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mgarch dcc — Dynamic conditional correlation multivariate GARCH models

Description Quick start Menu Syntax

Options Remarks and examples Stored results Methods and formulas

References Also see

# **Description**

mgarch dcc estimates the parameters of dynamic conditional correlation (DCC) multivariate generalized autoregressive conditionally heteroskedastic (MGARCH) models in which the conditional variances are modeled as univariate generalized autoregressive conditionally heteroskedastic (GARCH) models and the conditional covariances are modeled as nonlinear functions of the conditional variances. The conditional quasicorrelation parameters that weight the nonlinear combinations of the conditional variances follow the GARCH-like process specified in Engle (2002).

The DCC MGARCH model is about as flexible as the closely related varying conditional correlation MGARCH model (see [TS] **mgarch vcc**), more flexible than the conditional correlation MGARCH model (see [TS] **mgarch ccc**), and more parsimonious than the diagonal vech MGARCH model (see [TS] **mgarch dvech**).

### **Quick start**

Fit dynamic conditional correlation multivariate GARCH with first- and second-order ARCH components for dependent variables y1 and y2 using tsset data

```
mgarch dcc (y1 y2), arch(1 2)
```

Add regressors x1 and x2 and first-order GARCH component

mgarch dcc (y1 y2 = x1 x2), arch(1 2) garch(1)

Add z1 to the model for the conditional heteroskedasticity

mgarch dcc (y1 y2 = x1 x2), arch(1 2) garch(1) het(z1)

#### Menu

Statistics > Multivariate time series > Multivariate GARCH

### **Syntax**

```
mgarch dcc eq [eq \dots eq] [if] [in] [, options]
 where each eq has the form
      (depvars = [indepvars] [, eqoptions])
                              Description
 options
Model
 arch(numlist)
                               ARCH terms for all equations
 garch(numlist)
                               GARCH terms for all equations
 het(varlist)
                               include varlist in the specification of the conditional variance
                                 for all equations
 distribution(dist | # |)
                               use dist distribution for errors [may be gaussian
                                 (synonym normal) or t; default is gaussian]
 constraints(numlist)
                               apply linear constraints
SE/Robust
 vce(vcetype)
                               vcetype may be oim or robust
Reporting
                               set confidence level: default is level(95)
 level(#)
 nocnsreport
                              do not display constraints
 display_options
                              control columns and column formats, row spacing, line width,
                                 display of omitted variables and base and empty cells, and
                                 factor-variable labeling
Maximization
 maximize_options
                              control the maximization process; seldom used
 from (matname)
                               initial values for the coefficients; seldom used
                              display legend instead of statistics
 coeflegend
                               Description
 eqoptions
 noconstant
                               suppress constant term in the mean equation
 arch(numlist)
                               ARCH terms
 garch(numlist)
                              GARCH terms
 het(varlist)
                               include varlist in the specification of the conditional variance
 You must tsset your data before using mgarch dcc; see [TS] tsset.
```

indepvars and varlist may contain factor variables; see [U] 11.4.3 Factor variables.

depvars, indepvars, and varlist may contain time-series operators; see [U] 11.4.4 Time-series varlists.

by, collect, fp, rolling, and statsby are allowed; see [U] 11.1.10 Prefix commands.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

## **Options**

Model

arch(numlist) specifies the ARCH terms for all equations in the model. By default, no ARCH terms are specified.

garch(numlist) specifies the GARCH terms for all equations in the model. By default, no GARCH terms are specified.

het(varlist) specifies that varlist be included in the specification of the conditional variance for all equations. This varlist enters the variance specification collectively as multiplicative heteroskedasticity.

distribution(dist [#]) specifies the assumed distribution for the errors. dist may be gaussian, normal, or t.

gaussian and normal are synonyms; each causes mgarch dcc to assume that the errors come from a multivariate normal distribution. # may not be specified with either of them.

t causes mgarch dcc to assume that the errors follow a multivariate Student t distribution, and the degree-of-freedom parameter is estimated along with the other parameters of the model. If distribution(t #) is specified, then mgarch dcc uses a multivariate Student t distribution with # degrees of freedom. # must be greater than 2.

constraints (numlist) specifies linear constraints to apply to the parameter estimates.

SE/Robust

vce(vcetype) specifies the estimator for the variance—covariance matrix of the estimator.
vce(oim), the default, specifies to use the observed information matrix (OIM) estimator.
vce(robust) specifies to use the Huber/White/sandwich estimator.

