Description Remarks and examples References Also see

# Description

This entry contains more examples of lasso for prediction. It assumes you have already read [LASSO] Lasso intro and [LASSO] lasso.

# **Remarks and examples**

Remarks are presented under the following headings:

Overview Using vl to manage variables Using splitsample Lasso linear models Adaptive lasso Cross-validation folds BIC More potential variables than observations Factor variables in lasso Lasso logit and probit models Lasso Poisson models Lasso Cox models

### Overview

In the examples of this entry, we use a dataset of a realistic size for lasso. It has 1,058 observations and 172 variables. Still, it is a little on the small side for lasso. Certainly, you can use lasso on datasets of this size, but lasso can also be used with datasets that have thousands or tens of thousands of variables.

The number of variables can even be greater than the number of observations. What is essential for lasso is that the set of potential variables contains a subset of variables that are in the true model (or something close to it) or are correlated with the variables in the true model.

As to how many variables there can be in the true model, we can say that the number cannot be greater than something proportional to  $\sqrt{N}/\ln q$ , where N is the number of observations, p is the number of potential variables, and  $q = \max\{N, p\}$ . We cannot, however, say what the constant of proportionality is. That this upper bound decreases with q can be viewed as the cost of performing covariate selection.

### Using vI to manage variables

We will show how to use commands in the vl system to manage large numbers of variables. vl stands for "variable lists". The idea behind it is that we might want to run a lasso with hundreds or thousands or tens of thousands of variables specified as potential variables. We do not want to have to type all these variable names.

Many times, we will have a mix of different types of variables. Some we want to treat as continuous. Some we want to treat as categorical and use factor-variable operators with them to create indicator variables for their categories. See [U] 11.4.3 Factor variables.

The first goal of the vl system is to help us separate variables we want to treat as categorical from those we want to treat as continuous. The second goal of the system is to help us create named variable lists we can use as arguments to lasso or any other Stata command simply by referring to their names.

The purpose here is to illustrate the power of v1, not to explain in detail how it works or show all of its features. For that, see [D] vl.

We load the dataset we will use in these examples.

. use https://www.stata-press.com/data/r19/fakesurvey (Fictitious survey data)

It is simulated data designed to mimic survey data. It has 1,058 observations and 172 variables.

. describe

Contains data from htt Observations: Variables:		tps://www.stata-press.co 1,058 172		om/data/r19/fakesurvey.dta Fictitious survey data 14 Jun 2024 15:31	
Variable	Storage	Display	Value		
name	type	format	label	Variable label	
id	str8	%9s		Respondent ID	
gender	byte	%8.0g	gender	Gender	
age	byte	%8.0g		Age (y)	
q1	byte	%10.0g		Question 1	
q2	byte	%8.0g		Question 2	
q3	byte	%8.0g	yesno	Question 3	
(output omitt	ed)				
q160	byte	%8.0g	yesno	Question 160	
q161	byte	%8.0g	yesno	Question 161	
check8	byte	%8.0g	-	Check 8	

Sorted by: id

The variables are a mix. Some we know are integer-valued scales that we want to treat as continuous variables in our models. There are a lot of 0/1 variables, and there are some with only a few categories that we will want to turn into indicator variables. There are some with more categories that we do not yet know whether to treat as categorical or continuous.

The first vl subcommand we run is vl set. Nonnegative integer-valued variables are candidates for use as factor variables. Because factor variables cannot be negative, any variable with negative values is classified as continuous. Any variable with noninteger values is also classified as continuous.

vl set has two options, categorical(#) and uncertain(#), that allow us to separate out the nonnegative integer-valued variables into three named variable lists: vlcategorical, vluncertain, and vlcontinuous.

When the number of levels (distinct values), L, is

2 < L < categorical(#)

the variable goes in vlcategorical. When

 $categorical(#) < L \leq uncertain(#)$ 

the variable goes in vluncertain. When

L > uncertain(#)

the variable goes in vlcontinuous.

The defaults are categorical(10) and uncertain(100). For our data, we do not like the defaults, so we change them. We specify categorical(4) and uncertain(19). We also specify the option dummy to create a variable list, vldummy, consisting solely of 0/1 variables. Let's run vl set with these options.

. vl set, categorical(4) uncertain(19) d	lummy
--	-------

		Macro's contents
Macro	# Vars	Description
System		
\$vldummy	99	0/1 variables
<pre>\$vlcategorical</pre>	16	categorical variables
\$vlcontinuous	20	continuous variables
<pre>\$vluncertain</pre>	27	perhaps continuous, perhaps categorical variables
<pre>\$vlother</pre>	9	all missing or constant variables

Notes

- 1. Review contents of vlcategorical and vlcontinuous to ensure they are correct. Type vl list vlcategorical and type vl list vlcontinuous.
- If there are any variables in vluncertain, you can reallocate them to vlcategorical, vlcontinuous, or vlother. Type vl list vluncertain.
- 3. Use vl move to move variables among classifications. For example, type vl move (x50 x80) vlcontinuous to move variables x50 and x80 to the continuous classification.
- 4. vlnames are global macros. Type the vlname without the leading dollar sign (\$) when using vl commands. Example: vlcategorical not \$vlcategorical. Type the dollar sign with other Stata commands to get a varlist.

