Title stata.com

**xtgee** — GEE population-averaged panel-data models

Description Quick start Menu Syntax

Options Remarks and examples Stored results Methods and formulas

References Also see

# **Description**

xtgee fits population-averaged panel-data models. In particular, xtgee fits generalized linear models and allows you to specify the within-group correlation structure for the panels.

# **Quick start**

Population-averaged linear regression of y on x1 and x2

xtgee y x1 x2

Same as above, but estimate time-varying intragroup correlations

xtgee y x1 x2, corr(unstructured)

Same as above, but estimate a common second-order autoregression structure for the within-panel correlation

xtgee y x1 x2, corr(ar 2)

Population-averaged negative binomial regression of y2 on x3 and x4 equivalent to xtmbreg, pa xtgee y2 x3 x4, family(nbinomial 1)

Population-averaged logistic regression of y3 on x5 and x6 when y3 is the number of events out of 10 trials

xtgee y3 x5 x6, family(binomial 10)

### Menu

 ${\it Statistics} > {\it Longitudinal/panel data} > {\it Generalized estimating equations (GEE)} > {\it Generalized estimating equations (GEE)}$ 

# **Syntax**

```
xtgee depvar [indepvars] [if] [in] [weight] [, options]
```

options	Description
Model	
$\underline{\mathbf{f}}$ amily( $family$ )	distribution of depvar
$\underline{1}$ ink( $link$ )	link function
Model 2	
<pre>exposure(varname)</pre>	include ln(varname) in model with coefficient constrained to 1
offset(varname)	include varname in model with coefficient constrained to 1
<u>nocons</u> tant	suppress constant term
asis	retain perfect predictor variables
force	estimate even if observations unequally spaced in time
Correlation	
$\underline{\mathtt{c}}\mathtt{orr}(\mathit{correlation})$	within-group correlation structure
SE/Robust	
vce( <i>vcetype</i> )	vcetype may be conventional, robust, bootstrap, or jackknife
nmp	use divisor $N-P$ instead of the default $N$
rgf	multiply the robust variance estimate by $(N-1)/(N-P)$
$\underline{s}$ cale( $parm$ )	overrides the default scale parameter; parm may be x2, dev, phi, or #
Reporting	
<u>le</u> vel(#)	set confidence level; default is level(95)
<u>ef</u> orm	report exponentiated coefficients
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling
Optimization	
optimize_options	control the optimization process; seldom used
<u>nodis</u> play	suppress display of header and coefficients
$\underline{\mathtt{coefl}}\mathtt{egend}$	display legend instead of statistics
A 1 '11 '1	

A panel variable must be specified. Correlation structures other than exchangeable and independent require that a time variable also be specified. Use xtset; see [XT] xtset.

indepvars may contain factor variables; see [U] 11.4.3 Factor variables.

depvar and indepvars may contain time-series operators; see [U] 11.4 varname and varlists.

by, collect, mfp, mi estimate, and statsby are allowed; see [U] 11.1.10 Prefix commands.

vce(bootstrap) and vce(jackknife) are not allowed with the mi estimate prefix; see [MI] mi estimate.

iweights, fweights, and pweights are allowed; see [U] 11.1.6 weight. Weights must be constant within panel. nodisplay and coeflegend do not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

family	Description
gaussian	Gaussian (normal); family(normal) is a synonym
igaussian	inverse Gaussian
$\underline{\underline{b}}$ inomial $[\# varname]$	Bernoulli/binomial
poisson	Poisson
$\underline{\underline{nb}}$ inomial $[\#]$	negative binomial
gamma	gamma
link	Link function/definition
<u>i</u> dentity	identity; $y = y$
log	$\log; \ln(y)$
<u>logi</u> t	logit; $ln\{y/(1-y)\}$ , natural log of the odds
probit	probit; $\Phi^{-1}(y)$ , where $\Phi()$ is the normal cumulative distribution
<u>cl</u> oglog	cloglog; $ln\{-ln(1-y)\}$
power[#]	power; $y^k$ with $k = \#$ ; $\# = 1$ if not specified
opower #	odds power; $[\{y/(1-y)\}^k-1]/k$ with $k=\#; \#=1$ if not specified
<u>nb</u> inomial	negative binomial; $ln\{y/(y+\alpha)\}$
reciprocal	reciprocal; $1/y$
correlation	Description
<u>exc</u> hangeable	exchangeable
<u>ind</u> ependent	independent
$\underline{\mathtt{un}}\mathtt{structured}$	unstructured
<u>fix</u> ed matname	user-specified
ar#	autoregressive of order #
<pre>stationary #</pre>	stationary of order #
<pre>nonstationary #</pre>	nonstationary of order #

# **Options**

Model

family(family) specifies the distribution of depvar; family(gaussian) is the default.

link(link) specifies the link function; the default is the canonical link for the family() specified (except for family(nbinomial)).

```
Model 2
```

exposure(varname) and offset(varname) are different ways of specifying the same thing. exposure() specifies a variable that reflects the amount of exposure over which the *depvar* events were observed for each observation;  $\ln(varname)$  with coefficient constrained to be 1 is entered into the regression equation. offset() specifies a variable that is to be entered directly into the log-link function with its coefficient constrained to be 1; thus, exposure is assumed to be  $e^{varnam\overline{e}}$ . If you were fitting a Poisson regression model, family(poisson) link(log), for instance, you would account for exposure time by specifying offset() containing the log of exposure time.

- noconstant specifies that the linear predictor has no intercept term, thus forcing it through the origin on the scale defined by the link function.
- asis forces retention of perfect predictor variables and their associated, perfectly predicted observations and may produce instabilities in maximization; see [R] probit. This option is only allowed with option family(binomial) with a denominator of 1.
- force specifies that estimation be forced even though the time variable is not equally spaced. This is relevant only for correlation structures that require knowledge of the time variable. These correlation structures require that observations be equally spaced so that calculations based on lags correspond to a constant time change. If you specify a time variable indicating that observations are not equally spaced, the (time dependent) model will not be fit. If you also specify force, the model will be fit, and it will be assumed that the lags based on the data ordered by the time variable are appropriate.

Correlation

corr(correlation) specifies the within-group correlation structure; the default corresponds to the equal-correlation model, corr(exchangeable).

When you specify a correlation structure that requires a lag, you indicate the lag after the structure's name with or without a blank; for example, corr(ar 1) or corr(ar1).

If you specify the fixed correlation structure, you specify the name of the matrix containing the assumed correlations following the word fixed, for example, corr(fixed myr).

