Title

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intro 4c — Including a lag of a state variable

Description

Lags of state variables are not allowed to be included in the structural model specified in the `dsge` command. These lags do not fit into the form of a structural model that can be solved for the state-space form. However, this restriction does not prevent us from fitting models that involve the lag of a state variable. This entry shows how to define a new state variable and rewrite the equations in the required structural form.

Remarks and examples

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Remarks are presented under the following headings:

A model with a lagged state variable

The model in (1)–(5) is a standard monetary model in which the state driving output growth y_t is a second-order autoregressive AR(2) process instead of a first-order autoregressive process.

$$p_t = \beta E_t(p_{t+1}) + \kappa y_t \tag{1}$$

$$y_t = E_t(y_{t+1}) - \{r_t - E_t(p_{t+1}) - z_t\} \tag{2}$$

$$r_t = \frac{1}{\beta} p_t + u_t \tag{3}$$

$$z_{t+1} = \rho_{z1} z_t + \rho_{z2} z_{t-1} + \epsilon_{t+1} \tag{4}$$

$$u_{t+1} = \rho_u u_t + \xi_{t+1} \tag{5}$$

Equation (1) specifies the structural equation for inflation p_t. Inflation is a linear combination of expected future inflation $E_t(p_{t+1})$ and output growth y_t. Equation (2) specifies the structural equation for output growth. Output growth is a linear combination of expected future output growth $E_t(y_{t+1})$, the interest rate r_t, expected future inflation, and the state z_t. The state z_t is an AR(2) process that drives output growth. Equation (3) specifies the structural equation for the interest rate. The interest rate depends on the inflation rate and the state u_t. The state u_t is an AR(1) process that drives the interest rate. The control variables in this model are p_t, y_t, and r_t. The state variables are u_t and z_t.

The AR(2) term in (4) is a problematic term because a lag of a state variable does not fit into the structure required to solve the model for the state-space form. We accommodate this term by defining a new state variable Lz_t that equals z_{t-1} and replacing z_{t-1} in (4) with this new state variable. We define a new state instead of new control because the lagged state is exogenous. These changes yield the model in (6)–(11).

$$
\begin{aligned}
p_t &= \beta E_t(p_{t+1}) + \kappa y_t & (6)\\
y_t &= E_t(y_{t+1}) - \{r_t - E_t(p_{t+1}) - z_t\} & (7)\\
r_t &= \frac{1}{\beta} p_t + u_t & (8)\\
z_{t+1} &= \rho_{z1} z_t + \rho_{z2} L z_t + \epsilon_{t+1} & (9)\\
u_{t+1} &= \rho_u u_t + \xi_{t+1} & (10)\\
L z_{t+1} &= z_t & (11)
\end{aligned}
$$

We will estimate the parameters in the model in (6)–(11).

Parameter estimation

We estimate the parameters in the model in (6)–(11) using data on U.S. interest rate `r` and inflation `p`.

```
. use http://www.stata-press.com/data/r15/usmacro2
(Federal Reserve Economic Data - St. Louis Fed, 2017-01-15)
. dsge (p    = {beta}*E(F.p) + {kappa}*y)
>      (y    = E(F.y) -(r - E(f.p) - z), unobserved)
>      (r    = (1/{beta})*p + u)
>      (F.u  = {rhou}*u, state)
>      (F.z  = {rhoz1}*z + {rhoz2}*Lz, state)
>      (F.Lz = z, state noshock)
(setting technique to bfgs)
Iteration 0:   log likelihood = -9116.0985
Iteration 1:   log likelihood = -997.95396  (backed up)
Iteration 2:   log likelihood = -881.53897  (backed up)
Iteration 3:   log likelihood = -836.47478  (backed up)
Iteration 4:   log likelihood = -769.37111  (backed up)
(switching technique to nr)
Iteration 5:   log likelihood = -768.80775  (not concave)
Iteration 6:   log likelihood = -760.17544  (not concave)
Iteration 7:   log likelihood = -755.99765  (not concave)
Iteration 8:   log likelihood = -754.93526  (not concave)
Iteration 9:   log likelihood = -754.42155
Iteration 10:  log likelihood =  -753.8269
Iteration 11:  log likelihood = -753.20702
Iteration 12:  log likelihood = -753.08071
Iteration 13:  log likelihood = -753.07788
Iteration 14:  log likelihood = -753.07788
```

```
DSGE model

Sample: 1955q1 - 2015q4                         Number of obs     =        244
Log likelihood = -753.07788
```

	Coef.	OIM Std. Err.	z	P>|z|	[95% Conf.	Interval]
/structural						
beta	.5154874	.0770496	6.69	0.000	.364473	.6665019
kappa	.1662425	.0468473	3.55	0.000	.0744234	.2580616
rhou	.6979877	.0452071	15.44	0.000	.6093834	.7865921
rhoz1	.6735462	.2666028	2.53	0.012	.1510143	1.196078
rhoz2	.2710021	.2564596	1.06	0.291	-.2316495	.7736537
sd(e.u)	2.315262	.2992183			1.728805	2.901719
sd(e.z)	.7720678	.1716214			.4356961	1.10844

Looking at the confidence interval on `rhoz2`, we find no evidence that the AR(2) term is needed in this model.

Also see

[DSGE] **intro 2** — Learning the syntax

[DSGE] **intro 4** — Writing a DSGE in a solvable form