Reporting

level(#), nocnsreport; see [R] Estimation options.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(%fmt), and nolstretch; see [R] Estimation options.

Maximization

maximize\_options: difficult, technique(algorithm\_spec), iterate(#), [no]log, trace, gradient, showstep, hessian, showtolerance, tolerance(#), ltolerance(#), nrtolerance(#), nonrtolerance, and from(matname); see [R] Maximize for all options except from(), and see below for information on from(). These options are seldom used.

from (matname) specifies initial values for the coefficients. from (b0) causes mgarch dcc to begin the optimization algorithm with the values in b0. b0 must be a row vector, and the number of columns must equal the number of parameters in the model.

The following option is available with mgarch dcc but is not shown in the dialog box: coeflegend; see [R] Estimation options.

#### **Egoptions**

noconstant suppresses the constant term in the mean equation.

arch(numlist) specifies the ARCH terms in the equation. By default, no ARCH terms are specified. This option may not be specified with model-level arch().

garch(numlist) specifies the GARCH terms in the equation. By default, no GARCH terms are specified.
This option may not be specified with model-level garch().

het(varlist) specifies that varlist be included in the specification of the conditional variance. This varlist enters the variance specification collectively as multiplicative heteroskedasticity. This option may not be specified with model-level het().

### Remarks and examples

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We assume that you have already read [TS] **mgarch**, which provides an introduction to MGARCH models and the methods implemented in **mgarch** dcc.

MGARCH models are dynamic multivariate regression models in which the conditional variances and covariances of the errors follow an autoregressive-moving-average structure. The DCC MGARCH model uses a nonlinear combination of univariate GARCH models with time-varying cross-equation weights to model the conditional covariance matrix of the errors.

As discussed in [TS] **mgarch**, MGARCH models differ in the parsimony and flexibility of their specifications for a time-varying conditional covariance matrix of the disturbances, denoted by  $\mathbf{H}_t$ . In the conditional correlation family of MGARCH models, the diagonal elements of  $\mathbf{H}_t$  are modeled as univariate GARCH models, whereas the off-diagonal elements are modeled as nonlinear functions of the diagonal terms. In the DCC MGARCH model,

$$h_{ij,t} = \rho_{ij,t} \sqrt{h_{ii,t} h_{jj,t}}$$

where the diagonal elements  $h_{ii,t}$  and  $h_{jj,t}$  follow univariate GARCH processes and  $\rho_{ij,t}$  follows the dynamic process specified in Engle (2002) and discussed below.

Because the  $\rho_{ij,t}$  varies with time, this model is known as the DCC GARCH model.

#### □ Technical note

The DCC GARCH model proposed by Engle (2002) can be written as

$$\mathbf{y}_{t} = \mathbf{C}\mathbf{x}_{t} + \epsilon_{t}$$

$$\epsilon_{t} = \mathbf{H}_{t}^{1/2} \boldsymbol{\nu}_{t}$$

$$\mathbf{H}_{t} = \mathbf{D}_{t}^{1/2} \mathbf{R}_{t} \mathbf{D}_{t}^{1/2}$$

$$\mathbf{R}_{t} = \operatorname{diag}(\mathbf{Q}_{t})^{-1/2} \mathbf{Q}_{t} \operatorname{diag}(\mathbf{Q}_{t})^{-1/2}$$

$$\mathbf{Q}_{t} = (1 - \lambda_{1} - \lambda_{2}) \mathbf{R} + \lambda_{1} \widetilde{\epsilon}_{t-1} \widetilde{\epsilon}'_{t-1} + \lambda_{2} \mathbf{Q}_{t-1}$$
(1)

where

 $\mathbf{y}_t$  is an  $m \times 1$  vector of dependent variables;

 ${f C}$  is an  $m \times k$  matrix of parameters;

 $\mathbf{x}_t$  is a  $k \times 1$  vector of independent variables, which may contain lags of  $\mathbf{y}_t$ ;

 $\mathbf{H}_t^{1/2}$  is the Cholesky factor of the time-varying conditional covariance matrix  $\mathbf{H}_t$ ;  $\nu_t$  is an  $m \times 1$  vector of normal, independent, and identically distributed innovations;

