Title

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procrustes postestimation — Postestimation tools for procrustes

Postestimation commands

The following postestimation commands are of special interest after `procrustes`:

Command	Description
`estat compare`	fit statistics for orthogonal, oblique, and unrestricted transformations
`estat mvreg`	display multivariate regression resembling unrestricted transformation
`estat summarize`	display summary statistics over the estimation sample
`procoverlay`	produce a Procrustes overlay graph

The following standard postestimation commands are also available:

Command	Description
* `estimates`	cataloging estimation results
`predict`	compute fitted values and residuals

* All `estimates` subcommands except `table` and `stats` are available; see [R] **estimates**.

predict

Description for predict

`predict` creates new variables containing predictions such as fitted values, unstandardized residuals, and residual sum of squares.

Menu for predict

Statistics > Postestimation

Syntax for predict

`predict` [*type*] { *stub*`*` | *newvarlist* } [*if*] [*in*] , [*statistic*]

statistic	Description
Main	
`fitted`	fitted values $\mathbf{1}\,\mathbf{c}' + \rho\,\mathbf{X}\,\mathbf{A}$; the default (specify $\#_y$ vars)
`residuals`	unstandardized residuals (specify $\#_y$ vars)
`q`	residual sum of squares over the target variables (specify one var)

These statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample.

Options for predict

Main

`fitted`, the default, computes fitted values, that is, the least-squares approximations of the target (*varlist*$_y$) variables. You must specify the same number of new variables as there are target variables.

`residuals` computes the raw (unstandardized) residuals for each target (*varlist*$_y$) variable. You must specify the same number of new variables as there are target variables.

`q` computes the residual sum of squares over all variables, that is, the squared Euclidean distance between the target and transformed source points. Specify one new variable.

estat

Description for estat

`estat compare` displays a table with fit statistics of the three transformations provided by `procrustes`: `orthogonal`, `oblique`, and `unrestricted`. The two additional procrustes analyses are performed on the same sample as the original procrustes analysis and with the same options. F tests comparing the models are provided.

`estat mvreg` produces the `mvreg` (see [MV] **mvreg**) output related to the unrestricted Procrustes analysis (the `transform(unrestricted)` option of `procrustes`).

`estat summarize` displays summary statistics over the estimation sample of the target and source variables (*varlist*$_y$ and *varlist*$_x$).

Menu for estat

Statistics > Postestimation

Syntax for estat

Table of fit statistics

```
estat compare [, detail]
```

Comparison of `mvreg` *and* `procrustes` *output*

```
estat mvreg [, mvreg_options]
```

Display summary statistics

```
estat summarize [, labels noheader noweights]
```

Options for estat

`detail`, an option with `estat compare`, displays the standard `procrustes` output for the two additional transformations.

mvreg_options, allowed with `estat mvreg`, are any of the options allowed by `mvreg`; see [MV] **mvreg**. The constant is already suppressed if the Procrustes analysis suppressed it.

`labels`, `noheader`, and `noweights` are the same as for the generic `estat summarize` command; see [R] **estat summarize**.

procoverlay

Description for procoverlay

`procoverlay` displays a plot of the target variables overlaid with the fitted values derived from the source variables. If there are more than two target variables, multiple plots are shown in one graph.

Menu for procoverlay

Statistics > Multivariate analysis > Procrustes overlay graph

Syntax for procoverlay

`procoverlay` [*if*] [*in*] [, *procoverlay_options*]

procoverlay_options	Description
Main	
`autoaspect`	adjust aspect ratio on the basis of the data; default aspect ratio is 1
`targetopts(`*target_opts*`)`	affect the rendition of the target
`sourceopts(`*source_opts*`)`	affect the rendition of the source
Y axis, X axis, Titles, Legend, Overall	
twoway_options	any options other than `by()` documented in [G-3] ***twoway_options***
By	
`byopts(`*by_option*`)`	affect the rendition of combined graphs

target_opts	Description
Main	
`nolabel`	removes the default observation label from the target
marker_options	change look of markers (color, size, etc.)
marker_label_options	change look or position of marker labels

source_opts	Description
Main	
`nolabel`	removes the default observation label from the source
marker_options	change look of markers (color, size, etc.)
marker_label_options	change look or position of marker labels

Options for procoverlay

Main

`autoaspect` specifies that the aspect ratio be automatically adjusted based on the range of the data to be plotted. This option can make some `procoverlay` plots more readable. By default, `procoverlay` uses an aspect ratio of one, producing a square plot.

