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**stintcox** — Cox proportional hazards model for interval-censored survival-time data

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# **Description**

stintcox fits semiparametric Cox proportional hazards models to interval-censored survival-time data or, more precisely, event-time data, which may contain right-censored, left-censored, and interval-censored observations. stintcox can be used with single- or multiple-record interval-censored data. With interval-censored data, the event-time variables are specified with the stintcox command instead of using stset. All st settings are ignored by stintcox.

# **Quick start**

Single-record-per-subject interval-censored data

Cox proportional hazards model with covariates x1 and x2 fit to interval-censored data with lower and upper interval endpoints t1 and t2

```
stintcox x1 x2, interval(t1 t2)
```

Same as above, but estimate the baseline hazard function using all observed intervals instead of the default reduced set

```
stintcox x1 x2, interval(t1 t2) full
```

Use less stringent convergence criteria to explore initial results more quickly

```
stintcox x1 x2, interval(t1 t2) favorspeed
```

Fit a stratified Cox model with strata defined by levels of svar

```
stintcox x1 x2, interval(t1 t2) strata(svar)
```

Include a time-varying covariate by interacting x2 with the logarithm of analysis time

```
stintcox x1 x2, interval(t1 t2) tvc(x2) texp(ln(_t))
```

Report the log-likelihood model test instead of the default Wald model test, and report regression coefficients instead of hazard ratios

```
stintcox x1 x2, interval(t1 t2) lrmodel nohr
```

Report OIM standard errors instead of the default OPG standard errors

```
stintcox x1 x2, interval(t1 t2) vce(oim)
```

After estimation, report regression coefficients instead of hazard ratios

```
stintcox, nohr
```

After estimation, report OPG standard errors using fixed step size instead of the default adaptive step size

```
stintcox, vce(opg, stepsize(fixed))
```

After estimation, save estimated baseline hazard contributions to basehc.dta, and store estimation results as intcox

```
stintcox, saving(basehc)
estimates store intcox
```

Multiple-record-per-subject interval-censored data

Cox proportional hazards model with baseline (time-invariant) covariate x1 and time-varying covariate x2 fit to multiple-record-per-subject interval-censored data with subject identifier idvar, examination time tvar, and event status indicator status

```
stintcox x1 x2, id(idvar) time(tvar) status(status)
```

Same as above, but use the covariate values at the nearest examination time on the right instead of the default nearest examination time on the left to impute values of time-varying covariate x2 between two examination times

```
stintcox x1 x2, id(idvar) time(tvar) status(status) ///
    tvcovimpute(nearright)
```

Report robust standard errors instead of the default OPG standard errors

```
stintcox x1 x2, id(idvar) time(tvar) status(status) vce(robust)
```

Report cluster-robust standard errors with the cluster identifier clustvar

#### Menu

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# **Syntax**

```
Single-record-per-subject interval-censored data
```

```
stintcox [indepvars] [if] [in], interval(t_l t_u) [single\_options]
```

Multiple-record-per-subject interval-censored data

```
stintcox [indepvars] [if] [in], id(idvar) time(timevar) status(statusvar)
  [multiple_options]
```

single_options	Description
Model	
$^*$ $\underline{ ext{int}}$ erval $(t_l  t_u)$	specify lower and upper endpoints for the event-time interval
options	options for both single- and multiple-record interval-censored data

<sup>\*</sup>interval(t<sub>l</sub> t<sub>u</sub>) is required for single-record interval-censored data and cannot be combined with option id(), time(), status(), or tvcovimpute().

multiple_options	Description		
Model			
†id( <i>idvar</i> )	specify multiple-record ID variable		
†time(timevar)	specify examination time variable		
†status(statusvar)	specify event status indicator variable		
<pre>tvcovimpute(type)</pre>	specify how to impute unobserved covariate values between examination times for time-varying covariates; default is nearleft		
statussysmissok options	retain the observations that contain system missing values (.) options for both single- and multiple-record interval-censored data		

 $<sup>\</sup>dagger$ id(), time(), and status() are required for multiple-record interval-censored data and cannot be combined with option interval().

options	Description
Model	
strata(varlist)	specify strata variables
reduced	estimate baseline hazard function using a reduced set of time intervals; the default
full	estimate baseline hazard function using all time intervals
<u>favoracc</u> uracy	favor accuracy of results over speed; the default
favorspeed	favor speed possibly over accuracy of results
Time varying	
$tvc(varlist_t)$	specify covariates to be interacted with a function of time
texp(exp)	specify a function of time; default is texp(_t)
lrphtest	perform the likelihood-ratio test for covariates interacted with time; default is to perform Wald test
SE/Robust	
vce(vcetype)	<pre>vcetype may be one of opg (the default), oim, robust, or cluster clustvar; may be specified on replay of results</pre>
Reporting	
$\underline{1}$ evel(#)	set confidence level; default is level(95)
lrmodel	perform the likelihood-ratio model test instead of the default Wald model test
<pre>saving(filename[, replace])</pre>	save estimates of baseline hazard contributions to <i>filename</i> ; use replace to overwrite existing <i>filename</i>
nohr	report regression coefficients, not hazard ratios
<u>nohead</u> er	suppress header from coefficient table
[no]log	display or suppress EM and VCE iteration logs; default is log
dots	display all EM and VCE iterations as dots
[no]vcedots[(#)]	display or suppress VCE iteration dots; default is to display a dot every iteration, meaning vcedots or vcedots(1)
[no]emlog[(#)]	display or suppress EM iteration log; default is emlog(100), which displays the log-likelihood value every 100 iterations
[no]emdots $[(#)]$	display or suppress EM iteration dots; default is noemdots
display_options	control columns and column formats, row spacing, line width, display of omitted variables and base and empty cells, and factor-variable labeling

– N /I	options

vcetype

post

•	
<pre>emiterate(#)</pre>	maximum number of EM iterations; default is emiterate(5000)
<pre>emtolerance(#)</pre>	tolerance for the coefficient vector; default is emtolerance(1e-6)
<pre>emltolerance(#)</pre>	tolerance for the log likelihood; default is emltolerance(1e-7)
<pre>emhsgtolerance(#)</pre>	tolerance for the scaled gradient; default is emhsgtolerance(1e-5)
noemhsgtolerance	do not perform the scale-gradient convergence check
from(init_specs)	initial values for the regression coefficients
<u>coefl</u> egend	display legend instead of statistics

indepvars and  $varlist_t$  may contain factor variables; see [U] 11.4.3 Factor variables.

by, collect, and statsby are allowed; see [U] 11.1.10 Prefix commands.

coeflegend does not appear in the dialog box.

See [U] 20 Estimation and postestimation commands for more capabilities of estimation commands.

Description

rectype	2 estipation
opg[, vce_options]	outer product of the gradient (OPG) vectors; the default
$oim[, vce\_options]$	observed information matrix (OIM)
<pre>robust[ , vce_options ]</pre>	Huber/White/sandwich estimator
$\underline{\mathtt{cl}} \mathtt{uster} \ \mathit{clustvar} \big[ \ \textit{, vce\_options}  \big]$	clustered sandwich estimator
vce_options	Description
$\verb stepsize(adaptive fixed[\#]) $	use adaptive or fixed step size to compute VCE; default is adaptive step size
derivopts	options to control computation of numerical derivatives when adaptive step size is used
<pre>iterate(#)</pre>	maximum number of iterations to compute VCE; default is iterate(5000)
tolerance(#)	<pre>profile log-likelihood tolerance to compute VCE;   default is tolerance(1e-6)</pre>
[no]dots[(#)]	synonym for vcedots, vcedots(), and novcedots

or cluster

replace the current e(V) with the specified VCE type;

can be used only on replay with opg, oim, robust,

dots, dots(), nodots, and post do not appear in the dialog box.