SE/Robust

vce(vcetype) specifies the type of standard error reported, which includes types that are derived from asymptotic theory (conventional), that are robust to some kinds of misspecification (robust), and that use bootstrap or jackknife methods (bootstrap, jackknife); see [XT] vce\_options.

vce(conventional), the default, uses the conventionally derived variance estimator for generalized least-squares regression.

vce(robust) specifies that the Huber/White/sandwich estimator of variance is to be used in place of the default conventional variance estimator (see *Methods and formulas* below). Use of this option causes xtgee to produce valid standard errors even if the correlations within group are not as hypothesized by the specified correlation structure. Under a noncanonical link, it does, however, require that the model correctly specifies the mean. The resulting standard errors are thus labeled "semirobust" instead of "robust" in this case. Although there is no vce(cluster *clustvar*) option, results are as if this option were included and you specified clustering on the panel variable.

nmp; see [XT] vce\_options.

rgf specifies that the robust variance estimate is multiplied by (N-1)/(N-P), where N is the total number of observations and P is the number of coefficients estimated. This option can be used only with family(gaussian) when vce(robust) is either specified or implied by the use of pweights. Using this option implies that the robust variance estimate is not invariant to the scale of any weights used.

scale(x2 | dev | phi | #); see [XT] vce\_options.

Reporting

eform displays the exponentiated coefficients and corresponding standard errors and confidence intervals as described in [R] Maximize. For family (binomial) link (logit) (that is, logistic regression), exponentiation results in odds ratios; for family(poisson) link(log) (that is, Poisson regression), exponentiated coefficients are incidence-rate ratios.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, <u>allbase</u>levels, <u>nofvlab</u>el, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

Optimization

optimize\_options control the iterative optimization process. These options are seldom used.

iterate(#) specifies the maximum number of iterations. When the number of iterations equals #, the optimization stops and presents the current results, even if convergence has not been reached. The default is iterate(100).

tolerance (#) specifies the tolerance for the coefficient vector. When the relative change in the coefficient vector from one iteration to the next is less than or equal to #, the optimization process is stopped. tolerance(1e-6) is the default.

log and nolog specify whether to display the iteration log. The iteration log is displayed by default unless you used set iterlog off to suppress it; see set iterlog in [R] set iter.

trace specifies that the current estimates be printed at each iteration.

The following options are available with xtgee but are not shown in the dialog box: nodisplay is for programmers. It suppresses display of the header and coefficients. coeflegend; see [R] Estimation options.

# Remarks and examples

stata.com

For a thorough introduction to GEE in the estimation of GLM, see Hardin and Hilbe (2013). More information on linear models is presented in Nelder and Wedderburn (1972). Finally, there have been several illuminating articles on various applications of GEE in Zeger, Liang, and Albert (1988); Zeger and Liang (1986), and Liang (1987). Pendergast et al. (1996) surveys the current methods for analyzing clustered data in regard to binary response data. Our implementation follows that of Liang and Zeger (1986).

xtgee fits generalized linear models of  $y_{it}$  with covariates  $\mathbf{x}_{it}$ 

$$g\{E(y_{it})\} = \mathbf{x}_{it}\boldsymbol{\beta}, \quad y \sim F \text{ with parameters } \theta_{it}$$

for i = 1, ..., m and  $t = 1, ..., n_i$ , where there are  $n_i$  observations for each group identifier i. g()is called the link function, and F is the distributional family. Substituting various definitions for g()and F results in a wide array of models. For instance, if  $y_{it}$  is distributed Gaussian (normal) and q() is the identity function, we have

$$E(y_{it}) = \mathbf{x}_{it}\boldsymbol{\beta}, \quad y \sim N()$$

yielding linear regression, random-effects regression, or other regression-related models, depending on what we assume for the correlation structure.

If g() is the logit function and  $y_{it}$  is distributed Bernoulli (binomial), we have

$$logit\{E(y_{it})\} = \mathbf{x}_{it}\boldsymbol{\beta}, \quad y \sim Bernoulli$$

or logistic regression. If  $g(\cdot)$  is the natural log function and  $y_{it}$  is distributed Poisson, we have

$$ln\{E(y_{it})\} = \mathbf{x}_{it}\boldsymbol{\beta}, \quad y \sim Poisson$$

or Poisson regression, also known as the log-linear model. Other combinations are possible.

You specify the link function with the link() option, the distributional family with family(), and the assumed within-group correlation structure with corr().

The binomial distribution can be specified as case 1 family(binomial), case 2 family(binomial #), or case 3 family(binomial varname). In case 2, # is the value of the binomial denominator N, the number of trials. Specifying family(binomial 1) is the same as specifying family(binomial); both mean that y has the Bernoulli distribution with values 0 and 1 only. In case 3, varname is the variable containing the binomial denominator, thus allowing the number of trials to vary across observations.

The negative binomial distribution must be specified as family(nbinomial #), where # denotes the value of the parameter  $\alpha$  in the negative binomial distribution. The results will be conditional on this value.

You do not have to specify both family() and link(); the default link() is the canonical link for the specified family() (excluding family(nbinomial)):

Family	Default link
family(binomial)	link(logit)
family(gamma)	<pre>link(reciprocal)</pre>
family(gaussian)	link(identity)
family(igaussian)	link(power -2)
family(nbinomial)	link(log)
family(poisson)	link(log)

The canonical link for the negative binomial family is obtained by specifying link(nbinomial). If you specify both family() and link(), not all combinations make sense. You may choose among the following combinations:

	Gaussian	Inverse Gaussian	Binomial	Poisson	Negative Binomial	Gamma
Identity	x	X	X	х	X	х
Log	X	X	X	X	X	X
Logit			X			
Probit			X			
C. log-log			X			
Power	X	X	X	X	X	X
Odds Power			X			
Neg. binom.					X	
Reciprocal	X		X	X		X

You specify the assumed within-group correlation structure with the corr() option.

For example, call  $\mathbf R$  the working correlation matrix for modeling the within-group correlation, a square  $\max\{n_i\} \times \max\{n_i\}$  matrix. corr() specifies the structure of  $\mathbf R$ . Let  $\mathbf R_{t,s}$  denote the t,s element.

The independent structure is defined as

$$\mathbf{R}_{t,s} = \begin{cases} 1 & \text{if } t = s \\ 0 & \text{otherwise} \end{cases}$$

The corr(exchangeable) structure (corresponding to equal-correlation models) is defined as

$$\mathbf{R}_{t,s} = \begin{cases} 1 & \text{if } t = s \\ \rho & \text{otherwise} \end{cases}$$

The corr(ar g) structure is defined as the usual correlation matrix for an AR(g) model. This is sometimes called multiplicative correlation. For example, an AR(1) model is given by

$$\mathbf{R}_{t,s} = \begin{cases} 1 & \text{if } t = s \\ \rho^{|t-s|} & \text{otherwise} \end{cases}$$

The corr(stationary g) structure is a stationary(g) model. For example, a stationary(1) model is given by

$$\mathbf{R}_{t,s} = \begin{cases} 1 & \text{if } t = s \\ \rho & \text{if } |t - s| = 1 \\ 0 & \text{otherwise} \end{cases}$$

The corr (nonstationary g) structure is a nonstationary (g) model that imposes only the constraints that the elements of the working correlation matrix along the diagonal be 1 and the elements outside the gth band be zero,

$$\mathbf{R}_{t,s} = \begin{cases} 1 & \text{if } t = s \\ \rho_{ts} & \text{if } 0 < |t - s| \le g, \ \rho_{ts} = \rho_{st} \\ 0 & \text{otherwise} \end{cases}$$

corr(unstructured) imposes only the constraint that the diagonal elements of the working correlation matrix be 1.

$$\mathbf{R}_{t,s} = \begin{cases} 1 & \text{if } t = s \\ \rho_{ts} & \text{otherwise, } \rho_{ts} = \rho_{st} \end{cases}$$

The corr(fixed matname) specification is taken from the user-supplied matrix, such that

$$\mathbf{R} = matname$$

Here the correlations are not estimated from the data. The user-supplied matrix must be a valid correlation matrix with 1s on the diagonal.

