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xtnbreg — Fixed-effects, random-effects, & population-averaged negative binomial models

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Description

xtnbreg fits random-effects and conditional fixed-effects overdispersion models where the random effects or fixed effects apply to the distribution of the dispersion parameter. The dispersion is the same for all observations in the same panel. In the random-effects model, the dispersion varies randomly from group to group, such that the inverse of one plus the dispersion follows a Beta distribution. In the fixed-effects model, the dispersion parameter in a group can take on any value.

xtnbreg also fits a population-averaged negative binomial model for a nonnegative count dependent variable with overdispersion.

Quick start

Random-effects negative-binomial regression of y on x and indicators for levels of categorical variable a using xtset data

```
xtnbreg y x i.a
```

Same as above, but report incidence-rate ratios

```
xtnbreg y x i.a, irr
```

Conditional fixed-effects model with exposure variable evar

```
xtnbreg y x i.a, fe exposure(evar)
```

Population-averaged model with robust standard errors

xtnbreg y x i.a, pa vce(robust)

Menu

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Syntax

maximize_options

collinear

coeflegend

```
Random-effects (RE) and conditional fixed-effects (FE) overdispersion models
    xtnbreg depvar [indepvars] [if] [in] [weight] [, [re|fe] RE/FE_options]
 Population-averaged (PA) model
    xtnbreg depvar [indepvars] [if] [in] [weight], pa [PA_options]
 RE/FE_options
                               Description
Model
                               suppress constant term; not available with fe
 noconstant
                                use random-effects estimator; the default
 re
                                use fixed-effects estimator
 fe
 exposure(varname)
                               include ln(varname) in model with coefficient constrained to 1
                               include varname in model with coefficient constrained to 1
 offset(varname)
 constraints(constraints)
                               apply specified linear constraints
SE
 vce(vcetype)
                               vcetype may be oim, bootstrap, or jackknife
Reporting
                               set confidence level: default is level(95)
 level(#)
 irr
                               report incidence-rate ratios
                               perform the likelihood-ratio model test instead of the default Wald test
 lrmodel
                               do not display constraints
 nocnsreport
 display_options
                               control columns and column formats, row spacing, line width,
                                  display of omitted variables and base and empty cells, and
                                  factor-variable labeling
Maximization
```

control the maximization process; seldom used

keep collinear variables

display legend instead of statistics

PA_options	Description
Model	
<u>nocons</u> tant	suppress constant term
pa	use population-averaged estimator
exposure(<i>varname</i>)	include ln(<i>varname</i>) in model with coefficient constrained to 1
offset(varname)	include varname in model with coefficient constrained to 1
Correlation	
corr(correlation)	within-panel correlation structure
force	estimate even if observations unequally spaced in time
SE/Robust	
vce(vcetype)	vcetype may be conventional, robust, bootstrap, or jackknife
nmp	use divisor $N-P$ instead of the default N
<u>s</u> cale(parm)	overrides the default scale parameter; $parm$ may be x2, dev, phi, or #
Reporting	
$\underline{1}$ evel(#)	set confidence level; default is level(95)
<u>ir</u> r	report incidence-rate ratios
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
optimize_options	control the optimization process; seldom used
<u>coefl</u> egend	display legend instead of statistics
correlation	Description
<u>exc</u> hangeable	exchangeable
<u>ind</u> ependent	independent
$\underline{\mathtt{un}}\mathtt{structured}$	unstructured
<u>fix</u> ed matname	user-specified
ar#	autoregressive of order #
<u>sta</u> tionary #	stationary of order #
nonstationary #	nonstationary of order #
monstationary #	nonstationary of order #

A panel variable must be specified. For xtnbreg, pa, correlation structures other than exchangeable and independent require that a time variable also be specified. Use xtset; see [XT] xtset.