 $\mathbf{D}_t$  is a diagonal matrix of conditional variances,

$$\mathbf{D}_{t} = \begin{pmatrix} \sigma_{1,t}^{2} & 0 & \cdots & 0 \\ 0 & \sigma_{2,t}^{2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \sigma_{m,t}^{2} \end{pmatrix}$$

in which each  $\sigma_{i,t}^2$  evolves according to a univariate GARCH model of the form

$$\sigma_{i,t}^2 = s_i + \sum_{j=1}^{p_i} \alpha_j \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2$$

by default, or

$$\sigma_{i,t}^2 = \exp(\gamma_i \mathbf{z}_{i,t}) + \sum_{j=1}^{p_i} \alpha_j \epsilon_{i,t-j}^2 + \sum_{j=1}^{q_i} \beta_j \sigma_{i,t-j}^2$$

when the het() option is specified, where  $\gamma_t$  is a  $1 \times p$  vector of parameters,  $\mathbf{z}_i$  is a  $p \times 1$ vector of independent variables including a constant term, the  $\alpha_i$ 's are ARCH parameters, and the  $\beta_j$ 's are GARCH parameters;

 $\mathbf{R}_t$  is a matrix of conditional quasicorrelations,

$$\mathbf{R}_{t} = \begin{pmatrix} 1 & \rho_{12,t} & \cdots & \rho_{1m,t} \\ \rho_{12,t} & 1 & \cdots & \rho_{2m,t} \\ \vdots & \vdots & \ddots & \vdots \\ \rho_{1m,t} & \rho_{2m,t} & \cdots & 1 \end{pmatrix}$$

 $\widetilde{m{\epsilon}}_t$  is an m imes 1 vector of standardized residuals,  ${f D}_t^{-1/2} {m{\epsilon}}_t$ ; and

 $\lambda_1$  and  $\lambda_2$  are parameters that govern the dynamics of conditional quasicorrelations.  $\lambda_1$  and  $\lambda_2$  are nonnegative and satisfy  $0 \le \lambda_1 + \lambda_2 < 1$ .

When  $\mathbf{Q}_t$  is stationary, the  $\mathbf{R}$  matrix in (1) is a weighted average of the unconditional covariance matrix of the standardized residuals  $\tilde{\epsilon}_t$ , denoted by  $\overline{\mathbf{R}}$ , and the unconditional mean of  $\mathbf{Q}_t$ , denoted by  $\overline{\mathbf{Q}}$ . Because  $\overline{\mathbf{R}} \neq \overline{\mathbf{Q}}$ , as shown by Aielli (2009),  $\mathbf{R}$  is neither the unconditional correlation matrix nor the unconditional mean of  $\mathbf{Q}_t$ . For this reason, the parameters in  $\mathbf{R}$  are known as quasicorrelations; see Aielli (2009) and Engle (2009) for discussions.

# Some examples

### Example 1: Model with common covariates

We have daily data on the stock returns of three car manufacturers—Toyota, Nissan, and Honda, from January 2, 2003, to December 31, 2010—in the variables toyota, nissan and honda. We model the conditional means of the returns as a first-order vector autoregressive process and the conditional covariances as a DCC MGARCH process in which the variance of each disturbance term follows a GARCH(1,1) process.

```
. use https://www.stata-press.com/data/r18/stocks
(Data from Yahoo! Finance)
. mgarch dcc (toyota nissan honda = L.toyota L.nissan L.honda, noconstant),
> arch(1) garch(1)
Calculating starting values....
Optimizing log likelihood
(setting technique to bhhh)
Iteration 0: Log likelihood = 16902.435
Iteration 1: Log likelihood = 17005.448
Iteration 2: Log likelihood = 17157.958
Iteration 3: Log likelihood = 17267.363
Iteration 4: Log likelihood = 17318.29
Iteration 5: Log likelihood = 17353.029
Iteration 6: Log likelihood = 17369.115
Iteration 7: Log likelihood = 17388.035
Iteration 8: Log likelihood = 17401.254
Iteration 9: Log likelihood = 17435.556
(switching technique to nr)
Iteration 10: Log likelihood = 17451.739
Iteration 11: Log likelihood =
                                17467.815 (not concave)
Iteration 12: Log likelihood = 17476.592
Iteration 13: Log likelihood = 17481.979
Iteration 14: Log likelihood = 17484.838
Iteration 15: Log likelihood = 17484.95
Iteration 16: Log likelihood =
                                17484.95
Refining estimates
Iteration 0: Log likelihood =
                                 17484.95
Iteration 1: Log likelihood =
Dynamic conditional correlation MGARCH model
Sample: 2 thru 2015
                                                         Number of obs = 2.014
Distribution: Gaussian
                                                         Wald chi2(9) = 19.54
                                                        Prob > chi2 = 0.0210
Log likelihood = 17484.95
```