# **Options**

Model

interval(t<sub>1</sub> t<sub>n</sub>) is required with single-record-per-subject interval-censored data; see Single-versus multiple-record interval-censored data formats in Remarks and examples. It specifies two time variables that contain the endpoints of the event-time interval.  $t_l$  represents the lower endpoint, and  $t_u$  represents the upper endpoint. interval() may not be combined with option id(), time(), status(), or tvcovimpute().

The interval time variables  $t_l$  and  $t_u$  should have the following form:

Type of observations		$t_l$	$t_u$
interval-censored	(a,b]	a	$\overline{b}$
left-censored	(0,b]		b
left-censored	(0,b]	0	b
right-censored	$(a, +\infty)$	a	
missing			
missing		0	

In the table, a and b satisfy  $0 < a < b < \infty$ . Also note that  $t_l = t_u$  is not allowed with left-censored or interval-censored observations.

id(idvar) is required with multiple-record-per-subject interval-censored data; see Single- versus multiple-record interval-censored data formats in Remarks and examples. It specifies the subject-ID variable; observations with equal, nonmissing values of idvar are assumed to belong to the same subject. Observations for which idvar is missing are ignored.

When id() is not specified, each observation is assumed to represent a different subject and thus constitutes a single-record-per-subject dataset.

When you specify id(), the data are said to be multiple-record-per-subject data, even if it turns out that there is only one record per subject. Multiple-record-per-subject data can be used to accommodate time-varying covariates that exist in the dataset.

If you specify id(), stintcox requires that you also specify options time() and status(). id() may not be combined with the interval() option.

time(timevar) is required with multiple-record-per-subject interval-censored data; see Single-versus multiple-record interval-censored data formats in Remarks and examples. It specifies the examination times for the event of interest and may not be combined with the interval() option.

status(statusvar) is required with multiple-record-per-subject interval-censored data; see Singleversus multiple-record interval-censored data formats in Remarks and examples. It specifies a binary status indicator for the event of interest. For each examination time, statusvar indicates, by the value of 1 versus 0, whether the event has occurred between previous and current examination times. status() may not be combined with the interval() option.

In combination with option statussysmissok, observations with system missing values (.) in the statusvar variable will be used during estimation but will not be used to determine the event-time information for the corresponding subjects; see the description of option statussysmissok.

Recurrent events are not allowed. After the first event occurs, the subject is removed from the analysis, even if the subject has subsequent records in the data.

Options time() and status() together define the lower and upper endpoints of the event-time intervals ( $t_l$  and  $t_u$ ) and the censoring types in a multiple-record-per-subject interval-censored dataset. If the event of interest occurs before the first examination time for a subject, the subject is left-censored. In the first record for this subject, time is the first examination time, and the event-status indicator is 1. The corresponding event-time interval for this subject has 0 as the left lower endpoint,  $t_l$ , and the first examination time as the right upper endpoint,  $t_u$ . If the event occurs between two examination times, the subject is interval-censored. The event-time interval for this subject has the last examination time where the status indicator is 0 as  $t_l$  and the first examination time where the status indicator is 1 as  $t_u$ . If the event does not occur during the study, the subject is right-censored. The event-time interval for this subject has the last examination time as  $t_l$  and missing time (.) as  $t_u$ .

- strata(varlist) specifies the stratification variables. Observations with equal values of the strata variables are assumed to be in the same stratum. Stratified estimates (equal regression coefficients across strata but with a baseline hazard unique to each stratum) are then obtained.
- reduced, the default, specifies that the baseline hazard function be estimated using a reduced (innermost) set of time intervals. This allows the estimator of the cumulative baseline hazard function to change its values only at the endpoints of the innermost time intervals, which were originally used by Turnbull (1976) to estimate the survivor function in the one-sample case. This option may not be combined with full.
- full specifies that the baseline hazard function be estimated using all observed time intervals. In this case, the estimator of the cumulative baseline hazard function can potentially change its values at the endpoints of all the observed time intervals. This is the approach used by Zeng, Mao, and Lin (2016). It is more time consuming, but it may provide more accurate results. full may not be combined with reduced.
- favoraccuracy, the default, and favorspeed control the tradeoff between accuracy of the results and the execution speed. favoraccuracy specifies that the command run longer to obtain more accurate results, favorspeed specifies that the command run faster at the possible expense of reduced accuracy of the results. You can use favorspeed for a quick initial exploration of the results and favoraccuracy for final reporting of the results.
  - When you specify favorspeed, stintcox uses less stringent convergence criteria to obtain the results. Specifically, it assumes lower EM coefficient, likelihood, and VCE tolerances of 0.0001 and implies option noembsgtolerance. In addition, it uses a fixed step size with a multiplier of 5 instead of an adaptive step size when computing VCE. That is, specifying favorspeed is equivalent to specifying emtolerance (0.0001), emltolerance (0.0001), noembsgtolerance, and vce(, tolerance(0.0001) stepsize(fixed)).
- tvcovimpute(type) is used with multiple-record-per-subject interval-censored data and relevant only to variables included in the model whose values vary over time in the dataset. It specifies how to impute unobserved covariate values between two examination times for time-varying covariates in the dataset for each subject. type is one of nearleft, the default, nearright, nearest, or first. tvcovimpute() may not be combined with the interval() option.
  - stintcox requires covariate values for each subject at all distinct analysis times, but the data typically record covariates only at subject-specific examination times, and the covariate values at other analysis times need to be imputed. stintcox offers the following imputation methods.
  - nearleft, the default, uses covariate values at the nearest examination time on the left to impute covariate values at observation times that fall between two examination times for a given subject. This method is often preferred in practice because it does not use future covariate values to fill in the current values. This is also the method used with right-censored survival-time data.

nearright uses covariate values at the nearest examination time on the right to impute covariate values at analysis times that fall between two examination times for a given subject.

nearest uses covariate values at the nearest examination time, left or right, to impute covariate values at analysis times that fall between two examination times for a given subject.

For all three imputation methods above, if a subject is not examined at baseline (time 0), then the covariate value at the first examination time is used for all analysis times before the first examination time for this subject. And the covariate value at the last examination time is used for all observation times after the last examination time for a subject.

first replaces all covariate values for a subject with those at the first examination time, which is the same as using the baseline covariate values for all time-varying variables during estimation.

See Methods and formulas for details.

statussysmissok is used with multiple-record-per-subject format. It specifies that, during estimation, the observations that contain system missing values (.) in the event status variable specified in option status() be retained. These observations will not be used to determine the event-time intervals and censoring information for the corresponding subjects, but examination times and covariate values in these observations will be used during estimation. Without this option, stintcox omits system missing observations in status() from estimation, like any other missing value in any of the specified variables. Observations that contain extended missing values (.a through .z) are always omitted during estimation. Option statussysmissok is useful to create time-varying covariates in a multiple-record-per-subject format; see Single- versus multiple-record interval-censored data formats in Remarks and examples.

Time varying

tvc(varlist<sub>t</sub>) specifies the variables to be included in the model as an interaction with a function of time to form time-varying covariates. During estimation, these variables are interacted with analysis time or with a function of analysis time specified in the texp() option. This is a convenience option to include time-varying covariates that are deterministic functions of time. Using this option speeds up calculations and avoids having to split the data over many analysis times. tvc() in conjunction with texp() is also useful for testing the proportional-hazards assumption; see Testing the proportional-hazards assumption using option tvc() in Remarks and examples.

texp(exp) is used in conjunction with  $tvc(varlist_t)$  to specify the function of time that should be used to multiply covariates specified in the tvc() option to include in the model time-varying covariates that are deterministic functions of time. For example, specifying  $texp(ln(_t))$  would cause the covariates in option tvc() to be multiplied by the logarithm of analysis time. If  $tvc(varlist_t)$  is used without texp(exp), Stata understands that you mean  $texp(_t)$  and thus multiplies the covariates by the analysis time, denoted as  $_t$  here.

lrphtest is used in conjunction with  $tvc(varlist_t)$ . It performs the likelihood-ratio test between the full model and the model without covariates interacted with time, that is, without specifying option tvc(). By default, the Wald test of coefficients on time-varying covariates equal to zero is reported when option  $tvc(varlist_t)$  is specified.