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

depvar and indepvars may contain time-series operators; see [U] 11.4.4 Time-series varlists.

bayes, by, collect, mi estimate, and statsby are allowed; see [U] 11.1.10 Prefix commands. For more details, see [BAYES] bayes: xtnbreg. fp is allowed for the random-effects and fixed-effects models.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

iweights, fweights, and pweights are allowed for the population-averaged model, and iweights are allowed in the random-effects and fixed-effects models; see [U] 11.1.6 weight. Weights must be constant within panel.

collinear and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Options for RE/FE models

```
Model
    noconstant; see [R] Estimation options.
    re requests the random-effects estimator, which is the default.
    fe requests the conditional fixed-effects estimator.
    exposure(varname), offset(varname), constraints(constraints); see [R] Estimation options.
    vce(vcetype) specifies the type of standard error reported, which includes types that are derived from
      asymptotic theory (oim) and that use bootstrap or jackknife methods (bootstrap, jackknife);
      see [XT] vce_options.
         Reporting
    level(#); see [R] Estimation options.
   irr reports exponentiated coefficients e^b rather than coefficients b. For the negative binomial model,
      exponentiated coefficients have the interpretation of incidence-rate ratios.
    1rmodel, nocnsreport; see [R] Estimation options.
   display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels,
       <u>allbaselevels</u>, <u>nofvlab</u>el, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt),
      sformat(% fmt), and nolstretch; see [R] Estimation options.
         Maximization
   maximize_options: difficult, technique(algorithm_spec), iterate(#), [no]log, trace,
       gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#),
      <u>nrtolerance(#)</u>, nonrtolerance, and from(init_specs); see [R] Maximize. These options are
      seldom used.
   The following options are available with xtnbreg but are not shown in the dialog box:
    collinear, coeflegend; see [R] Estimation options.
Options for PA model
         Model
    noconstant; see [R] Estimation options.
   pa requests the population-averaged estimator.
    exposure(varname), offset(varname); see [R] Estimation options.
          Correlation
    corr(correlation) specifies the within-panel correlation structure; the default corresponds to the
```

When you specify a correlation structure that requires a lag, you indicate the lag after the structure's

name with or without a blank; for example, corr(ar 1) or corr(ar1).

equal-correlation model, corr(exchangeable).

If you specify the fixed correlation structure, you specify the name of the matrix containing the assumed correlations following the word fixed, for example, corr(fixed myr).

force specifies that estimation be forced even though the time variable is not equally spaced. This is relevant only for correlation structures that require knowledge of the time variable. These correlation structures require that observations be equally spaced so that calculations based on lags correspond to a constant time change. If you specify a time variable indicating that observations are not equally spaced, the (time dependent) model will not be fit. If you also specify force, the model will be fit, and it will be assumed that the lags based on the data ordered by the time variable are appropriate.

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce_options.

vce (conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

nmp, scale(x2 | dev | phi | #); see [XT] vce_options.

Reporting

level(#); see [R] Estimation options.

irr reports exponentiated coefficients e^b rather than coefficients b. For the negative binomial model, exponentiated coefficients have the interpretation of incidence-rate ratios.

display_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, <u>allbaselevels</u>, <u>nofvlabel</u>, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Optimization

optimize_options control the iterative optimization process. These options are seldom used.

iterate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimization stops and presents the current results, even if convergence has not been reached. The default is iterate(100).

tolerance (#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

trace specifies that the current estimates be printed at each iteration.

The following option is available with xtnbreg but is not shown in the dialog box: coeflegend; see [R] Estimation options.

Remarks and examples

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xtnbreg fits random-effects overdispersion models, conditional fixed-effects overdispersion models, and population-averaged negative binomial models. Here "random effects" and "fixed effects" apply to the distribution of the dispersion parameter, not to the $\mathbf{x}\boldsymbol{\beta}$ term in the model. In the random-effects and fixed-effects overdispersion models, the dispersion is the same for all elements in the same group (that is, elements with the same value of the panel variable). In the random-effects model, the dispersion varies randomly from group to group, such that the inverse of one plus the dispersion follows a Beta(r,s) distribution. In the fixed-effects model, the dispersion parameter in a group can take on any value, because a conditional likelihood is used in which the dispersion parameter drops out of the estimation.

By default, the population-averaged model is an equal-correlation model; xtnbreg, pa assumes corr(exchangeable). Thus, xtnbreg is a convenience command for fitting the population-averaged using xtgee; see [XT] xtgee. Typing

```
. xtnbreg ..., ... pa exposure(time)
is equivalent to typing
. xtgee ..., ... family(nbinomial) link(log) corr(exchangeable) exposure(time)
```

See also [XT] **xtgee** for information about xtnbreg.

