mixed postestimation — Postestimation tools for mixed

Postestimation commands

The following postestimation commands are of special interest after `mixed`:

Command	Description
`estat df`	calculate and display degrees of freedom for fixed effects
`estat group`	summarize the composition of the nested groups
`estat icc`	estimate intraclass correlations
`estat recovariance`	display the estimated random-effects covariance matrices
`estat sd`	display variance components as standard deviations and correlations
`estat wcorrelation`	display within-cluster correlations and standard deviations

The following standard postestimation commands are also available:

Command	Description
`contrast`	contrasts and ANOVA-style joint tests of parameters
`estat ic`	Akaike's, consistent Akaike's, corrected Akaike's, and Schwarz's Bayesian information criteria (AIC, CAIC, AICc, and BIC, respectively)
`estat summarize`	summary statistics for the estimation sample
`estat vce`	variance–covariance matrix of the estimators (VCE)
`estimates`	cataloging estimation results
`etable`	table of estimation results
`hausman`	Hausman's specification test
`lincom`	point estimates, standard errors, testing, and inference for linear combinations of parameters
`lrtest`	likelihood-ratio test
`margins`	marginal means, predictive margins, marginal effects, and average marginal effects
`marginsplot`	graph the results from margins (profile plots, interaction plots, etc.)
`nlcom`	point estimates, standard errors, testing, and inference for nonlinear combinations of parameters
`predict`	predictions and their SEs, residuals, etc.
`predictnl`	point estimates, standard errors, testing, and inference for generalized predictions
`pwcompare`	pairwise comparisons of parameters
`test`	Wald tests of simple and composite linear hypotheses
`testnl`	Wald tests of nonlinear hypotheses

predict

Description for predict

`predict` creates a new variable containing predictions such as linear predictions, standard errors, fitted values, residuals, and standardized residuals.

Menu for predict

Statistics > Postestimation

Syntax for predict

Syntax for obtaining predictions of the outcome and other statistics

`predict` [*type*] *newvar* [*if*] [*in*] [, *statistic* `relevel`(*levelvar*)]

Syntax for obtaining BLUPs of random effects and the BLUPs' standard errors

`predict` [*type*] {*stub*∗ | *newvarlist*} [*if*] [*in*], `reffects` [`relevel`(*levelvar*)
`reses`(*stub*∗ | *newvarlist*)]

Syntax for obtaining scores after ML estimation

`predict` [*type*] *stub*∗ [*if*] [*in*], `scores`

statistic	Description
Main	
`xb`	linear prediction for the fixed portion of the model only; the default
`stdp`	standard error of the fixed-portion linear prediction
`fitted`	fitted values, fixed-portion linear prediction plus contributions based on predicted random effects
`residuals`	residuals, response minus fitted values
* `rstandard`	standardized residuals

Unstarred statistics are available both in and out of sample; type `predict ... if e(sample) ...` if wanted only for the estimation sample. Starred statistics are calculated only for the estimation sample, even when `if e(sample)` is not specified.

Options for predict

Main

`xb`, the default, calculates the linear prediction $\mathbf{x}\boldsymbol{\beta}$ based on the estimated fixed effects (coefficients) in the model. This is equivalent to fixing all random effects in the model to their theoretical mean value of 0.

`stdp` calculates the standard error of the linear predictor $\mathbf{x}\boldsymbol{\beta}$.

`fitted` calculates fitted values, which are equal to the fixed-portion linear predictor *plus* contributions based on predicted random effects, or in mixed-model notation, $\mathbf{x}\boldsymbol{\beta}+\mathbf{Zu}$. By default, the fitted values take into account random effects from all levels in the model; however, if the `relevel(`*levelvar*`)` option is specified, then the fitted values are fit beginning with the topmost level down to and including level *levelvar*. For example, if `classes` are nested within `schools`, then typing

```
. predict yhat_school, fitted relevel(school)
```

would produce school-level predictions. That is, the predictions would incorporate school-specific random effects but not those for each class nested within each school.

`residuals` calculates residuals, equal to the responses minus fitted values. By default, the fitted values take into account random effects from all levels in the model; however, if the `relevel(`*levelvar*`)` option is specified, then the fitted values are fit beginning at the topmost level down to and including level *levelvar*.

`rstandard` calculates standardized residuals, equal to the residuals multiplied by the inverse square root of the estimated error covariance matrix.

`reffects` calculates best linear unbiased predictions (BLUPs) of the random effects. By default, BLUPs for all random effects in the model are calculated. However, if the `relevel(`*levelvar*`)` option is specified, then BLUPs for only level *levelvar* in the model are calculated. For example, if `classes` are nested within `schools`, then typing

```
. predict b*, reffects relevel(school)
```

would produce BLUPs at the school level. You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub*`*` and let Stata name the variables *stub*1, *stub*2, …, *stubq* for you.

Rabe-Hesketh and Skrondal (2022, sec. 2.11.2) discuss the link between the empirical Bayes predictions and BLUPs and how these predictions are unbiased. They are unbiased when the groups associated with the random effects are expected to vary in repeated samples. If you expect the groups to be fixed in repeated samples, then these predictions are no longer unbiased.

`scores` calculates the parameter-level scores, one for each parameter in the model including regression coefficients and variance components. The score for a parameter is the first derivative of the log likelihood (or log pseudolikelihood) with respect to that parameter. One score per highest-level group is calculated, and it is placed on the last record within that group. Scores are calculated in the estimation metric as stored in `e(b)`.

