Title

> **tsfilter bw** — Butterworth time-series filter

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Description

tsfilter bw uses the Butterworth high-pass filter to separate a time series into trend and cyclical components. The trend component may contain a deterministic or a stochastic trend. The stationary cyclical component is driven by stochastic cycles at the specified periods.

See [TS] **tsfilter** for an introduction to the methods implemented in tsfilter bw.

Quick start

Use the Butterworth filter for y, and obtain cyclical component ct using tsset data
    tsfilter bw ct=y

Same as above, but filter periods larger than 20
    tsfilter bw ct=y, maxperiod(20)

Same as above, and save the trend component in the variable trendvar
    tsfilter bw ct=y, maxperiod(20) trend(trendvar)

Same as above, and save gain and angular frequency with the names gain and angle
    tsfilter bw ct=y, maxperiod(20) trend(trendvar) gain(gain angle)

Set the order of the filter to 4, and save the trend component in the variable trendvar
    tsfilter bw ct=y, order(4) trend(trendvar)

Use Butterworth filter for variables y1, y2, and y3 to obtain cyclical components with prefix cycl
    tsfilter bw cycl* = y1 y2 y3

Note: The above commands can also be used to apply the filter separately to each panel of a panel dataset when a *panelvar* has been specified by using tsset or xtset.

Menu

Statistics > Time series > Filters for cyclical components > Butterworth

Syntax

Filter one variable

    tsfilter bw [*type*] *newvar* = *varname* [*if*] [*in*] [, *options*]

Filter multiple variables, unique names

    tsfilter bw [*type*] *newvarlist* = *varlist* [*if*] [*in*] [, *options*]

Filter multiple variables, common name stub

    tsfilter bw [*type*] *stub** = *varlist* [*if*] [*in*] [, *options*]

options	Description
Main	
<u>maxp</u>eriod(*#*)	filter out stochastic cycles at periods larger than *#*
<u>or</u>der(*#*)	set the order of the filter; default is order(2)
Trend	
<u>tr</u>end(*newvar* | *newvarlist* | *stub**)	save the trend component(s) in new variable(s)
Gain	
<u>ga</u>in(*gainvar anglevar*)	save the gain and angular frequency

You must tsset or xtset your data before using tsfilter; see [TS] **tsset** and [XT] **xtset**.
varname and *varlist* may contain time-series operators; see [U] **11.4.4 Time-series varlists**.
collect is allowed; see [U] **11.1.10 Prefix commands**.

Options

    � Main

maxperiod(*#*) filters out stochastic cycles at periods larger than *#*, where *#* must be greater than 2.
    By default, if the units of the time variable are set to daily, weekly, monthly, quarterly, half-yearly,
    or yearly, then *#* is set to the number of periods equivalent to 8 years; otherwise, the default value
    is maxperiod(32).

order(*#*) sets the order of the Butterworth filter, which must be an integer. The default is order(2).

    ◻ Trend

trend(*newvar* | *newvarlist* | *stub**) saves the trend component(s) in the new variable(s) specified by
    newvar, *newvarlist*, or *stub**.

    ◻ Gain

gain(*gainvar anglevar*) saves the gain in *gainvar* and its associated angular frequency in *anglevar*.
    Gains are calculated at the N angular frequencies that uniformly partition the interval $(0, \pi]$, where
    N is the sample size.

Remarks and examples

We assume that you have already read [TS] **tsfilter**, which provides an introduction to filtering and the methods implemented in `tsfilter bw`, more examples using `tsfilter bw`, and a comparison of the four filters implemented by `tsfilter`. In particular, an understanding of gain functions as presented in [TS] **tsfilter** is required to understand these remarks.

`tsfilter bw` uses the Butterworth high-pass filter to separate a time-series y_t into trend and cyclical components:

$$y_t = \tau_t + c_t$$

where τ_t is the trend component and c_t is the cyclical component. τ_t may be nonstationary; it may contain a deterministic or a stochastic trend, as discussed below.

The primary objective is to estimate c_t, a stationary cyclical component that is driven by stochastic cycles within a specified range of periods. The trend component τ_t is calculated by the difference $\tau_t = y_t - c_t$.

Although the Butterworth high-pass filter implemented in `tsfilter bw` has been widely applied by macroeconomists and engineers, it is a general time-series method and may be of interest to other researchers.

Engineers have used Butterworth filters for a long time because they are "maximally flat". The gain functions of these filters are as close as possible to being a flat line at 0 for the unwanted periods and a flat line at 1 for the desired periods; see Butterworth (1930) and Bianchi and Sorrentino (2007, 17–20). (See [TS] **tsfilter** for an introduction to gain functions.)