Full formulas for all the correlation structures are provided in the Methods and formulas below.

#### □ Technical note

Some family(), link(), and corr() combinations result in models already fit by Stata:

<pre>family()</pre>	link()	corr()	Other Stata estimation command
gaussian	identity	independent	regress
gaussian	identity	exchangeable	xtreg, re
gaussian	identity	exchangeable	xtreg, pa
binomial	cloglog	independent	cloglog (see note 1)
binomial	cloglog	exchangeable	xtcloglog, pa
binomial	logit	independent	logit or logistic
binomial	logit	exchangeable	xtlogit, pa
binomial	probit	independent	probit (see note 2)
binomial	probit	exchangeable	xtprobit, pa
nbinomial	log	independent	nbreg (see note 3)
poisson	log	independent	poisson
poisson	log	exchangeable	xtpoisson, pa
gamma	log	independent	streg, dist(exp) nohr (see note 4)
family	link	independent	glm, irls (see note 5)

#### Notes:

- 1. For cloglog estimation, xtgee with corr(independent) and cloglog (see [R] cloglog) will produce the same coefficients, but the standard errors will be only asymptotically equivalent because cloglog is not the canonical link for the binomial family.
- 2. For probit estimation, xtgee with corr(independent) and probit will produce the same coefficients, but the standard errors will be only asymptotically equivalent because probit is not the canonical link for the binomial family. If the binomial denominator is not 1, the equivalent maximum-likelihood command is glm with options family(binomial #) or family(binomial varname) and link(probit); see [R] probit and [R] glm.
- 3. Fitting a negative binomial model by using xtgee (or using glm) will yield results conditional on the specified value of α. The nbreg command, however, estimates that parameter and provides unconditional estimates; see [R] nbreg.
- 4. xtgee with corr(independent) can be used to fit exponential regressions, but this requires specifying scale(1). As with probit, the xtgee-reported standard errors will be only asymptotically equivalent to those produced by streg, dist(exp) nohr (see [ST] streg) because log is not the canonical link for the gamma family. xtgee cannot be used to fit exponential regressions on censored data.
  - Using the independent correlation structure, the xtgee command will fit the same model fit with the glm, irls command if the family—link combination is the same.
- 5. If the xtgee command is equivalent to another command, using corr(independent) and the vce(robust) option with xtgee corresponds to using the vce(cluster *clustvar*) option in the equivalent command, where *clustvar* corresponds to the panel variable.

xtgee is a generalization of the glm, irls command and gives the same output when the same family and link are specified together with an independent correlation structure. What makes xtgee useful is

- the number of statistical models that it generalizes for use with panel data, many of which are not otherwise available in Stata;
- the richer correlation structure xtgee allows, even when models are available through other xt commands; and
- the availability of robust standard errors (see [U] 20.22 Obtaining robust variance estimates), even when the model and correlation structure are available through other xt commands.

In the following examples, we illustrate the relationships of xtgee with other Stata estimation commands. Remember that, although xtgee generalizes many other commands, the computational algorithm is different; therefore, the answers you obtain will not be identical. The dataset we are using is a subset of the nlswork data (see [XT] xt); we are looking at observations before 1980.

# ▶ Example 1

We can use xtgee to perform ordinary least squares by regress:

- . use https://www.stata-press.com/data/r18/nlswork2 (National Longitudinal Survey of Young Women, 14-24 years old in 1968)
- . regress ln\_w grade age c.age#c.age

Source	SS	df	MS			= 16,085 = 1413.68
Model Residual	597.54468 2265.74584	3 16,081	199.18156 .14089583	Prob R-sq	> F uared	= 0.0000 = 0.2087 = 0.2085
Total	2863.29052	16,084	.178021047		i bquarca	= 0.2085
ln_wage	Coefficient	Std. err.	t	P> t	[95% conf	. interval]
grade age	.0724483 .1064874	.0014229 .0083644	50.91 12.73	0.000 0.000	.0696592 .0900922	.0752374 .1228825
c.age#c.age	0016931	.0001655	-10.23	0.000	0020174	0013688
_cons	8681487	.1024896	-8.47	0.000	-1.06904	6672577

. xtgee ln\_w grade age c.age#c.age, corr(indep) nmp

Iteration 1: Tolerance = 8.684e-13

GEE population-averaged model Number of obs 16,085 Group variable: idcode Number of groups = 3,913 Family: Gaussian Obs per group: Link: Identity min = Correlation: independent 4.1 avg = 9 max =Wald chi2(3) 4241.04 = Scale parameter = .1408958Prob > chi2 Pearson chi2(16081) = 2265.75Deviance 2265.75 Dispersion (Pearson) = .1408958Dispersion = .1408958

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
grade age	.0724483 .1064874	.0014229 .0083644	50.91 12.73	0.000	.0696594 .0900935	.0752372
c.age#c.age	0016931	.0001655	-10.23	0.000	0020174	0013688
_cons	8681487	.1024896	-8.47	0.000	-1.069025	6672728

When nmp is specified, the coefficients and the standard errors produced by the estimators are the same. Moreover, the scale parameter estimate from the xtgee command equals the MSE calculation from regress; both are estimates of the variance of the residuals.

4

#### Example 2

The identity link and Gaussian family produce regression-type models. With the independent correlation structure, we reproduce ordinary least squares. With the exchangeable correlation structure, we produce an equal-correlation linear regression estimator.

xtgee, fam(gauss) link(ident) corr(exch) is asymptotically equivalent to the weighted-GLS estimator provided by xtreg, re and to the full maximum-likelihood estimator provided by xtreg, mle. In balanced data, xtgee, fam(gauss) link(ident) corr(exch) and xtreg, mle produce the same results. With unbalanced data, the results are close but differ because the two estimators handle unbalanced data differently. For both balanced and unbalanced data, the results produced by xtgee, fam(gauss) link(ident) corr(exch) and xtreg, mle differ from those produced by xtreg, re. Below we demonstrate the use of the three estimators with unbalanced data. We begin with xtgee; show the maximum likelihood estimator xtreg, mle; show the GLS estimator xtreg, re; and finally show xtgee with the vce(robust) option.