`scores` is not available after restricted maximum-likelihood (REML) estimation.

`relevel(`*levelvar*`)` specifies the level in the model at which predictions involving random effects are to be obtained; see the options above for the specifics. *levelvar* is the name of the model level and is either the name of the variable describing the grouping at that level or is `_all`, a special designation for a group comprising all the estimation data.

`reses(`*stub*`*` | *newvarlist*`)` calculates the standard errors of the BLUPs of the random effects. By default, standard errors for all BLUPs in the model are calculated. However, if the `relevel(`*levelvar*`)` option is specified, then standard errors for only level *levelvar* in the model are calculated; see the `reffects` option.

You must specify q new variables, where q is the number of random-effects terms in the model (or level). However, it is much easier to just specify *stub*`*` and let Stata name the variables *stub*`1`, *stub*`2`, …, *stubq* for you. The new variables will have the same storage type as the corresponding random-effects variables.

The `reffects` and `reses()` options often generate multiple new variables at once. When this occurs, the random effects (or standard errors) contained in the generated variables correspond to the order in which the variance components are listed in the output of `mixed`. Still, examining the variable labels of the generated variables (with the `describe` command, for instance) can be useful in deciphering which variables correspond to which terms in the model.

margins

Description for margins

`margins` estimates margins of response for linear predictions.

Menu for margins

Statistics > Postestimation

Syntax for margins

`margins` [*marginlist*] [, *options*]

`margins` [*marginlist*] , `predict(`*statistic* ...`)` [*options*]

statistic	Description
`xb`	linear predictor for the fixed portion of the model only; the default
`stdp`	not allowed with `margins`
`fitted`	not allowed with `margins`
`residuals`	not allowed with `margins`
`rstandard`	not allowed with `margins`
`reffects`	not allowed with `margins`
`scores`	not allowed with `margins`

Statistics not allowed with `margins` are functions of stochastic quantities other than `e(b)`.

For the full syntax, see [R] **margins**.

test and testparm

Description for test and testparm

test and testparm, by default, perform χ^2 tests of simple and composite linear hypotheses about the parameters for the most recently fit mixed model. They also support F tests with a small-sample adjustment for fixed effects.

Menu for test and testparm

Statistics > Postestimation

Syntax for test and testparm

test (*spec*) [(*spec*) ...] [, *test_options* small]

testparm *varlist* [, *testparm_options* small]

Options for test and testparm

Options

test_options; see [R] **test** options. Options df(), common, and nosvyadjust may not be specified together with small.

testparm_options; see options of testparm in [R] **test**. Options df() and nosvyadjust may not be specified together with small.

small specifies that F tests for fixed effects be carried out with the denominator degrees of freedom (DDF) obtained by the same method used in the most recently fit mixed model. If option dfmethod() is not specified in the previous mixed command, option small is not allowed. For certain methods, the DDF for some tests may not be available. See *Small-sample inference for fixed effects* in [ME] **mixed** for more details.

lincom

Description for lincom

lincom, by default, computes point estimates, standard errors, z statistics, p-values, and confidence intervals for linear combinations of parameters after mixed. lincom also provides t statistics for linear combinations of the fixed effects, with the degrees of freedom calculated by the DF method specified in option dfmethod() of mixed.

Menu for lincom

Statistics > Postestimation

Syntax for lincom

lincom *exp* [, *lincom_options* small]

Options for lincom

lincom_options; see [R] **lincom** options. Option df() may not be specified together with small.

small specifies that t statistics for linear combinations of fixed effects be displayed with the degrees of freedom obtained by the same method used in the most recently fit mixed model. If option dfmethod() is not specified in the previous mixed command, option small is not allowed. For certain methods, the degrees of freedom for some linear combinations may not be available. See *Small-sample inference for fixed effects* in [ME] **mixed** for more details.

contrast

Description for contrast

contrast, by default, performs χ^2 tests of linear hypotheses and forms contrasts involving factor variables and their interactions for the most recently fit mixed model. contrast also supports tests with small-sample adjustments after mixed, dfmethod().

Menu for contrast

Statistics > Postestimation

Syntax for contrast

contrast *termlist* [, *contrast_options* small]

Options for contrast

contrast_options; see [R] **contrast** options. Options df() and nosvyadjust may not be specified together with small.

small specifies that tests for contrasts be carried out with the DDF obtained by the same method used in the most recently fit mixed model. If option dfmethod() is not specified in the previous mixed command, option small is not allowed. For certain methods, the DDF for some contrasts may not be available. See *Small-sample inference for fixed effects* in [ME] **mixed** for more details.

pwcompare

Description for pwcompare

pwcompare performs pairwise comparisons across the levels of factor variables from the most recently fit mixed model. pwcompare, by default, reports the comparisons as contrasts (differences) of margins along with z tests or confidence intervals for the pairwise comparisons. pwcompare also supports t tests with small-sample adjustments after mixed, dfmethod().

Menu for pwcompare

Statistics > Postestimation

Syntax for pwcompare

pwcompare *marginlist* [, *pwcompare_options* small]

Options for pwcompare

pwcompare_options; see [R] **pwcompare** options. Option df() may not be specified together with small.

small specifies that t tests for pairwise comparisons be carried out with the degrees of freedom obtained by the same method used in the most recently fit mixed model with the dfmethod() option. If option dfmethod() is not specified in the previous mixed command, option small is not allowed. For certain methods, the degrees of freedom for some pairwise comparisons may not be available. See *Small-sample inference for fixed effects* in [ME] **mixed** for more details.