	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
toyota						
toyota						
L1.	0510871	.0339825	-1.50	0.133	1176915	.0155173
nissan						
L1.	.0297835	.0247455	1.20	0.229	0187169	.0782838
honda						
L1.	0162823	.0300323	-0.54	0.588	0751446	.04258
ARCH_toyota						
arch						
L1.	.0608226	.0086687	7.02	0.000	.0438322	.0778131
garch						
L1.	.92222	.0111056	83.04	0.000	.9004535	.9439865
_cons	4.47e-06	1.15e-06	3.90	0.000	2.22e-06	6.72e-06

1					
0056724	.0389348	-0.15	0.884	0819832	.0706384
					.0319276
.0154978	.0358802	0.43	0.666	0548261	.0858217
.0844234	.0128191	6.59	0.000	.0592984	.1095484
.8994214	.0151124	59.52	0.000	.8698017	.9290412
7.21e-06	1.93e-06	3.74	0.000	3.43e-06	.000011
0272424	.0361819	-0.75	0.451	0981576	.0436728
.0617496	.0271378	2.28	0.023	.0085604	.1149388
0635069	.0332919	-1.91	0.056	1287578	.0017439
.0490136	.0073696	6.65	0.000	.0345695	.0634576
.9331123	.0103687	89.99	0.000	.9127901	.9534346
5.35e-06	1.35e-06	3.95	0.000	2.69e-06	8.00e-06
. 6689555	.0168021	39.81	0.000	. 6360241	.7018869
.7259628	.0140156	51.80	0.000	.6984927	.753433
. 6335666	.0180412	35.12	0.000	.5982065	.6689266
.0315259 .8704241	.0088382	3.57 14.19	0.000	.0142034 .7502146	.0488484
	0287095 .0154978 .0844234 .8994214 7.21e-060272424 .06174960635069 .0490136 .9331123 5.35e-06 .6689555 .7259628 .6335666 .0315259	0287095 .0309379 .0154978 .0358802 .0844234 .0128191 .8994214 .0151124 7.21e-06 1.93e-060272424 .0361819 .0617496 .02713780635069 .0332919 .0490136 .0073696 .9331123 .0103687 5.35e-06 1.35e-06 .6689555 .0168021 .7259628 .0140156 .6335666 .0180412 .0315259 .0088382	0287095 .0309379 -0.93 .0154978 .0358802 0.43 .0844234 .0128191 6.59 .8994214 .0151124 59.52 7.21e-06 1.93e-06 3.74 0272424 .0361819 -0.75 .0617496 .0271378 2.280635069 .0332919 -1.91  .0490136 .0073696 6.65 .9331123 .0103687 89.99 5.35e-06 1.35e-06 3.95  .6689555 .0168021 39.81 .7259628 .0140156 51.80 .6335666 .0180412 35.12	0287095 .0309379 -0.93 0.353 .0154978 .0358802 0.43 0.666  .0844234 .0128191 6.59 0.000 .8994214 .0151124 59.52 0.000 7.21e-06 1.93e-06 3.74 0.000 0272424 .0361819 -0.75 0.451 .0617496 .0271378 2.28 0.0230635069 .0332919 -1.91 0.056  .0490136 .0073696 6.65 0.000 .9331123 .0103687 89.99 0.000 .933123 .0103687 89.99 0.000 5.35e-06 1.35e-06 3.95 0.000 .6689555 .0168021 39.81 0.000 .7259628 .0140156 51.80 0.000 .6335666 .0180412 35.12 0.000	0287095 .0309379 -0.93 0.3530893466 .0154978 .0358802 0.43 0.6660548261 .0844234 .0128191 6.59 0.000 .0592984 .8994214 .0151124 59.52 0.000 .8698017 7.21e-06 1.93e-06 3.74 0.000 3.43e-06 0272424 .0361819 -0.75 0.4510981576 .0617496 .0271378 2.28 0.023 .00856040635069 .0332919 -1.91 0.0561287578  .0490136 .0073696 6.65 0.000 .0345695 .9331123 .0103687 89.99 0.000 .9127901 5.35e-06 1.35e-06 3.95 0.000 2.69e-06 .6689555 .0168021 39.81 0.000 .6360241 .7259628 .0140156 51.80 0.000 .6984927 .6335666 .0180412 35.12 0.000 .5982065

The iteration log has three parts: the dots from the search for initial values, the iteration log from optimizing the log likelihood, and the iteration log from the refining step. A detailed discussion of the optimization methods is in Methods and formulas.