The high-pass Butterworth filter is a two-parameter filter. The `maxperiod()` option specifies the maximum period; the stochastic cycles of all higher periodicities are filtered out. The `maxperiod()` option sets the location of the cutoff period in the gain function. The `order()` option specifies the order of the filter, which determines the slope of the gain function at the cutoff frequency.

For a given cutoff period, the slope of the gain function at the cutoff period increases with filter order. For a given filter order, the slope of the gain function at the cutoff period increases with the cutoff period.

We cannot obtain a vertical slope at the cutoff frequency, which is the ideal, because the computation becomes unstable; see Pollock (2000). The filter order for which the computation becomes unstable depends on the cutoff period.

Among economists, the high-pass Butterworth filter is commonly used for investigating business cycles. Burns and Mitchell (1946) defined business cycles as stochastic cycles in business data corresponding to periods between 1.5 and 8 years. For this reason, the default value for `maxperiod()` is the number of periods in 8 years, if the time variable is formatted as daily, weekly, monthly, quarterly, half-yearly, or yearly; see [D] **format**. The default value for `maxperiod()` is 32 for all other time formats.
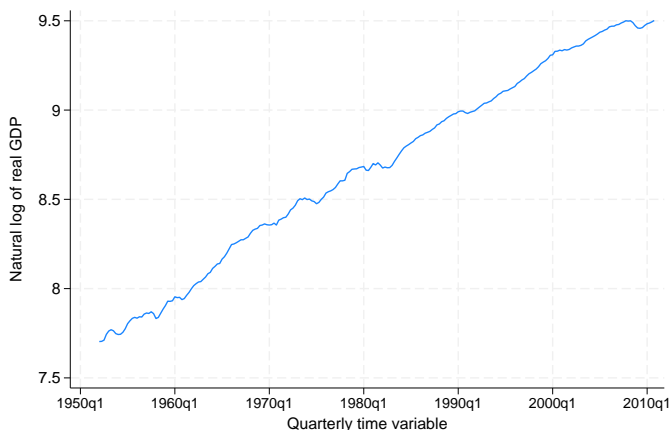
For each variable, the high-pass Butterworth filter estimate of c_t is put in the corresponding new variable, and when the `trend()` option is specified, the estimate of τ_t is put in the corresponding new variable.

`tsfilter bw` automatically detects panel data from the information provided when the dataset was `tsset` or `xtset`. All calculations are done separately on each panel. Missing values at the beginning and end of the sample are excluded from the sample. The sample may not contain gaps.

▷ Example 1: Estimating a business-cycle component

In this and the subsequent examples, we use `tsfilter bw` to estimate the business-cycle component of the natural log of the real gross domestic product (GDP) of the United States. Our sample of quarterly data goes from 1952q1 to 2010q4. Below we read in and plot the data.

```
. use https://www.stata-press.com/data/r18/gdp2
(Federal Reserve Economic Data, St. Louis Fed)
. tsline gdp_ln
```


The series is nonstationary. Pollock (2000) shows that the high-pass Butterworth filter can estimate the components driven by the stochastic cycles at the specified frequencies when the original series is nonstationary.

Below we use `tsfilter bw` to filter `gdp_ln` and use `pergram` (see [TS] **pergram**) to compute and to plot the periodogram of the estimated cyclical component.

```
. tsfilter bw gdp_bw = gdp_ln
. pergram gdp_bw, xline(.03125 .16667)
```
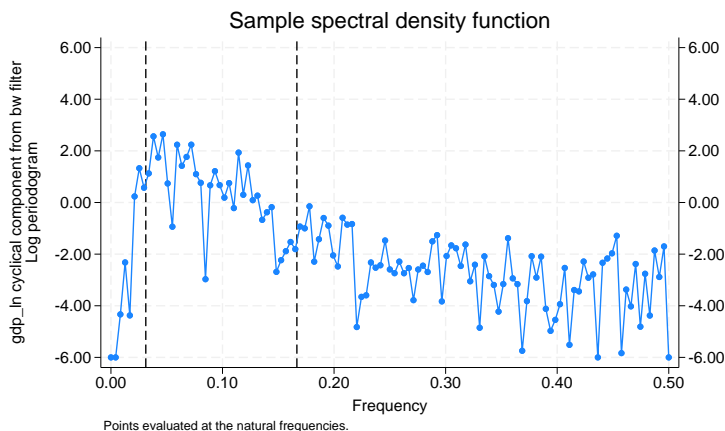

`tsfilter bw` used the default value of `maxperiod(32)` because our sample is of quarterly data. In the periodogram, we added vertical lines at the natural frequencies corresponding to the conventional Burns and Mitchell (1946) values for business-cycle components. `pergram` displays the results in

natural frequencies, which are the standard frequencies divided by 2π. We use option `xline()` to draw vertical lines at the lower natural-frequency cutoff ($1/32 = 0.03125$) and the upper natural-frequency cutoff ($1/6 \approx 0.16667$).