Remarks and examples

Various predictions, statistics, and diagnostic measures are available after fitting a mixed model using mixed. For the most part, calculation centers around obtaining BLUPs of the random effects. Random effects are not estimated when the model is fit but instead need to be predicted after estimation. Calculation of intraclass correlations, estimating the dependence between responses for different levels of nesting, may also be of interest.

▷ Example 1: Obtaining predictions of random effects and checking model fit

In example 3 of [ME] **mixed**, we modeled the weights of 48 pigs measured on nine successive weeks as

$$\texttt{weight}_{ij} = \beta_0 + \beta_1 \texttt{week}_{ij} + u_{0j} + u_{1j}\texttt{week}_{ij} + \epsilon_{ij} \tag{1}$$

for $i = 1, \dots, 9$, $j = 1, \dots, 48$, $\epsilon_{ij} \sim N(0, \sigma^2_\epsilon)$, and u_{0j} and u_{1j} normally distributed with mean 0 and variance–covariance matrix

$$\boldsymbol{\Sigma} = \mathrm{Var}\begin{bmatrix} u_{0j} \\ u_{1j} \end{bmatrix} = \begin{bmatrix} \sigma^2_{u0} & \sigma_{01} \\ \sigma_{01} & \sigma^2_{u1} \end{bmatrix}$$

```
. use https://www.stata-press.com/data/r19/pig
(Longitudinal analysis of pig weights)

. mixed weight week || id: week, covariance(unstructured)

Performing EM optimization ...

Performing gradient-based optimization:
Iteration 0:  Log likelihood = -868.96185
Iteration 1:  Log likelihood = -868.96185

Computing standard errors ...

Mixed-effects ML regression                     Number of obs     =        432
Group variable: id                              Number of groups  =         48
                                                Obs per group:
                                                              min =          9
                                                              avg =        9.0
                                                              max =          9
                                                Wald chi2(1)      =    4649.17
Log likelihood = -868.96185                     Prob > chi2       =     0.0000

------------------------------------------------------------------------------
      weight | Coefficient  Std. err.      z    P>|z|     [95% conf. interval]
-------------+----------------------------------------------------------------
        week |   6.209896   .0910745    68.18   0.000     6.031393    6.388399
       _cons |   19.35561   .3996387    48.43   0.000     18.57234    20.13889
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                   var(week) |   .3715251   .0812958      .2419532     .570486
                  var(_cons) |   6.823363   1.566194      4.351297    10.69986
            cov(week,_cons)  |  -.0984378   .2545767     -.5973991    .4005234
-----------------------------+------------------------------------------------
               var(Residual) |   1.596829    .123198      1.372735    1.857505
------------------------------------------------------------------------------
LR test vs. linear model: chi2(3) = 764.58                Prob > chi2 = 0.0000

Note: LR test is conservative and provided only for reference.
```

Rather than see the estimated variance components listed as variance and covariances as above, we can instead see them as correlations and standard deviations using `estat sd`; see [ME] **estat sd**.

```
. estat sd

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
id: Unstructured             |
                    sd(week) |   .6095286   .0666874      .4918874    .7553052
                   sd(_cons) |   2.612157   .2997895      2.085976    3.271064
             corr(week,_cons)|  -.0618257   .1575911     -.3557072     .243182
-----------------------------+------------------------------------------------
                sd(Residual) |   1.263657   .0487466      1.171638    1.362903
------------------------------------------------------------------------------
```

We can use `estat recovariance` to display the estimated variance components $\widehat{\Sigma}$ as a correlation matrix; see [ME] **estat recovariance**.

```
. estat recovariance, correlation
Random-effects correlation matrix for level id

             |      week       _cons
-------------+-----------------------
        week |         1
       _cons |  -.0618257          1
```

Finally, we can use `estat wcorrelation` to display the within-cluster marginal standard deviations and correlations for one of the clusters; see [ME] **estat wcorrelation**.

```
. estat wcorrelation, format(%4.2g)
Standard deviations and correlations for id = 1:
Standard deviations:

         obs |     1     2     3     4     5     6     7     8     9
-------------+------------------------------------------------------
          sd |   2.9   3.1   3.3   3.7   4.1   4.5     5   5.5   6.1
Correlations:

         obs |     1     2     3     4     5     6     7     8     9
-------------+------------------------------------------------------
           1 |     1
           2 |    .8     1
           3 |   .77   .83     1
           4 |   .72   .81   .86     1
           5 |   .67   .78   .85   .89     1
           6 |   .63   .75   .83   .88   .91     1
           7 |   .59   .72   .81   .87   .91   .93     1
           8 |   .55   .69   .79   .86    .9   .93   .94     1
           9 |   .52   .66   .77   .85   .89   .92   .94   .95     1
```

Because within-cluster correlations can vary between clusters, `estat wcorrelation` by default displays the results for the first cluster. In this example, each cluster (pig) has the same number of observations, and the timings of measurements (`week`) are the same between clusters. Thus the within-cluster correlations are the same for all the clusters. In example 1 of [ME] **estat wcorrelation**, we fit a model where different clusters have different within-cluster correlations and show how to display these correlations.