The header describes the estimation sample and reports a Wald test against the null hypothesis that all the coefficients on the independent variables in the mean equations are zero. Here the null hypothesis is rejected at the 5% level.

The output table first presents results for the mean or variance parameters used to model each dependent variable. Subsequently, the output table presents results for the conditional quasicorrelations.

For example, the conditional quasicorrelation between the standardized residuals for Toyota and Nissan is estimated to be 0.67. Finally, the output table presents results for the adjustment parameters  $\lambda_1$ and  $\lambda_2$ . In the example at hand, the estimates for both  $\lambda_1$  and  $\lambda_2$  are statistically significant.

The DCC MGARCH model reduces to the CCC MGARCH model when  $\lambda_1 = \lambda_2 = 0$ . The output below shows that a Wald test rejects the null hypothesis that  $\lambda_1 = \lambda_2 = 0$  at all conventional levels.

```
. test _b[/Adjustment:lambda1] = _b[/Adjustment:lambda2] = 0
      [/Adjustment]lambda1 - [/Adjustment]lambda2 = 0
      [/Adjustment]lambda1 = 0
          chi2(2) = 1102.43
        Prob > chi2 =
```

These results indicate that the assumption of time-invariant conditional correlations maintained in the CCC MGARCH model is too restrictive for these data.

1

Example 2: Model with covariates that differ by equation

We improve the previous example by removing the insignificant parameters from the model. To remove these parameters, we specify the honda equation separately from the toyota and nissan equations:

```
. mgarch dcc (toyota nissan = , noconstant) (honda = L.nissan, noconstant),
> arch(1) garch(1)
Calculating starting values....
Optimizing log likelihood
(setting technique to bhhh)
Iteration 0: Log likelihood = 16884.502
Iteration 1: Log likelihood = 16970.755
Iteration 2: Log likelihood = 17140.318
Iteration 3: Log likelihood = 17237.807
Iteration 4: Log likelihood = 17306.12
Iteration 5: Log likelihood = 17342.533
Iteration 6: Log likelihood = 17363.511
Iteration 7: Log likelihood = 17392.501
Iteration 8: Log likelihood = 17407.242
Iteration 9: Log likelihood = 17448.702
(switching technique to nr)
Iteration 10: Log likelihood = 17472.199
Iteration 11: Log likelihood =
                                 17475.84
Iteration 12: Log likelihood = 17476.345
Iteration 13: Log likelihood =
                                17476.35
Iteration 14: Log likelihood =
                                 17476.35
Refining estimates
Iteration 0: Log likelihood =
                                  17476.35
Iteration 1: Log likelihood =
                                  17476.35
```

Dynamic conditional correlation MGARCH model

Sample: 2 thru 2015 Number of obs = 2,014Distribution: Gaussian Wald chi2(1) = 2.21Log likelihood = 17476.35 Prob > chi2 = 0.1374

	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
ARCH_toyota						
arch L1.	.0608188	.0086675	7.02	0.000	.0438308	.0778067
garch L1.	.9219957	.0111066	83.01	0.000	.9002271	.9437643
_cons	4.49e-06	1.14e-06	3.95	0.000	2.27e-06	6.72e-06
ARCH_nissan						
arch L1.	.0876162	.01302	6.73	0.000	.0620974	.1131349
garch L1.	.8950964	.0152908	58.54	0.000	.865127	.9250657
_cons	7.69e-06	1.99e-06	3.86	0.000	3.79e-06	.0000116
honda						
nissan L1.	.019978	.0134488	1.49	0.137	0063811	.0463371
ARCH_honda						
arch L1.	.0488799	.0073767	6.63	0.000	.0344218	.063338
garch L1.	.9330047	.0103944	89.76	0.000	.912632	.9533774
_cons	5.42e-06	1.36e-06	3.98	0.000	2.75e-06	8.08e-06
corr(toyota, nissan)	. 6668433	.0163209	40.86	0.000	. 6348548	.6988317
corr(toyota, honda)	.7258101	.0137072	52.95	0.000	.6989446	.7526757
corr(nissan, honda)	.6313515	.0175454	35.98	0.000	.5969631	.6657399
/Adjustment lambda1 lambda2	.0324493 .8574681	.0074013 .0476273	4.38 18.00	0.000	.0179429 .7641202	.0469556

It turns out that the coefficient on L1.nissan in the honda equation is now statistically insignificant. We could further improve the model by removing L1.nissan from the model.