The vluncertain variable list contains all the variables we are not sure whether we want to treat as categorical or continuous. We use vl list to list the variables in vluncertain.

ariable	Macro	Values	Levels
q12	<pre>\$vluncertain</pre>	integers >=(	) 5
q18	<pre>\$vluncertain</pre>	integers >=(	) 7
q23	<pre>\$vluncertain</pre>	integers >=(	) 10
q27	<pre>\$vluncertain</pre>	integers >=(	) 8
q28	<pre>\$vluncertain</pre>	integers >=(	) 15
q35	<pre>\$vluncertain</pre>	integers >=(	) 7
q39	<pre>\$vluncertain</pre>	integers >=(	) 5
q54	<pre>\$vluncertain</pre>	integers >=(	) 10
q63	<pre>\$vluncertain</pre>	integers >=(	) 7
q66	<pre>\$vluncertain</pre>	integers >=(	) 5
q80	<pre>\$vluncertain</pre>	integers >=(	) 5
q81	<pre>\$vluncertain</pre>	integers >=(	) [
q92	<pre>\$vluncertain</pre>	integers >=(	) [
q93	<pre>\$vluncertain</pre>	integers >=(	) 7
q99	<pre>\$vluncertain</pre>	integers >=(	) [
q103	<pre>\$vluncertain</pre>	integers >=(	) 7
q107	<pre>\$vluncertain</pre>	integers >=(	) 18
q111	<pre>\$vluncertain</pre>	integers >=(	) 7
q112	<pre>\$vluncertain</pre>	integers >=(	) 7
q119	<pre>\$vluncertain</pre>	integers >=(	) (
q120	<pre>\$vluncertain</pre>	integers >=(	) 7
q124	<pre>\$vluncertain</pre>	integers >=(	) 14
q127	<pre>\$vluncertain</pre>	integers >=(	) 5
q132	<pre>\$vluncertain</pre>	integers >=(	) 7
q135	<pre>\$vluncertain</pre>	integers >=(	) 10
q141	<pre>\$vluncertain</pre>	integers >=(	) 12
q157	<pre>\$vluncertain</pre>	integers >=(	) 7

. vl list vluncertain

We are going to have to go through these variables one by one and reclassify them. We know we have several seven-level Likert scales in these data. We tabulate one of them.

. tabulate q18			
Question 18	Freq.	Percent	Cum.
Very strongly disagree	139	13.15	13.15
Strongly disagree	150	14.19	27.34
Disagree	146	13.81	41.15
Neither agree nor disagree	146	13.81	54.97
Agree	174	16.46	71.43
Strongly agree	146	13.81	85.24
Very strongly agree	156	14.76	100.00
Total	1,057	100.00	

We look at all the variables with seven levels, and they are all Likert scales. We want to treat them as continuous in our models, so we move them out of vluncertain and into vlcontinuous.

. vl move (q18 q35 q63 q93 q103 q111 q112 q120 q132 q157) vlcontinuous note: 10 variables specified and 10 variables moved.

Macro	# Added/Removed
\$vldummy	0
\$vlcategorical	0
\$vlcontinuous	10
\$vluncertain	-10
\$vlother	0

When variables are moved into a new vl system-defined variable list, they are automatically moved out of their current system-defined variable list.

In our examples, we have three variables we want to predict: q104, a continuous variable; q106, a 0/1 variable; and q107, a count variable. Because we are going to use the variables in vlcategorical and vlcontinuous as potential variables to select in our lassos, we do not want these dependent variables in these variable lists. We move them into vlother, which is intended as a place to put variables we do not want in our models.

. vl move (q104 q106 q107) vlother note: 3 variables specified and 3 variables moved.

Macro	#	Added/Removed
\$vldummy		-1
<pre>\$vlcategorical</pre>		0
\$vlcontinuous		-1
\$vluncertain		-1
\$vlother		3

Notice the parentheses around the variable names when we used vl move. The rule for vl is to use parentheses around variable names and to not use parentheses for variable-list names.

The system-defined variable lists are good for a general division of variables. But we need further subdivision for our models. We have four demographic variables, which are all categorical, but we want them included in all lasso models. So we create a user-defined variable list containing these variables.

```
. vl create demographics = (gender q3 q4 q5) note: $demographics initialized with 4 variables.
```

We want to convert the variables in vldummy and vlcategorical into indicator variables. We create a new variable list, factors, containing the union of these lists. Because we want to handle the variables in demographics separately, we remove them from factors.

```
. vl create factors = vldummy + vlcategorical
note: $factors initialized with 114 variables.
. vl modify factors = factors - demographics
note: 4 variables removed from $factors.
```

The vl substitute command allows us to apply factor-variable operators to a variable list. We turn the variables in demographics and factors into factor variables.

```
. vl substitute idemographics = i.demographics
```

. vl substitute ifactors = i.factors

We are done using vl and we save our dataset. One nice feature of vl is that the variable lists are saved with the data.

```
. label data "Fictitious survey data with vl"
. save fakesurvey_vl
file fakesurvey_vl.dta saved
```

We are now ready to run some lassos.

## Using splitsample

Well, almost ready. We want to evaluate our lasso predictions on a sample that we did not use to fit the lasso. So we decide to randomly split our data into two samples of equal sizes. We will fit models on one, and we will use the other to test their predictions.

Let's load the version of our dataset that contains our variable lists. We first increase maxvar because we are going to create thousands of interactions in a later example.

```
. clear all
. set maxvar 10000
. use https://www.stata-press.com/data/r19/fakesurvey_vl
(Fictitious survey data with vl)
```

Variable lists are not automatically restored. We have to run vl rebuild to make them active.

```
. vl rebuild
Rebuilding vl macros ...
```

	Macro's contents			
Macro	# Vars	Description		
System				
\$vldummy	98	0/1 variables		
<pre>\$vlcategorical</pre>	16	categorical variables		
\$vlcontinuous	29	continuous variables		
<pre>\$vluncertain</pre>	16	perhaps continuous, perhaps categorical variables		
<pre>\$vlother</pre>	12	all missing or constant variables		
User		C C		
\$demographics	4	variables		
\$factors	5 I			
\$idemographics		factor-variable list		
\$ifactors		factor-variable list		

We now use splitsample to generate a variable indicating the two subsamples.

- . set seed 1234
- . splitsample, generate(sample) nsplit(2)
- . label define svalues 1 "Training" 2 "Testing"
- . label values sample svalues

# Lasso linear models

When fitting our lasso model, we can now specify variables succinctly using our vl variable lists. Variable lists are really global macros—we bet you already guessed this. Listing them under the header "Macro" in vl output was a real tip-off, right? Because they are global macros, when we use them as arguments in commands, we put a \$ in front of them.