We can also obtain BLUPs of the pig-level random effects (u_{0j} and u_{1j}). We need to specify the variables to be created in the order `u1 u0` because that is the order in which the corresponding variance components are listed in the output (`week _cons`). We obtain the predictions and list them for the first 10 pigs.

```
. predict u1 u0, reffects
. by id, sort: generate tolist = (_n==1)
. list id u0 u1 if id <=10 & tolist

     +----------------------------+
     | id          u0          u1 |
     |----------------------------|
  1. |  1    .2369444   -.3957636 |
 10. |  2   -1.584127     .510038 |
 19. |  3   -3.526551    .3200372 |
 28. |  4    1.964378   -.7719702 |
 37. |  5    1.299236   -.9241479 |
     |----------------------------|
 46. |  6   -1.147302   -.5448151 |
 55. |  7   -2.590529    .0394454 |
 64. |  8   -1.137067   -.1696566 |
 73. |  9   -3.189545   -.7365507 |
 82. | 10    1.160324    .0030772 |
     +----------------------------+
```

If you forget how to order your variables in `predict`, or if you use `predict` *stub*`*`, remember that `predict` labels the generated variables for you to avoid confusion.

```
. describe u0 u1
Variable      Storage   Display    Value
    name         type    format    label      Variable label
-------------------------------------------------------------------------------
u0              float   %9.0g                 BLUP r.e. for id: _cons
u1              float   %9.0g                 BLUP r.e. for id: week
```

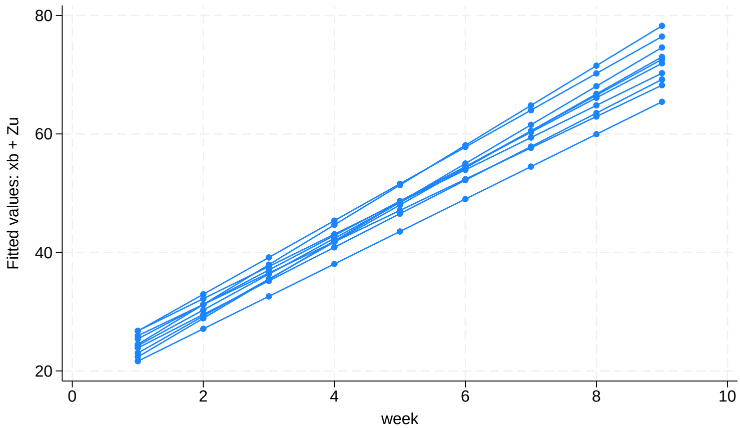
Examining (1), we see that within each pig, the successive weight measurements are modeled as simple linear regression with intercept $\beta_0 + u_{j0}$ and slope $\beta_1 + u_{j1}$. We can generate estimates of the pig-level intercepts and slopes with

```
. generate intercept = _b[_cons] + u0
. generate slope = _b[week] + u1
. list id intercept slope if id<=10 & tolist

     +--------------------------+
     | id   interc~t      slope |
     |--------------------------|
  1. |  1   19.59256   5.814132 |
 10. |  2   17.77149   6.719934 |
 19. |  3   15.82906   6.529933 |
 28. |  4   21.31999   5.437926 |
 37. |  5   20.65485   5.285748 |
     |--------------------------|
 46. |  6   18.20831   5.665081 |
 55. |  7   16.76509   6.249341 |
 64. |  8   18.21855   6.040239 |
 73. |  9   16.16607   5.473345 |
 82. | 10   20.51594   6.212973 |
     +--------------------------+
```

Thus we can plot estimated regression lines for each of the pigs. Equivalently, we can just plot the fitted values because they are based on both the fixed and the random effects:

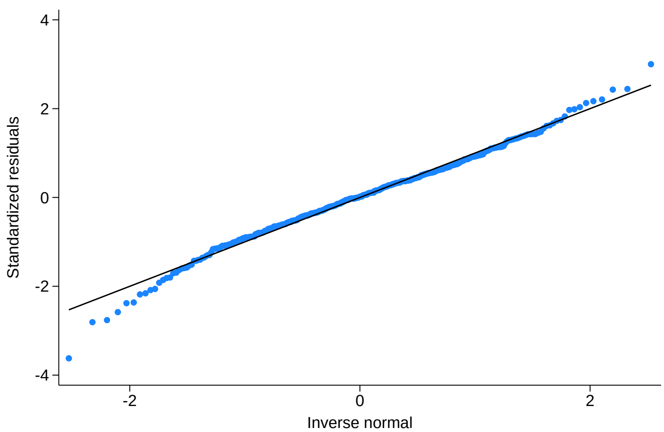
```
. predict fitweight, fitted
. twoway connected fitweight week if id<=10, connect(L)
```



We can also generate standardized residuals and see whether they follow a standard normal distribution, as they should in any good-fitting model:

```
. predict rs, rstandard
. summarize rs

    Variable |        Obs        Mean    Std. dev.       Min        Max
-------------+---------------------------------------------------------
          rs |        432    1.01e-09    .8929356  -3.621446   3.000929
. qnorm rs
```



▷ Example 2: Estimating the intraclass correlation

Following Rabe-Hesketh and Skrondal (2022, chap. 2), we fit a two-level random-effects model for human peak-expiratory-flow rate. The subjects were each measured twice with the Mini-Wright peak-flow meter. It is of interest to determine how reliable the meter is as a measurement device. The intraclass correlation provides a measure of reliability. Formally, in a two-level random-effects model, the intraclass correlation corresponds to the correlation of measurements within the same individual and also to the proportion of variance explained by the individual random effect.