As an alternative to `autoaspect`, the *twoway_option* `aspectratio()` can be used to override the default aspect ratio. `procoverlay` accepts the `aspectratio()` option as a suggestion only and will override it when necessary to produce plots with balanced axes, that is, where distance on the x axis equals distance on the y axis.

twoway_options, such as `xlabel()`, `xscale()`, `ylabel()`, and `yscale()`, should be used with caution. These *axis_options* are accepted but may have unintended side effects on the aspect ratio. See [G-3] ***twoway_options***.

`targetopts(`*target_opts*`)` affects the rendition of the target plot. The following *target_opts* are allowed:

`nolabel` removes the default target observation label from the graph.

marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] ***marker_options***.

marker_label_options specify if and how the markers are to be labeled; see [G-3] ***marker_label_options***.

`sourceopts(`*source_opts*`)` affects the rendition of the source plot. The following *source_opts* are allowed:

`nolabel` removes the default source observation label from the graph.

marker_options affect the rendition of markers drawn at the plotted points, including their shape, size, color, and outline; see [G-3] ***marker_options***.

marker_label_options specify if and how the markers are to be labeled; see [G-3] ***marker_label_options***.

Y axis, X axis, Titles, Legend, Overall

twoway_options are any of the options documented in [G-3] ***twoway_options***, excluding `by()`. These include options for titling the graph (see [G-3] ***title_options***) and for saving the graph to disk (see [G-3] ***saving_option***). See `autoaspect` above for a warning against using options such as `xlabel()`, `xscale()`, `ylabel()`, and `yscale()`.

By

`byopts(`*by_option*`)` is documented in [G-3] ***by_option***. This option affects the appearance of the combined graph and is ignored, unless there are more than two target variables specified in `procrustes`.

Remarks and examples

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The examples in [MV] **procrustes** demonstrated a Procrustes transformation of a historical map, produced by John Speed in 1610, to a modern map. Here we demonstrate the use of `procrustes` postestimation tools in assessing the accuracy of Speed's map. Example 1 of [MV] **procrustes** performed the following analysis:

```
. use http://www.stata-press.com/data/r15/speed_survey
(Data on Speed's Worcestershire map (1610))
. procrustes (survey_x survey_y) (speed_x speed_y)
  (output omitted)
```

See example 1 of [MV] **procrustes**. The following examples are based on this `procrustes` analysis.

Example 1: Predictions

Did John Speed get the coordinates of the towns right—up to the location, scale, and orientation of his map relative to the modern map? In example 1 of [MV] **procrustes**, we demonstrated how the optimal transformation from the historical coordinates to the modern (true) coordinates can be estimated by `procrustes`.

It is possible to "predict" the configuration of 20 cities on Speed's historical map, optimally transformed (rotated, dilated, and translated) to approximate the true configuration. `predict` with the `fitted` option expects the same number of variables as the number of target (dependent) variables (`survey_x` and `survey_y`).

```
. predict fitted_x fitted_y
(fitted assumed)
```

We omitted the `fitted` option because it is the default.

It is often useful to also compute the (squared) distance between the true location and the transformed location of the historical map. This can be seen as a quality measure—the larger the value, the more Speed erred in the location of the respective town.

```
. predict q, q
```

We now list the target data (`survey_x` and `survey_y`, the values from the modern map), the fitted values (`fitted_x` and `fitted_y`, produced by `predict`), and the squared distance between them (`q`, produced by `predict` with the `q` option).

```
. list name survey_x survey_y fitted_x fitted_y q, sep(0) noobs
```