By default, or when re is specified, xtnbreg fits a maximum-likelihood random-effects overdispersion model.

▶ Example 1

You have (fictional) data on injury "incidents" incurred among 20 airlines in each of 4 years. (Incidents range from major injuries to exceedingly minor ones.) The government agency in charge of regulating airlines has run an experimental safety training program, and, in each of the years, some airlines have participated and some have not. You now wish to analyze whether the "incident" rate is affected by the program. You choose to estimate using random-effects negative binomial regression, as the dispersion might vary across the airlines for unidentified airline-specific reasons. Your measure of exposure is passenger miles for each airline in each year.

```
. use https://www.stata-press.com/data/r18/airacc
. xtnbreg i_cnt inprog, exposure(pmiles) irr
Fitting negative binomial (constant dispersion) model:
Iteration 0: Log likelihood = -293.57997
Iteration 1: Log likelihood = -293.57997
 (output omitted)
Fitting full model:
Iteration 0: Log likelihood = -295.72633
Iteration 1: Log likelihood = -270.49929 (not concave)
 (output omitted)
Random-effects negative binomial regression
                                                       Number of obs
                                                                               80
Group variable: airline
                                                       Number of groups =
Random effects u_i ~ Beta
                                                       Obs per group:
                                                                    min =
                                                                              4.0
                                                                    avg =
                                                                    max =
                                                                                4
                                                       Wald chi2(1)
                                                                             2.04
Log likelihood = -265.38202
                                                       Prob > chi2
                                                                        = 0.1532
                       IRR
                             Std. err.
                                                  P>|z|
                                                            [95% conf. interval]
       i_cnt
                                            7.
      inprog
                   .911673
                             .0590277
                                         -1.43
                                                  0.153
                                                            .8030206
                                                                         1.035027
       _cons
                  .0367524
                             .0407032
                                         -2.98
                                                  0.003
                                                            .0041936
                                                                         .3220983
  ln(pmiles)
                            (exposure)
       /ln_r
                  4.794991
                              .951781
                                                            2.929535
                                                                        6.660448
                  3.268052
                             .4709033
                                                            2.345098
                                                                        4.191005
       /ln_s
                  120.9033
                             115.0735
                                                            18.71892
                                                                        780.9007
           r
                  26.26013
                             12.36598
                                                             10.4343
                                                                        66.08918
```

Note: Estimates are transformed only in the first equation to incidence-rate

Note: _cons estimates baseline incidence rate (conditional on zero random effects).

LR test vs. pooled: chibar2(01) = 19.03 Prob >= chibar2 = 0.000

In the output above, the $/\ln_r$ and $/\ln_s$ lines refer to $\ln(r)$ and $\ln(s)$, where the inverse of one plus the dispersion is assumed to follow a Beta(r,s) distribution. The output also includes a likelihood-ratio test, which compares the panel estimator with the pooled estimator (that is, a negative binomial estimator with constant dispersion).

You find that the incidence rate for accidents is not significantly different for participation in the program and that the panel estimator is significantly different from the pooled estimator.

We may alternatively fit a fixed-effects overdispersion model:

. xtnbreg i_cnt inprog, exposure(pmiles) irr fe nolog

Conditional FE negative binomial regression Number of obs Group variable: airline Number of groups = 20

Obs per group:

4 min =avg = 4.0 max =

Wald chi2(1) = 2.11 Prob > chi2 = 0.1463

Log likelihood = -174.25143

i_cnt	IRR	Std. err.	z	P> z	[95% conf.	interval]
inprog _cons ln(pmiles)	.9062669 .0329025 1	.0613917 .0331262 (exposure)	-1.45 -3.39	0.146 0.001	.793587 .0045734	1.034946 .2367111

Note: _cons estimates baseline incidence rate (conditional on zero random effects).