First, we fit the two-level model with `mixed`:

```
. use https://www.stata-press.com/data/r19/pefrate, clear
(Peak-expiratory-flow rate)
. mixed wm || id:
Performing EM optimization ...
Performing gradient-based optimization:
Iteration 0:  Log likelihood = -184.57839
Iteration 1:  Log likelihood = -184.57839
Computing standard errors ...
Mixed-effects ML regression                     Number of obs     =   34
Group variable: id                              Number of groups  =   17
                                                Obs per group:
                                                              min =    2
                                                              avg =  2.0
                                                              max =    2
                                                Wald chi2(0)      =    .
Log likelihood = -184.57839                     Prob > chi2       =    .
```

wm	Coefficient	Std. err.	z	P>|z|	[95% conf. interval]	
_cons	453.9118	26.18617	17.33	0.000	402.5878	505.2357

Random-effects parameters	Estimate	Std. err.	[95% conf. interval]	
id: Identity				
var(_cons)	11458.94	3998.952	5782.176	22708.98
var(Residual)	396.441	135.9781	202.4039	776.4942

```
LR test vs. linear model: chibar2(01) = 46.27         Prob >= chibar2 = 0.0000
```

Now we use `estat icc` to estimate the intraclass correlation:

```
. estat icc
Intraclass correlation
```

Level	ICC	Std. err.	[95% conf. interval]	
id	.9665602	.0159495	.9165853	.9870185

This correlation is close to 1, indicating that the Mini-Wright peak-flow meter is reliable. But as noted by Rabe-Hesketh and Skrondal (2022), the reliability is not only a characteristic of the instrument but also of the between-subject variance. Here we see that the between-subject standard deviation, `sd(_cons)`, is much larger than the within-subject standard deviation, `sd(Residual)`.

In the presence of fixed-effects covariates, `estat icc` reports the residual intraclass correlation, the correlation between measurements conditional on the fixed-effects covariates. This is equivalent to the correlation of the model residuals.

In the presence of random-effects covariates, the intraclass correlation is no longer constant and depends on the values of the random-effects covariates. In this case, `estat icc` reports conditional intraclass correlations assuming 0 values for all random-effects covariates. For example, in a two-level model, this conditional correlation represents the correlation of the residuals for two measurements on the same subject, which both have random-effects covariates equal to 0. Similarly to the interpretation of intercept variances in random-coefficients models (Rabe-Hesketh and Skrondal 2022, chap. 4), interpretation of this conditional intraclass correlation relies on the usefulness of the 0 baseline values of random-effects covariates. For example, mean centering of the covariates is often used to make a 0 value a useful reference.

See [ME] **estat icc** for more information.

◁

▷ Example 3: Estimating residual intraclass correlations

In example 4 of [ME] **mixed**, we estimated a Cobb–Douglas production function with random intercepts at the region level and at the state-within-region level:

$$\mathbf{y}_{jk} = \mathbf{X}_{jk}\boldsymbol{\beta} + u_k^{(3)} + u_{jk}^{(2)} + \boldsymbol{\epsilon}_{jk}$$

```
. use https://www.stata-press.com/data/r19/productivity
(Public capital productivity)
. mixed gsp private emp hwy water other unemp || region: || state:
  (output omitted)
```

We can use `estat group` to see how the data are broken down by state and region:

```
. estat group
```

Group variable	No. of groups	Observations per group Minimum	Average	Maximum
region	9	51	90.7	136
state	48	17	17.0	17

We are reminded that we have balanced productivity data for 17 years for each state.

We can use `predict, fitted` to get the fitted values

$$\widehat{\mathbf{y}}_{jk} = \mathbf{X}_{jk}\widehat{\boldsymbol{\beta}} + \widehat{u}_k^{(3)} + \widehat{u}_{jk}^{(2)}$$

but if we instead want fitted values at the region level, that is,

$$\widehat{\mathbf{y}}_{jk} = \mathbf{X}_{jk}\widehat{\boldsymbol{\beta}} + \widehat{u}_k^{(3)}$$

we need to use the `relevel()` option:

```
. predict gsp_region, fitted relevel(region)
. list gsp gsp_region in 1/10
```

	gsp	gsp_re~n
1.	10.25478	10.40529
2.	10.2879	10.42336
3.	10.35147	10.47343
4.	10.41721	10.52648
5.	10.42671	10.54947
6.	10.4224	10.53537
7.	10.4847	10.60781
8.	10.53111	10.64727
9.	10.59573	10.70503
10.	10.62082	10.72794

❑ Technical note

Out-of-sample predictions are permitted after `mixed`, but if these predictions involve BLUPs of random effects, the integrity of the estimation data must be preserved. If the estimation data have changed since the mixed model was fit, `predict` will be unable to obtain predicted random effects that are appropriate for the fitted model and will give an error. Thus to obtain out-of-sample predictions that contain random-effects terms, be sure that the data for these predictions are in observations that augment the estimation data.

❑

We can use `estat icc` to estimate residual intraclass correlations between productivity years in the same region and in the same state and region.

```
. estat icc
Residual intraclass correlation
```

Level	ICC	Std. err.	[95% conf. interval]	
region	.159893	.127627	.0287143	.5506202
state|region	.8516265	.0301733	.7823466	.9016272

`estat icc` reports two intraclass correlations for this three-level nested model. The first is the level-3 intraclass correlation at the region level, the correlation between productivity years in the same region. The second is the level-2 intraclass correlation at the state-within-region level, the correlation between productivity years in the same state and region.