Options  $tvc(varlist_t)$ , texp(exp), and lrphtest are explained more in Testing the proportional-hazards assumption using option tvc() in Remarks and examples.

SE/Robust

- vce(vcetype) specifies the type of standard error estimate reported. vce() may be specified at the time of estimation or when replaying results. If specified when replaying results, vce()-related stored results are not updated unless the post suboption is specified. vcetype may be one of the following:
  - vce(opg[, vce\_options]) uses the sum of the OPG vectors based on the profile log likelihood; see Methods and formulas. vce(opg) is the default.
  - vce(oim[, vce\_options]) uses the sum of the OIM vectors based on the profile log likelihood; see Methods and formulas.
  - vce(robust[, vce\_options]) uses the robust or sandwich estimator of variance based on the
    profile log likelihood; see Methods and formulas.
  - vce(cluster clustvar [, vce\_options]) specifies that the standard errors allow for intragroup correlation, relaxing the usual requirement that the observations be independent. That is, the observations are independent across groups (clusters) but not necessarily within groups. clustvar specifies to which group each observation belongs. The clustered sandwich estimator of variance is based on the profile log likelihood; see Methods and formulas.
  - vce\_options may be stepsize(), derivopts with adaptive step size, iterate(#), tolerance(#),
    dots, dots(), nodots, and post.
    - stepsize(adaptive | fixed  $\lfloor \# \rfloor$ ) specifies the step size for computing numerical derivatives with methods opg, oim, robust, or cluster. The default is stepsize(adaptive), which uses adaptive step size in computations; see [M-5] **deriv**(). stepsize(fixed) uses a fixed step size equal to  $\delta_n = 5n^{-1/2}$ , where n is the number of subjects or the number of clusters. stepsize(fixed #) uses a fixed step size equal to  $\# \times n^{-1/2}$ .
    - derivopts are allowed only with stepsize(adaptive) and may be search(), h(), scale(),
      and bounds().
      - search(search\_type) specifies the approach used to search for an optimal step size
        for computing the numerical derivatives; three approaches are offered: bracket,
        interpolate, and off; see deriv\_init\_search() in [M-5] deriv(). The default
        is search(interpolate). In some cases, such as when factor variables have highly
        unbalanced levels, the search may lead to the step size that is too small or too large, which
        may lead to the error message that the estimates of baseline hazard contributions cannot be
        computed because the VCE matrix is close to being singular. Trying search(bracket)
        may be helpful in this case.
      - h(#|matname) specifies the h values, which are multipliers for step size used to compute numerical derivatives; see  $deriv\_init\_h()$  in [M-5] deriv(). You can specify the same h value, #, for all parameters or parameter-specific h values as a Stata matrix (vector) matname.
      - scale(# | matname) specifies the starting scale values used to compute numerical derivatives;
        see deriv\_init\_scale() in [M-5] deriv(). You can specify the same initial scale value,
        #, for all parameters or parameter-specific initial scale values as a Stata matrix (vector)
        matname.
      - bounds(#<sub>1</sub> #<sub>2</sub>) specifies the minimum and maximum values used to search for optimal scale values; see deriv\_init\_bounds() in [M-5] deriv(). The default is bounds(1e-6 1e-5).
    - iterate(#) specifies the maximum number of iterations to compute the VCE based on the profile log likelihood. The default is iterate(5000).

- tolerance(#) specifies the tolerance for the profile log likelihood used to compute the VCE. The default is tolerance(1e-6).
- dots, dots(#), and nodots display or suppress iteration dots showing the progress of the variance estimation. The dots are displayed by default, but you can use nodots to suppress them. By default, the dot is displayed every iteration, but you can change this by specifying dots(#).

When a fixed step size is used, an iteration corresponds to one derivative computation with respect to a regression coefficient. When an adaptive step size is used, an iteration corresponds to one call of the Mata deriv() function, which may be called multiple times to compute one derivative with respect to one regression coefficient. Thus, you will typically see more iteration dots with VCE estimation using an adaptive step size than using a fixed step size.

These options do not appear in the dialog box.

post can be used only on replay with vce(opg), vce(oim), vce(robust), or vce(cluster clustvar). It replaces the current e(V) with the specified vcetype. When vce(vcetype) is used on replay without post, the coefficient table will display the standard error of the specified vcetype, but e(V) will remain unchanged. This option does not appear in the dialog box.

Reporting

level(#), lrmodel; see [R] Estimation options.

saving(filename[, replace]) saves the estimated baseline hazard contributions in filename.dta. The replace option specifies to overwrite filename.dta if it exists. If option saving() is not specified, stintcox saves estimation results in a temporary file for later access by postestimation commands. This temporary file will be overridden every time stintcox is run and will also be erased if the current estimation results are cleared. saving() may be specified during estimation or on replay.

Because the file containing the baseline hazard contributions is considered to be part of estimation results, you must use option saving() before storing or saving your estimation results using estimates store or estimates save.

nohr specifies that regression coefficients be displayed rather than exponentiated regression coefficients or hazard ratios. This option affects only how results are displayed and not how they are estimated. nohr may be specified at estimation time or when replaying results.

noheader suppresses the output header, either at estimation or upon replay.

- log and nolog display or suppress stintcox's iteration log, which includes both the EM and VCE iterations. The EM iteration log displays the log-likelihood value every 100 iterations (option emlog). The VCE iteration log displays iterations as dots (option vcedots). log, the default, implies emlog and vcedots. Use nolog to suppress both EM and VCE iteration logs, which is equivalent to specifying noemlog and novcedots. If log or nolog is specified, any other options that control an iteration log are ignored. log and nolog may not be combined with dots.
- dots implies option emdots and vcedots to display both the EM and VCE iteration logs as dots. The VCE iteration log is always displayed as dots. However, the EM iteration log, by default, displays the log-likelihood value every 100 iterations (option emlog). To display the EM iterations as dots, you can specify dots or emdots. If dots is specified, any other options that control an iteration log are ignored. dots may not be combined with log and nolog.
- vcedots, vcedots(#), and novcedots are synonyms for vce(, dots), vce(, dots(#)), and vce(, nodots), respectively.

emlog, emlog(#), and noemlog display or suppress an iteration log showing the progress of the EM algorithm. The log is displayed by default, and noemlog suppresses it; see set iterlog in [R] set iter. emlog, the default, displays the log-likelihood value every 100 iterations and is equivalent to emlog(100). emlog(#) displays the log-likelihood value every #th iterations.

noemdots, emdots(#), and emdots control the display of the EM iteration log as dots. By default, the EM iteration log displays the log-likelihood value every 100 iterations; that is, noemdots is implied. Instead, you can specify emdots to display every 100 iterations as a dot or emdots (#) to display every # iterations as a dot. This is a useful alternative for long EM iteration logs.

display\_options: noci, nopvalues, noomitted, vsquish, noemptycells, baselevels, allbaselevels, nofvlabel, fvwrap(#), fvwrapon(style), cformat(%fmt), pformat(%fmt), sformat(% fmt), and nolstretch; see [R] Estimation options.

emiterate(#), emtolerance(#), emltolerance(#), emhsgtolerance(#), noemhsgtolerance, and from(); see iterate(), tolerance(), ltolerance(), nrtolerance(), nonrtolerance, and from() in [R] Maximize. These options control the EM optimization process. The defaults are emiterate(5000), emtolerance(1e-6), emltolerance(1e-7), and emhsgtolerance(1e-5).

The following option is available with stintcox but is not shown in the dialog box: coeflegend; see [R] Estimation options.