We put parentheses around idemographics. This notation means that we want to force these variables into the model regardless of whether lasso wants to select them. See *Syntax* in [LASSO] **lasso**.

**.**...

We also set the random-number seed using the rseed() option so that we can reproduce our results.

. .

We fit lasso on the first subsample.

. . . . . . .

	. lasso linear q104 (\$idemographics) \$ifactors \$vlcontinuous > if sample == 1, rseed(1234)						
10-fold cross-validation with 100 lambdas Grid value 1: lambda = .8978025 no. of nonzero coef. = 4 Folds: 1510 CVF = 16.93341							
(output on	nitted)						
Grid value	e 23: lambda = .	1159557 no	. of nonzer	co coef. =	74		
		2.17933					
cross-	-validation complet	e minimu	m found				
Lasso line	ear model		No. of		= 458		
				covariates			
Selection	Cross-validation		No. of	CV folds	= 10		
			No. of	Out-of-	CV mean		
			nonzero	sample	prediction		
ID	Description	lambda	coef.	R-squared	error		
1	first lambda	.8978025	4	0.0147	16.93341		
18	lambda before	.1846342	42	0.2953	12.10991		
* 19	selected lambda	.1682318	49	0.2968	12.08516		
20	lambda after		55	0.2964			
23	last lambda	.1159557	74	0.2913	12.17933		

\* lambda selected by cross-validation.

. estimates store linearcv

After the command finished, we used estimates store to store the results in memory so that we can later compare these results with those from other lassos. Note, however, that estimates store only saves them in memory. To save the results to disk, use

. estimates save filename

#### See [LASSO] estimates store.

The minimum of the cross-validation (CV) function was found to be at  $\lambda = 0.1682318$ . It selects  $\lambda^*$  as this  $\lambda$ , which corresponds to 49 variables in the model, out of 277 potential variables.

After fitting a lasso using CV to select  $\lambda$ , it is a good idea to plot the CV function and look at the shape of the curve around the minimum.

. cvplot



By default, the lasso command stops when it has identified a minimum. Computation time increases as  $\lambda$ 's get smaller, so computing the CV function for smaller  $\lambda$ 's is computationally expensive. We could specify the option selection(cv, alllambdas) to compute models for more small  $\lambda$ 's. See [LASSO] **lasso** and [LASSO] **lasso fitting** for details and a description of less computationally intensive options to get more assurance that lasso has identified a minimum.

We can also get a plot of the size of the coefficients as they become nonzero and change as  $\lambda$  gets smaller. Typically, they get larger as  $\lambda$  gets smaller. But they can sometimes return to 0 after being nonzero.



We see four lines that do not start at 0. These are lines corresponding to the four variables in idemographics that we forced into the model.

#### Adaptive lasso

We are now going to run an adaptive lasso, which we do by specifying the option selection(adaptive).

```
. lasso linear q104 ($idemographics) $ifactors $vlcontinuous
> if sample == 1, rseed(4321) selection(adaptive)
Lasso step 1 of 2:
10-fold cross-validation with 100 lambdas ...
Grid value 1:
                  lambda = .8978025
                                      no. of nonzero coef. =
                                                                4
Folds: 1...5....10 CVF =
                           17.012
 (output omitted)
Grid value 24:
                  lambda = .1056545
                                      no. of nonzero coef. = 78
Folds: 1...5....10 CVF = 12.40012
... cross-validation complete ... minimum found
Lasso step 2 of 2:
Evaluating up to 100 lambdas in grid ...
                  lambda = 48.55244
Grid value 1:
                                     no. of nonzero coef. = 4
 (output omitted)
Grid value 100:
                  lambda = .0048552
                                      no. of nonzero coef. = 59
10-fold cross-validation with 100 lambdas ...
Fold 1 of 10: 10....20....30....40....50....60....70....80....90....100
 (output omitted)
Fold 10 of 10: 10....20....30....40....50....60....70....80....90....100
... cross-validation complete
Lasso linear model
                                           No. of obs
                                                                        458
                                                               _
                                           No. of covariates =
                                                                        277
                                           No. of lasso steps =
Selection: Adaptive
                                                                          2
Final adaptive step results
```

ID	Description	lambda	No. of nonzero coef.	Out-of- sample R-squared	CV mean prediction error
25	first lambda	48.55244	4	0.0101	17.01083
77	lambda before	.3847698	46	0.3985	10.33691
* 78	selected lambda	.3505879	46	0.3987	10.33306
79	lambda after	.3194427	47	0.3985	10.33653
124	last lambda	.0048552	59	0.3677	10.86697

\* lambda selected by cross-validation in final adaptive step.

. estimates store linearadaptive

Adaptive lasso performs multiple lassos. In the first lasso, a  $\lambda^*$  is selected, and penalty weights are constructed from the coefficient estimates. Then these weights are used in a second lasso, where another  $\lambda^*$  is selected. We did not specify how many lassos should be performed, so we got the default of two. We could specify more, but typically the selected  $\lambda^*$  does not change after the second lasso, or it changes little. See the selection(adaptive) option in [LASSO] lasso.

We can see details of the two lassos by using lassoknots and specifying the option steps to see all steps of the adaptive lasso.