Conditional on the fixed-effects covariates, we find that annual productivity is only slightly correlated within the same region, but it is highly correlated within the same state and region. We estimate that state and region random effects compose approximately 85% of the total residual variance.

◁

▷ Example 4: Small-sample adjusted tests for fixed effects

To illustrate the use of test and testparm with the small option for small-sample adjusted tests for fixed effects, we refit the dental veneer data from example 14 of [ME] **mixed** using the Satterthwaite method (option dfmethod(satterthwaite)) to compute the DF for fixed effects.

```
. use https://www.stata-press.com/data/r19/veneer, clear
(Dental veneer data)

. mixed gcf followup base_gcf cda age
> || patient: followup, covariance(unstructured)
> || tooth:, reml nolog dfmethod(satterthwaite)

Mixed-effects REML regression                       Number of obs =    110

        Grouping information
        -------------------------------------------------------------
                        |     No. of       Observations per group
         Group variable |     groups    Minimum    Average    Maximum
        ----------------+--------------------------------------------
                patient |         12          2        9.2         12
                  tooth |         55          2        2.0          2
        -------------------------------------------------------------

DF method: Satterthwaite                            DF:         min =  10.41
                                                                avg =  28.96
                                                                max =  50.71
                                                    F(4, 16.49)   =   1.87
Log restricted-likelihood = -420.92761              Prob > F      = 0.1638

------------------------------------------------------------------------------
         gcf | Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
-------------+----------------------------------------------------------------
    followup |   .3009815   1.936863     0.16   0.879    -3.963754    4.565717
    base_gcf |  -.0183127   .1433094    -0.13   0.899    -.3065704     .269945
         cda |   -.329303   .5292525    -0.62   0.537     -1.39197    .7333636
         age |  -.5773932   .2139656    -2.70   0.022    -1.051598   -.1031885
       _cons |   45.73862   12.55497     3.64   0.001     19.90352    71.57372
------------------------------------------------------------------------------

------------------------------------------------------------------------------
  Random-effects parameters  |   Estimate   Std. err.     [95% conf. interval]
-----------------------------+------------------------------------------------
patient: Unstructured        |
                var(followup)|   41.88772   18.79997      17.38009    100.9535
                   var(_cons)|   524.9851   253.0205      204.1287    1350.175
          cov(followup,_cons)|  -140.4229   66.57623     -270.9099   -9.935904
-----------------------------+------------------------------------------------
tooth: Identity              |
                   var(_cons)|   47.45738   16.63034       23.8792     94.3165
-----------------------------+------------------------------------------------
                var(Residual)|   48.86704   10.50523      32.06479    74.47382
------------------------------------------------------------------------------
LR test vs. linear model: chi2(4) = 91.12                 Prob > chi2 = 0.0000
Note: LR test is conservative and provided only for reference.
```

Now we can, for example, test the hypotheses that all fixed effects are zero by typing

```
. testparm *, small
 ( 1)  [gcf]followup = 0
 ( 2)  [gcf]base_gcf = 0
 ( 3)  [gcf]cda = 0
 ( 4)  [gcf]age = 0

       F(  4, 16.49) =    1.87
            Prob > F =    0.1638
```

The F statistic for the overall test is 1.87, and the DDF is estimated to be 16.49. These results are different from the model test using the Kenward–Roger DDF method reported in the header of the estimation output in example 1 of [ME] **estat df** (the F statistic is 1.47, and the model DDF is 27.96).

The results differ because the Kenward–Roger method uses an adjusted F-test statistic and adjusts the fixed-effects variance–covariance estimator for a small sample. Both methods, however, lead to the same conclusion of no joint significance of the fixed effects.

Without option `small`, the commands `test` and `testparm` report large-sample χ^2 Wald tests. We can compare the small-sample and large-sample tests of the joint hypotheses that the coefficient on `followup` and the coefficient on `age` equal zero.

```
. test followup = age = 0, small

 ( 1)  [gcf]followup - [gcf]age = 0
 ( 2)  [gcf]followup = 0

       F(  2, 10.75) =    3.65
            Prob > F =    0.0617

. test followup = age = 0

 ( 1)  [gcf]followup - [gcf]age = 0
 ( 2)  [gcf]followup = 0

           chi2(  2) =    7.30
         Prob > chi2 =    0.0260
```

The DDF of the F test, which is computed using the Satterthwaite method from our posted results, is 10.75. The p-values are very different (0.0617 versus 0.0260), and they lead to different conclusions of whether we should reject the null hypotheses at the $\alpha = 0.05$ level.

Similarly, you can use the `small` option with `lincom` to perform small-sample inference for linear combinations of fixed effects.