. xtgee ln\_w grade age c.age#c.age, nolog

GEE population-averaged model Group variable: idcode

Family: Gaussian Link: Identity

Correlation: exchangeable

Scale parameter = .1416586

Number of obs = 16,085 Number of groups = 3,913

Obs per group: min = 1 avg = 4.1

max = 9 Wald chi2(3) = 2918.26 Prob > chi2 = 0.0000

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
grade age	.0717731 .1077645	.00211	34.02 15.65	0.000	.0676377 .0942701	.0759086 .1212589
c.age#c.age	0016381	.0001362	-12.03	0.000	001905	0013712
_cons	9480449	.0869277	-10.91	0.000	-1.11842	7776698

. xtreg ln\_w grade age c.age#c.age, mle

Fitting constant-only model:

Iteration 0: Log likelihood = -5868.3483 Iteration 1: Log likelihood = -5858.8833 Iteration 2: Log likelihood = -5858.8244

Fitting full model:

Iteration 0: Log likelihood = -4591.9241 Iteration 1: Log likelihood = -4562.4406 Iteration 2: Log likelihood = -4562.3526 Iteration 3: Log likelihood = -4562.3525

Random-effects ML regression

Group variable: idcode

Random effects u\_i ~ Gaussian

Number of obs = 16,085Number of groups = 3,913

Obs per group:

Prob > chi2

min = 1 avg = 4.1 max =

= 0.0000

= 2592.94 LR chi2(3)

Log likelihood = -4562.3525

ln_wage	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
grade age	.0717747 .1077899	.002142 .0068266	33.51 15.79	0.000	.0675765 .0944101	.075973 .1211697
c.age#c.age	0016364	.000135	-12.12	0.000	0019011	0013718
_cons	9500833	.086384	-11.00	0.000	-1.119393	7807737
/sigma_u /sigma_e rho	.2689639 .2669944 .5036748	.0040854 .0017113 .0086449			.2610748 .2636613 .4867329	.2770915 .2703696 .52061

LR test of  $sigma_u=0$ : chibar2(01) = 4996.22

Prob >= chibar2 = 0.000

```
. xtreg ln_w grade age c.age#c.age, re
Random-effects GLS regression
                                                  Number of obs
                                                                           16,085
                                                  Number of groups =
Group variable: idcode
                                                                            3,913
R-squared:
                                                  Obs per group:
     Within = 0.0983
                                                                 min =
                                                                                1
     Between = 0.2946
                                                                              4.1
                                                                 avg =
     Overall = 0.2076
                                                                 max =
                                                  Wald chi2(3)
                                                                          2875.02
corr(u_i, X) = 0 (assumed)
                                                  Prob > chi2
                                                                           0.0000
     ln_wage
               Coefficient
                             Std. err.
                                             z
                                                  P>|z|
                                                             [95% conf. interval]
       grade
                  .0717757
                             .0021666
                                          33.13
                                                  0.000
                                                             .0675294
                                                                         .0760221
                             .0068125
                                                                         .1211566
                  .1078042
                                          15.82
                                                  0.000
                                                             .0944519
         age
 c.age#c.age
                 -.0016355
                             .0001347
                                         -12.14
                                                  0.000
                                                           -.0018996
                                                                        -.0013714
                -.9512118
                             .0863139
                                         -11.02
                                                  0.000
                                                           -1.120384
                                                                        -.7820397
       _cons
     sigma_u
                 .27383747
     sigma_e
                 .26624266
                 .51405959
                             (fraction of variance due to u_i)
         rho
. xtgee ln_w grade age c.age#c.age, vce(robust) nolog
GEE population-averaged model
                                                      Number of obs
                                                                           16,085
```

Group variable: idcode Number of groups = 3,913 Family: Gaussian Obs per group: min = Link: Identity Correlation: exchangeable avg = 4.1 max = 9 Wald chi2(3) = 2031.28 Scale parameter = .1416586 Prob > chi2 0.0000

(Std. err. adjusted for clustering on idcode)

ln_wage	Coefficient	Robust std. err.	z	P> z	[95% conf.	interval]
grade age	.0717731 .1077645	.0023341	30.75 10.99	0.000	.0671983 .0885379	.0763479
c.age#c.age	0016381	.0001964	-8.34	0.000	002023	0012532
_cons	9480449	.1195009	-7.93	0.000	-1.182262	7138274

In [R] regress, regress, vce(cluster *clustvar*) may produce inefficient coefficient estimates with valid standard errors for random-effects models. These standard errors are robust to model misspecification. The vce(robust) option of xtgee, on the other hand, requires that the model correctly specify the mean and the link function when the noncanonical link is used.