# Remarks and examples

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Remarks are presented under the following headings:

Introduction

Single- versus multiple-record interval-censored data formats

Case II interval-censored data

Time-varying covariates

Standard error estimation with interval-censored data

Current status or case I interval-censored data

Testing the proportional-hazards assumption using option tvc()

#### Introduction

stintcox fits the Cox proportional hazards model to interval-censored survival-time data. In the context of interval-censored data, the term "failure-time data" or "event-time data" is more appropriate, so we will use it in that context.

Interval-censoring occurs when the failure time or the event time of interest is not exactly observed but is known only to lie within some interval. See Introduction in [ST] stintreg for details about interval-censored data. If you have right-censored data, see [ST] stcox. See [ST] stintreg for fitting parametric models to interval-censored data.

The Cox proportional hazards model was first introduced by Cox (1972) for right-censored survival data. For an introduction to interval-censored data, see Finkelstein and Wolfe (1985), Odell, Anderson, and D'Agostino (1992), Rabinowitz, Tsiatis, and Aragon (1995), Huang and Wellner (1997), Lindsey (1998), Lindsey and Ryan (1998), Sun (2006), and Sun and Li (2014).

The Cox proportional hazards model specifies that the hazard function of the event time conditional on a p-vector of baseline (time-invariant or time-independent) covariates  $\mathbf{x} = (x_1, \dots, x_p)$  takes the form

$$h(t; \mathbf{x}) = h_0(t) \exp(\beta_1 x_1 + \dots + \beta_p x_p)$$

where  $\beta_1,\ldots,\beta_p$  are unknown regression coefficients and  $h_0(t)$  is an arbitrary baseline hazard function. Under the proportional-hazards assumption, the hazard ratios, or exponentiated regression coefficients  $\exp(\beta_1),\ldots,\exp(\beta_p)$ , are constant over time. As with right-censored data, the Cox proportional hazards model is appealing for interval-censored data because it does not require parameterization of the baseline hazard function and, for low event rates, the exponentiated regression parameters approximate the relative risks.

The partial-likelihood approach (Cox 1972, 1975) is used to estimate parameters of the Cox model with right-censored data, in which some of the event times are observed exactly, while others are known to be longer than the duration of follow-up. Under interval-censoring, however, none of the event times are observed exactly. Thus, it is much more challenging to deal with interval-censored data than right-censored data, both theoretically and computationally. In particular, the traditional partial-likelihood approach is not applicable.

Several authors (Cai and Betensky 2003; Zhang, Hua, and Huang 2010; Wang et al. 2016) have proposed spline methods to fit the Cox proportional hazards model to interval-censored data. Spline methods have limitations, however. First, the choices for the type of spline and the number and positions of knots are arbitrary, and different choices may yield conflicting results. Second, the analysis will be biased if the event-time distribution is not well approximated by the chosen spline function. Finally, the variance estimation is difficult given the data-dependent choices of spline functions (Zhang, Hua, and Huang 2010).

Direct maximum-likelihood optimization for the Cox model with interval-censored data using, for instance, the Newton-Raphson algorithm is highly unstable (Sun 2006; Finkelstein 1986).

Zeng, Mao, and Lin (2016) developed a novel EM algorithm for efficient nonparametric maximum-likelihood estimation (NPMLE) of the Cox proportional hazards model with interval-censored data. It allows a completely arbitrary event-time distribution and results in consistent, asymptotically normal, and asymptotically efficient estimators of the regression parameters. And it reduces to the classical maximum partial-likelihood estimation in the special case of right-censored data. For more details about this method, see *Methods and formulas*.

Unlike with right-censored data, the estimation of regression coefficients must be performed jointly with estimation of the baseline cumulative hazard function for interval-censored data. Stata provides two ways to estimate the baseline cumulative hazard function. One is to use all distinct lower and upper interval endpoints as time points for estimating the baseline cumulative hazard function. This is available by specifying the full option.

For large datasets with many distinct time points, this approach may become time consuming. An alternative is to estimate the baseline cumulative hazard at fewer time points. Turnbull (1976) proposed a method for estimating the one-sample survivor function at a subset of time intervals, known as Turnbull's intervals, or innermost intervals, or regions of the maximal cliques. Thus, one can allow the baseline cumulative hazard function to change its values only at the endpoints of those time intervals and set baseline hazard contributions to zero for the other times. This is available via the reduced option and, for computational reasons, is the default in stintcox.

As mentioned above, NPMLE is a computationally intensive approach, so stintcox may take some time to run, especially for large datasets. The speed of the command depends on the desired accuracy of the computations, among other things. The higher the accuracy, the more iterations are needed to achieve that accuracy, and thus the longer the command runs. It is important to have high accuracy for the final reporting of the results, but the speed may become an issue during the exploratory stage of the project. Thus, you may consider using the favorspeed option to expedite

the command execution. When you specify this option, stintcox uses less stringent convergence criteria to produce the results more quickly; see the description of option favorspeed for details.

Unlike many st commands, stintcox requires that you specify event-time ("survival") information directly with the command instead of using stset. Typing stset is unnecessary, and stintcox will ignore any settings of stset for the usual trivariate response variable  $(t_0,t,d)$ . Event-time information can be specified with stintcox in two different ways depending on the storage format of the interval-censored event-time data. The two different storage formats are single-record-per-subject format (or time-intervals format) and multiple-record-per-subject format (or examination-times format). With single-record-per-subject data, you must specify two variables containing time intervals for each subject in stintcox's interval() option. With multiple-record-per-subject data, you must specify subjects' identifiers in option id(), examination times in option time(), and event-status indicators at each examination time in option status(). See Single- versus multiple-record interval-censored data formats for details.

stintcox supports time-varying (time-dependent) covariates. You can use option tvc() to more easily include time-varying covariates that are formed by multiplying covariates specified in tvc() with a deterministic function of time specified in option texp(). In a multiple-record-per-subject format, you can include more general time-varying covariates; see <u>Single-versus multiple-record interval-censored data formats</u> and <u>Time-varying covariates</u>. In the presence of general time-varying covariates, stintcox offers several methods for imputing the values of these covariates between the examination times; see the description of option tvcovimpute() for details.

stintcox does not support data exhibiting delayed entry, gaps, and multiple failures.

## Single- versus multiple-record interval-censored data formats

Interval-censored event-time data can be recorded in two different formats. In one format, the event-time information is recorded as interval data, with one record per subject containing lower and upper endpoints of the event-time interval. We call data stored in this format "single-record-per-subject interval-censored event-time data" or "single-record interval-censored data" for short.

In the other format, the event-time information is recorded by a pair of an examination time and an event status at that time. The dataset typically contains multiple records (multiple examination times) for a subject. We call data stored in this format "multiple-record-per-subject interval-censored event-time data" or "multiple-record interval-censored data". This format is common for what is called "case I interval-censored data or current status data" and is often used to accommodate time-varying covariates.

stintcox supports both formats.

**Single-record interval-censored data**. Consider the following dataset:

id	ltime	rtime	x1	x2	х3
101	0	6	17	22	0
102	4	9	12	22	1
103	13		13	22	0

Here variables ltime and rtime record the respective lower,  $t_l$ , and upper,  $t_u$ , endpoints of the eventtime interval for each subject. stintcox requires that we specify these variables in the interval() option:

```
. stintcox x1 x2 x3, interval(ltime rtime) ...
```

In this format, if the data are left-censored, the lower endpoint is zero and may be represented in  $t_l$  by either a missing value (.) or zero. If the data are right-censored, the upper endpoint is  $+\infty$  and is represented in  $t_u$  by a missing value. Uncensored data are represented by the two endpoints that are

equal. If  $0 < t_l < t_u < \infty$ , the data are interval-censored. Truly missing values must be represented by missing values in both  $t_l$  and  $t_u$  or by a 0 in  $t_l$  and a missing value in  $t_u$ . Uncensored observations, with  $t_l = t_u$ , are not allowed in the presence of left-censored or interval-censored observations.

