

## Description

`matrix svd` produces the singular value decomposition (SVD) of **A**.

Also see [M-5] `svd()` for alternative routines for obtaining the singular value decomposition.

## Menu

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## Syntax

`matrix svd U w V = A`

where **U**, **w**, and **V** are matrix names (the matrices may exist or not) and **A** is the name of an existing  $m \times n$  matrix,  $m \geq n$ .

## Remarks and examples

The singular value decomposition of  $m \times n$  matrix **A**,  $m \geq n$ , is defined as

$$\mathbf{A} = \mathbf{U} \text{diag}(\mathbf{w}) \mathbf{V}'$$

**U**:  $m \times n$ , **w**:  $1 \times n$ ,  $\text{diag}(\mathbf{w})$ :  $n \times n$ , and **V**:  $n \times n$ , where **U** is column orthogonal ( $\mathbf{U}'\mathbf{U} = \mathbf{I}$  if  $m = n$ ), all the elements of **w** are positive or zero, and  $\mathbf{V}'\mathbf{V} = \mathbf{I}$ .

Singular value decomposition can be used to obtain a g2-inverse of **A** ( $\mathbf{A}^*$ :  $n \times m$ , such that  $\mathbf{A}\mathbf{A}^*\mathbf{A} = \mathbf{A}$  and  $\mathbf{A}^*\mathbf{A}\mathbf{A}^* = \mathbf{A}^*$ —the first two Moore–Penrose conditions) via  $\mathbf{A}^* = \mathbf{V}\{\text{diag}(1/w_j)\}\mathbf{U}'$ , where  $1/w_j$  refers to individually taking the reciprocal of the elements of **w** and substituting 0 if  $w_j = 0$  or is small. If **A** is square and of full rank,  $\mathbf{A}^* = \mathbf{A}^{-1}$ .

## ► Example 1

Singular value decomposition is used to obtain accurate inverses of nearly singular matrices and to obtain g2-inverses of matrices that are singular, to construct orthonormal bases, and to develop approximation matrices. Our example will prove that `matrix svd` works:

```
. matrix A = (1,2,9\2,7,5\2,4,18)
. matrix svd U w V = A
. matrix list U
U[3,3]
      c1      c2      c3
r1  .42313293  .89442719  -.1447706
r2  .3237169  -6.016e-17  .94615399
r3  .84626585  -.4472136  -.2895412
. matrix list w
w[1,3]
      c1      c2      c3
r1  21.832726  2.612e-16  5.5975071
. matrix list V
V[3,3]
      c1      c2      c3
c1  .12655765  -.96974658  .2087456
c2  .29759672  .23786237  .92458514
c3  .94626601  .05489132  -.31869671
. matrix newA = U*diag(w)*V'
. matrix list newA
newA[3,3]
      c1  c2  c3
r1   1   2   9
r2   2   7   5
r3   2   4  18
```

As claimed, **newA** is equal to our original **A**.

The g2-inverse of **A** is computed below. The second element of **w** is small, so we decide to set the corresponding element of `diag(1/wj)` to zero. We then show that the resulting **Ainv** matrix has the properties of a g2-inverse for **A**.

```
. matrix Winv = J(3,3,0)
. matrix Winv[1,1] = 1/w[1,1]
. matrix Winv[3,3] = 1/w[1,3]
. matrix Ainv = V*Winv*U'
. matrix list Ainv
Ainv[3,3]
      r1      r2      r3
c1  -.0029461  .03716103  -.0058922
c2  -.0181453  .16069635  -.03629059
c3  .02658185  -.0398393  .05316371
```

```

. matrix AAiA = A*Ainv*A
. matrix list AAiA
AAiA[3,3]
      c1  c2  c3
r1      1   2   9
r2      2   7   5
r3      2   4  18

. matrix AiAAi = Ainv*A*Ainv
. matrix list AiAAi
AiAAi[3,3]
      r1      r2      r3
c1  -.0029461  .03716103  -.0058922
c2  -.0181453  .16069635  -.03629059
c3  .02658185  -.0398393  .05316371

```

◀

## Methods and formulas

Stewart (1993) surveys the contributions of five mathematicians—Beltrami, Jordan, Sylvester, Schmidt, and Weyl—who established the existence of the singular value decomposition and developed its theory.

## Reference

Stewart, G. W. 1993. On the early history of the singular value decomposition. *SIAM Review* 35: 551–566. <https://doi.org/10.1137/1035134>.

## Also see

[P] **matrix** — Introduction to matrix commands

[P] **matrix define** — Matrix definition, operators, and functions

[M-4] **Matrix** — Matrix functions

[M-5] **svd()** — Singular value decomposition

[U] **14 Matrix expressions**

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