			No. of nonzero	CV mean pred.	Variables (A)dded, (R)emoved,
Step	ID	lambda	coef.	error	or left (U)nchanged
1					
-	1	.8978025	4	17.012	A 1.q3 1.q4
					1.q5 1.gender
	2	.8180442	7	16.91096	A 0.q19 0.q85
	2	7450714	0	16 66200	3.q156
	3 4	.7453714 .6791547	8 9	16.66328 16.33224	A 0.q101 A 0.q88
	-		9	10.33224	A 0.000
(outpu	t omitte	ed)			
	23	.1159557	74	12.35715	A 3.q6 0.q40
					0.q82 0.q98
					0.q128 2.q134
	0.4	1050545	70	10 10010	0.q148 q157
	24	.1056545	78	12.40012	A 2.q6 0.q9
					1.q34 4.q155
2					
	25	48.55244	4	17.01083	A 1.q3 1.q4
					1.q5 1.gender
	26	44.23918	6	16.94087	A 0.q19 0.q85
(outpu	t omitte	ed)			
	76	.4222844	45	10.33954	A 0.q44
	77	.3847698	46	10.33691	A q111
	* 78	.3505879	46	10.33306	U
	79	.3194427	47	10.33653	A 0.q97
	80	.2910643	48	10.3438	A 0.q138
(outpu	t omitte	ed)			
	112	.0148272	59	10.7663	A q70
	124	.0048552	59	10.86697	U

. lassoknots, steps

\* lambda selected by cross-validation in final adaptive step.

Notice how the scale of  $\lambda$  changes in the second lasso. That is because of the penalty weights generated by the first lasso.

The ordinary lasso selected 49 variables, and the adaptive lasso selected 46. It is natural to ask how much these two groups of variables overlap. When the goal is prediction, however, we are not supposed to care about this. Ordinary lasso might select one variable, and adaptive lasso might instead select another that is highly correlated to it. So it is wrong to place importance on any particular variable selected or not selected. It is the group of variables selected as a whole that matters.

Still, we cannot resist looking, and the lassocoef command was designed especially for this purpose. We specify lassocoef with the option sort(coef, standardized). This sorts the listing by the absolute values of the standardized coefficients with the largest displayed first. lassocoef can list different types of coefficients and display them in different orderings. See [LASSO] lassocoef.

	linearcv	linearadaptive				
q19 No	x	x				
q85 No	x	x				
q5 Yes	x	x				
3.q156	x	x				
q101 No	x	x				
(output omitted)	)					
q160 No	x	x				
age	х	х				
q53	x	х				
2.q105	х					
q102 No	x	x				
q154 No	x	x				
q111	x	х				
q142 No 0.q55 0.q97	x x x	x				
q65 4	x	x				
1.q110 q70	x x	x				
q44 No		x				
(output omitted	(output omitted)					
Legend: b - base level e - empty cell						

o - omitted
x - estimated

. lassocoef linearcv linearadaptive, sort(coef, standardized)

We see that the adaptive lasso did not select four variables that the lasso did, and it selected one that the lasso did not. All the differences occurred among the variables with smaller standardized coefficients.

504

494

The most important question to ask is which performed better for out-of-sample prediction. lassogof is the command for that. We specify the over () option with the name of our sample indicator variable, sample. We specify the postselection option because for linear models, postselection coefficients are theoretically slightly better for prediction than the penalized coefficients (which lassogof uses by default).

·						
Postselection coefficients						
Name	sample	MSE	R-squared	Obs		
linearcv						
	Training	8.652771	0.5065	503		
	Testing	14.58354	0.2658	493		

8.637575

14.70756

. lassogof linearcv linearadaptive, over(sample) postselection

The ordinary lasso did a little better in this case than the adaptive lasso.

Training Testing

#### Cross-validation folds

linearadaptive

CV works by dividing the data randomly into K folds. One fold is chosen, and then a linear regression is fit on the other K-1 folds using the variables in the model for that  $\lambda$ . Then using these new coefficient estimates, a prediction is computed for the data of the chosen fold. The mean squared error (MSE) of the prediction is computed. This process is repeated for the other K-1 folds. The K MSEs are then averaged to give the value of the CV function.

0.5057

0.2595

Let's increase the number of folds from the default of 10 to 20 by specifying selection(cv, folds(20)).

```
. lasso linear q104 ($idemographics) $ifactors $vlcontinuous
> if sample == 1, selection(cv, folds(20)) rseed(9999)
20-fold cross-validation with 100 lambdas ...
Grid value 1:
                  lambda = .8978025
                                      no. of nonzero coef. =
                                                                 4
Folds: 1...5....10....15....20
                                CVF = 17.08362
 (output omitted)
Grid value 23:
                  lambda = .1159557
                                       no. of nonzero coef. = 74
Folds: 1...5....10....15....20
                                  CVF = 12.12667
... cross-validation complete ... minimum found
Lasso linear model
                                              No. of obs
                                                                          458
                                             No. of covariates =
                                                                          277
Selection: Cross-validation
                                             No. of CV folds
                                                                           20
                                           No. of
                                                        Out-of-
                                                                      CV mean
                                          nonzero
                                                         sample
                                                                  prediction
      TD
               Description
                                 lambda
                                            coef.
                                                      R-squared
                                                                        error
       1
              first lambda
                               .8978025
                                                4
                                                         0.0059
                                                                    17.08362
      19
             lambda before
                               .1682318
                                               49
                                                         0.2999
                                                                    12.03169
      20
           selected lambda
                               .1532866
                                                         0.3002
                                                                    12.02673
                                                55
                                                         0.2988
      21
              lambda after
                                .139669
                                                62
                                                                    12.05007
      23
               last lambda
                               .1159557
                                                74
                                                         0.2944
                                                                    12.12667
```

\* lambda selected by cross-validation.

. estimates store linearcv2

Which performs better for out-of-sample prediction?

Name	sample	MSE	R-squared	Obs
linearcv				
	Training	8.652771	0.5065	503
	Testing	14.58354	0.2658	493
linearcv2				
	Training	8.545785	0.5126	502
	Testing	14.7507	0.2594	488

. lassogof linearcv linearcv2, over(sample) postselection Postselection coefficients

The first lasso with 10 folds did better than the lasso with 20 folds. This is generally true. More than 10 folds typically does not yield better predictions.

We should mention again that CV is a randomized procedure. Changing the random-number seed can result in a different  $\lambda^*$  being selected and so give different predictions.