In our example, subject 101 is left-censored, subject 102 is interval-censored, and subject 103 is right-censored.

This format is convenient for storing data containing interval-censored observations and covariates that are constant over time.

Multiple-record interval-censored data. We can provide the same event-time information as in the previous dataset in the following format:

id	time	status	x1	x2	x3
101	6	1	17	22	0
102	4	0	12	22	1
102	9	1	12	22	1
103	13	0	13	22	0

Here variable time records examination times, and variable status records the event status indicator for each examination time. And subjects may have multiple examination times. In this format, stintcox requires that we specify the subject identifier (id) in option id(), examination time (time) in option time(), and the event status indicator (status) in option status().

```
. stintcox x1 x2 x3, id(id) time(time) status(status) ...
```

Examination times and event status at each examination time can be used to determine the censoring type and the event-time interval for each subject. In our example, subject 101 has only one examination time, 6, and the event has already occurred by that time. Subject 101 is thus left-censored with the time interval  $(t_l, t_u) = (0, 6]$ . Subject 102 has two examination times, 4 and 9, and the event occurred by time 9. Subject 102 is interval-censored with the time interval (4, 9]. Subject 103 has one examination at time 13, and the event has not occurred by that time. This subject is right-censored with the time interval  $(13, +\infty)$ . Uncensored observations are not allowed in this format.

In this format, we can easily record time-varying covariates. For instance, suppose that subject 102 had an additional examination at time 6 with a status of 0 and the value for covariate x3 was recorded to be 0. x3 is no longer constant within subject 102 but varies discretely with time. (The event-time interval for this subject also changed to (6,9]! More about this below.) We can accommodate varying x3 by adding an extra record for subject 102 at time 6:

id	time st	atus	x1	x2	x3
101	6	1	17	22	0
102	4	0	12	22	1
102	6	0	12	22	0
102	9	1	12	22	1
103	13	0	13	22	0

Potential caveat when creating time-varying covariates with interval-censored data. In the above, when we added an extra record for subject 102 at time 6 with status equal 0, we modified the actual event-time interval for this subject from the original (4,9] to (6,9]. This is not a problem if this new record we added corresponds to the actual examination time for this subject. Sometimes, however, we may need to add extra records to specify varying covariate values at intermediate times without changing the event-time information for subjects. This is useful, for instance, when we create more complicated functions of time and covariates to assess the proportional-hazards assumption.

Returning to our previous example, how can we incorporate the varying values of covariate x3 without changing the event-time interval for subject 102? We can replace the zero status with a system missing value (.) in the new observation and use the statussysmissok option with stintcox to specify that this observation should be included in the estimation but should not be used to determine the subject's event-time interval and censoring information:

id	time st	atus	x1	x2	x3
101	6	1	17	22	0
102	4	0	12	22	1
102	6		12	22	0
102	9	1	12	22	1
103	13	0	13	22	0

and

. stintcox x1 x2 x3, id(id) time(time) status(status) statussysmissok ...

Without the statussysmissok option, the record with a system missing status value would have been omitted from the analysis entirely, like any missing value in any of the specified variables. If you need to omit any values in the status() variable from your analysis in the presence of statussysmissok, you should record them as extended system missing values (.a through .z).

For an example of fitting stintcox to multiple-record interval-censored data, see *Time-varying covariates*. The multiple-record-per-subject format is also convenient for fitting current status data; see *Current status or case I interval-censored data*.

#### Case II interval-censored data

Case II interval-censored data arise when there are potentially two or more examination times for each study subject. With baseline covariates, case II interval-censored data are typically recorded in a single-record-per-subject format. In the presence of time-varying covariates in the dataset, a multiple-record-per-subject format is used; see *Time-varying covariates*.

In a single-record-per-subject format, the interval that brackets the event time of interest, the event-time interval, is recorded for each subject. The event of interest may occur before the first examination time, resulting in a left-censored observation; after the last examination time, resulting in a right-censored observation; or between two examination times, resulting in a truly interval-censored observation.

# ▶ Example 1: Single-record-per-subject case II interval-censored data

Zeng, Mao, and Lin (2016) considered a cohort study of injecting drug users in Thailand. Subjects were initially seronegative for the HIV-1 virus. They were followed and assessed for HIV-1 seropositivity through blood tests approximately every four months. The event of interest was time to HIV-1 seropositivity. Because the subjects were tested approximately every four months, the exact time of HIV-1 seropositivity was not observed but was known to fall only in the interval between blood tests.

The data used in this example are the data provided in supplementary materials of Zeng, Mao, and Lin (2016), which are based on the study described above. The dataset contains 1,124 subjects: 76 are females and 1,048 are males. We wish to identify the factors that influence HIV-1 infection. The covariates that we are interested in are age at recruitment (age), sex (male), history of needle sharing (needle), history of drug injection before recruitment (inject), and whether a subject has been in jail at the time of recruitment (jail). The dataset also contains two variables, ltime and rtime, that record, respectively, the last time of blood test when the HIV-1 was seronegative and the first time of blood test when the HIV-1 was seropositive.

. use https://www.stata-press.com/data/r18/idu (Modified Bangkok IDU Preparatory Study)

Contains data from https://www.stata-press.com/data/r18/idu.dta

Observations: 1,124 Modified Bangkok IDU Preparatory Study Variables: 15 Dec 2022 13:34 (\_dta has notes)

Variable name	Storage type	Display format	Value label	Variable label
age male needle jail inject ltime rtime age_mean	byte byte byte byte byte double double	%8.0g %8.0g %8.0g %8.0g %8.0g %10.0g %10.0g	yesno yesno yesno yesno	Age (in years) Male Shared needles Imprisoned at recruitment Injected drugs before recruitment Last time seronegative for HIV-1 First time seropositive for HIV-1 Centered age (in years)

Sorted by:

We want to use stintcox to fit the Cox proportional hazards model in which the time to HIV-1 infection depends on age, male, needle, inject, and jail. To make the interpretation of the baseline hazard function more meaningful, we will use the centered age variable, age\_mean. (Remember that a baseline hazard function corresponds to all covariates equal to zero, and age of zero would not make sense for our sample of subjects. See Making baseline reasonable in [ST] stcox **postestimation** for more details.)

Unlike stcox's specification, in which the survival variables are set using stset and do not appear in the command, for single-record-per-subject, the interval time variables ltime and rtime must be specified in the stintcox's option interval(). Also, recall that the command implements an NPMLE method, which is computationally intensive, so it may take a little longer to run on our dataset of 1,124 observations:

Number of obs = 1.124

. stintcox age\_mean i.male i.needle i.inject i.jail, interval(ltime rtime) note: using adaptive step size to compute derivatives.

Performing EM optimization (showing every 100 iterations):

Iteration 0: Log likelihood = -1086.2564 Iteration 100: Log likelihood = -597.65634 Iteration 200: Log likelihood = -597.57555 Iteration 295: Log likelihood = -597.56443

Computing standard errors: ..... done

Interval-censored Cox regression Baseline hazard: Reduced intervals Uncensored = Ω Left-censored = 41 Event-time interval: Right-censored = 991 Lower endpoint: ltime Interval-cens. =

Upper endpoint: rtime

Wald chi2(5) = 17.10= 0.0043Log likelihood = -597.56443Prob > chi2

	T					
		OPG				
	Haz. ratio	std. err.	Z	P> z	[95% conf.	interval]
age_mean	.9684341	.0126552	-2.45	0.014	.9439452	.9935582
male Yes	. 6846949	.1855907	-1.40	0.162	.4025073	1.164717
needle Yes	1.275912	.2279038	1.36	0.173	.8990401	1.810768
inject Yes	1.250154	. 2414221	1.16	0.248	.8562184	1.825334
jail Yes	1.567244	.3473972	2.03	0.043	1.014982	2.419998

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

The header above the coefficient table summarizes censored observations. Because this is a singlerecord-per-subject dataset, the number of observations equals the number of subjects. There are 991 subjects who did not test positive for HIV-1 by the last visit, resulting in right-censored observations. There are 41 subjects who tested positive for HIV-1 at their first follow-up, resulting in left-censored observations. The remaining 92 subjects are interval-censored.