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Example 2

We rerun our previous example, but this time we fit a robust equal-correlation population-averaged model:

. xtnbreg i_cnt inprog, exposure(pmiles) irr vce(robust) pa

Iteration 1: Tolerance = .02499392 Iteration 2: Tolerance = .0000482 Iteration 3: Tolerance = 2.929e-07

GEE population-averaged model Number of obs 80 Group variable: airline Number of groups = 20

Family: Negative binomial(k=1) Obs per group:

Link: Log min =

Correlation: exchangeable avg = 4.0 4 max =

Wald chi2(1) 1.28 Scale parameter = 1 Prob > chi2 = 0.2571

(Std. err. adjusted for clustering on airline)

i_c	nt	IRR	Semirobust std. err.	z	P> z	[95% conf.	interval]
inproco: ln(pmile:	ns	.927275 .0080211 1	.0617857 .0004117 (exposure)	-1.13 -94.02	0.257 0.000	.8137513 .0072535	1.056636 .00887

Note: _cons estimates baseline incidence rate (conditional on zero random effects).

We compare this with a pooled estimator with clustered robust-variance estimates:

```
. nbreg i_cnt inprog, exposure(pmiles) irr vce(cluster airline)
```

Fitting Poisson model:

Iteration 0: Log pseudolikelihood = -293.57997 Iteration 1: Log pseudolikelihood = -293.57997

Fitting constant-only model:

Iteration 0: Log pseudolikelihood = -335.13615 Iteration 1: Log pseudolikelihood = -279.43327 Iteration 2: Log pseudolikelihood = -276.09296 Iteration 3: Log pseudolikelihood = -274.84036 Iteration 4: Log pseudolikelihood = -274.81076 Iteration 5: Log pseudolikelihood = -274.81075

Fitting full model:

Iteration 0: Log pseudolikelihood = -274.56985 Iteration 1: Log pseudolikelihood = -274.55077 Iteration 2: Log pseudolikelihood = -274.55077

Negative binomial regression

Wald chi2(1) =0.60 Dispersion: mean Prob > chi2 = 0.4369Log pseudolikelihood = -274.55077Pseudo R2 = 0.0009

(Std. err. adjusted for 20 clusters in airline)

Number of obs =

80

i_cnt	IRR	Robust std. err.	z	P> z	[95% conf.	interval]
inprog _cons ln(pmiles)	.9429015 .007956	.0713091 .0004237 (exposure)	-0.78 -90.77	0.437 0.000	.8130032 .0071674	1.093555 .0088314
/lnalpha	-2.835089	.3351784			-3.492027	-2.178151
alpha	.0587133	.0196794			.0304391	.1132507

Note: Estimates are transformed only in the first equation to incidence-rate ratios.

Note: _cons estimates baseline incidence rate.

Stored results

xtnbreg, re stores the following in e():

```
Scalars
    e(N)
                              number of observations
    e(N_g)
                              number of groups
    e(k)
                              number of parameters
                              number of auxiliary parameters
    e(k_aux)
    e(k_eq)
                              number of equations in e(b)
                              number of equations in overall model test
    e(k_eq_model)
    e(k_dv)
                              number of dependent variables
    e(df_m)
                              model degrees of freedom
    e(11)
                              log likelihood
    e(11_0)
                              log likelihood, constant-only model
    e(11_c)
                              log likelihood, comparison model
    e(chi2)
                              \chi^2 for comparison test
    e(chi2_c)
    e(g_min)
                              smallest group size
    e(g_avg)
                              average group size
    e(g_max)
                              largest group size
    e(r)
                              value of r in Beta(r, s)
                              value of s in Beta(r, s)
    e(s)
                              p-value for model test
    e(p)
                              rank of e(V)
    e(rank)
    e(rank0)
                              rank of e(V) for constant-only model
    e(ic)
                              number of iterations
    e(rc)
                              return code
    e(converged)
                              1 if converged, 0 otherwise
Macros
    e(cmd)
                              xtnbreg
    e(cmdline)
                              command as typed
    e(depvar)
                              name of dependent variable
    e(ivar)
                              variable denoting groups
    e(model)
                              re
    e(wtype)
                              weight type
                              weight expression
    e(wexp)
    e(title)
                              title in estimation output
    e(offset)
                              linear offset variable
                              Wald or LR; type of model \chi^2 test Wald or LR; type of model \chi^2 test corresponding to e(chi2_c)
    e(chi2type)
    e(chi2_ct)
                              vcetype specified in vce()
    e(vce)
    e(method)
                              estimation method
    e(distrib)
                              Beta; the distribution of the random effect
    e(opt)
                              type of optimization
    e(which)
                              max or min; whether optimizer is to perform maximization or minimization
    e(ml_method)
                              type of ml method
    e(user)
                              name of likelihood-evaluator program
    e(technique)
                              maximization technique
    e(properties)
                              program used to implement predict
    e(predict)
    e(asbalanced)
                              factor variables fvset as asbalanced
    e(asobserved)
                              factor variables fyset as asobserved
Matrices
    e(b)
                              coefficient vector
    e(Cns)
                              constraints matrix
    e(ilog)
                              iteration log
    e(gradient)
                              gradient vector
    e(V)
                              variance-covariance matrix of the estimators
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
                                 matrix containing the coefficients with their standard errors, test statistics, p-values,
    r(table)
                                     and confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtnbreg, fe stores the following in e():

