, and the null mean. For power and sample-size analysis in a cluster randomized design, see [PSS] **power onemean, cluster**. Also see [PSS] **power** for a general introduction to the power command using hypothesis tests.

Quick start

Sample size for two-sided test of H_0: $\mu = 10$ versus H_a: $\mu \neq 10$ with null mean $m_0 = 10$, alternative mean $m_a = 15$, and standard deviation of 12 using default power of 0.8 and significance level $\alpha = 0.05$

    power onemean 10 15, sd(12)

As above, but for a one-sided test with power of 0.9

    power onemean 10 15, sd(12) power(.9) onesided

Same as above, but specified as m_0 and difference $m_a - m_0 = 5$

    power onemean 10, sd(12) power(.9) onesided diff(5)

Power for a sample size of 75

    power onemean 10 15, sd(12) n(75)

Power for sample sizes of 50, 60, 70, and 80

    power onemean 10 15, sd(12) n(50(10)80)

As above, but display results in a graph of power versus sample size

    power onemean 10 15, sd(12) n(50(10)80) graph

Effect size and target mean for $m_0 = 10$ with standard deviation of 4, for a sample size of 40, power of 0.9, and $\alpha = 0.01$

    power onemean 10, sd(4) n(40) power(.9) alpha(.01)

Menu

Statistics > Power and sample size

Syntax

Compute sample size

> `power onemean` m_0 m_a [`,` `power(`*numlist*`)` *options*]

Compute power

> `power onemean` m_0 m_a `,` `n(`*numlist*`)` [*options*]

Compute effect size and target mean

> `power onemean` m_0 `,` `n(`*numlist*`)` `power(`*numlist*`)` [*options*]

where m_0 is the null (hypothesized) mean or the value of the mean under the null hypothesis and m_a is the alternative (target) mean or the value of the mean under the alternative hypothesis. m_0 and m_a may each be specified either as one number or as a list of values in parentheses (see [U] **11.1.8 numlist**).

options	Description
[Main]	
* <u>a</u>lpha(*numlist*)	significance level; default is alpha(0.05)
* <u>pow</u>er(*numlist*)	power; default is power(0.8)
*beta(*numlist*)	probability of type II error; default is beta(0.2)
*n(*numlist*)	sample size; required to compute power or effect size
<u>nfrac</u>tional	allow fractional sample size
*diff(*numlist*)	difference between the alternative mean and the null mean, $m_a - m_0$; specify instead of the alternative mean m_a
*sd(*numlist*)	standard deviation; default is sd(1)
<u>knownsd</u>	request computation assuming a known standard deviation; default is to assume an unknown standard deviation
*fpc(*numlist*)	finite population correction (FPC) as a sampling rate or as a population size
<u>dir</u>ection(<u>upper</u>|<u>lower</u>)	direction of the effect for effect-size determination; default is direction(upper), which means that the postulated value of the parameter is larger than the hypothesized value
<u>onesided</u>	one-sided test; default is two sided
<u>parallel</u>	treat number lists in starred options or in command arguments as parallel when multiple values per option or argument are specified (do not enumerate all possible combinations of values)
[Table]	
[<u>no</u>]<u>table</u>[(*tablespec*)]	suppress table or display results as a table; see [PSS] **power, table**
<u>saving</u>(*filename*[, replace])	save the table data to *filename*; use replace to overwrite existing *filename*
[Graph]	
<u>graph</u>[(*graphopts*)]	graph results; see [PSS] **power, graph**
[Iteration]	
<u>init</u>(#)	initial value for sample size or mean; default is to use normal approximation
<u>iter</u>ate(#)	maximum number of iterations; default is iterate(500)
<u>tol</u>erance(#)	parameter tolerance; default is tolerance(1e-12)
<u>ftol</u>erance(#)	function tolerance; default is ftolerance(1e-12)
[<u>no</u>]log	suppress or display iteration log
[<u>no</u>]dots	suppress or display iterations as dots
cluster	perform computations for a CRD; see [PSS] **power onemean, cluster**
<u>notitle</u>	suppress the title