There is no mean equation for Toyota or Nissan. In [TS] mgarch dcc postestimation, we discuss prediction from models without covariates.

#### Example 3: Model with constraints

Iteration 1: Log likelihood =

Here we fit a bivariate DCC MGARCH model for the Toyota and Nissan shares. We believe that the shares of these car manufacturers follow the same process, so we impose the constraints that the ARCH coefficients are the same for the two companies and that the GARCH coefficients are also the same.

```
. constraint 1 _b[ARCH_toyota:L.arch] = _b[ARCH_nissan:L.arch]
. constraint 2 _b[ARCH_toyota:L.garch] = _b[ARCH_nissan:L.garch]
. mgarch dcc (toyota nissan = , noconstant), arch(1) garch(1) constraints(1 2)
Calculating starting values....
Optimizing log likelihood
(setting technique to bhhh)
Iteration 0: Log likelihood = 10307.609
Iteration 1: Log likelihood = 10656.153
Iteration 2: Log likelihood = 10862.137
Iteration 3: Log likelihood =
                               10987.457
Iteration 4: Log likelihood = 11062.347
Iteration 5: Log likelihood =
                               11135.207
Iteration 6: Log likelihood = 11245.619
Iteration 7: Log likelihood =
                               11253.56
Iteration 8: Log likelihood =
                                   11294
Iteration 9: Log likelihood =
                               11296.364
(switching technique to nr)
Iteration 10: Log likelihood =
                                11296.76
Iteration 11: Log likelihood =
                               11297.087
Iteration 12: Log likelihood =
                               11297.091
Iteration 13: Log likelihood =
                               11297.091
Refining estimates
Iteration 0: Log likelihood = 11297.091
```

Dynamic conditional correlation MGARCH model

Sample: 1 thru 2015 Distribution: Gaussian Log likelihood = 11297.09 Number of obs = 2,015 Wald chi2(.) = . Prob > chi2 = .

- ( 1) [ARCH\_toyota]L.arch [ARCH\_nissan]L.arch = 0
- ( 2) [ARCH\_toyota]L.garch [ARCH\_nissan]L.garch = 0

	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
ARCH_toyota arch L1.	.080889	.0103229	7.84	0.000	.0606565	.1011216
garch L1.	.9060711	.011911	76.07	0.000	.882726	.9294163
_cons	4.21e-06	1.10e-06	3.83	0.000	2.05e-06	6.36e-06
ARCH_nissan arch L1.	.080889	.0103229	7.84	0.000	.0606565	.1011216
garch L1.	.9060711	.011911	76.07	0.000	.882726	.9294163
_cons	5.92e-06	1.47e-06	4.03	0.000	3.04e-06	8.80e-06
corr(toyota, nissan)	. 6646283	.0187794	35.39	0.000	.6278214	.7014351
/Adjustment lambda1 lambda2	.0446559 .8686055	.0123017 .0510884	3.63 17.00	0.000	.020545 .7684741	.0687668

We could test our constraints by fitting the unconstrained model and performing a likelihood-ratio test. The results indicate that the restricted model is preferable.

# Example 4: Model with a GARCH term

In this example, we have data on fictional stock returns for the Acme and Anvil corporations, and we believe that the movement of the two stocks is governed by different processes. We specify one ARCH and one GARCH term for the conditional variance equation for Acme and two ARCH terms for the conditional variance equation for Anvil. In addition, we include the lagged value of the stock return for Apex, the main subsidiary of Anvil corporation, in the variance equation of Anvil. For Acme, we have data on the changes in an index of futures prices of products related to those produced by Acme in afrelated. For Anvil, we have data on the changes in an index of futures prices of inputs used by Anvil in afinputs.

4

- . use https://www.stata-press.com/data/r18/acmeh
- . mgarch dcc (acme = afrelated, noconstant arch(1) garch(1))
- > (anvil = afinputs, arch(1/2) het(L.apex))

Calculating starting values....