```
Scalars
    e(N)
                              number of observations
                              number of groups
    e(N_g)
    e(k)
                             number of parameters
                              number of equations in e(b)
    e(k_eq)
                              number of equations in overall model test
    e(k_eq_model)
                              number of dependent variables
    e(k_dv)
    e(df_m)
                              model degrees of freedom
    e(11)
                              log likelihood
    e(11_0)
                              log likelihood, constant-only model
    e(chi2)
                              \chi^2
    e(g_min)
                             smallest group size
    e(g_avg)
                             average group size
    e(g_max)
                              largest group size
                             p-value for model test
    e(p)
    e(rank)
                             rank of e(V)
    e(ic)
                             number of iterations
    e(rc)
                             return code
    e(converged)
                              1 if converged, 0 otherwise
Macros
    e(cmd)
                             xtnbreg
                             command as typed
    e(cmdline)
                             name of dependent variable
    e(depvar)
    e(ivar)
                              variable denoting groups
    e(model)
    e(wtype)
                             weight type
    e(wexp)
                              weight expression
    e(title)
                             title in estimation output
    e(offset)
                             linear offset variable
                             LR; type of model \chi^2 test
    e(chi2type)
                              vcetype specified in vce()
    e(vce)
                             requested estimation method
    e(method)
                             type of optimization
    e(opt)
                             max or min; whether optimizer is to perform maximization or minimization
    e(which)
    e(ml_method)
                             type of ml method
                             name of likelihood-evaluator program
    e(user)
    e(technique)
                             maximization technique
                             b V
    e(properties)
    e(predict)
                             program used to implement predict
    e(asbalanced)
                              factor variables fyset as asbalanced
    e(asobserved)
                             factor variables fyset, as asobserved
Matrices
    e(b)
                             coefficient vector
    e(Cns)
                             constraints matrix
    e(ilog)
                             iteration log
    e(gradient)
                             gradient vector
                             variance-covariance matrix of the estimators
    e(V)
Functions
    e(sample)
                             marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
r(table) matrix containing the coefficients with their standard errors, test statistics, p-values,
and confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

xtnbreg, pa stores the following in e():

```
Scalars
    e(N)
                              number of observations
                              number of groups
    e(N_g)
    e(df_m)
                             model degrees of freedom
    e(chi2)
    e(p)
                              p-value for model test
                             degrees of freedom for Pearson \chi^2
    e(df_pear)
    e(chi2_dev)
                              \chi^2 test of deviance
                             \chi^2 test of deviance dispersion
    e(chi2_dis)
    e(deviance)
                             deviance
    e(dispers)
                             deviance dispersion
                             scale parameter
    e(phi)
    e(g_min)
                             smallest group size
    e(g_avg)
                             average group size
    e(g_max)
                             largest group size
                             rank of e(V)
    e(rank)
    e(tol)
                             target tolerance
    e(dif)
                             achieved tolerance
    e(rc)
                             return code
Macros
    e(cmd)
                             xtgee
    e(cmd2)
                             xtnbreg
    e(cmdline)
                             command as typed
    e(depvar)
                             name of dependent variable
    e(ivar)
                             variable denoting groups
                             variable denoting time within groups
    e(tvar)
    e(model)
    e(family)
                             negative binomial (k=1)
    e(link)
                             log; link function
    e(corr)
                             correlation structure
                             x2, dev, phi, or #; scale parameter
    e(scale)
                             weight type
    e(wtype)
                             weight expression
    e(wexp)
                             linear offset variable
    e(offset)
                             Wald; type of model \chi^2 test
    e(chi2type)
                             vcetype specified in vce()
    e(vce)
                             title used to label Std. err.
    e(vcetype)
                             nmp, if specified
    e(nmp)
    e(nbalpha)
                             \alpha
    e(properties)
    e(predict)
                             program used to implement predict
    e(marginsnotok)
                              predictions disallowed by margins
    e(asbalanced)
                              factor variables fvset as asbalanced
                              factor variables fyset as asobserved
    e(asobserved)
Matrices
    e(b)
                             coefficient vector
                             estimated working correlation matrix
    e(R)
                             variance-covariance matrix of the estimators
    e(V)
    e(V_modelbased)
                             model-based variance
```