*Specifying a list of values in at least two starred options, or at least two command arguments, or at least one starred option and one argument results in computations for all possible combinations of the values; see [U] **11.1.8 numlist**. Also see the parallel option.

cluster and notitle do not appear in the dialog box.

where *tablespec* is

   column[:*label*] [*column*[:*label*] [. . .]] [, *tableopts*]

column is one of the columns defined below, and *label* is a column label (may contain quotes and compound quotes).

column	Description	Symbol
alpha	significance level	α
power	power	$1 - \beta$
beta	type II error probability	β
N	number of subjects	N
delta	effect size	δ
m0	null mean	μ_0
ma	alternative mean	μ_a
diff	difference between the alternative and null means	$\mu_a - \mu_0$
sd	standard deviation	σ
fpc	FPC as population size	N_{pop}
	FPC as sampling rate	γ
target	target parameter; synonym for ma	
_all	display all supported columns	

Column beta is shown in the default table in place of column power if specified.

Columns diff and fpc are shown in the default table if specified.

Options

> [Main]

alpha(), power(), beta(), n(), nfractional; see [PSS] **power**. The nfractional option is allowed only for sample-size determination.

diff(*numlist*) specifies the difference between the alternative mean and the null mean, $m_a - m_0$. You can specify either the alternative mean m_a as a command argument or the difference between the two means in diff(). If you specify diff(*#*), the alternative mean is computed as $m_a = m_0 +$ *#*. This option is not allowed with the effect-size determination.

sd(*numlist*) specifies the sample standard deviation or the population standard deviation. The default is sd(1). By default, sd() specifies the sample standard deviation. If knownsd is specified, sd() specifies the population standard deviation.

knownsd requests that the standard deviation be treated as known in the computation. By default, the standard deviation is treated as unknown, and the computation is based on a t test, which uses a Student's t distribution as a sampling distribution of the test statistic. If knownsd is specified, the computation is based on a z test, which uses a normal distribution as the sampling distribution of the test statistic.

fpc(*numlist*) requests that a finite population correction be used in the computation. If fpc() has values between 0 and 1, it is interpreted as a sampling rate, n/N, where N is the total number of units in the population. If fpc() has values greater than n, it is interpreted as a population size. It is an error for fpc() to have values between 1 and n or to have a mixture of sampling rates and population sizes.

direction(), onesided, parallel; see [PSS] **power**.

Table

table, table(), notable; see [PSS] **power, table**.

saving(); see [PSS] **power**.

Graph

graph, graph(); see [PSS] **power, graph**. Also see the *column* table for a list of symbols used by the graphs.

Iteration

init(#) specifies the initial value of the sample size for the sample-size determination or the initial value of the mean for the effect-size determination. The default is to use a closed-form normal approximation to compute an initial value of the sample size or mean.

iterate(), tolerance(), ftolerance(), log, nolog, dots, nodots; see [PSS] **power**.

The following options are available with power onemean but are not shown in the dialog box:

cluster; see [PSS] **power onemean, cluster**.

notitle; see [PSS] **power**.

Remarks and examples   stata.com

Remarks are presented under the following headings:

> *Introduction*
> *Using power onemean*
> *Computing sample size*
> *Computing power*
> *Computing effect size and target mean*
> *Performing hypothesis tests on mean*
> *Video examples*

This entry describes the power onemean command and the methodology for power and sample-size analysis for a one-sample mean test. See [PSS] **intro** for a general introduction to power and sample-size analysis, and see [PSS] **power** for a general introduction to the power command using hypothesis tests. Also see [PSS] **power onemean, cluster** for power and sample-size analysis in a cluster randomized design.

Introduction

There are many examples of studies where a researcher would like to compare an observed mean with a hypothesized mean. A company that provides preparatory classes for a standardized exam would like to see if the mean score of students who take its classes is higher than the national average. A fitness center would like to know if its average clients' weight loss is greater than zero after six months. Or a government agency would like to know if a job training program results in higher wages than the national average.