Optimizing log likelihood

(setting technique to bhhh)

Iteration 0: Log likelihood = -13260.522

(output omitted)

Iteration 9: Log likelihood = -12362.876

(switching technique to nr)

Iteration 10: Log likelihood = -12362.876

Refining estimates

Iteration 0: Log likelihood = -12362.876 Iteration 1: Log likelihood = -12362.876 Dynamic conditional correlation MGARCH model

Sample: 2 thru 2500 Distribution: Gaussian

Log likelihood = -12362.88

Number of obs = 2,499 Wald chi2(2) = 2596.18Prob > chi2 = 0.0000

	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
acme afrelated	.950805	.0557082	17.07	0.000	.841619	1.059991
ARCH_acme arch	.1063295	.015716	6.77	0.000	.0755266	.1371324
garch L1.	.7556294	.0391568	19.30	0.000	. 6788836	.8323752
_cons	2.197566	.458343	4.79	0.000	1.29923	3.095901
anvil afinputs _cons	-1.015657 .0808653	.0209959 .019445	-48.37 4.16	0.000	-1.056808 .0427538	9745054 .1189767
ARCH_anvil arch L1. L2.	.5261675 .2866454	.0281586 .0196504	18.69 14.59	0.000	.4709777 .2481314	.5813572 .3251595
apex L1. _cons	1.953173 0062964	.0594862	32.83	0.000	1.836582 1456188	2.069763
corr(acme, anvil)	5600358	.0326358	-17.16	0.000	6240008	4960708
/Adjustment lambda1 lambda2	.1904321 .7147267	.0154449 .0226204	12.33 31.60	0.000	.1601607 .6703916	.2207035

The results indicate that increases in the futures prices for related products lead to higher returns on the Acme stock, and increased input prices lead to lower returns on the Anvil stock. In the conditional variance equation for Anvil, the coefficient on L1.apex is positive and significant, which indicates that an increase in the return on the Apex stock leads to more variability in the return on the Anvil stock.

### Stored results

mgarch dcc stores the following in e():

```
Scalars
    e(N)
                             number of observations
    e(k)
                             number of parameters
    e(k_extra)
                             number of extra estimates added to _b
                             number of equations in e(b)
    e(k_eq)
    e(k_dv)
                             number of dependent variables
    e(df_m)
                             model degrees of freedom
    e(11)
                             log likelihood
                             \chi^2
    e(chi2)
    e(p)
                             p-value for model test
    e(estdf)
                             1 if distribution parameter was estimated, 0 otherwise
    e(usr)
                             user-provided distribution parameter
    e(tmin)
                             minimum time in sample
    e(tmax)
                             maximum time in sample
    e(N_gaps)
                             number of gaps
    e(rank)
                             rank of e(V)
                             number of iterations
    e(ic)
    e(rc)
                             return code
    e(converged)
                             1 if converged, 0 otherwise
Macros
    e(cmd)
                             mgarch
    e(model)
                             dcc
    e(cmdline)
                             command as typed
    e(depvar)
                             names of dependent variables
    e(covariates)
                             list of covariates
    e(dv_eqs)
                             dependent variables with mean equations
                             independent variables in each equation
    e(indeps)
    e(tvar)
                             time variable
    e(hetvars)
                             variables included in the conditional variance equations
    e(title)
                             title in estimation output
                             Wald; type of model \chi^2 test
    e(chi2type)
                             vcetype specified in vce()
    e(vce)
    e(vcetype)
                             title used to label Std. err.
    e(tmins)
                             formatted minimum time
    e(tmaxs)
                             formatted maximum time
                             distribution for error term: gaussian or t
    e(dist)
                             specified ARCH terms
    e(arch)
                             specified GARCH terms
    e(garch)
    e(technique)
                             maximization technique
    e(properties)
    e(estat_cmd)
                             program used to implement estat
    e(predict)
                             program used to implement predict
                             predictions allowed by margins
    e(marginsok)
    e(marginsnotok)
                             predictions disallowed by margins
    e(marginsdefault)
                             default predict() specification for margins
    e(asbalanced)
                             factor variables fyset as asbalanced
                             factor variables fvset as asobserved
    e(asobserved)
```

```
Matrices
    e(b)
                              coefficient vector
    e(Cns)
                              constraints matrix
    e(ilog)
                              iteration log (up to 20 iterations)
    e(gradient)
                              gradient vector
    e(hessian)
                              Hessian matrix
    e(V)
                              variance-covariance matrix of the estimators
    e(V_modelbased)
                              model-based variance
    e(pinfo)
                              parameter information, used by predict
Functions
    e(sample)
                              marks estimation sample
```

In addition to the above, the following is stored in r():

```
Matrices
r(table) matrix containing the coefficients with their standard errors, test statistics, p-values,
and confidence intervals
```