### Stored results

Scalars

```
xtgee stores the following in e():
```

```
e(N)
                                 number of observations
                                 number of groups
       e(N_g)
                                 model degrees of freedom
       e(df_m)
       e(chi2)
       e(p)
                                 p-value for model test
       e(df_pear)
                                 degrees of freedom for Pearson \chi^2
       e(chi2_dev)
                                 \chi^2 test of deviance
                                 \chi^2 test of deviance dispersion
       e(chi2_dis)
                                 deviance
       e(deviance)
       e(dispers)
                                 deviance dispersion
       e(phi)
                                 scale parameter
       e(g_min)
                                 smallest group size
                                 average group size
       e(g_avg)
       e(g_max)
                                 largest group size
       e(tol)
                                 target tolerance
       e(dif)
                                 achieved tolerance
       e(rank)
                                 rank of e(V)
       e(rc)
                                 return code
   Macros
       e(cmd)
                                 xtgee
       e(cmdline)
                                 command as typed
       e(depvar)
                                 name of dependent variable
       e(ivar)
                                 variable denoting groups
                                 variable denoting time within groups
       e(tvar)
       e(model)
       e(family)
                                 distribution family
       e(link)
                                 link function
       e(corr)
                                 correlation structure
                                 x2, dev, phi, or #; scale parameter
       e(scale)
       e(wtype)
                                 weight type
       e(wexp)
                                 weight expression
                                 linear offset variable
       e(offset)
                                 Wald; type of model \chi^2 test
       e(chi2type)
       e(vce)
                                 vcetype specified in vce()
                                 title used to label Std. err.
       e(vcetype)
       e(nmp)
                                 nmp, if specified
       e(properties)
                                 b V
       e(estat_cmd)
                                 program used to implement estat
       e(predict)
                                 program used to implement predict
       e(marginsnotok)
                                 predictions disallowed by margins
       e(asbalanced)
                                 factor variables fyset as asbalanced
       e(asobserved)
                                 factor variables fyset as asobserved
   Matrices
                                 coefficient vector
       e(b)
                                 estimated working correlation matrix
       e(R)
       e(V)
                                 variance-covariance matrix of the estimators
                                 model-based variance
       e(V_modelbased)
   Functions
       e(sample)
                                 marks estimation sample
In addition to the above, the following is stored in r():
   Matrices
                                 matrix containing the coefficients with their standard errors, test statistics, p-values,
       r(table)
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

and confidence intervals

### Methods and formulas

Methods and formulas are presented under the following headings:

Introduction
Calculating GEE for GLM
Correlation structures
Nonstationary and unstructured

#### Introduction

xtgee fits generalized linear models for panel data with the GEE approach described in Liang and Zeger (1986). A related method, referred to as GEE2, is described in Zhao and Prentice (1990) and Prentice and Zhao (1991). The GEE2 method attempts to gain efficiency in the estimation of  $\beta$  by specifying a parametric model for  $\alpha$  and then assumes that the models for both the mean and dependency parameters are correct. Thus there is a tradeoff in robustness for efficiency. The preliminary work of Liang, Zeger, and Qaqish (1992), however, indicates that there is little efficiency gained with this alternative approach.

In the GLM approach (see McCullagh and Nelder [1989]), we assume that

$$\begin{split} h(\boldsymbol{\mu}_{i,j}) &= \boldsymbol{x}_{i,j}^{\mathrm{T}} \boldsymbol{\beta} \\ \operatorname{Var}(\boldsymbol{y}_{i,j}) &= g(\boldsymbol{\mu}_{i,j}) \boldsymbol{\phi} \\ \boldsymbol{\mu}_i &= E(\mathbf{y}_i) = \{h^{-1}(\boldsymbol{x}_{i,1}^{\mathrm{T}} \boldsymbol{\beta}), \dots, h^{-1}(\boldsymbol{x}_{i,n_i}^{\mathrm{T}} \boldsymbol{\beta})\}^{\mathrm{T}} \\ \mathbf{A}_i &= \operatorname{diag}\{g(\boldsymbol{\mu}_{i,1}), \dots, g(\boldsymbol{\mu}_{i,n_i})\} \\ \operatorname{Cov}(\mathbf{y}_i) &= \boldsymbol{\phi} \mathbf{A}_i \quad \text{for independent observations.} \end{split}$$

In the absence of a convenient likelihood function with which to work, we can rely on a multivariate analog of the quasiscore function introduced by Wedderburn (1974):

$$\mathbf{S}_{\beta}(\beta, \alpha) = \sum_{i=1}^{m} \left( \frac{\partial \mu_i}{\partial \beta} \right)^{\mathrm{T}} \mathrm{Var}(\mathbf{y}_i)^{-1} (\mathbf{y}_i - \mu_i) = 0$$

We can solve for correlation parameters  $\alpha$  by simultaneously solving

$$\mathbf{S}_{\alpha}(\boldsymbol{\beta}, \boldsymbol{\alpha}) = \sum_{i=1}^{m} \left( \frac{\partial \boldsymbol{\eta}_{i}}{\partial \boldsymbol{\alpha}} \right)^{\mathrm{T}} \mathbf{H}_{i}^{-1} (\mathbf{W}_{i} - \boldsymbol{\eta}_{i}) = 0$$

In the GEE approach to GLM, we let  $\mathbf{R}_i(\alpha)$  be a "working" correlation matrix depending on the parameters in  $\alpha$  (see the *Correlation structures* section for the number of parameters), and we estimate  $\beta$  by solving the GEE,