This entry describes power and sample-size analysis for the inference about the population mean performed using hypothesis testing. Specifically, we consider the null hypothesis H_0: $\mu = \mu_0$ versus the two-sided alternative hypothesis H_a: $\mu \neq \mu_0$, the upper one-sided alternative H_a: $\mu > \mu_0$, or the lower one-sided alternative H_a: $\mu < \mu_0$.

The considered one-sample mean tests rely on the assumption that a random sample is normally distributed or that the sample size is large. Different test statistics can be based on whether the variance of the sampling process is known a priori. In the case of a known variance, the test statistic follows a standard normal distribution under the null hypothesis, and the corresponding test is known as a z test. In the case of an unknown variance, an estimate of the population variance is used to form a test statistic, which follows a Student's t distribution under the null hypothesis, and the corresponding test is known as a t test.

The random sample is typically drawn from an infinite population. When the sample is drawn from a population of a fixed size, sampling variability must be adjusted for a finite population size.

The `power onemean` command provides power and sample-size analysis for the comparison of a mean with a reference value using a t test or a z test.

Using power onemean

`power onemean` computes sample size, power, or target mean for a one-sample mean test. All computations are performed for a two-sided hypothesis test where, by default, the significance level is set to 0.05. You may change the significance level by specifying the `alpha()` option. You can specify the `onesided` option to request a one-sided test.

By default, all computations are based on a t test, which assumes an unknown standard deviation, and use the default value of 1 as the estimate of the standard deviation. You may specify other values for the standard deviation in the `sd()` option. For a known standard deviation, you can specify the `knownsd` option to request a z test.

To compute sample size, you must specify the means under the null and alternative hypotheses, m_0 and m_a, respectively, and, optionally, the power of the test in the `power()` option. The default power is set to 0.8.

To compute power, you must specify the sample size in the `n()` option and the means under the null and alternative hypotheses, m_0 and m_a, respectively.

Instead of the alternative mean, m_a, you may specify the difference $m_a - m_0$ between the alternative mean and the null mean in the `diff()` option when computing sample size or power.

To compute effect size, the standardized difference between the alternative and null means, and the corresponding target mean, you must specify the sample size in the `n()` option, the power in the `power()` option, the null mean m_0, and, optionally, the direction of the effect. The direction is upper by default, `direction(upper)`, which means that the target mean is assumed to be larger than the specified null mean value. This is also equivalent to the assumption of a positive effect size. You can change the direction to lower, which means that the target mean is assumed to be smaller than the specified null value, by specifying the `direction(lower)` option. This is equivalent to assuming a negative effect size.

By default, the computed sample size is rounded up. You can specify the `nfractional` option to see the corresponding fractional sample size; see *Fractional sample sizes* in [PSS] **unbalanced designs** for an example. The `nfractional` option is allowed only for sample-size determination.

Some of `power onemean`'s computations require iteration. For example, when the standard deviation is unknown, computations use a noncentral Student's t distribution. Its degrees of freedom depends on the sample size, and the noncentrality parameter depends on the sample size and effect size. Therefore, the sample-size and effect-size determinations require iteration. The default initial values of the estimated parameters are obtained by using a closed-form normal approximation. They may be changed by specifying the `init()` option. See [PSS] **power** for the descriptions of other options that control the iteration procedure.

All computations assume an infinite population. For a finite population, use the `fpc()` option to specify a sampling rate or a population size. When this option is specified, a finite population correction is applied to the population standard deviation. The correction factor depends on the sample size; therefore, computing sample size for a finite population requires iteration even for a known standard deviation. The initial value for the sample size is based on the corresponding sample size assuming an infinite population.

In the following sections, we describe the use of `power onemean` accompanied by examples for computing sample size, power, and target mean.

Computing sample size

To compute sample size, you must specify the means under the null and alternative hypotheses, m_0 and m_a, respectively, and, optionally, the power of the test in the `power()` option. A default power of 0.8 is assumed if `power()` is not specified.