# BIC

We are now going to select  $\lambda^*$  by minimizing the BIC function, which we do by specifying the option selection(bic).

<pre>. lasso linear q1 &gt; if sample == 1,</pre>		\$ifactors \$vlcontinuous
Evaluating up to	100 lambdas in grid	
		no. of nonzero coef. = 4
	BIC = 2618.642	
Grid value 2:	lambda = .8180442	no. of nonzero coef. = 7
	BIC = 2630.961	
Grid value 3:	lambda = .7453714	no. of nonzero coef. = 8
	BIC = 2626.254	
Grid value 4:	lambda = .6791547	no. of nonzero coef. = 9
Grid value 5:	BIC = 2619.727	no, of nonzero coef. = 10
Grid value 5:	lambda = .6188205 BIC = 2611.577	no. of nonzero coef. = 10
Grid value 6:	lambda = .5638462	no. of nonzero coef. = 13
dilu value o.	BIC = 2614.155	
Grid value 7:	lambda = .5137556	no. of nonzero coef. = 13
	BIC = 2597.164	
Grid value 8:	lambda = .468115	no. of nonzero coef. = 14
	BIC = 2588.189	
Grid value 9:	lambda = .4265289	no. of nonzero coef. = 16
	BIC = 2584.638	
Grid value 10:	lambda = .3886373	no. of nonzero coef. = 18
<i></i>	BIC = 2580.891	
Grid value 11:	lambda = .3541118	no. of nonzero coef. = 22
Grid value 12:	BIC = 2588.984 lambda = .3226535	no, of nonzero coef. = 26
GIIG Value 12:	BIC = 2596.792	no. of nonzero coef 20
Grid value 13:	lambda = .2939899	no. of nonzero coef. = 27
dila valao io.	BIC = 2586.521	
Grid value 14:	lambda = .2678726	no. of nonzero coef. = 28
	BIC = 2578.211	
Grid value 15:	lambda = .2440755	no. of nonzero coef. = 32
	BIC = 2589.632	

Grid value 16:	lambda = .2223925 BIC = 2593.753	no. of nonzero coef. =	35
Grid value 17:	lambda = .2026358 BIC = 2592.923	no. of nonzero coef. =	37
Grid value 18:	lambda = .1846342 BIC = 2609.975	no. of nonzero coef. =	42
Grid value 19:	lambda = .1682318 BIC = 2639.437	no. of nonzero coef. =	49
selection BIC	complete minimu	um found	
Lasso linear mode	1	No. of obs No. of covariates	100

Selection: Bayesian information criterion

ID	Description	lambda	No. of nonzero coef.	In-sample R-squared	BIC
1	first lambda	.8978025	4	0.0308	2618.642
13	lambda before	.2939899	27	0.3357	2586.521
* 14	selected lambda	.2678726	28	0.3563	2578.211
15	lambda after	.2440755	32	0.3745	2589.632
19	last lambda	.1682318	49	0.4445	2639.437

\* lambda selected by Bayesian information criterion.

. estimates store linearbic

The minimum of the BIC function was found to be at  $\lambda = 0.268$ . It selects  $\lambda^*$  as this  $\lambda$ , which corresponds to 28 variables in the model out of 277 potential variables.

After fitting a lasso using BIC, it is a good idea to plot the BIC function and look at the shape of the curve around the minimum.

. bicplot



We see that the BIC function rises sharply once it hits the minimum. By default, the lasso command stops when it has identified a minimum.

So far, we have fit lasso linear models using CV, an adaptive lasso, and BIC. Which one performs better in the out-of-sample prediction?

Name	sample	MSE	R-squared	Obs	
linearcv					
	Training	8.652771	0.5065	503	
	Testing	14.58354	0.2658	493	
linearadapt:	ive				
_	Training	8.637575	0.5057	504	
	Testing	14.70756	0.2595	494	
linearbic					
	Training	9.740229	0.4421	508	
	Testing	13.44496	0.3168	503	

. lassogof linearcv linearadaptive linearbic, over(sample) postselection Postselection coefficients

The BIC lasso performs the best.

#### More potential variables than observations

Lasso has no difficulty fitting models when the number of potential variables exceeds the number of observations.

We use vl substitute to create interactions of all of our factor-variable indicators with our continuous variables.

```
. vl substitute interact = i.factors##c.vlcontinuous
```

We fit the lasso.

```
. lasso linear q104 ($idemographics) $interact if sample == 1, rseed(1234)
note: 1.q32#c.q70 omitted because of collinearity with another variable.
note: 2.q34#c.q63 omitted because of collinearity with another variable.
 (output omitted)
10-fold cross-validation with 100 lambdas ...
Grid value 1:
                 lambda = 1.020288
                                     no. of nonzero coef. =
                                                                  Δ
Folds: 1...5....10 CVF = 16.93478
 (output omitted)
Grid value 34:
                  lambda = .2198144
                                      no. of nonzero coef. =
                                                                106
Folds: 1...5....10 CVF = 12.91285
... cross-validation complete ... minimum found
Lasso linear model
                                                                        458
                                            No. of obs
                                                               =
                                            No. of covariates =
                                                                      7,227
Selection: Cross-validation
                                            No. of CV folds
                                                               =
                                                                         10
```

ID	Description	lambda	No. of nonzero coef.	Out-of- sample R-squared	CV mean prediction error
1	first lambda	1.020288	4	0.0146	16.93478
29	lambda before	.2773743	80	0.2531	12.83525
* 30	selected lambda	.2647672	85	0.2545	12.81191
31	lambda after	.2527331	89	0.2541	12.81893
34	last lambda	.2198144	106	0.2486	12.91285

\* lambda selected by cross-validation.

. estimates store big

There were 7,227 potential covariates in our model, of which lasso selected 85. That seems significantly more than the 49 selected by our earlier lasso.