If the filter completely removed the stochastic cycles at the unwanted frequencies, the periodogram would be a flat line at the minimum value of -6 outside the range identified by the vertical lines.

The periodogram reveals two issues. First, it indicates that the default value of `order(2)` did not do a good job of filtering out the high-periodicity stochastic cycles, because there are too many points above -6.00 to the left of the left-hand vertical line. Second, it reveals the high-pass nature of the filter, because none of the low-period (high-frequency) stochastic cycles have been filtered out.

We cope with these two issues in the remaining examples.

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▷ Example 2: Changing the order of the filter

In this example, we change the order of the filter so that it will remove more of the unwanted low-frequency stochastic cycles. As previously mentioned, increasing the order of the filter increases the slope of the gain function at the cutoff period.

For orders 2 and 8, we compute the filtered series, compute the gain functions, and label the gain variables. We also generate `ideal`, the gain function of the ideal band-pass filter at the frequencies `f`. Then we plot the gain function of the ideal band-pass filter and the gain functions of the high-pass Butterworth filters of orders 2 and 8.

```
. tsfilter bw gdp_bw2 = gdp_ln, gain(g1 a1)
. label variable g1 "BW order 2"
. tsfilter bw gdp_bw8 = gdp_ln, gain(g8 a8) order(8)
. label variable g8 "BW order 8"
. generate f = _pi*(_n-1)/_N
. generate ideal = cond(f<_pi/16, 0, cond(f<_pi/3, 1,0))
. label variable ideal "Ideal filter"
. twoway line ideal f || line g1 a1 || line g8 a8
```
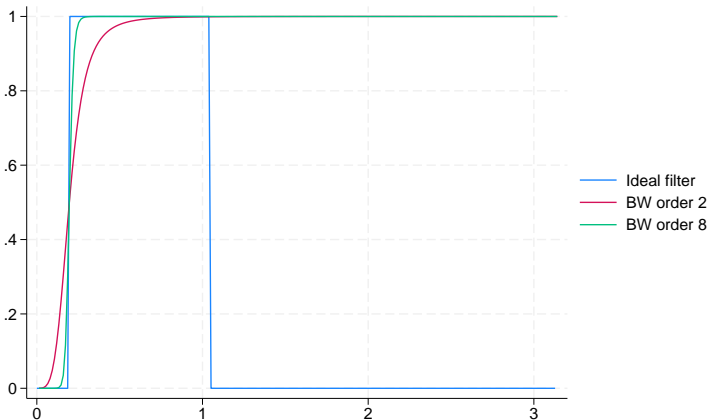

As discussed in [TS] **tsfilter**, the gain function of the ideal filter is a square wave with a value of 0 at the frequencies corresponding to unwanted frequencies and a value of 1 at the desired frequencies.

The vertical lines in the gain function of the ideal filter occur at $\pi/16$, corresponding to 32 periods, and at $\pi/3$, corresponding to 6 periods. (Given that $p = 2\pi/\omega$, where p is the period corresponding to frequency ω, the frequency is given by $2\pi/p$.)

The distance between the gain function of the filter with order 2 and the gain function of the ideal band-pass filter at $\pi/16$ is the root of the first issue mentioned at the end of example 1. The filter with order 8 is much closer to the gain function of the ideal band-pass filter at $\pi/16$ than is the filter with order 2. That both gain functions are 1 to the right of the vertical line at $\pi/3$ reveals the high-pass nature of the filter.

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▷ Example 3: Removing the high-frequency component

In this example, we use a common trick to resolve the second issue mentioned at the end of example 1. Keeping the trend produced by a high-pass filter turns that high-pass filter into a low-pass filter. Because we want to remove the high-frequency stochastic cycles still in the previously filtered series gdp_bw8, we need to run gdp_bw8 through a low-pass filter. So we keep the trend produced by refiltering the previously filtered series.

