qrsolve( ) — Solve AX=B for X using QR decomposition

Description

qrsolve(*A*, *B*, ...) uses QR decomposition to solve $AX = B$ and returns *X*. When *A* is singular or nonsquare, qrsolve() computes a least-squares generalized solution. When *rank* is specified, in it is placed the rank of *A*.

_qrsolve(*A*, *B*, ...), does the same thing, except that it destroys the contents of *A* and it overwrites *B* with the solution. Returned is the rank of *A*.

In both cases, *tol* specifies the tolerance for determining whether *A* is of full rank. *tol* is interpreted in the standard way—as a multiplier for the default if $tol > 0$ is specified and as an absolute quantity to use in place of the default if $tol \leq 0$ is specified; see [M-1] **tolerance**.

Syntax

```
numeric matrix    qrsolve(A, B)
numeric matrix    qrsolve(A, B, rank)
numeric matrix    qrsolve(A, B, rank, tol)

real scalar      _qrsolve(A, B)
real scalar      _qrsolve(A, B, tol)
```

where

A:	*numeric matrix*
B:	*numeric matrix*
rank:	irrelevant; *real scalar* returned
tol:	*real scalar*

Remarks and examples

qrsolve(*A*, *B*, ...) is suitable for use with square and possibly rank-deficient matrix *A*, or when *A* has more rows than columns. When *A* is square and full rank, qrsolve() returns the same solution as lusolve() (see [M-5] **lusolve( )**), up to roundoff error. When *A* is singular, qrsolve() returns a generalized (least-squares) solution.

Remarks are presented under the following headings:

Derivation
Relationship to inversion
Tolerance

Derivation

We wish to solve for X

$$AX = B \tag{1}$$

Perform QR decomposition on A so that we have $A = QRP'$. Then (1) can be rewritten as

$$QRP'X = B$$

Premultiplying by Q' and remembering that $Q'Q = QQ' = I$, we have

$$RP'X = Q'B \tag{2}$$

Define

$$Z = P'X \tag{3}$$

Then (2) can be rewritten as

$$RZ = Q'B \tag{4}$$

It is easy to solve (4) for Z because R is upper triangular. Having Z, we can obtain X via (3), because $Z = P'X$, premultiplied by P (and if we remember that $PP' = I$), yields

$$X = PZ$$

For more information on QR decomposition, see [M-5] **qrd( )**.

Relationship to inversion

For a general discussion, see *Relationship to inversion* in [M-5] **lusolve( )**.

For an inverse based on QR decomposition, see [M-5] **qrinv( )**. `qrinv(`*A*`)` amounts to `qrsolve(`*A*`, I(rows(`*A*`)))`, although it is not actually implemented that way.

Tolerance

The default tolerance used is

$$eta = \texttt{1e-13} * \texttt{trace(abs(}R\texttt{))/rows(}R\texttt{)}$$

where R is the upper-triangular matrix of the QR decomposition; see *Derivation* above. When A is less than full rank, by, say, d degrees of freedom, then R is also rank deficient by d degrees of freedom and the bottom d rows of R are essentially zero. If the ith diagonal element of R is less than or equal to *eta*, then the ith row of Z is set to zero. Thus if the matrix is singular, `qrsolve()` provides a generalized solution.

If you specify $tol > 0$, the value you specify is used to multiply *eta*. You may instead specify $tol \leq 0$, and then the negative of the value you specify is used in place of *eta*; see [M-1] **tolerance**.

Conformability

`qrsolve(`A`, `B`, `*rank*`, `*tol*`):`

input:

A:	$m \times n,$	$m \geq n$
B:	$m \times k$	
tol:	1×1	(optional)

output:

rank:	1×1	(optional)
result:	$n \times k$	

`_qrsolve(`A`, `B`, `*tol*`):`

input:

A:	$m \times n,$	$m \geq n$
B:	$m \times k$	
tol:	1×1	(optional)

output:

A:	0×0
B:	$n \times k$
result:	1×1

Diagnostics

`qrsolve(`A`, `B`, ...)` and `_qrsolve(`A`, `B`, ...)` return a result containing missing if A or B contain missing values.

`_qrsolve(`A`, `B`, ...)` aborts with error if A or B are views.

Also see

[M-5] **qrinv()** — Generalized inverse of matrix via QR decomposition

[M-5] **qrd()** — QR decomposition

[M-5] **solvelower()** — Solve AX=B for X, A triangular

[M-5] **cholsolve()** — Solve AX=B for X using Cholesky decomposition

[M-5] **lusolve()** — Solve AX=B for X using LU decomposition

[M-5] **svsolve()** — Solve AX=B for X using singular value decomposition

[M-5] **solve_tol()** — Tolerance used by solvers and inverters

[M-4] **matrix** — Matrix functions

[M-4] **solvers** — Functions to solve AX=B and to obtain A inverse