Functions
e(sample) marks estimation sample

In addition to the above, the following is stored in r():

Matrices

r(table) matrix containing the coefficients with their standard errors, test statistics, p-values, and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

Methods and formulas

xtnbreg, pa reports the population-averaged results obtained by using xtgee, family(nbinomial) link(log) to obtain estimates. See [XT] xtgee for details on the methods and formulas.

For the random-effects and fixed-effects overdispersion models, let y_{it} be the count for the tth observation in the ith group. We begin with the model $y_{it} \mid \gamma_{it} \sim \text{Poisson}(\gamma_{it})$, where $\gamma_{it} \mid \delta_i \sim \text{gamma}(\lambda_{it}, \delta_i)$ with $\lambda_{it} = \exp(\mathbf{x}_{it}\boldsymbol{\beta} + \text{offset}_{it})$ and δ_i is the dispersion parameter. This yields the model

$$\Pr(Y_{it} = y_{it} \mid \mathbf{x}_{it}, \delta_i) = \frac{\Gamma(\lambda_{it} + y_{it})}{\Gamma(\lambda_{it})\Gamma(y_{it} + 1)} \left(\frac{1}{1 + \delta_i}\right)^{\lambda_{it}} \left(\frac{\delta_i}{1 + \delta_i}\right)^{y_{it}}$$

(See Hausman, Hall, and Griliches [1984, eq. 3.1, 922]; our δ is the inverse of their δ .) Looking at within-panel effects only, we find that this specification yields a negative binomial model for the *i*th group with dispersion (variance divided by the mean) equal to $1 + \delta_i$, that is, constant dispersion within group. This parameterization of the negative binomial model differs from the default parameterization of nbreg, which has dispersion equal to $1 + \alpha \exp(\mathbf{x}\beta + \text{offset})$; see [R] nbreg.

For a random-effects overdispersion model, we allow δ_i to vary randomly across groups; namely, we assume that $1/(1+\delta_i) \sim \text{Beta}(r,s)$. The joint probability of the counts for the *i*th group is

$$\Pr(Y_{i1} = y_{i1}, \dots, Y_{in_i} = y_{in_i} | \mathbf{X}_i) = \int_0^\infty \prod_{t=1}^{n_i} \Pr(Y_{it} = y_{it} | \mathbf{x}_{it}, \delta_i) f(\delta_i) d\delta_i$$

$$= \frac{\Gamma(r+s)\Gamma(r + \sum_{t=1}^{n_i} \lambda_{it})\Gamma(s + \sum_{t=1}^{n_i} y_{it})}{\Gamma(r)\Gamma(s)\Gamma(r + s + \sum_{t=1}^{n_i} \lambda_{it} + \sum_{t=1}^{n_i} y_{it})} \prod_{t=1}^{n_i} \frac{\Gamma(\lambda_{it} + y_{it})}{\Gamma(\lambda_{it})\Gamma(y_{it} + 1)}$$

for $\mathbf{X}_i = (\mathbf{x}_{i1}, \dots, \mathbf{x}_{in_i})$ and where f is the probability density function for δ_i . The resulting log likelihood is