As seen in the table, age is associated with lower risk of HIV-1 infection, and being in jail at enrollment is associated with higher risk of HIV-1 infection. Being a female, needle sharing, and history of drug injection are associated with the higher risk of HIV-1 infection, although we do not have strong evidence that the hazard ratios for these variables are different from 1.

The command displays a note following the command that an adaptive step size is used to compute derivatives during VCE computation. The command also displays a note following the output table about potential variability of the standard error estimates. We address all of this in more detail in Standard error estimation with interval-censored data.

stintcox uses the EM algorithm to estimate parameters. EM algorithms are known to require many iterations. Thus, by default, stintcox displays log-likelihood values every 100 iterations. You can change how often to display the iteration log by specifying the emlog(#) option. For example, emlog(1) will display every iteration. You can also use the noemlog option to suppress the iteration log.

Similarly, the progress of the VCE computation is displayed with a dot for each iteration. With more regression coefficients and with larger datasets, you may see many more dots. In this case, you may consider displaying a dot every # iterations by specifying vcedots(#). To suppress the dots, use novcedots.

## Example 2: Speed versus accuracy

By default, stintcox uses Stata's standard convergence rules for estimation of parameters. For instance, the parameter tolerance of 1e-6 is used as a stopping rule, and the Hessian scale-gradient tolerance of 1e-5 is used to check for convergence. More stringent criteria typically require more iterations and thus lead to longer execution times.

Although high accuracy of the results is important for final reporting, it may be reasonable to consider less stringent criteria during exploratory analysis in favor of speed. stintcox provides the favorspeed option for this.

```
. stintcox age_mean i.male i.needle i.inject i.jail, interval(ltime rtime)
```

> favorspeed

note: using fixed step size with a multiplier of 5 to compute derivatives.

note: using EM and VCE tolerances of 0.0001.

note: option noemhsgtolerance assumed.

Performing EM optimization (showing every 100 iterations):

Iteration 0: Log likelihood = -1086.2564
Iteration 32: Log likelihood = -598.60293

Computing standard errors: ..... done

Interval-censored Cox regression

Baseline hazard: Reduced intervals

Uncensored = 0

Left-censored = 41

Event-time interval:

Right-censored = 991

Interval-cens. =

Lower endpoint: ltime
Upper endpoint: rtime

Wald chi2(5) = 16.95 Log likelihood = -598.60293 Prob > chi2 = 0.0046

	Haz. ratio	OPG std. err.	z	P> z	[95% conf.	interval]
age_mean	.9684228	.0126481	-2.46	0.014	.9439476	.9935326
male Yes	.6853044	.1873617	-1.38	0.167	.4010197	1.17112
needle Yes	1.275045	.2272609	1.36	0.173	.899103	1.80818
inject Yes	1.251637	. 2414583	1.16	0.245	.8575707	1.826784
jail Yes	1.566873	.3479667	2.02	0.043	1.013914	2.421398

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

The above takes only a few seconds to run. And the results happen to be very similar to the results from the previous two examples.

1

1

Following the command specification, we now see additional notes about the updated convergence criteria. The EM and VCE tolerances are now 0.0001 compared with the defaults of 1e–6. The tolerance check for the Hessian scaled gradient is suppressed. And a fixed step size with the multiplier of 5 is used to compute derivatives during the VCE computation.

### Example 3: Full versus reduced sets of time intervals

By default, stintcox uses a reduced set of intervals (option reduced) to estimate the baseline hazard function. You can specify the full option to estimate the baseline hazard using all time intervals. This approach may be much more time consuming, especially for large datasets. For demonstration purposes, we will also use the favorspeed option from example 2 to speed up execution.

```
. stintcox age_mean i.male i.needle i.inject i.jail, interval(ltime rtime) full
> favorspeed
note: using fixed step size with a multiplier of 5 to compute derivatives.
note: using EM and VCE tolerances of 0.0001.
note: option noemhsgtolerance assumed.
Performing EM optimization (showing every 100 iterations):
Iteration 0:
              Log likelihood = -951.11659
Iteration 42: Log likelihood = -599.05659
Computing standard errors: ..... done
Interval-censored Cox regression
                                                    Number of obs
                                                                    = 1,124
Baseline hazard: All intervals
                                                          Uncensored =
                                                                             0
                                                                            41
                                                       Left-censored =
Event-time interval:
                                                       Right-censored =
                                                                           991
  Lower endpoint: ltime
                                                       Interval-cens. =
                                                                            92
  Upper endpoint: rtime
                                                    Wald chi2(5)
                                                                     = 16.94
Log likelihood = -599.05659
                                                    Prob > chi2
                                                                      = 0.0046
```

	Haz. ratio	OPG std. err.	z	P> z	[95% conf.	interval]
age_mean	.9685754	.0126454	-2.45	0.014	.9441053	.9936797
male Yes	. 6845339	. 1871941	-1.39	0.166	.4005194	1.169947
needle Yes	1.276357	.2276023	1.37	0.171	. 8998792	1.810339
inject Yes	1.252501	. 2416295	1.17	0.243	.8581566	1.828058
jail Yes	1.566614	.3479196	2.02	0.043	1.013734	2.42103

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

The results are very similar between the two ways of estimating the baseline hazard in this example. And EM optimization iterations change from 32 to 42 when the full option is specified.

4

## Time-varying covariates

There are two ways to incorporate time-varying covariates with stintcox. If a time-varying covariate can be represented as an interaction between a baseline covariate and a deterministic function of time, you can use the tvc() and texp() options to include such a covariate in the model. You can do this with either single-record or multiple-record interval-censored data. When you specify tvc(), stintcox provides a more memory-efficient computation and automatically incorporates the specified function of time in various calculations during estimation and postestimation.

But you may already have data in which some variables vary with time. These data will have multiple examination times for some subjects. You can use stintcox's multiple-record-per-subject syntax to analyze data with existing time-varying covariates. And you can still use the tvc() option to additionally include time-varying covariates that are deterministic functions of time by specifying baseline or existing time-varying variables in that option.

## Example 4: Multiple-record-per-subject case II interval-censored data

The dataset used in this example is an extended version of the dataset described in example 1; it has the imprisonment indicator (jail\_vary) that varies with time. The data are recorded in the multiple-record-per-subject format, where each observation records a subject identifier (id), the examination time of the blood test (time), whether the blood test changes to positive for HIV-1 since last examination time (is\_seropos), and whether the subject has been imprisoned since the last clinic visit (jail\_vary), which is a time-varying covariate. Each observation also records baseline (time-invariant) factors, such as centered age at recruitment (age\_mean) and sex (male), as described in example 1.