Let's see how they do for out-of-sample prediction.

lease of linear him and (see 1.) westerlastic

. lassogof linearcv big, over(sample) postselection							
Postselection coefficients							
Name	sample	MSE	R-squared	Obs			
linearcv							
	Training	8.652771	0.5065	503			
	Testing	14.58354	0.2658	493			
big							
	Training	6.705183	0.6117	490			
	Testing	17.00972	0.1403	478			

Our model with thousands of potential covariates did better for in-sample prediction but significantly worse for out-of-sample prediction.

### Factor variables in lasso

It is important to understand how lasso handles factor variables. Let's say we have a variable, region, that has four categories representing four different regions of the country. Other Stata estimation commands handle factor variables by setting one of the categories to be the base level; it then makes indicator variables for the other three categories, and they become covariates for the estimation.

Lasso does not set a base level. It creates indicator variables for all levels (1.region, 2.region, 3.region, and 4.region) and adds these to the set of potential covariates. The reason for this should be clear. What if 1.region versus the other three categories is all that matters for prediction? Lasso would select 1.region and not select the other three indicators. If, however, 1.region was set as a base level and omitted from the set of potential covariates, then lasso would have to select 2.region, 3.region, and 4.region to pick up the 1.region effect. It might be wasting extra penalty on three coefficients when only one was needed.

```
See [LASSO] Collinear covariates.
```

# Lasso logit and probit models

lasso will also fit logit, probit, Poisson, and Cox models.

We fit a logit model.

```
. lasso logit q106 $idemographics $ifactors $vlcontinuous
> if sample == 1, rseed(1234)
10-fold cross-validation with 100 lambdas ...
Grid value 1:
                   lambda = .1155342
                                        no. of nonzero coef. =
                                                                  0
Folds: 1...5....10
                      CVF = 1.384878
 (output omitted)
Grid value 27:
                   lambda =
                            .010285
                                        no. of nonzero coef. = 88
                     CVF = 1.147343
Folds: 1...5....10
... cross-validation complete ... minimum found
Lasso logit model
                                              No. of obs
                                                                           458
                                                                 =
                                              No. of covariates =
                                                                           277
Selection: Cross-validation
                                              No. of CV folds
                                                                 =
                                                                            10
                                            No. of
                                                         Out-of-
                                           nonzero
                                                                      CV mean
                                                          sample
      ID
               Description
                                             coef.
                                                      dev. ratio
                                                                     deviance
                                 lambda
       1
              first lambda
                                .1155342
                                                 0
                                                         -0.0004
                                                                     1.384878
                                                                     1.127315
      22
             lambda before
                               .0163767
                                                65
                                                          0.1857
      23
           selected lambda
                                                          0.1871
                                                                     1.125331
                               .0149218
                                                69
      24
              lambda after
                               .0135962
                                                73
                                                          0.1864
                                                                     1.126333
      27
               last lambda
                                 .010285
                                                88
                                                          0.1712
                                                                     1.147343
```

\* lambda selected by cross-validation.

. estimates store logit

Logit and probit lasso models are famous for having CV functions that are more wiggly than those for linear models.

. cvplot



This curve is not as smoothly convex as was the CV function for the linear lasso shown earlier. But it is not as bad as some logit CV functions. Because the CV functions for nonlinear models are not as smooth, lasso has a stricter criterion for declaring that a minimum of the CV function is found than it has for linear models. lasso requires that five smaller  $\lambda$ 's to the right of a nominal minimum be observed with larger CV function values by a relative difference of cvtolerance(#) or more. Linear models only require three such  $\lambda$ 's be found before declaring a minimum and stopping.

Let's now fit a probit model.

```
. lasso probit q106 $idemographics $ifactors $vlcontinuous
> if sample == 1, rseed(1234)
10-fold cross-validation with 100 lambdas ...
Grid value 1:
                 lambda = .1844415
                                     no. of nonzero coef. =
                                                               Ο
Folds: 1...5....10 CVF = 1.384877
 (output omitted)
Grid value 26:
                 lambda = .0180201
                                      no. of nonzero coef. = 87
Folds: 1...5....10 CVF = 1.152188
... cross-validation complete ... minimum found
Lasso probit model
                                            No. of obs
                                                                        458
                                                              =
                                            No. of covariates =
                                                                        277
Selection: Cross-validation
                                            No. of CV folds
                                                             =
                                                                        10
```

ID	Description	lambda	No. of nonzero coef.	Out-of- sample dev. ratio	CV mean deviance
1	first lambda	.1844415	0	-0.0004	1.384877
21	lambda before	.0286931	61	0.1820	1.132461
* 22	selected lambda	.0261441	64	0.1846	1.128895
23	lambda after	.0238215	70	0.1841	1.129499
26	last lambda	.0180201	87	0.1677	1.152188

\* lambda selected by cross-validation.

. estimates store probit

lassocoef can be used to display coefficient values. Obviously, logit and probit coefficient values cannot be compared directly. But we do see similar relative scales.

. lassocoef logit probit, sort(coef, standardized) display(coef, standardized)

	logit	probit
q142 No	50418	3065817
q154 No	3875702	2344515
q90 No	3771052	2288992
q8 No	3263827	200673
(output omitted	!)	
q37 No 2.q158 3.q65 3.q110 q120 0.q146	0128537 .0065661 0062113 0055616 .0044864 004312	
q95 3	.0030261	
Legend: b - base lev e - empty ce		

```
o - omitted
```

The probit lasso selected five fewer variables than logit, and they were the five variables with the smallest absolute values of standardized coefficients.

We look at how they did for out-of-sample prediction.

```
. lassogof logit probit, over(sample)
Penalized coefficients
```

Name	sample	Deviance	Deviance ratio	Obs
logit	Training	.8768969	0.3674	499
	Testing	1.268346	0.0844	502
probit	Training	.8833892	0.3627	500
	Testing	1.27267	0.0812	503

Neither did very well. The out-of-sample deviance ratios were notably worse than the in-sample values. The deviance ratio for nonlinear models is analogous to  $R^2$  for linear models. See *Methods and formulas* for [LASSO] **lassogof** for the formal definition.