◁

▷ Example 5: Small-sample adjusted contrasts

As we did with `test`, after fitting a mixed model with the `dfmethod()` option for small-sample adjustment, we can use the `small` option with `contrast` to adjust for a small sample when estimating contrasts. Suppose we have collected data on a vigilance performance test. This experiment has been designed to test the response latency scores of two modes of signal during a four-hour monitoring period. This is a split-plot factorial design where `signal` is the whole-plot factor, `hour` is the subplot factor, and `subject` is the block factor. The whole-plot factor and the subplot factor are fixed; the block factor is random. Also, suppose that two measurements are missing in this dataset.

```
. use https://www.stata-press.com/data/r19/vptscores, clear
(Vigilance performance test scores with missing data)

. tabdisp subject hour, cellvar(score) by(signal) concise missing

─────────────────────────────────────
Signal    │
and       │
Subject   │     Monitoring period
ID        │     1      2      3      4
──────────┼──────────────────────────
Auditory  │
        1 │     3      4      7      7
        2 │     6      5      .      8
        3 │     3      4      7      9
        4 │     3      3      6      8
──────────┼──────────────────────────
Visual    │
        5 │     1      2      5     10
        6 │     2      3      6      .
        7 │     2      4      5      9
        8 │     2      3      6     11
─────────────────────────────────────
```

We start by fitting a mixed model. Because the dataset is small and unbalanced, we apply the Kenward–Roger method for small-sample adjustment:

```
. mixed score signal##hour || subject:, reml dfmethod(kroger) nolog nogroup

Mixed-effects REML regression                       Number of obs =     30
DF method: Kenward-Roger                            DF:       min =  16.02
                                                              avg =  16.76
                                                              max =  18.29
                                                    F(7, 16.08)   =  43.84
Log restricted-likelihood =   -32.9724              Prob > F      = 0.0000

──────────────────────────────────────────────────────────────────────────────
       score │ Coefficient  Std. err.      t    P>|t|     [95% conf. interval]
─────────────┼────────────────────────────────────────────────────────────────
      signal │
     Visual  │         -2   .6288677    -3.18   0.005    -3.319693   -.6803071
             │
        hour │
          2  │        .25   .5359916     0.47   0.647    -.8861371    1.386137
          3  │   3.108222   .5911044     5.26   0.000     1.859163    4.357281
          4  │       4.25   .5359916     7.93   0.000     3.113863    5.386137
             │
 signal#hour │
   Visual#2  │          1   .7580066     1.32   0.206    -.6067405    2.606741
   Visual#3  │   .6417778   .7979294     0.80   0.433    -1.046666    2.330221
   Visual#4  │   4.044205   .7979294     5.07   0.000     2.355762    5.732649
             │
       _cons │       3.75   .4446766     8.43   0.000     2.816836    4.683164
──────────────────────────────────────────────────────────────────────────────

──────────────────────────────────────────────────────────────────────────────
  Random-effects parameters  │   Estimate   Std. err.     [95% conf. interval]
─────────────────────────────┼────────────────────────────────────────────────
subject: Identity            │
                  var(_cons) │   .2163751   .2345718      .0258477    1.811312
─────────────────────────────┼────────────────────────────────────────────────
               var(Residual) │    .574574   .2062107      .2843515    1.161011
──────────────────────────────────────────────────────────────────────────────
LR test vs. linear model: chibar2(01) = 1.55          Prob >= chibar2 = 0.1069
```

We can test the main effects and the interaction effects by typing the `contrast` command. With the `small` option, `contrast` reports small-sample adjusted F tests. Without the `small` option, `contrast` performs large-sample χ^2 Wald tests. Below is the comparison of the small-sample and the large-sample contrasts:

```
. contrast signal##hour, small
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	ddf	F	P>F
score				
signal	1	5.95	1.78	0.2307
hour	3	16.35	100.62	0.0000
signal#hour	3	16.35	9.66	0.0007

```
. contrast signal##hour
Contrasts of marginal linear predictions
Margins: asbalanced
```

	df	chi2	P>chi2
score			
signal	1	1.79	0.1810
hour	3	304.95	0.0000
signal#hour	3	29.35	0.0000

From these results, we can see that the p-values for the main effect of `signal` and the interaction effect vary between small-sample and large-sample tests. However, both tests indicate that the `hour` effect and the interaction effects are significant. We can decompose the interaction effect into separate interaction contrasts for further investigation.

```
. contrast r.signal#ar.hour, small
Contrasts of marginal linear predictions
Margins: asbalanced

------------------------------------------------------------------------
                                 |     df        ddf          F        P>F
---------------------------------+--------------------------------------
score                            |
                    signal#hour  |
(Visual vs Auditory) (2 vs 1)    |      1      16.02       1.74     0.2056
(Visual vs Auditory) (3 vs 2)    |      1      16.37       0.20     0.6594
(Visual vs Auditory) (4 vs 3)    |      1      16.66      16.57     0.0008
                         Joint   |      3      16.35       9.66     0.0007
------------------------------------------------------------------------

------------------------------------------------------------------------
             |   Contrast   Std. err.          df     [95% conf. interval]
-------------+----------------------------------------------------------
score        |
 signal#hour |
    (Visual  |
          vs |
  Auditory)  |
    (2 vs 1) |          1    .7580066        16.0    -.6067405    2.606741
    (Visual  |
          vs |
  Auditory)  |
    (3 vs 2) |  -.3582222    .7979294        16.4    -2.046666    1.330221
    (Visual  |
          vs |
  Auditory)  |
    (4 vs 3) |   3.402427    .8359478        16.7     1.635991    5.168863
------------------------------------------------------------------------
```

From previous analysis, we already knew the overall interaction was significant. From the decomposition, we can easily see that the overall significance is driven by differences in the third and fourth hours; the change in response latency from hour three to hour four is greater for visual signals than for auditory signals.

We can also calculate the pairwise differences of the hourly marginal means by typing the `pwcompare` command. With the `small` option, `pwcompare` reports small-sample adjusted pairwise comparisons along with the degrees of freedom for each pairwise comparison.