$$\begin{aligned} \mathbf{U}(\boldsymbol{\beta}) &= \sum_{i=1}^m \left(\frac{\partial \boldsymbol{\mu}_i}{\partial \boldsymbol{\beta}}\right)^{\mathrm{T}} \mathbf{V}_i^{-1}(\boldsymbol{\alpha}) (\mathbf{y}_i - \boldsymbol{\mu}_i) = 0 \\ \text{where} \quad \mathbf{V}_i(\boldsymbol{\alpha}) &= \mathbf{A}_i^{1/2} \mathbf{R}_i(\boldsymbol{\alpha}) \mathbf{A}_i^{1/2} \end{aligned}$$

To solve this equation, we need only a crude approximation of the variance matrix, which we can obtain from a Taylor series expansion, where

$$Cov(\mathbf{y}_i) = \mathbf{L}_i \mathbf{Z}_i \mathbf{D}_i \mathbf{Z}_i^{\mathrm{T}} \mathbf{L}_i + \phi \mathbf{A}_i = \widetilde{\mathbf{V}}_i$$
$$\mathbf{L}_i = \operatorname{diag} \{ \partial h^{-1}(u) / \partial u, u = x_{i,j}^{\mathrm{T}} \boldsymbol{\beta}, j = 1, \dots, n_i \}$$

which allows that

$$\widehat{\mathbf{D}}_{i} \approx (\mathbf{Z}_{i}^{\mathrm{T}}\mathbf{Z}_{i})^{-1}\mathbf{Z}_{i}\widehat{\mathbf{L}}_{i}^{-1} \left\{ (\mathbf{y}_{i} - \widehat{\boldsymbol{\mu}}_{i})(\mathbf{y}_{i} - \widehat{\boldsymbol{\mu}}_{i})^{\mathrm{T}} - \widehat{\boldsymbol{\phi}}\widehat{\mathbf{A}}_{i} \right\} \widehat{\mathbf{L}}_{i}^{-1}\mathbf{Z}_{i}^{\mathrm{T}}(\mathbf{Z}_{i}'\mathbf{Z}_{i})^{-1}$$

$$\widehat{\boldsymbol{\phi}} = \sum_{i=1}^{m} \sum_{i=1}^{n_{i}} \frac{(y_{i,j} - \widehat{\boldsymbol{\mu}}_{i,j})^{2} - (\widehat{\mathbf{L}}_{i,j})^{2}\mathbf{Z}_{i,j}^{\mathrm{T}}\widehat{\mathbf{D}}_{i}\mathbf{Z}_{i,j}}{g(\widehat{\boldsymbol{\mu}}_{i,j})}$$

## Calculating GEE for GLM

Using the notation from Liang and Zeger (1986), let  $\mathbf{y}_i = (y_{i,1}, \dots, y_{i,n_i})^{\mathrm{T}}$  be the  $n_i \times 1$  vector of outcome values, and let  $\mathbf{X}_i = (x_{i,1}, \dots, x_{i,n_i})^{\mathrm{T}}$  be the  $n_i \times p$  matrix of covariate values for the ith subject  $i = 1, \dots, m$ . We assume that the marginal density for  $y_{i,j}$  may be written in exponential family notation as

$$f(y_{i,j}) = \exp \left[ \left\{ y_{i,j} \theta_{i,j} - a(\theta_{i,j}) + b(y_{i,j}) \right\} \phi \right]$$

where  $\theta_{i,j} = h(\eta_{i,j}), \eta_{i,j} = x_{i,j}\beta$ . Under this formulation, the first two moments are given by

$$E(y_{i,j}) = a'(\theta_{i,j}), \quad \operatorname{Var}(y_{i,j}) = a''(\theta_{i,j})/\phi$$

In what follows, we let  $n_i = n$  without loss of generality. We define the quantities, assuming that we have an  $n \times n$  working correlation matrix  $\mathbf{R}(\alpha)$ ,

$$\begin{aligned} & \mathbf{\Delta}_i = \operatorname{diag}(d\theta_{i,j}/d\eta_{i,j}) & n \times n \text{ matrix} \\ & \mathbf{A}_i = \operatorname{diag}\{a''(\theta_{i,j})\} & n \times n \text{ matrix} \\ & \mathbf{S}_i = \mathbf{y}_i - a'(\theta_i) & n \times 1 \text{ matrix} \\ & \mathbf{D}_i = \mathbf{A}_i \mathbf{\Delta}_i \mathbf{X}_i & n \times p \text{ matrix} \\ & \mathbf{V}_i = \mathbf{A}_i^{1/2} \mathbf{R}(\boldsymbol{\alpha}) \mathbf{A}_i^{1/2} & n \times n \text{ matrix} \end{aligned}$$

such that the GEE becomes

$$\sum_{i=1}^{m} \mathbf{D}_i^{\mathrm{T}} \mathbf{V}_i^{-1} \mathbf{S}_i = 0$$

We then have that

$$\widehat{\boldsymbol{\beta}}_{j+1} = \widehat{\boldsymbol{\beta}}_j - \left\{ \sum_{i=1}^m \mathbf{D}_i^{\mathrm{T}}(\widehat{\boldsymbol{\beta}}_j) \widetilde{\mathbf{V}}_i^{-1}(\widehat{\boldsymbol{\beta}}_j) \mathbf{D}_i(\widehat{\boldsymbol{\beta}}_j) \right\}^{-1} \left\{ \sum_{i=1}^m \mathbf{D}_i^{\mathrm{T}}(\widehat{\boldsymbol{\beta}}_j) \widetilde{\mathbf{V}}_i^{-1}(\widehat{\boldsymbol{\beta}}_j) \mathbf{S}_i(\widehat{\boldsymbol{\beta}}_j) \right\}$$

where the term

$$\left\{\sum_{i=1}^m \mathbf{D}_i^{\mathrm{T}}(\widehat{\boldsymbol{\beta}}_j) \widetilde{\mathbf{V}}_i^{-1}(\widehat{\boldsymbol{\beta}}_j) \mathbf{D}_i(\widehat{\boldsymbol{\beta}}_j) \right\}^{-1}$$

is what we call the conventional variance estimate. It is used to calculate the standard errors if the vce(robust) option is not specified. This command supports the clustered version of the Huber/White/sandwich estimator of the variance with panels treated as clusters when vce(robust) is specified. See [P] \_robust, particularly Maximum likelihood estimators and Methods and formulas. Liang and Zeger (1986) also discuss the calculation of the robust variance estimator.

Define the following:

$$\begin{split} \mathbf{D} &= (\mathbf{D}_1^{\mathrm{T}}, \dots, \mathbf{D}_m^{\mathrm{T}}) \\ \mathbf{S} &= (\mathbf{S}_1^{\mathrm{T}}, \dots, \mathbf{S}_m^{\mathrm{T}})^{\mathrm{T}} \\ \widetilde{\mathbf{V}} &= nm \times nm \text{ block diagonal matrix with } \widetilde{\mathbf{V}}_i \\ \mathbf{Z} &= \mathbf{D}\boldsymbol{\beta} - \mathbf{S} \end{split}$$

At a given iteration, the correlation parameters  $\alpha$  and scale parameter  $\phi$  can be estimated from the current Pearson residuals, defined by

$$\widehat{r}_{i,j} = \{y_{i,j} - a'(\widehat{\theta}_{i,j})\}/\{a''(\widehat{\theta}_{i,j})\}^{1/2}$$

where  $\widehat{\theta}_{i,j}$  depends on the current value for  $\widehat{\beta}$ . We can then estimate  $\phi$  by

$$\widehat{\phi}^{-1} = \sum_{i=1}^{m} \sum_{j=1}^{n_i} \widehat{r}_{i,j}^2 / (N - p)$$

As this general derivation is complicated, let's follow the derivation of the Gaussian family with the identity link (regression) to illustrate the generalization. After making appropriate substitutions, we will see a familiar updating equation. First, we rewrite the updating equation for  $\beta$  as

$$\widehat{\boldsymbol{\beta}}_{i+1} = \widehat{\boldsymbol{\beta}}_i - \mathbf{Z}_1^{-1} \mathbf{Z}_2$$

and then derive  $\mathbf{Z}_1$  and  $\mathbf{Z}_2$ .