▷ Example 1: Sample size for a one-sample mean test

Consider an example from Tamhane and Dunlop (2000, 209) that discusses the effectiveness of coaching programs in improving the verbal part of SAT scores. Previous studies found that students retaking the SAT exams without any coaching program improve their scores by 15 points on average with a standard deviation of about 40 points. A new coaching program claims to improve the SAT scores by 40 points above the average. The changes in scores are assumed to be approximately normally distributed. The parameter of interest in this example is the mean change in the test scores. To test the claim, investigators wish to conduct another study and compute the sample size that is required to detect a mean change in scores of 40 points with 80% power using a 5%-level two-sided test. We assume that the true population standard deviation is unknown and use its estimate from previous studies to compute the sample size:

```
. power onemean 15 40, sd(40)

Performing iteration ...

Estimated sample size for a one-sample mean test
t test
Ho: m = m0  versus  Ha: m != m0

Study parameters:

        alpha =    0.0500
        power =    0.8000
        delta =    0.6250
           m0 =   15.0000
           ma =   40.0000
           sd =   40.0000

Estimated sample size:

            N =        23
```

We find that a sample of 23 subjects is required to detect a shift of 40 points in average SAT scores given the standard deviation of 40 points with 80% power using a 5%-level two-sided test.

As we mentioned in *Using power onemean* and as is also indicated in the output, sample-size computation requires iteration when the standard deviation is unknown. The iteration log is suppressed by default, but you can display it by specifying the `log` option.

◁

▷ Example 2: Specifying difference between means

Instead of the alternative mean change of 40 as in example 1, we can specify the difference of 25 between the mean changes in scores under the alternative and null hypotheses in the diff() option and obtain the same results.

```
. power onemean 15, diff(25) sd(40)

Performing iteration ...

Estimated sample size for a one-sample mean test
t test
Ho: m = m0   versus   Ha: m != m0

Study parameters:

        alpha =     0.0500
        power =     0.8000
        delta =     0.6250
           m0 =    15.0000
           ma =    40.0000
         diff =    25.0000
           sd =    40.0000

Estimated sample size:

            N =         23
```

When we specify the diff() option, the difference between the alternative and null values is also reported in the output.

◁

▷ Example 3: Known variance

If we know the population standard deviation, we can use the knownsd option to request a z test.

```
. power onemean 15 40, sd(40) knownsd

Performing iteration ...

Estimated sample size for a one-sample mean test
z test
Ho: m = m0   versus   Ha: m != m0

Study parameters:

        alpha =     0.0500
        power =     0.8000
        delta =     0.6250
           m0 =    15.0000
           ma =    40.0000
           sd =    40.0000

Estimated sample size:

            N =         21
```

The output now indicates that the computation is based on a z test instead of a t test. We find that a smaller sample of 21 subjects is required to detect the same effect size as in example 1 when the standard deviation is known.

◁

Computing power

To compute power, you must specify the sample size in the n() option and the means under the null and alternative hypotheses, m_0 and m_a, respectively.

▷ Example 4: Power of a one-sample mean test

Continuing with example 1, we will suppose that we are designing a new study and anticipate to obtain a sample of 30 subjects. To compute the power corresponding to this sample size given the study parameters from example 1, we specify the sample size of 30 in the n() option:

```
. power onemean 15 40, n(30) sd(40)
Estimated power for a one-sample mean test
t test
Ho: m = m0  versus  Ha: m != m0
Study parameters:
        alpha =    0.0500
            N =        30
        delta =    0.6250
           m0 =   15.0000
           ma =   40.0000
           sd =   40.0000
Estimated power:
        power =    0.9112
```

With a larger sample size, the power of the test increases to about 91.12%.

◁

▷ Example 5: Multiple values of study parameters

To investigate the effect of a finite population size on power, we can specify a list of population sizes in the fpc() option:

```
. power onemean 15 40, n(30) sd(40) fpc(100 500 1000)
Estimated power for a one-sample mean test
t test
Ho: m = m0  versus  Ha: m != m0
```

alpha	power	N	delta	m0	ma	sd	fpc
.05	.9769	30	.625	15	40	40	100
.05	.9267	30	.625	15	40	40	500
.05	.919	30	.625	15	40	40	1000

As expected, when the population size increases, the power tends to get closer to that obtained by assuming an infinite population size.

