Title

intro 4d — Including an expectation of a control dated by more than one period ahead

Description   Remarks and examples   Also see

Description

Expectations of control variables dated by more than one period ahead are not allowed in the equations specified in dsge because they do not appear in the required form of a structural model that can be solved for the state-space form. In this entry, we demonstrate how to fit models with these types of expectations by defining a new variable and rewriting our equations. Because the expectation dated more than one period ahead is endogenous, the new variable that we define is a new control variable.

Remarks and examples

Remarks are presented under the following headings:

The model

Equations (1)–(4) specify a model of consumption growth and growth in hours worked.

$$c_t = (1-h)w_t + hE_t c_{t+2} + r_t \tag{1}$$
$$n_t = w_t - \gamma c_t \tag{2}$$
$$w_{t+1} = \rho_w w_t + \xi_{t+1} \tag{3}$$
$$r_{t+1} = \rho_r r_t + \epsilon_{t+1} \tag{4}$$

Equation (1) specifies that consumption growth c_t is a linear combination of wage growth w_t, the expected value of consumption growth two periods ahead $E_t c_{t+2}$, and the interest rate r_t. Equation (2) specifies that the growth rate of hours worked n_t depends on wage growth and consumption growth. Equations (3) and (4) specify a first-order autoregressive process for wage growth and for the interest rate, respectively. The control variables are c_t and n_t, and the state variables are w_t and r_t.

The expected value of consumption growth two periods ahead in (1) is a problematic term because it does not fit into the structure required to solve for the state-space form. The structure requires that expectations be only of one-period ahead values. We accommodate this term by defining a new control variable Fc_t that equals $E_t(c_{t+1})$ and replacing c_{t+1} in (1) with this new control variable. We define a new control variable instead of a new state variable because the expected future consumption is endogenous instead of exogenous. These changes yield the model in (5)–(9).

$$c_t = (1-h)w_t + hE_t(Fc_{t+1}) + r_t \tag{5}$$
$$n_t = w_t - \gamma c_t \tag{6}$$
$$Fc_t = E_t(c_{t+1}) \tag{7}$$
$$w_{t+1} = \rho_w w_t + \xi_{t+1} \tag{8}$$
$$r_{t+1} = \rho_r r_t + \epsilon_{t+1} \tag{9}$$

The new (7) defines the new control variable Fc_t as the expected value of next period's consumption. Equation (5) is (1) but with c_{t+2} replaced with the new control Fc_{t+1}.

There are now three control variables and only two shocks, so we treat the new control variable Fc_t as unobserved. There is a logic to this process. The one-step ahead structure of state-space models makes it impossible to solve for terms like $E_t(c_{t+2})$. In contrast, it is possible to solve for terms like $E_t\{E_t(c_{t+1})\}$ as long as the unobserved $E_t(c_{t+1})$ is determined by another equation, which in this case is (7). In fact, this example illustrates a part of the recursive dynamic modeling approach at the heart of the DSGE approach to macroeconomics.

Parameter estimation

We estimate the parameters of the model in (5)–(9) using U.S. data on consumption growth and growth in hours worked.

```
. use http://www.stata-press.com/data/r15/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
. dsge (c   = (1-{h})*(w) + {h}*E(F.fc) + r)
>      (n   = w - {gamma}*c)
>      (fc  = E(F.c), unobserved)
>      (F.w = {rho_w}*w, state)
>      (F.r = {rho_r}*r, state)
(setting technique to bfgs)
Iteration 0:   log likelihood = -2423.7325
Iteration 1:   log likelihood = -1284.9295  (backed up)
Iteration 2:   log likelihood = -1193.2234  (backed up)
Iteration 3:   log likelihood = -1180.1787  (backed up)
Iteration 4:   log likelihood = -1175.3563  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -1171.0676  (backed up)
Iteration 6:   log likelihood = -1152.0355
Iteration 7:   log likelihood = -1134.7089
Iteration 8:   log likelihood = -1130.0791
Iteration 9:   log likelihood = -1129.9372
Iteration 10:  log likelihood = -1129.9357
Iteration 11:  log likelihood = -1129.9357
```

```
DSGE model

Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -1129.9357
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
h	.661791	.0427917	15.47	0.000	.5779208	.7456611
gamma	.273381	.1120734	2.44	0.015	.0537212	.4930408
rho_w	.6545228	.0485628	13.48	0.000	.5593415	.7497041
rho_r	.1050387	.0638171	1.65	0.100	-.0200406	.230118
sd(e.w)	2.905807	.2023204			2.509267	3.302348
sd(e.r)	2.049915	.1437862			1.7681	2.331731

The estimate of h is greater than 0.5, so we conclude that slightly more of the unobserved wage state is allocated to the expected growth in consumption two periods ahead than to current consumption.

Also see

[DSGE] **intro 2** — Learning the syntax

[DSGE] **intro 4** — Writing a DSGE in a solvable form