```
. pwcompare hour, small
Pairwise comparisons of marginal linear predictions
Margins: asbalanced

------------------------------------------------------------------------
             |                                            Unadjusted
             |   Contrast   Std. err.          df     [95% conf. interval]
-------------+----------------------------------------------------------
score        |
        hour |
     2 vs 1  |        .75    .3790033        16.0    -.0533703     1.55337
     3 vs 1  |   3.429111    .3989647        16.4     2.584889    4.273333
     4 vs 1  |   6.272103    .3989647        16.4     5.427881    7.116324
     3 vs 2  |   2.679111    .3989647        16.4     1.834889    3.523333
     4 vs 2  |   5.522103    .3989647        16.4     4.677881    6.366324
     4 vs 3  |   2.842991    .4179739        16.7     1.959774    3.726209
------------------------------------------------------------------------
```

When we compare these results with the large-sample results below, we can see that the confidence interval of hour 2 versus hour 1 changes to include 0. Therefore, after adjusting for small-sample size, we would not reject the hypothesis that the means for hour 1 and hour 2 are equivalent at the 5% significance level.

```
. pwcompare hour
Pairwise comparisons of marginal linear predictions
Margins: asbalanced
```

	Contrast	Std. err.	Unadjusted [95% conf.	interval]
score				
hour				
2 vs 1	.75	.3790033	.0071672	1.492833
3 vs 1	3.429111	.3971529	2.650706	4.207516
4 vs 1	6.272103	.3971529	5.493697	7.050508
3 vs 2	2.679111	.3971529	1.900706	3.457516
4 vs 2	5.522103	.3971529	4.743697	6.300508
4 vs 3	2.842991	.4145085	2.03057	3.655413

◁

Stored results

`pwcompare` with option `small` stores the following in `r()`:

Matrices

`r(L_df)`	degrees of freedom for each margin difference
`r(M_df)`	degrees of freedom for each margin estimate

`pwcompare` with options `post` and `small` stores the following in `e()`:

Matrices

`e(L_df)`	degrees of freedom for each margin difference
`e(M_df)`	degrees of freedom for each margin estimate

Methods and formulas

Methods and formulas are presented under the following headings:

Prediction
Small-sample inference

Prediction

Following the notation defined throughout [ME] **mixed**, BLUPs of random effects $\mathbf{u}$ are obtained as

$$\widetilde{\mathbf{u}} = \widetilde{\mathbf{G}}\mathbf{Z}'\widetilde{\mathbf{V}}^{-1}\left(\mathbf{y} - \mathbf{X}\widehat{\boldsymbol{\beta}}\right)$$

where $\widetilde{\mathbf{G}}$ and $\widetilde{\mathbf{V}}$ are $\mathbf{G}$ and $\mathbf{V} = \mathbf{ZGZ}' + \sigma^2_\epsilon \mathbf{R}$ with maximum likelihood (ML) or REML estimates of the variance components plugged in. Standard errors for BLUPs are calculated based on the iterative technique of Bates and Pinheiro (1998, sec. 3.3) for estimating the BLUPs themselves. If estimation is done by REML, these standard errors account for uncertainty in the estimate of $\boldsymbol{\beta}$, while for ML the standard errors treat $\boldsymbol{\beta}$ as known. As such, standard errors of REML-based BLUPs will usually be larger.

Fitted values are given by $\mathbf{X}\widehat{\boldsymbol{\beta}} + \mathbf{Z}\tilde{\mathbf{u}}$, residuals as $\hat{\boldsymbol{\epsilon}} = \mathbf{y} - \mathbf{X}\widehat{\boldsymbol{\beta}} - \mathbf{Z}\tilde{\mathbf{u}}$, and standardized residuals as

$$\hat{\boldsymbol{\epsilon}}_* = \hat{\sigma}_\epsilon^{-1}\widehat{\mathbf{R}}^{-1/2}\hat{\boldsymbol{\epsilon}}$$

If the `relevel(`*levelvar*`)` option is specified, fitted values, residuals, and standardized residuals consider only those random-effects terms up to and including level *levelvar* in the model.

For details concerning the calculation of scores, see *Methods and formulas* in [ME] **mixed**.

Small-sample inference

For small-sample computations performed when the `small` option is used with `test`, `testparm`, `lincom`, `contrast`, or `pwcompare`, see *Denominator degrees of freedom* in *Methods and formulas* of [ME] **mixed**.

References

Baldwin, S. 2019. *Psychological Statistics and Psychometrics Using Stata*. College Station, TX: Stata Press.

Bates, D. M., and J. C. Pinheiro. 1998. "Computational methods for multilevel modelling". In *Technical Memorandum BL0112140-980226-01TM*. Murray Hill, NJ: Bell Labs, Lucent Technologies.

Mitchell, M. N. 2015. *Stata for the Behavioral Sciences*. College Station, TX: Stata Press.

Rabe-Hesketh, S., and A. Skrondal. 2022. *Multilevel and Longitudinal Modeling Using Stata*. 4th ed. College Station, TX: Stata Press.

Also see

[ME] **mixed** — Multilevel mixed-effects linear regression

[U] 20 Estimation and postestimation commands