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

#### Methods and formulas

mgarch dcc estimates the parameters of the DCC MGARCH model by maximum likelihood. The log-likelihood function based on the multivariate normal distribution for observation t is

$$l_t = -0.5m\log(2\pi) - 0.5\log\left\{\det\left(\mathbf{R}_t\right)\right\} - \log\left\{\det\left(\mathbf{D}_t^{1/2}\right)\right\} - 0.5\widetilde{\epsilon}_t\mathbf{R}_t^{-1}\widetilde{\epsilon}_t'$$

where  $\tilde{\boldsymbol{\epsilon}}_t = \mathbf{D}_t^{-1/2} \boldsymbol{\epsilon}_t$  is an  $m \times 1$  vector of standardized residuals,  $\boldsymbol{\epsilon}_t = \mathbf{y}_t - \mathbf{C}\mathbf{x}_t$ . The log-likelihood function is  $\sum_{t=1}^T l_t$ .

If we assume that  $\nu_t$  follow a multivariate t distribution with degrees of freedom (df) greater than 2, then the log-likelihood function for observation t is

$$\begin{split} l_t &= \log \Gamma \left(\frac{\mathrm{df} + m}{2}\right) - \log \Gamma \left(\frac{\mathrm{df}}{2}\right) - \frac{m}{2} \log \left\{ (\mathrm{df} - 2)\pi \right\} \\ &- 0.5 \mathrm{log} \left\{ \det \left(\mathbf{R}_t\right) \right\} - \log \left\{ \det \left(\mathbf{D}_t^{1/2}\right) \right\} - \frac{\mathrm{df} + m}{2} \mathrm{log} \left(1 + \frac{\widetilde{\boldsymbol{\epsilon}}_t \mathbf{R}_t^{-1} \widetilde{\boldsymbol{\epsilon}}_t'}{\mathrm{df} - 2} \right) \end{split}$$

The starting values for the parameters in the mean equations and the initial residuals  $\hat{\epsilon}_t$  are obtained by least-squares regression. The starting values for the parameters in the variance equations are obtained by a procedure proposed by Gouriéroux and Monfort (1997, sec. 6.2.2). The starting values for the quasicorrelation parameters are calculated from the standardized residuals  $\tilde{\epsilon}_t$ . Given the starting values for the mean and variance equations, the starting values for the parameters  $\lambda_1$  and  $\lambda_2$  are obtained from a grid search performed on the log likelihood.

The initial optimization step is performed in the unconstrained space. Once the maximum is found, we impose the constraints  $\lambda_1 \geq 0$ ,  $\lambda_2 \geq 0$ , and  $0 \leq \lambda_1 + \lambda_2 < 1$ , and maximize the log likelihood in the constrained space. This step is reported in the iteration log as the refining step.

GARCH estimators require initial values that can be plugged in for  $\epsilon_{t-i}\epsilon'_{t-i}$  and  $\mathbf{H}_{t-j}$  when t-i < 1 and t-j < 1. mgarch dcc substitutes an estimator of the unconditional covariance of the disturbances

$$\widehat{\Sigma} = T^{-1} \sum_{t=1}^{T} \widehat{\hat{\epsilon}}_t \widehat{\hat{\epsilon}}_t'$$
 (2)

for  $\epsilon_{t-i}\epsilon'_{t-i}$  when t-i < 1 and for  $\mathbf{H}_{t-j}$  when t-j < 1, where  $\widehat{\hat{\epsilon}}_t$  is the vector of residuals calculated using the estimated parameters.

mgarch dcc uses numerical derivatives in maximizing the log-likelihood function.

### References

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Engle, R. F. 2002. Dynamic conditional correlation: A simple class of multivariate generalized autoregressive conditional heteroskedasticity models. Journal of Business and Economic Statistics 20: 339-350. https://doi.org/10.1198/073500102288618487.

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Gouriéroux, C. S., and A. Monfort. 1997. Time Series and Dynamic Models. Trans. ed. G. M. Gallo. Cambridge: Cambridge University Press.

#### Also see

- [TS] mgarch dcc postestimation Postestimation tools for mgarch dcc
- [TS] arch Autoregressive conditional heteroskedasticity (ARCH) family of estimators
- [TS] mgarch Multivariate GARCH models
- [TS] **tsset** Declare data to be time-series data
- [TS] var Vector autoregressive models<sup>+</sup>
- [U] 20 Estimation and postestimation commands

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