$$\begin{split} \mathbf{Z}_{1} &= \sum_{i=1}^{m} \mathbf{D}_{i}^{\mathrm{T}}(\widehat{\boldsymbol{\beta}}_{j}) \widetilde{\mathbf{V}}_{i}^{-1}(\widehat{\boldsymbol{\beta}}_{j}) \mathbf{D}_{i}(\widehat{\boldsymbol{\beta}}_{j}) = \sum_{i=1}^{m} \mathbf{X}_{i}^{\mathrm{T}} \boldsymbol{\Delta}_{i}^{\mathrm{T}} \mathbf{A}_{i}^{\mathrm{T}} \{ \mathbf{A}_{i}^{1/2} \mathbf{R}(\boldsymbol{\alpha}) \mathbf{A}_{i}^{1/2} \}^{-1} \mathbf{A}_{i} \boldsymbol{\Delta}_{i} \mathbf{X}_{i} \\ &= \sum_{i=1}^{m} \mathbf{X}_{i}^{\mathrm{T}} \operatorname{diag} \left\{ \frac{\partial \theta_{i,j}}{\partial (\mathbf{X}\boldsymbol{\beta})} \right\} \operatorname{diag} \left\{ a''(\theta_{i,j}) \right\} \left[ \operatorname{diag} \left\{ a''(\theta_{i,j}) \right\}^{1/2} \mathbf{R}(\boldsymbol{\alpha}) \operatorname{diag} \left\{ a''(\theta_{i,j}) \right\}^{1/2} \right]^{-1} \\ & \operatorname{diag} \left\{ a''(\theta_{i,j}) \right\} \operatorname{diag} \left\{ \frac{\partial \theta_{i,j}}{\partial (\mathbf{X}\boldsymbol{\beta})} \right\} \mathbf{X}_{i} \\ &= \sum_{i=1}^{m} \mathbf{X}_{i}^{\mathrm{T}} \mathbf{II} (\mathbf{III})^{-1} \mathbf{II} \mathbf{X}_{i} = \sum_{i=1}^{m} \mathbf{X}_{i}^{\mathrm{T}} \mathbf{X}_{i} = \mathbf{X}^{\mathrm{T}} \mathbf{X} \end{split}$$

$$\begin{split} \mathbf{Z}_2 &= \sum_{i=1}^m \mathbf{D}_i^{\mathrm{T}}(\widehat{\boldsymbol{\beta}}_j) \widetilde{\mathbf{V}}_i^{-1}(\widehat{\boldsymbol{\beta}}_j) \mathbf{S}_i(\widehat{\boldsymbol{\beta}}_j) = \sum_{i=1}^m \mathbf{X}_i^{\mathrm{T}} \mathbf{\Delta}_i^{\mathrm{T}} \mathbf{A}_i^{\mathrm{T}} \{ \mathbf{A}_i^{1/2} \mathbf{R}(\boldsymbol{\alpha}) \mathbf{A}_i^{1/2} \}^{-1} \left( \mathbf{y}_i - \mathbf{X}_i \widehat{\boldsymbol{\beta}}_j \right) \\ &= \sum_{i=1}^m \mathbf{X}_i^{\mathrm{T}} \operatorname{diag} \left\{ \frac{\partial \theta_{i,j}}{\partial (\mathbf{X}\boldsymbol{\beta})} \right\} \operatorname{diag} \left\{ a''(\theta_{i,j}) \right\} \left[ \operatorname{diag} \left\{ a''(\theta_{i,j}) \right\}^{1/2} \mathbf{R}(\boldsymbol{\alpha}) \operatorname{diag} \left\{ a''(\theta_{i,j}) \right\}^{1/2} \right]^{-1} \\ & \left( \mathbf{y}_i - \mathbf{X}_i \widehat{\boldsymbol{\beta}}_j \right) \\ &= \sum_{i=1}^m \mathbf{X}_i^{\mathrm{T}} \mathbf{II} (\mathbf{III})^{-1} (\mathbf{y}_i - \mathbf{X}_i \widehat{\boldsymbol{\beta}}_j) = \sum_{i=1}^m \mathbf{X}_i^{\mathrm{T}} (\mathbf{y}_i - \mathbf{X}_i \widehat{\boldsymbol{\beta}}_j) = \mathbf{X}^{\mathrm{T}} \widehat{\boldsymbol{s}}_j \end{split}$$

So, we may write the update formula as

$$\widehat{\boldsymbol{\beta}}_{i+1} = \widehat{\boldsymbol{\beta}}_i - (\mathbf{X}^{\mathrm{T}}\mathbf{X})^{-1}\mathbf{X}^{\mathrm{T}}\widehat{\boldsymbol{s}}_j$$

which is the same formula for GLS in regression.

#### **Correlation structures**

The working correlation matrix  $\mathbf{R}$  is a function of  $\alpha$  and is more accurately written as  $\mathbf{R}(\alpha)$ . Depending on the assumed correlation structure,  $\alpha$  might be

Independent	no parameters to estimate
Exchangeable	lpha is a scalar
Autoregressive	$\alpha$ is a vector
Stationary	$\alpha$ is a vector
Nonstationary	lpha is a matrix
Unstructured	lpha is a matrix

Also, throughout the estimation of a general unbalanced panel, it is more proper to discuss  $\mathbf{R}_i$ , which is the upper left  $n_i \times n_i$  submatrix of the ultimately stored matrix in  $e(\mathbf{R})$ ,  $\max\{n_i\} \times \max\{n_i\}$ .

The only panels that enter into the estimation for a lag-dependent correlation structure are those with  $n_i > g$  (assuming a lag of g). xtgee drops panels with too few observations (and mentions when it does so).

# Independent

The working correlation matrix  $\mathbf{R}$  is an identity matrix.

# Exchangeable

$$\alpha = \frac{\sum_{i=1}^{m} \left(\sum_{j=1}^{n_i} \sum_{k=1}^{n_i} \widehat{r}_{i,j} \widehat{r}_{i,k} - \sum_{j=1}^{n_i} \widehat{r}_{i,j}^2\right)}{\sum_{i=1}^{m} \left\{n_i(n_i-1)\right\}} \Bigg/ \frac{\sum_{i=1}^{m} \left(\sum_{j=1}^{n_i} \widehat{r}_{i,j}^2\right)}{\sum_{i=1}^{m} n_i}$$

and the working correlation matrix is given by

$$\mathbf{R}_{s,t} = \begin{cases} 1 & s = t \\ \alpha & \text{otherwise} \end{cases}$$

### Autoregressive and stationary

These two structures require g parameters to be estimated so that  $\alpha$  is a vector of length g+1 (the first element of  $\alpha$  is 1).

$$\alpha = \sum_{i=1}^{m} \left( \frac{\sum_{j=1}^{n_i} \widehat{r}_{i,j}^2}{n_i} , \frac{\sum_{j=1}^{n_i-1} \widehat{r}_{i,j} \widehat{r}_{i,j+1}}{n_i} , \dots , \frac{\sum_{j=1}^{n_i-g} \widehat{r}_{i,j} \widehat{r}_{i,j+g}}{n_i} \right) \middle/ \left( \sum_{i=1}^{m} \frac{\sum_{j=1}^{n_i} \widehat{r}_{i,j}^2}{n_i} \right)$$

The working correlation matrix for the AR model is calculated as a function of Toeplitz matrices formed from the  $\alpha$  vector; see Newton (1988). The working correlation matrix for the stationary model is given by

$$\mathbf{R}_{s,t} = \begin{cases} \alpha_{1,|s-t|} & \text{if } |s-t| \le g\\ 0 & \text{otherwise} \end{cases}$$

## Nonstationary and unstructured

These two correlation structures require a matrix of parameters.  $\alpha$  is estimated (where we replace  $\hat{r}_{i,j} = 0$  whenever  $i > n_i$  or  $j > n_i$ ) as

$$\alpha = \sum_{i=1}^{m} m \begin{pmatrix} N_{1,1}^{-1} \hat{r}_{i,1}^2 & N_{1,2}^{-1} \hat{r}_{i,1} \hat{r}_{i,2} & \cdots & N_{1,n}^{-1} \hat{r}_{i,1} \hat{r}_{i,n} \\ N_{2,1}^{-1} \hat{r}_{i,2} \hat{r}_{i,1} & N_{2,2}^{-1} \hat{r}_{i,2}^2 & \cdots & N_{2,n}^{-1} \hat{r}_{i,2} \hat{r}_{i,n} \\ \vdots & \vdots & \ddots & \vdots \\ N_{n,1}^{-1} \hat{r}_{i,n_i} \hat{r}_{i,1} & N_{n,2}^{-1} \hat{r}_{i,n_i} \hat{r}_{i,2} & \cdots & N_{n,n}^{-1} \hat{r}_{i,n}^2 \end{pmatrix} / \begin{pmatrix} \sum_{i=1}^{m} \frac{\sum_{j=1}^{n_i} \hat{r}_{i,j}^2}{n_i} \end{pmatrix}$$

where  $N_{p,q} = \sum_{i=1}^{m} I(i, p, q)$  and

$$I(i,p,q) = \left\{ \begin{matrix} 1 & \text{if panel } i \text{ has valid observations at times p and q} \\ 0 & \text{otherwise} \end{matrix} \right.$$

where  $N_{i,j} = \min(N_i, N_j)$ ,  $N_i = \text{number of panels observed at time } i$ , and  $n = \max(n_1, n_2, \dots, n_m)$ .

The working correlation matrix for the nonstationary model is given by