$$\begin{split} \ln L &= \sum_{i=1}^n w_i \bigg[\ln \Gamma(r+s) + \ln \Gamma \bigg(r + \sum_{k=1}^{n_i} \lambda_{ik} \bigg) + \ln \Gamma \bigg(s + \sum_{k=1}^{n_i} y_{ik} \bigg) - \ln \Gamma(r) - \ln \Gamma(s) \\ &- \ln \Gamma \bigg(r + s + \sum_{k=1}^{n_i} \lambda_{ik} + \sum_{k=1}^{n_i} y_{ik} \bigg) + \sum_{t=1}^{n_i} \bigg\{ \ln \Gamma(\lambda_{it} + y_{it}) - \ln \Gamma(\lambda_{it}) - \ln \Gamma(y_{it} + 1) \bigg\} \bigg] \end{split}$$

where $\lambda_{it} = \exp(\mathbf{x}_{it}\boldsymbol{\beta} + \text{offset}_{it})$ and w_i is the weight for the *i*th group (Hausman, Hall, and Griliches 1984, eq. 3.5, 927).

For the fixed-effects overdispersion model, we condition the joint probability of the counts for each group on the sum of the counts for the group (that is, the observed $\sum_{t=1}^{n_i} y_{it}$). This yields

$$\Pr(Y_{i1} = y_{i1}, \dots, Y_{in_i} = y_{in_i} \mid \mathbf{X}_i, \sum_{t=1}^{n_i} Y_{it} = \sum_{t=1}^{n_i} y_{it})$$

$$= \frac{\Gamma(\sum_{t=1}^{n_i} \lambda_{it}) \Gamma(\sum_{t=1}^{n_i} y_{it} + 1)}{\Gamma(\sum_{t=1}^{n_i} \lambda_{it} + \sum_{t=1}^{n_i} y_{it})} \prod_{t=1}^{n_i} \frac{\Gamma(\lambda_{it} + y_{it})}{\Gamma(\lambda_{it}) \Gamma(y_{it} + 1)}$$

The conditional log likelihood is

$$\begin{split} \ln L &= \sum_{i=1}^n w_i \Bigg[\ln \Gamma \left(\sum_{t=1}^{n_i} \lambda_{it} \right) + \ln \Gamma \left(\sum_{t=1}^{n_i} y_{it} + 1 \right) - \ln \Gamma \left(\sum_{t=1}^{n_i} \lambda_{it} + \sum_{t=1}^{n_i} y_{it} \right) \\ &+ \sum_{t=1}^{n_i} \bigg\{ \ln \Gamma (\lambda_{it} + y_{it}) - \ln \Gamma (\lambda_{it}) - \ln \Gamma (y_{it} + 1) \bigg\} \Bigg] \end{split}$$

See Hausman, Hall, and Griliches (1984) for a more thorough development of the random-effects and fixed-effects models. Also see Cameron and Trivedi (2013) for a good textbook treatment of this model.

References

Cameron, A. C., and P. K. Trivedi. 2013. Regression Analysis of Count Data. 2nd ed. New York: Cambridge University Press.

Guimarães, P. 2005. A simple approach to fit the beta-binomial model. Stata Journal 5: 385-394.

Hausman, J. A., B. H. Hall, and Z. Griliches. 1984. Econometric models for count data with an application to the patents–R & D relationship. *Econometrica* 52: 909–938. https://doi.org/10.2307/1911191.

Liang, K.-Y., and S. L. Zeger. 1986. Longitudinal data analysis using generalized linear models. *Biometrika* 73: 13–22. https://doi.org/10.1093/biomet/73.1.13.

Also see

[XT] xtnbreg postestimation — Postestimation tools for xtnbreg

[XT] **xtgee** — GEE population-averaged panel-data models

[XT] xtpoisson — Fixed-effects, random-effects, and population-averaged Poisson models

[XT] **xtset** — Declare data to be panel data

[BAYES] bayes: xtnbreg — Bayesian random-effects negative binomial model

[ME] menbreg — Multilevel mixed-effects negative binomial regression

[MI] Estimation — Estimation commands for use with mi estimate

[R] **nbreg** — Negative binomial regression

[U] 20 Estimation and postestimation commands

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