For multiple values of parameters, the results are automatically displayed in a table, as we see above. For more examples of tables, see [PSS] **power, table**. If you wish to produce a power plot, see [PSS] **power, graph**.

◁

▷ Example 6: Reproducing published results from a text book

We can also reproduce the example from Tamhane and Dunlop (2000, 213–214). The authors consider a one-sided test with a 0.132 significance level and a known standard deviation and compute the power to be 95.3%. We can replicate their example by typing

```
. power onemean 15 40, n(20) sd(40) alpha(0.132) onesided knownsd
Estimated power for a one-sample mean test
z test
Ho: m = m0  versus  Ha: m > m0
Study parameters:
        alpha =    0.1320
            N =        20
        delta =    0.6250
           m0 =   15.0000
           ma =   40.0000
           sd =   40.0000
Estimated power:
        power =    0.9533
```

◁

Computing effect size and target mean

Effect size δ for a one-sample mean test is defined as the ratio of the difference between the alternative and null values of the mean to the standard deviation, $\delta = (\mu_a - \mu_0)/\sigma$.

Sometimes, we may be interested in determining the smallest effect and the corresponding alternative or target mean that yield a statistically significant result for prespecified sample size and power. In this case, power, sample size, and null mean must be specified. In addition, you must also decide on the direction of the effect: upper, which means $\mu_a > \mu_0$, or lower, which means $\mu_a < \mu_0$. The direction may be specified in the direction() option; direction(upper) is the default.

▷ Example 7: Minimum detectable value of the mean change in SAT scores

Continuing with example 4, we may also be interested to find the smallest mean change in SAT scores that can be detected with a power of 80% given a sample of 30 subjects. To compute this, we specify only the null value of 15 as the command argument and also specify the sample size and power in the n(30) and power(0.8) options, respectively. We use the same value of 40 for the standard deviation as in example 4.

```
. power onemean 15, n(30) power(0.8) sd(40)
Performing iteration ...
Estimated target mean for a one-sample mean test
t test
Ho: m = m0  versus  Ha: m != m0; ma > m0
Study parameters:
        alpha =    0.0500
        power =    0.8000
            N =        30
           m0 =   15.0000
           sd =   40.0000
Estimated effect size and target mean:
        delta =    0.5292
           ma =   36.1694
```

The estimated smallest mean change in SAT scores is 36.17, which corresponds to the effect size of 0.53. Compared with example 1, for the same power of 80%, this example shows a smaller difference between the mean SAT scores of the two programs for a larger sample of 30 subjects.

In the above, we assumed the effect to be in the upper direction. By symmetry, the effect size in the lower direction will be -0.53, which can also be obtained by specifying direction(lower) in the above example.

◁

Performing hypothesis tests on mean

In this section, we briefly demonstrate the use of the ttest command for testing hypotheses about means; see [R] **ttest** for details. Suppose we wish to test the hypothesis that the mean is different from a reference value on the collected data. We can use the ttest command to do this. Below we demonstrate the use of this command for the analysis of sat.dta.

▷ Example 8: Testing for mean

Suppose that we wish to test whether the mean verbal SAT score is equal to 600. We use the ttest command to do this as follows:

```
. use http://www.stata-press.com/data/r15/sat
(Fictional SAT data)
. ttest score == 600
One-sample t test
```

Variable	Obs	Mean	Std. Err.	Std. Dev.	[95% Conf. Interval]
score	75	504.8	15.24616	132.0356	474.4214 535.1786

```
    mean = mean(score)                                     t =  -6.2442
Ho: mean = 600                              degrees of freedom =       74

   Ha: mean < 600             Ha: mean != 600              Ha: mean > 600
 Pr(T < t) = 0.0000        Pr(|T| > |t|) = 0.0000        Pr(T > t) = 1.0000
```

We find statistical evidence to reject the null hypothesis of H_0: $\mu_{\text{SAT}} = 600$ versus a two-sided alternative H_a: $\mu_{\text{SAT}} \neq 600$ at the 5% significance level; the p-value < 0.0000.