Here is a subset of the dataset:

- . use https://www.stata-press.com/data/r18/idu2 (Modified Bangkok IDU Preparatory Study with time-varying variable jail\_vary)
- . format time age\_mean %6.2f
- . list id time is\_seropos age\_mean male needle inject jail\_vary
- > if id >= 271 & id <= 274, sepby(id) noobs compress abbreviate(10)

id	time	is_seropos	age_mean	male	needle	inject	jail_vary
271	4.89	No	-6.46	Yes	Yes	No	No
271	9.31	No	-6.46	Yes	Yes	No	No
271	13.38	No	-6.46	Yes	Yes	No	Yes
271	17.97	No	-6.46	Yes	Yes	No	Yes
271	22.00	No	-6.46	Yes	Yes	No	No
272	3.80	No	8.54	No	No	No	Yes
272	9.41	Yes	8.54	No	No	No	No
273	3.93	No	-11.46	Yes	Yes	No	No
273	8.00	No	-11.46	Yes	Yes	No	No
273	12.07	No	-11.46	Yes	Yes	No	Yes
273	15.97	No	-11.46	Yes	Yes	No	Yes
273	20.66	No	-11.46	Yes	Yes	No	Yes
274	3.87	Yes	-4.46	Yes	Yes	Yes	Yes

There are subjects with multiple records. The examination time and event status is\_seropos define subjects' event-time intervals and censoring types. For example, subject 271 never tested positive for HIV-1 through all his examinations. He is right-censored with an event-time interval  $(22, +\infty)$ . Subject 272 tested negative at her first clinic visit but then tested positive at her second clinic visit. She is interval-censored with an event-time interval (3.80, 9.41]. Like subject 271, subject 273 is right-censored with an event-time interval  $(20.66, +\infty]$ . Finally, subject 274 tested positive at his first clinic visit, so he is left-censored with an event-time interval (0, 3.87].

We use stintcox to fit a Cox proportional hazards model in which the time to HIV-1 infection depends on baseline covariates age\_mean, male, needle, inject, and time-varying covariate jail\_vary. In a multiple-record-per-subject format, we must specify options id(), time(), and status() with stintcox.

```
. stintcox age_mean i.male i.needle i.inject i.jail_vary, id(id) time(time)
> status(is seropos)
note: time-varying covariates detected in the data; using method nearleft to
      impute their values between examination times.
note: using adaptive step size to compute derivatives.
Performing EM optimization (showing every 100 iterations):
Iteration 0:
             Log likelihood = -1086.2564
Iteration 100: Log likelihood = -598.45375
Iteration 200: Log likelihood = -598.35872
Iteration 285: Log likelihood = -598.34887
Interval-censored Cox regression
                                                Number of obs
                                                                     1,124
Baseline hazard: Reduced intervals
                                                Number of subjects =
                                                        Uncensored =
ID variable: id
                                                     Left-censored =
                                                                        41
Examination time: time
                                                    Right-censored =
                                                                       991
Status indicator: is_seropos
                                                    Interval-cens. =
                                                                        92
                                                Wald chi2(5)
                                                                     17.03
Log likelihood = -598.34887
                                                Prob > chi2
                                                                  = 0.0044
                             OPG
       time
              Haz. ratio
                          std. err.
                                         7.
                                             P>|z|
                                                       [95% conf. interval]
    age_mean
                .9714605
                            .012757
                                      -2.20
                                              0.027
                                                       .9467762
                                                                   .9967884
       male
       Yes
                .6678044
                           .1816576
                                      -1.48
                                              0.138
                                                       .3918353
                                                                   1.138138
```

Time varying: jail\_vary

needle

jail\_vary Yes

Yes inject Yes

1.271409

1.370672

1.440966

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

.2275426

.2575405

.2916178

There are 6,453 observations on 1,124 subjects, but the censoring information (as expected) is the same as in example 1. stintcox identified one time-varying covariate in this dataset—jail\_vary.

1.34

1.68

1.81

0.180

0.093

0.071

.8952546

.9484142

.9691488

1.805609

1.980928

2.142481

Compared with example 1, after we account for time-varying imprisonment, the hazard ratio for drug injection inject increases from 1.25 to 1.37, but the effect of imprisonment decreases from 1.57 for baseline jail to 1.44 for time-varying jail\_vary.

#### Standard error estimation with interval-censored data

With interval-censored data, the NPMLE approach estimates regression coefficients jointly with the contributions to the baseline cumulative hazard function, which is infinite dimensional. The inverse of the entire information matrix for all the parameters, which is typically used to estimate standard errors, does not provide a valid estimator of the variance—covariance matrix in this case.

Murphy and van der Vaart (2000) and Zeng, Mao, and Lin (2016) propose to estimate the VCE for regression coefficients using the profile log likelihood, which is obtained by maximizing the likelihood by holding the regression coefficients fixed. Four different methods based on this profile log likelihood are offered to calculate the VCE for regression coefficients. The vce(opg) method, the default, uses the first-order numerical derivatives, whereas the vce(oim) method uses the second-order numerical derivatives. When there are sufficient data to estimate the second-order derivatives reliably, the OIM method will generally provide more accurate results. With small samples, however, Zeng, Gao, and Lin (2017) found that the OIM method may lead to a negative definite matrix of second-order derivatives, which is not invertible. In addition, vce(robust) provides a VCE that is robust to some types of misspecification, and vce(cluster clustvar) provides a VCE that allows for observations to be correlated within groups defined by clustvar.

For some datasets, the choice of the step size used to compute numerical derivatives may also affect the estimates. By default, stintcox uses an adaptive step size, stepsize(adaptive), which is calculated from the data and updated during the computation of numerical derivatives. Zeng, Mao, and Lin (2016) and Zeng, Gao, and Lin (2017) found a fixed ad hoc step size of  $\delta_n = 5n^{-1/2}$  to work well in the examples they considered. You can specify the stepsize(fixed) option to use that step size. And you can also provide your own multiplier instead of the above 5 by specifying stepsize(fixed #).

When an adaptive step size is used, stintcox searches for an optimal step size to use to compute the numerical derivatives. In some cases, such as in the presence of covariates of different magnitudes, the search may lead to step sizes that are too large or too small such that the VCE matrix becomes close to being singular. In that case, you may consider trying a different search method, for example, vce(, search(bracket)), or using a fixed step size, vce(, stepsize(fixed)).

For small datasets or datasets with low proportions of interval-censored observations, the standard error estimates may be more variable between different estimation methods. In that case, you may want to compare several VCE estimation methods. stintcox provides the OPG, OIM, robust, and cluster—robust methods on replay so that you do not need to rerun the estimation command.

## Example 5: Variance estimation

Continuing with example 1, we note that the dataset contains only 92 interval-censored observations out of a total of 1,124 observations. Let's compare the OPG and OIM methods using both an adaptive and a fixed step size.

Let's refit our model using the default settings and save the estimation results for later comparisons.

```
. stintcox age_mean i.male i.needle i.inject i.jail, interval(ltime rtime)
  (output omitted)
```

To save estimation results after stintcox, we must save the estimated baseline hazard contributions in a dataset before using estimates store because they are an integral part of the estimation results. This can be done after estimation, on replay, or we could have specified the saving() option during estimation above.

```
. stintcox, saving(basehc, replace) note: file basehc.dta not found; file saved.
```

. estimates store opg\_adapt

We do not need to refit the model to compute OPG and OIM VCEs. To compute OIM estimates using an adaptive step size, we simply type

. stintcox, vce(oim, post) note: using adaptive step size to compute derivatives.

Computing standard errors: ...... done Number of obs = 1,124Interval-censored Cox regression

Baseline hazard: Reduced intervals Uncensored = Left-censored = 41

Event-time interval: Right-censored = 991 Lower endpoint: ltime Interval-cens. = 92 Upper endpoint: rtime

Wald chi2(5) = 17.10 Prob > chi2 Log likelihood = -597.56443= 0.0043

	Haz. ratio	OIM std. err.	z	P> z	[95% conf.	interval]
age_mean	.9684341	.0142537	-2.18	0.029	.9408965	.9967776
male Yes	.6846949	.2175931	-1.19	0.233	.3672728	1.276455
needle Yes	1.275912	.2688546	1.16	0.248	.8442274	1.928334
inject Yes	1.250154	.3363611	0.83	0.407	.7378069	2.118284
jail Yes	1.567244	.4679699	1.50	0.132	.8729166	2.813847

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

. estimates store oim\_adapt

We also specified the post suboption with vce(oim) to store the oim estimates in e(V) and stored the updated estimation results.