To determine an order for the filter, we run the filter with order(8), then with order(15), and then we plot the gain functions along with the gain function of the ideal filter.

```
. tsfilter bw gdp_bwn8  = gdp_bw8, gain(gc8 ac8) order(8)
> maxperiod(6) trend(gdp_bwc8)
. label variable gc8 "BW order 8"
. tsfilter bw gdp_bwn15 = gdp_bw8, gain(gc15 ac15) order(15)
> maxperiod(6) trend(gdp_bwc15)
. label variable gc15 "BW order 15"
. twoway line ideal f || line gc8 ac8 || line gc15 ac15
```
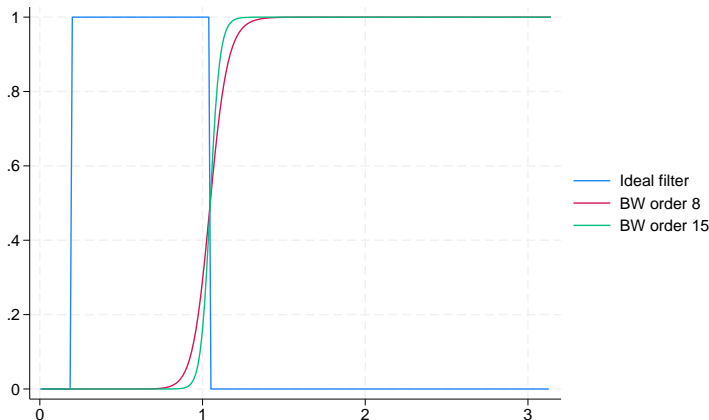

We specified much higher orders for the filter in this example because the cutoff period is 6 instead of 32. (As previously mentioned, holding the order of the filter constant, the slope of the gain function at the cutoff period decreases when the period decreases.) The above graph indicates that the filter with order(15) is reasonably close to the gain function of the ideal filter.

Now we compute and plot the periodogram of the estimated business-cycle component.

```
. pergram gdp_bwc15, xline(.03125 .16667)
```


Points evaluated at the natural frequencies.
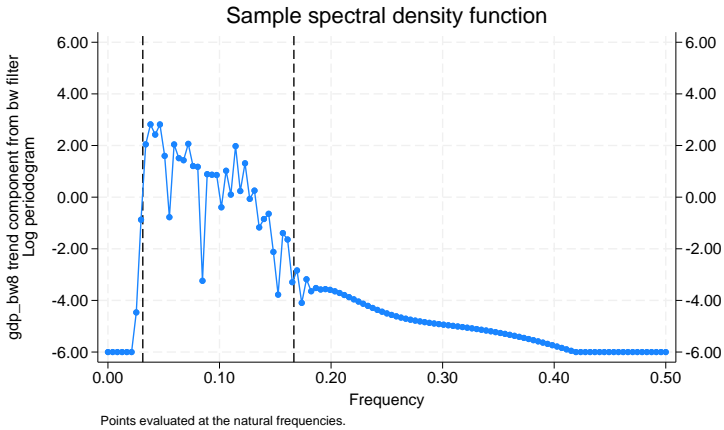
The graph indicates that the above applications of the Butterworth filter did a reasonable job of filtering out the high-periodicity stochastic cycles but that the low-periodicity stochastic cycles have not been completely removed.

Below we plot the estimated business-cycle component with recessions identified by the shaded areas.
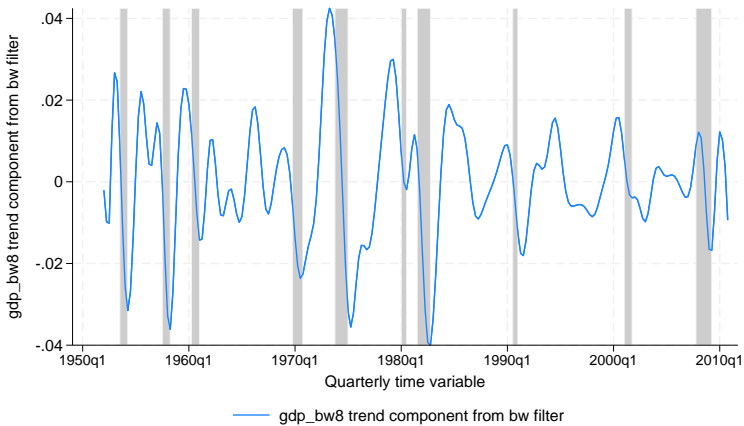
Stored results

tsfilter bw stores the following in r():

Scalars
    r(order)   order of the filter
    r(maxperiod) maximum period of stochastic cycles

Macros
    r(varlist)   original time-series variables
    r(filterlist) variables containing estimates of the cyclical components
    r(trendlist)   variables containing estimates of the trend components, if trend() was specified
    r(method)   Butterworth
    r(unit)   units of time variable set using tsset or xtset

Methods and formulas

tsfilter bw uses the computational methods described in Pollock (2000) to implement the filter.