We use the estimates based on this study to perform a sample-size analysis we would have conducted before the study.

```
. power onemean 600 505, sd(132)
Performing iteration ...
Estimated sample size for a one-sample mean test
t test
Ho: m = m0  versus  Ha: m != m0
Study parameters:
        alpha =    0.0500
        power =    0.8000
        delta =   -0.7197
           m0 =  600.0000
           ma =  505.0000
           sd =  132.0000
Estimated sample size:
            N =        18
```

We find that the sample size required to detect a mean score of 505 with 80% power using a 5%-level two-sided test is only 18. The current sample contains 75 subjects, which would allow us to detect a potentially smaller (in absolute value) difference between the alternative and null means.

◁

Video examples

Sample-size calculation for comparing a sample mean to a reference value

Power calculation for comparing a sample mean to a reference value

Minimum detectable effect size for comparing a sample mean to a reference value

Stored results

power onemean stores the following in r():

Scalars
r(alpha)	significance level
r(power)	power
r(beta)	probability of a type II error
r(delta)	effect size
r(N)	sample size
r(nfractional)	1 if nfractional is specified; 0 otherwise
r(onesided)	1 for a one-sided test; 0 otherwise
r(m0)	mean under the null hypothesis
r(ma)	mean under the alternative hypothesis
r(diff)	difference between the alternative and null means
r(sd)	standard deviation
r(knownsd)	1 if option knownsd is specified; 0 otherwise
r(fpc)	finite population correction (if specified)
r(separator)	number of lines between separator lines in the table
r(divider)	1 if divider is requested in the table; 0 otherwise
r(init)	initial value for sample size or mean
r(maxiter)	maximum number of iterations
r(iter)	number of iterations performed
r(tolerance)	requested parameter tolerance
r(deltax)	final parameter tolerance achieved
r(ftolerance)	requested distance of the objective function from zero
r(function)	final distance of the objective function from zero
r(converged)	1 if iteration algorithm converged; 0 otherwise

Macros
r(type)	test
r(method)	onemean
r(direction)	upper or lower
r(columns)	displayed table columns
r(labels)	table column labels
r(widths)	table column widths
r(formats)	table column formats

Matrices
r(pss_table)	table of results

Methods and formulas

Let $x_1, \ldots, x_n$ be a sequence of n independent and identically distributed random variables drawn from a normal population with mean μ and variance σ^2. Let

$$\overline{x} = \frac{1}{n} \sum_{i=1}^{n} x_i \qquad \text{and} \qquad s^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \overline{x})^2$$

denote the sample mean and the sample variance, respectively. Let μ_0 and μ_a denote the null and alternative values of the mean parameter, respectively.

A one-sample mean test involves testing the null hypothesis $H_0\colon \mu = \mu_0$ versus the two-sided alternative hypothesis $H_a\colon \mu \neq \mu_0$, the upper one-sided alternative $H_a\colon \mu > \mu_0$, or the lower one-sided alternative $H_a\colon \mu < \mu_0$.

If the `nfractional` option is not specified, the computed sample size is rounded up.

The following formulas are based on Chow, Shao, and Wang (2008).

Methods and formulas are presented under the following headings:

> *Known standard deviation*
> *Unknown standard deviation*
> *Finite population size*

Known standard deviation

In the case of a known standard deviation, the sampling distribution of the test statistic $z = \sqrt{n}(\overline{x}-\mu_0)/\sigma$ under the null hypothesis follows the standard normal distribution, and the corresponding test is known as a z test.

Let α be the significance level, β be the probability of a type II error, and $z_{1-\alpha}$ and z_β be the $(1 - \alpha)$th and the βth quantiles of the standard normal distribution.

The power $\pi = 1 - \beta$ is computed using

$$\pi = \begin{cases} \Phi\left(\sqrt{n}\delta - z_{1-\alpha}\right) & \text{for an upper one-sided test} \\ \Phi\left(-\sqrt{n}\delta - z_{1-\alpha}\right) & \text{for a lower one-sided test} \\ \Phi\left(\sqrt{n}\delta - z_{1-\alpha/2}\right) + \Phi\left(-\sqrt{n}\delta - z_{1-\alpha/2}\right) & \text{for a two-sided test} \end{cases} \qquad (1)$$

where $\Phi(\cdot)$ is the cdf of the standard normal distribution and $\delta = (\mu_a - \mu_0)/\sigma$ is the effect size.