$$\mathbf{R}_{s,t} = \begin{cases} 1 & \text{if } s = t \\ \alpha_{s,t} & \text{if } 0 < |s - t| \le g \\ 0 & \text{otherwise} \end{cases}$$

The working correlation matrix for the unstructured model is given by

$$\mathbf{R}_{s,t} = \begin{cases} 1 & \text{if } s = t \\ \boldsymbol{\alpha}_{s,t} & \text{otherwise} \end{cases}$$

such that the unstructured model is equal to the nonstationary model at lag g = n - 1, where the panels are balanced with  $n_i = n$  for all i.

### References

- Aloisio, K. M., N. Micali, S. A. Swanson, A. Field, and N. J. Horton. 2014. Analysis of partially observed clustered data using generalized estimating equations and multiple imputation. Stata Journal 14: 863–883.
- Cabanillas, O. B., J. D. Michler, A. Michuda, and E. Tjernström. 2018. Fitting and interpreting correlated random-coefficient models using Stata. Stata Journal 18: 159–173.
- Caria, M. P., M. R. Galanti, R. Bellocco, and N. J. Horton. 2011. The impact of different sources of body mass index assessment on smoking onset: An application of multiple-source information models. Stata Journal 11: 386–402.
- Cui, J. 2007. QIC program and model selection in GEE analyses. Stata Journal 7: 209-220.
- Earnest, A. 2017. Essentials of a Successful Biostatistical Collaboration. Boca Raton, FL: CRC Press.
- Gallis, J. A., F. Li, and E. L. Turner. 2020. xtgeebcv: A command for bias-corrected sandwich variance estimation for GEE analyses of cluster randomized trials. Stata Journal 20: 363–381.
- Hardin, J. W., and J. M. Hilbe. 2013. Generalized Estimating Equations. 2nd ed. Boca Raton, FL: Chapman and Hall/CRC.
- Hosmer, D. W., Jr., S. A. Lemeshow, and R. X. Sturdivant. 2013. Applied Logistic Regression. 3rd ed. Hoboken, NJ: Wiley.
- Lenzi, J., and S. Pildava. 2019. Tips for calculating and displaying risk-standardized hospital outcomes in Stata. Stata Journal 19: 477–496.
- Liang, K.-Y. 1987. Estimating functions and approximate conditional likelihood. *Biometrika* 4: 695–702. https://doi.org/10.2307/2336463.
- Liang, K.-Y., and S. L. Zeger. 1986. Longitudinal data analysis using generalized linear models. *Biometrika* 73: 13–22. https://doi.org/10.1093/biomet/73.1.13.
- Liang, K.-Y., S. L. Zeger, and B. Qaqish. 1992. Multivariate regression analyses for categorical data. *Journal of the Royal Statistical Society, Series B* 54: 3–40. https://doi.org/10.1111/j.2517-6161.1992.tb01862.x.
- McCullagh, P., and J. A. Nelder. 1989. Generalized Linear Models. 2nd ed. London: Chapman and Hall/CRC.
- Nelder, J. A., and R. W. M. Wedderburn. 1972. Generalized linear models. *Journal of the Royal Statistical Society*, Series A 135: 370–384. https://doi.org/10.2307/2344614.
- Newton, H. J. 1988. TIMESLAB: A Time Series Analysis Laboratory. Belmont, CA: Wadsworth.
- Pendergast, J. F., S. J. Gange, M. A. Newton, M. J. Lindstrom, M. Palta, and M. R. Fisher. 1996. A survey of methods for analyzing clustered binary response data. *International Statistical Review* 64: 89–118. https://doi.org/10.2307/1403425.
- Prentice, R. L., and L. P. Zhao. 1991. Estimating equations for parameters in means and covariances of multivariate discrete and continuous responses. *Biometrics* 47: 825–839. https://doi.org/10.2307/2532642.
- Rabe-Hesketh, S., A. Skrondal, and A. Pickles. 2002. Reliable estimation of generalized linear mixed models using adaptive quadrature. *Stata Journal* 2: 1–21.
- Shults, J., and J. M. Hilbe. 2014. Quasi-Least Squares Regression. Boca Raton, FL: Chapman and Hall/CRC.
- Shults, J., S. J. Ratcliffe, and M. Leonard. 2007. Improved generalized estimating equation analysis via xtqls for quasi-least squares in Stata. Stata Journal 7: 147–166.
- Twisk, J. W. R. 2013. Applied Longitudinal Data Analysis for Epidemiology: A Practical Guide. 2nd ed. Cambridge: Cambridge University Press.
- Wedderburn, R. W. M. 1974. Quasi-likelihood functions, generalized linear models, and the Gauss–Newton method. *Biometrika* 61: 439–447. https://doi.org/10.2307/2334725.
- Zeger, S. L., and K.-Y. Liang. 1986. Longitudinal data analysis for discrete and continuous outcomes. *Biometrics* 42: 121–130. https://doi.org/10.2307/2531248.
- Zeger, S. L., K.-Y. Liang, and P. S. Albert. 1988. Models for longitudinal data: A generalized estimating equation approach. *Biometrics* 44: 1049–1060. https://doi.org/10.2307/2531734.
- Zhao, L. P., and R. L. Prentice. 1990. Correlated binary regression using a quadratic exponential model. *Biometrika* 77: 642–648. https://doi.org/10.2307/2337004.

# Also see

- [XT] **xtgee postestimation** Postestimation tools for xtgee
- [XT] **xtcloglog** Random-effects and population-averaged cloglog models
- [XT] **xtlogit** Fixed-effects, random-effects, and population-averaged logit models
- [XT] xtnbreg Fixed-effects, random-effects, & population-averaged negative binomial models
- [XT] **xtpoisson** Fixed-effects, random-effects, and population-averaged Poisson models
- [XT] **xtprobit** Random-effects and population-averaged probit models
- [XT] **xtreg** Fixed-, between-, and random-effects and population-averaged linear models<sup>+</sup>
- [XT] xtregar Fixed- and random-effects linear models with an AR(1) disturbance
- [XT] **xtset** Declare data to be panel data
- [MI] Estimation Estimation commands for use with mi estimate
- [R] **glm** Generalized linear models
- [R] logistic Logistic regression, reporting odds ratios
- [R] regress Linear regression
- [U] 20 Estimation and postestimation commands

Stata, Stata Press, and Mata are registered trademarks of StataCorp LLC. Stata and Stata Press are registered trademarks with the World Intellectual Property Organization of the United Nations. StataNow and NetCourseNow are trademarks of StataCorp LLC. Other brand and product names are registered trademarks or trademarks of their respective companies. Copyright © 1985–2023 StataCorp LLC, College Station, TX, USA. All rights reserved.



For suggested citations, see the FAQ on citing Stata documentation.