We did not specify the postselection option in this case because there are no theoretical grounds for using postselection coefficients for prediction with nonlinear models.

## Lasso Poisson models

Next, we fit a Poisson model.

```
. lasso poisson q107 $idemographics $ifactors $vlcontinuous
> if sample == 1, rseed(1234)
10-fold cross-validation with 100 lambdas ...
Grid value 1: lambda = .5745539
                                    no. of nonzero coef. =
                                                              0
Folds: 1...5....10 CVF = 2.049149
 (output omitted)
Grid value 21:
                 lambda = .089382
                                     no. of nonzero coef. = 66
Folds: 1...5....10 CVF = 1.653376
... cross-validation complete ... minimum found
Lasso Poisson model
                                           No. of obs
                                                             =
                                                                      458
                                           No. of covariates =
                                                                      277
Selection: Cross-validation
                                           No. of CV folds =
                                                                       10
```

ID	Description	lambda	No. of nonzero coef.	Out-of- sample dev. ratio	CV mean deviance
1	first lambda	.5745539	0	-0.0069	2.049149
16	lambda before	.1423214	37	0.1995	1.629222
* 17	selected lambda	.129678	45	0.1999	1.628315
18	lambda after	.1181577	48	0.1993	1.62962
21	last lambda	.089382	66	0.1876	1.653376

\* lambda selected by cross-validation.

We see how it does for out-of-sample prediction.

```
. lassogof, over(sample)
```

```
Penalized coefficients
```

sample	Deviance	Deviance ratio	Obs
Training	1.289175	0.3515	510
Testing	1.547816	0.2480	502

Its in-sample and out-of-sample predictions are fairly close. Much closer than they were for the logit and probit models.

#### Lasso Cox models

lasso will also fit Cox proportional hazards models. We illustrate lasso cox with an example that predicts risk of death for stage I lung adenocarcinoma patients. Lung adenocarcinoma is one of the most common non-small-cell lung cancers.

Stage I adenocarcinoma indicates that the tumor size is relatively small, and cancer has not spread to other distant organs. Stage I adenocarcinoma patients usually have varied survival outcomes even though they are in the early cancer development stage. For example, Yu et al. (2016) show that, in one cohort, more than 50% of stage I adenocarcinoma patients died within 5 years after the initial diagnosis, while about 15% of the patients survived for more than 10 years.

Histopathology image features are indispensable for prognostic analysis. Examples of the histopathology image features include image granularity, image intensity, cell size and shape, pixel intensity of the cell, cell texture, area occupied by cells, neighboring relation of the cells, nucleus size and shape, and nucleus texture. We can use lasso cox to extract the top histopathology image features that distinguish short-term survivors from long-term survivors.

We have a fictitious survival dataset (lungcancer.dta) inspired by Yu et al. (2016). The variable t records either the time of death or censoring in months for stage I adenocarcinoma lung cancer patients. The indicator variable died is 1 or 0 if the patient died or is censored, respectively. There are 500 histopathology image features, histfeature1 to hisfeature500, and only 250 patients. The analysis aims to classify a new patient into a low-risk or high-risk group, given the histopathology image features.

We first load the dataset and then type stset to show it has already been stset.

```
. use https://www.stata-press.com/data/r19/lungcancer
(Fictitious data on stage I adenocarcinoma lung cancer)
. stset
-> stset t, failure(died)
Survival-time data settings
         Failure event: died!=0 & died<.
Observed time interval: (0, t]
    Exit on or before: failure
        250 total observations
         0 exclusions
        250 observations remaining, representing
        211 failures in single-record/single-failure data
 18,465.093 total analysis time at risk and under observation
                                                At risk from t =
                                                                          0
                                     Earliest observed entry t =
                                                                          0
                                          Last observed exit t =
                                                                        260
```

Next, we need to split the entire sample into training and testing data. The training data will be used for estimation, and the testing data will be used to measure the prediction performance. These steps are typically used in the microarray survival literature; for an application to the performance of a Cox model with lasso, see Sohn et al. (2009).

We use splitsample to split the data into two parts. The generate(group) option creates a new variable group for the identification of the training and testing data. That is, group equals 1 if it belongs to the training data or 0 if it belongs to the testing data. The split(0.60.4) option specifies that 60% of the entire data be used as training data and 40% of them be used as testing data. To make the results reproducible, we specify the rseed() option.

. splitsample, generate(group) split(0.6 0.4) rseed(12345)

For the convenience of later use, we separately save the training data (lungcancer\_training.dta) and the testing data (lungcancer\_testing.dta).

8.922501

8.042941

8.039609

8.05246 8.105045

```
. preserve
. keep if group == 1
(100 observations deleted)
. save lungcancer_training
file lungcancer_training.dta saved
. restore
. preserve
. keep if group == 2
(150 observations deleted)
. save lungcancer testing
file lungcancer_testing.dta saved
. restore
```