name	survey_x	survey_y	fitted_x	fitted_y	q
Alve	1027	725	1037.117	702.9464	588.7149
Arro	1083	565	1071.682	562.6791	133.4802
Astl	787	677	783.0652	674.5216	21.62482
Beck	976	358	978.8665	366.3761	78.37637
Beng	1045	435	1055.245	431.6015	116.51
Crad	736	471	725.8594	476.5895	134.075
Droi	893	633	890.5839	633.6066	6.205747
Ecki	922	414	929.4932	411.1757	64.12465
Eves	1037	437	1036.887	449.2707	150.5827
Hall	828	579	825.1494	575.9836	17.22464
Hanb	944	637	954.6189	643.6107	156.4629
Inkb	1016	573	1004.869	577.1111	140.7917
Kemp	848	490	845.7215	490.8959	5.994327
Kidd	826	762	836.8665	760.5699	120.1264
Mart	756	598	745.2623	597.5585	115.4937
Stud	1074	632	1072.622	634.3164	7.264294
Tewk	891	324	898.4571	318.632	84.42448
UpSn	943	544	939.3932	545.8247	16.33858
Upto	852	403	853.449	400.9419	6.335171
Worc	850	545	848.7917	547.7881	9.233305

We see that Speed especially erred in the location of Alvechurch—it is off by no less than $\sqrt{588} = 24$ miles, whereas the average error is about 8 miles. In a serious analysis of this dataset, we would check the data on Alvechurch, and, if we found it to be in order, consider whether we should actually drop Alvechurch from the analysis. In this illustration, we ignore this potential problem.

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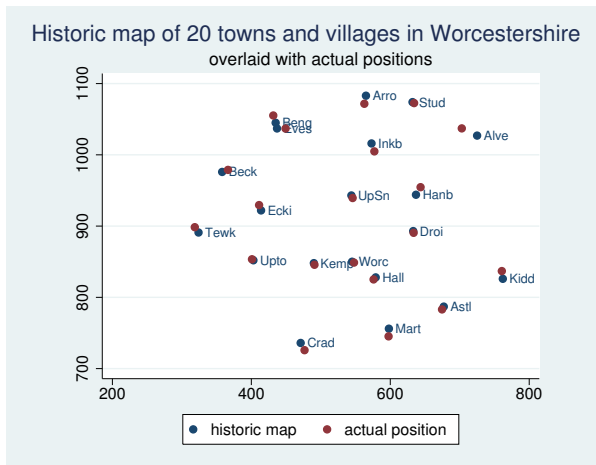
▷ Example 2: Procrustes overlay graph

Although the numerical information convinces us that Speed's map is generally accurate, a plot will convey this message more convincingly. `procoverlay` produces a plot that contains the target (survey) coordinates and the Procrustes-transformed historical coordinates. We could just type

```
. procoverlay
```

However, we decide to set several options to produce a presentation-quality graph. The suboption `mlabel()` of `target()` (or of `source()`) adds labels, identifying the towns. Because the target and source points are so close, there can be no confusing how they are matched. Displaying the labels twice in the plot is not helpful for this dataset. Therefore, we choose to label the target points, but not the source points using the `nolabel` suboption of `source()`. We preserve the equivalence of the x and y scale while using as much of the graphing region as possible with the `autoaspect` option. The `span` suboption of `title()` allows the long title to extend beyond the graph region if needed. We override the default legend by using the `legend()` option.

```
. procoverlay, target(mlabel(name)) source(nolabel) autoaspect
> title(Historic map of 20 towns and villages in Worcestershire, span)
> subtitle(overlaid with actual positions)
> legend(label(1 historic map) label(2 actual position))
```



◁

▷ Example 3: estat

`estat` offers three specific facilities after `procrustes`. These can all be seen as convenience tools that accomplish simple analyses, ensuring that the same variables and the same observations are used as in the Procrustes analysis.

The variables involved in the Procrustes analysis can be summarized over the estimation sample, for instance, to gauge differences in scales and location of the target and source variables.

```
. estat summarize
Estimation sample procrustes                Number of obs =         20
```

Variable	Mean	Std. Dev.	Min	Max
target				
survey_x	916.7	106.6993	736	1083
survey_y	540.1	121.1262	324	762
source				
speed_x	153.95	46.76084	78	220
speed_y	133.9	49.90401	40	220

From the summarization, the two maps have different origins and scale.