The sample size n for a one-sided test is computed by inverting a one-sided power equation from (1):

$$n = \left(\frac{z_{1-\alpha} - z_\beta}{\delta}\right)^2 \qquad (2)$$

Similarly, the absolute value of the effect size for a one-sided test is computed as follows:

$$|\delta| = \frac{(z_{1-\alpha} - z_\beta)}{\sqrt{n}} \qquad (3)$$

Note that the magnitude of the effect size is the same regardless of the direction of the test.

The minimum detectable value of the mean for a one-sided test is computed as $\mu_a = \mu_0 + (z_{1-\alpha} - z_\beta)\sigma/\sqrt{n}$ when $\mu_a > \mu_0$ and as $\mu_a = \mu_0 - (z_{1-\alpha} - z_\beta)\sigma/\sqrt{n}$ when $\mu_a < \mu_0$.

Sample size and minimum detectable value of the mean for a two-sided test are computed iteratively using the two-sided power equation from (1). The initial values are obtained from (2) and (3), correspondingly, with $\alpha/2$ in place of α.

Unknown standard deviation

In the case of an unknown standard deviation, an unbiased estimator s is used in place of σ in the definition of a z test statistic. The sampling distribution of the test statistic $t = \sqrt{n}(\overline{x} - \mu_0)/s$ under the null hypothesis follows a Student's t distribution with $n - 1$ degrees of freedom, and the corresponding test is known as a t test.

Let $t_{n-1,\alpha}$ denote the αth quantile of a Student's t distribution with $n - 1$ degrees of freedom. Under the alternative hypothesis, the test statistic follows a noncentral Student's t distribution, and the power is computed using

$$
\pi = \begin{cases}
1 - T_{n-1,\lambda}\left(t_{n-1,1-\alpha}\right) & \text{for an upper one-sided test} \\
T_{n-1,\lambda}\left(-t_{n-1,1-\alpha}\right) & \text{for a lower one-sided test} \\
1 - T_{n-1,\lambda}\left(t_{n-1,1-\alpha/2}\right) + T_{n-1,\lambda}\left(-t_{n-1,1-\alpha/2}\right) & \text{for a two-sided test}
\end{cases} \tag{4}
$$

where $T_{n-1,\lambda}\left(\cdot\right)$ is the cumulative noncentral Student's t distribution with a noncentrality parameter $\lambda = \sqrt{n}\delta$.

Sample size and minimum detectable value of the mean are obtained by iteratively solving nonlinear equations in (4), for n and δ, respectively. The default initial values for the iterative procedure are calculated from (2) and (3), respectively, assuming a normal distribution.

Finite population size

The above formulas assume that the random sample is drawn from an infinite population. In cases when the size of the population is known, we need to make the following adjustment to the standard deviation,

$$
\sigma_{\text{fpc}} = \sigma\sqrt{\left(1 - \frac{n}{N}\right)}
$$

where σ_{fpc} is the population standard deviation adjusted for finite population size. The correction factor depends on the sample size; therefore, computing sample size in this case requires iteration. The initial value for the sample size is based on the corresponding normal approximation with infinite population size.

References

Chow, S.-C., J. Shao, and H. Wang. 2008. *Sample Size Calculations in Clinical Research*. 2nd ed. New York: Dekker.

Tamhane, A. C., and D. D. Dunlop. 2000. *Statistics and Data Analysis: From Elementary to Intermediate*. Upper Saddle River, NJ: Prentice Hall.

Also see

[PSS] **power onemean, cluster** — Power analysis for a one-sample mean test, CRD

[PSS] **power** — Power and sample-size analysis for hypothesis tests

[PSS] **power, graph** — Graph results from the power command

[PSS] **power, table** — Produce table of results from the power command

[PSS] **Glossary**

[R] **ttest** — t tests (mean-comparison tests)