We are now ready to fit a lasso cox model using only the training data. By default, we use crossvalidation. We specify rseed() to make the results reproducible.

```
. use lungcancer_training, clear
(Fictitious data on stage I adenocarcinoma lung cancer)
. lasso cox histfeature*, rseed(12345671)
        Failure _d: died
  Analysis time _t: t
10-fold cross-validation with 100 lambdas ...
                 lambda = .3539123
                                      no. of nonzero coef. =
Grid value 1:
                                                               Λ
Folds: 1...5....10 CVF = 8.922501
                                     no. of nonzero coef. =
Grid value 2:
                 lambda = .3378265
                                                               1
Folds: 1...5....10
                    CVF = 8.917438
 (output omitted)
Grid value 30:
                  lambda = .0918411
                                      no. of nonzero coef. = 45
Folds: 1...5....10
                    CVF = 8.042941
Grid value 31:
                 lambda = .0876668
                                      no. of nonzero coef. =
                                                              48
Folds: 1...5....10
                    CVF = 8.039609
Grid value 32:
                 lambda = .0836822
                                      no. of nonzero coef. =
                                                              52
Folds: 1...5....10 CVF = 8.05246
Grid value 33:
                 lambda = .0798787
                                      no. of nonzero coef. =
                                                              57
Folds: 1...5....10
                    CVF = 8.070293
Grid value 34:
                 lambda = .0762481
                                      no. of nonzero coef. =
                                                              63
                    CVF = 8.105045
Folds: 1...5....10
... cross-validation complete ... minimum found
Lasso Cox model
                                            No. of obs
                                                              =
                                                                       150
                                            No. of covariates =
                                                                       500
Selection: Cross-validation
                                            No. of CV folds
                                                                        10
                                          No. of
                                         nonzero
                                                    In-sample
                                                                   CV mean
      ID
               Description
                                lambda
                                           coef.
                                                   dev. ratio
                                                                  deviance
```

.3539123

.0918411

.0876668

.0836822

.0762481

0

45

48

52

63

0.0000

0.2199

0.2306

0.2419

0.2662

last lambda \* lambda selected by cross-validation.

first lambda

lambda after

lambda before

selected lambda

1

30

32

34

\* 31

lasso cox selects 48 of the 500 features. We can now predict the relative-hazard ratio, which we will call riskscore\_training, and evaluate risk scores. We will use the median of riskscore\_training as a threshold to classify a patient as low risk or high risk. We store the median value in a global macro (median) for later use.

```
. predict riskscore_training
(options hr penalized assumed; predicted hazard ratio with penalized
coefficients)
. summarize riskscore training, detail
              Predicted hazard ratio, penalized
      Percentiles
                       Smallest
1%
         .054982
                       .0414753
5%
        .0838301
                        .054982
10%
        .1308778
                       .0702972
                                                            150
                                       Obs
25%
        .3676802
                       .0727958
                                       Sum of wgt.
                                                            150
50%
        .9458244
                                       Mean
                                                      1.998198
                                       Std. dev.
                                                       3.75226
                        Largest
75%
        2.368032
                       9.962103
90%
        4.912702
                       11.13334
                                       Variance
                                                      14.07945
95%
        6.651043
                        12.4411
                                       Skewness
                                                      7.054249
99%
         12.4411
                       39.40631
                                                      67.68195
                                       Kurtosis
. global median = r(p50)
```

Based on the median of the predicted risk ratio in the training data, we now use the testing data to validate the model. First, we predict the risk ratio in the testing sample, which we will call riskscore\_testing. Then, we compare riskscore\_testing with the median of the risk ratio obtained in the training data (\$median). If the predicted risk score is greater than or equal to the median, the patient is labeled as high risk. If the predicted risk score is less than the median, the patient is classified as low risk.

```
. use lungcancer_testing, clear
(Fictitious data on stage I adenocarcinoma lung cancer)
. predict riskscore_testing
(options hr penalized assumed; predicted hazard ratio with penalized
coefficients)
. generate byte risk = (riskscore_testing >= $median)
. label define risk_lb 1 "High risk" 0 "Low risk"
. label values risk risk lb
```

To evaluate the effectiveness of risk classification, we first look at the Kaplan–Meier plot, which draws the survival curve for both low-risk and high-risk groups.



The graph shows that the predicted high-risk patients have a more steeply falling survival curve than the predicted low-risk patients. To confirm this conjecture, we do a log-rank test.

```
. sts test risk
        Failure _d: died
  Analysis time _t: t
Equality of survivor functions
Log-rank test
             Observed
                              Expected
risk
                events
                                events
Low risk
                    39
                                 68.17
High risk
                    51
                                 21.83
    Total
                    90
                                 90.00
                     chi2(1) =
                                 61.50
                     Pr>chi2 = 0.0000
```

The log-rank test rejects the hypothesis that the predicted low-risk and high-risk patients have the same survival functions. Both the Kaplan–Meier plot and the log-rank test show that using the predicted hazard ratios' median can effectively distinguish a low-risk patient from a high-risk patient. We can now make prognostic predictions given new data.

The dataset (newlungcancer.dta) contains histopathology image features for some new stage I adenocarcinoma patients, but their survival time is not recorded because they are still alive. Based on the prediction model from lasso cox, we want to classify these new patients as low risk or high risk. To achieve this objective, we need to predict the new patients' hazard ratios and compare them with the median level of risk score obtained in the training data.

. use https://www.stata-press.com/data/r19/newlungcancer, clear (Fictitious new data on stage I adenocarcinoma lung cancer)						
<pre>. predict riskscore_new (options hr penalized assumed; predicted hazard ratio with penalized coefficients)</pre>						
. generate risk = (riskscore_new >= \$median)						
. label define risk_lb 1 "High risk" 0 "Low risk"						
. label values risk risk_lb						
. tabulate risk						
risk	Freq.	Percent	Cum.			
Low risk	27	54.00	54.00			
High risk	23	46.00	100.00			
Total	50	100.00				

The table of the predicted risk level shows that 27 patients are classified as low risk, while 23 patients are classified as high risk.

# References

- Sohn, I., J. Kim, S.-H. Jung, and C. Park. 2009. Gradient lasso for Cox proportional hazards model. *Bioinformatics* 25: 1775–1781. https://doi.org/10.1093/bioinformatics/btp322.
- Yu, K., C. Zhang, G. J. Berry, R. B. Altman, C. Ré, D. L. Rubin, and M. Snyder. 2016. Predicting non-small cell lung cancer prognosis by fully automated microscopic pathology image features. *Nature Communications* 7(12474). https: //doi.org/10.1038/ncomms12474.

# Also see

- [LASSO] lasso Lasso for prediction and model selection
- [LASSO] lasso fitting The process (in a nutshell) of fitting lasso models

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