As pointed out in [MV] **procrustes**, orthogonal and oblique Procrustes analyses can be thought of as special cases of multivariate regression (see [MV] **mvreg**), subject to nonlinear restrictions on the coefficient matrix. Comparing the Procrustes statistics and the transformations for each of the three classes of transformations is helpful in selecting a transformation. The `compare` subcommand of `estat` provides summary information for the optimal transformations in each of the three classes.

```
. estat compare
Summary statistics for three transformations
```

	Procrustes	df_m	df_r	rmse
orthogonal	0.0040	4	36	7.403797
oblique	0.0040	5	35	7.498294
unrestricted	0.0037	6	34	7.343334

```
(F tests comparing the models suppressed)
```

The Procrustes statistic is ensured to decrease (not increase) from orthogonal to oblique to unrestricted because the associated classes of transformations are getting less restrictive. The model degrees of freedom (`df_m`) of the three transformation classes are the dimension of the classes, that is, the number of "free parameters". For instance, with orthogonal transformations between two source and two target variables, there is 1 degree of freedom for the rotation (representing the rotation angle), 2 degrees of freedom for the translation, and 1 degree of freedom for dilation (uniform scaling), that is, four in total. The residual degrees of freedom (df_r) are the number of observations (number of target variables times the number of observations) minus the model degrees of freedom. The root mean squared error RMSE, defined as

$$\mathrm{RMSE} = \sqrt{\frac{\mathrm{RSS}}{\mathrm{df}_r}}$$

does not, unlike the Procrustes statistic, surely become smaller with the less restrictive models. In this example, in fact, the RMSE of the orthogonal transformation is smaller than that of the oblique transformation. This indicates that the additional degree of freedom allowing for skew rotations does not produce a closer fit. In this example, we see little reason to relax orthogonal transformations; very little is gained in terms of the Procrustes statistic (an illness-of-fit measure) or the RMSE. In this interpretation, we used our intuition to guide us whether a difference in fit is substantively and statistically meaningful—formal significance tests are not provided.

Finally, the unrestricted transformation can be estimated with `procrustes ..., transform(unrestricted)`. This analysis is related to a multivariate regression with the target variables as the dependent variables and the source variables as the independent variables. Although the unrestricted Procrustes analysis assumes spherical (uncorrelated homoskedastic) residuals, this restrictive assumption is not made in multivariate regression as estimated by the `mvreg` command. The comparable multivariate regression over the same estimation sample can be viewed simply by typing

```
. estat mvreg
Multivariate regression, similar to "procrustes ..., transform(unrestricted)"
```

Equation	Obs	Parms	RMSE	"R-sq"	F	P
survey_x	20	3	7.696981	0.9953	1817.102	0.0000
survey_y	20	3	6.971772	0.9970	2859.068	0.0000

	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
survey_x						
speed_x	2.27584	.0379369	59.99	0.000	2.1958	2.35588
speed_y	.4147244	.0355475	11.67	0.000	.3397257	.489723
_cons	510.8028	8.065519	63.33	0.000	493.7861	527.8196
survey_y						
speed_x	-.4129564	.0343625	-12.02	0.000	-.485455	-.3404579
speed_y	2.355725	.0321982	73.16	0.000	2.287793	2.423658
_cons	288.243	7.305587	39.46	0.000	272.8296	303.6564

This analysis is seen as postestimation after a Procrustes analysis, so it does not change the "last estimation results". We may still replay `procrustes` and use other `procrustes` postestimation commands.

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Stored results

`estat compare` after `procrustes` stores the following in `r()`:

Matrices
- `r(cstat)` Procrustes statistics, degrees of freedom, and RMSEs
- `r(fstat)` F statistics, degrees of freedom, and p-values

`estat mvreg` does not return results.

`estat summarize` after `procrustes` stores the following in `r()`:

Matrices
- `r(stats)` means, standard deviations, minimums, and maximums

Methods and formulas

The predicted values for the jth variable are defined as

$$\widehat{y}_j = \widehat{c}_j + \widehat{\rho}\,\mathbf{X}\,\widehat{\mathbf{A}}[.,j]$$

The residual for y_j is simply $y_j - \widehat{y}_j$. The "rowwise" quality q of the approximation is defined as the residual sum of squares:

$$q = \sum_j (y_j - \widehat{y}_j)^2$$

The entries of the summary table produced by `estat compare` are described in *Methods and formulas* of [MV] **procrustes**. The F tests produced by `estat compare` are similar to standard nested model tests in linear models.

References

See *References* in [MV] **procrustes**.

Also see

[MV] **procrustes** — Procrustes transformation

[MV] **mvreg** — Multivariate regression

[U] 20 Estimation and postestimation commands