Pollock (2000) shows that the gain of the Butterworth high-pass filter is given by

$$\psi(\omega) = \left[1 + \left\{ \frac{\tan(\omega_c/2)}{\tan(\omega/2)} \right\}^{2m} \right]^{-1}$$

where m is the order of the filter, $\omega_c = 2\pi/p_h$ is the cutoff frequency, and p_h is the maximum period.

Here is an outline of the computational procedure that Pollock (2000) derived.

Pollock (2000) showed that the Butterworth filter corresponds to a particular model. Actually, his model is more general than the Butterworth filter, but tsfilter bw restricts the computations to the case in which the model corresponds to the Butterworth filter.

The model represents the series to be filtered, y_t, in terms of zero mean, covariance stationary, and independent and identically distributed shocks ν_t and ε_t:

$$y_t = \frac{(1+L)^m}{(1-L)^m}\nu_t + \varepsilon_t$$

From this model, Pollock (2000) shows that the optimal estimate for the cyclical component is given by
$$\mathbf{c} = \lambda \mathbf{Q}(\mathbf{\Omega_L} + \lambda\mathbf{\Omega_H})^{-1}\mathbf{Q'}\mathbf{y}$$

where $\text{Var}\{\mathbf{Q'}(\mathbf{y} - \mathbf{c})\} = \sigma_\nu^2 \mathbf{\Omega_L}$ and $\text{Var}\{\mathbf{Q'c}\} = \sigma_\varepsilon^2 \mathbf{\Omega_H}$. Here $\mathbf{\Omega_L}$ and $\mathbf{\Omega_H}$ are symmetric Toeplitz matrices with $2m+1$ nonzero diagonal bands and generating functions $(1+z)^m(1+z^{-1})^m$ and $(1-z)^m(1-z^{-1})^m$, respectively.

The parameter λ in this expression is a function of p_h (the maximum period of stochastic cycles filtered out) and the order of the filter:

$$\lambda = \{\tan(\pi/p_h)\}^{-2m}$$

The matrix $\mathbf{Q}'$ in this expression is a function of the coefficients in the polynomial $(1 - L)^d = 1 + \delta_1 L + \cdots + \delta_d L^d$:

$$\mathbf{Q}' = \begin{pmatrix} \delta_d & \ldots & \delta_1 & 1 & \ldots & 0 & 0 & \ldots & 0 & 0 \\ \vdots & \ddots & \vdots & \vdots & \ddots & \vdots & \vdots & & \vdots & \vdots \\ 0 & \ldots & \delta_d & \delta_{d-1} & \ldots & 1 & 0 & \ldots & 0 & 0 \\ 0 & \ldots & 0 & \delta_d & \ldots & \delta_1 & 1 & \ldots & 0 & 0 \\ \vdots & & \vdots & \vdots & \ddots & & & \ddots & \vdots & \vdots \\ 0 & \ldots & 0 & 0 & \ldots & \delta_d & \delta_{d-1} & \ldots & 1 & 0 \\ 0 & \ldots & 0 & 0 & \ldots & 0 & \delta_d & \ldots & \delta_1 & 1 \end{pmatrix}^{(T-d)\times T}$$

It can be shown that $\Omega_H = \mathbf{Q}'\mathbf{Q}$ and $\boldsymbol{\Omega}_{\mathbf{L}} = |\Omega_H|$, which simplifies the calculation of the cyclical component to

$$\mathbf{c} = \lambda\mathbf{Q}\{|\mathbf{Q}'\mathbf{Q}| + \lambda(\mathbf{Q}'\mathbf{Q})\}^{-1}\mathbf{Q}'\mathbf{y}$$

References

Bianchi, G., and R. Sorrentino. 2007. *Electronic Filter Simulation and Design*. New York: McGraw–Hill.

Burns, A. F., and W. C. Mitchell. 1946. *Measuring Business Cycles*. New York: National Bureau of Economic Research.

Butterworth, S. 1930. On the theory of filter amplifiers. *Experimental Wireless and the Wireless Engineer* 7: 536–541.

Pollock, D. S. G. 1999. *A Handbook of Time-Series Analysis, Signal Processing and Dynamics*. London: Academic Press.

——. 2000. Trend estimation and de-trending via rational square-wave filters. *Journal of Econometrics* 99: 317–334. https://doi.org/10.1016/S0304-4076(00)00028-2.

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Also see

[TS] **tsfilter** — Filter a time series for cyclical components

[TS] **tsset** — Declare data to be time-series data

[TS] **tssmooth** — Smooth and forecast univariate time-series data

[D] **format** — Set variables' output format

[XT] **xtset** — Declare data to be panel data