We repeat the same steps using a fixed step size for oim,

- . stintcox, vce(oim, stepsize(fixed) post) (output omitted)
- . estimates store oim\_fixed

#### and for opg,

- . stintcox, vce(opg, stepsize(fixed) post) (output omitted)
- . estimates store opg\_fixed

We compare the results using estimates table:

. е	stimates	table	opg*	oim*,	b(%9.4f)	se(%9.4f)	t	р
-----	----------	-------	------	-------	----------	-----------	---	---

Variable	opg_adapt	opg_fixed	oim_adapt	oim_fixed
age_mean	-0.0321	-0.0321	-0.0321	-0.0321
	0.0131	0.0131	0.0147	0.0120
	-2.45	-2.46	-2.18	-2.67
	0.0141	0.0141	0.0293	0.0077
male				
Yes	-0.3788	-0.3788	-0.3788	-0.3788
	0.2711	0.2736	0.3178	0.2994
	-1.40	-1.38	-1.19	-1.27
	0.1623	0.1662	0.2333	0.2058
needle				
Yes	0.2437	0.2437	0.2437	0.2437
102	0.1786	0.1784	0.2107	0.1824
	1.36	1.37	1.16	1.34
	0.1725	0.1719	0.2475	0.1817
inject				
Yes	0.2233	0.2233	0.2233	0.2233
105	0.1931	0.1933	0.2691	0.1961
	1.16	1.16	0.83	1.14
	0.2476	0.2480	0.4066	0.2548
4.47				
jail	0 4400	0 4400	0 4400	0 4400
Yes	0.4493	0.4493	0.4493	0.4493
	0.2217	0.2221	0.2986	0.2379
	2.03 0.0427	2.02 0.0431	1.50 0.1324	1.89 0.0589
	0.0427	0.0431	0.1324	0.0589

Legend: b/se/t/p

As expected, the regression coefficient estimates are the same for all VCE methods. The standard error estimates are fairly similar across all methods but a little more variable for oim\_adapt. This is not surprising because the OIM method is based on the second-order derivatives, which are estimated numerically and thus require higher tolerances to produce more accurate estimates. For instance, we can specify a slightly lower tolerance (1e-7 instead of the default 1e-6) for the oim method using an adaptive step size:

4

```
. stintcox, vce(oim, tolerance(1e-7)) nohr
note: using adaptive step size to compute derivatives.
Computing standard errors: .....
> .... done
Interval-censored Cox regression
                                           Number of obs = 1,124
Baseline hazard: Reduced intervals
                                                             0
                                                Uncensored =
                                             Left-censored =
```

Event-time interval: Right-censored = 991 Lower endpoint: ltime Interval-cens. = 92 Upper endpoint: rtime

Wald chi2(5) = 17.10 Prob > chi2 = 0.0043 Log likelihood = -597.56443

	Coefficient	OIM std. err.	z	P> z	[95% conf.	interval]
age_mean	0320749	.012257	-2.62	0.009	0560982	0080516
male Yes	378782	. 288278	-1.31	0.189	9437965	.1862326
needle Yes	. 2436616	. 1848383	1.32	0.187	1186149	.6059381
inject Yes	. 2232666	.1981754	1.13	0.260	1651501	.6116832
jail Yes	.4493186	. 2299995	1.95	0.051	0014721	.9001094

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

The standard errors are now closer to those of the other methods.

#### Current status or case I interval-censored data

Current status data or case I interval-censored data arise when each study subject is examined only once, such that the event of interest is known to occur before or after the examination time, resulting in a left- or right-censored observation. These data can be viewed as a special case of intervalcensored data without truly interval-censored observations. Current status data can be recorded in either single-record-per-subject format or multiple-record-per-subject format, but the latter is more common.

## Example 6: Case I interval-censoring

Sun (2006) investigated a dataset about calcification of the hydrogel intraocular lenses (IOL), a rarely reported complication of cataract treatment. The dataset contains 379 patients who had IOL implantation and were examined by an ophthalmologist. The status variable indicates the degree of severity of IOL calcification: 0 means no or little calcification, and 1 means mild or serious calcification. The study contains 237 females and 142 males. We want to test whether the IOL calcification is different between males and females.

Current status data usually contain two variables: one that records the examination time (time) and one that records the status of the event of interest (status). So we can analyze these data using the multiple-record-per-subject format, even though each subject has only one observation.

Alternatively, we can create two interval time variables based on examination times and event statuses and analyze the data using the single-record-per-subject format. We already have the respective interval variables, 1time and rtime, in our dataset, but see example 3 of [ST] stintreg on how to generate these variables.

In the following, we will demonstrate how to analyze current status data using both formats.

Let us fit a Cox proportional hazards model on gender using the multiple-record-per-subject format first:

```
. use https://www.stata-press.com/data/r18/iol
(Hydrogel Intraocular Lenses (IOL) Study)
. stintcox i.gender, id(id) time(time) status(status) nohr
note: using adaptive step size to compute derivatives.
Performing EM optimization (showing every 100 iterations):
              Log likelihood = -191.3847
Iteration 0:
Iteration 100: Log likelihood = -137.70653
Iteration 200: Log likelihood = -137.65739
Iteration 300: Log likelihood = -137.64799
Iteration 400: Log likelihood = -137.64509
Iteration 425: Log likelihood = -137.64472
Computing standard errors: ..... done
Interval-censored Cox regression
                                                   Number of obs
                                                                           379
                                                                           379
Baseline hazard: Reduced intervals
                                                   Number of subjects =
                                                           Uncensored =
ID variable: id
                                                        Left-censored =
                                                                            48
Examination time: time
                                                                           331
                                                       Right-censored =
Status indicator: status
                                                       Interval-cens. =
                                                   Wald chi2(1)
                                                                          0.52
Log likelihood = -137.64472
                                                   Prob > chi2
                                                                      = 0.4719
                               OPG
                                                          [95% conf. interval]
        time
              Coefficient std. err.
                                                P>|z|
      gender
      Male
                -.2242553
                            .3117387
                                        -0.72
                                                0.472
                                                          -.835252
                                                                       .3867415
```

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

We specified the nohr option to report the regression coefficient estimates instead of the hazard ratios for the independent variables to more easily compare results from the literature.

The above table reports  $\hat{\beta} = -0.2243$  and its estimated standard error of 0.312. This yields a test of  $\beta = 0$  with a p-value of 0.47, which suggests that there is no difference between males and females in terms of the time to IOL calcification.

Sun (2006) reports  $\beta = -0.2241$  with its standard error of 0.295, which are obtained by direct maximum-likelihood optimization. Our EM-based estimates are comparable.

Let's fit the same model but now use the single-record-per-subject format, where we specify the event-time intervals ltime and rtime in the interval() option:

4

```
. stintcox i.gender, interval(ltime rtime) nohr
note: using adaptive step size to compute derivatives.
Performing EM optimization (showing every 100 iterations):
Iteration 0:
              Log likelihood = -191.3847
Iteration 100: Log likelihood = -137.70653
Iteration 200: Log likelihood = -137.65739
Iteration 300: Log likelihood = -137.64799
Iteration 400: Log likelihood = -137.64509
Iteration 425: Log likelihood = -137.64472
Computing standard errors: ..... done
Interval-censored Cox regression
                                                                            379
                                                     Number of obs
Baseline hazard: Reduced intervals
                                                            Uncensored =
                                                                              0
                                                         Left-censored =
                                                                             48
Event-time interval:
                                                        Right-censored =
                                                                            331
  Lower endpoint: ltime
                                                        Interval-cens. =
  Upper endpoint: rtime
                                                     Wald chi2(1)
                                                                           0.52
Log likelihood = -137.64472
                                                     Prob > chi2
                                                                       = 0.4719
                               OPG
               Coefficient std. err.
                                            z
                                                P>|z|
                                                           [95% conf. interval]
      gender
       Male
                -.2242553
                             .3117387
                                         -0.72
                                                 0.472
                                                           -.835252
                                                                       .3867415
```

Note: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

We obtained the same results in the multiple-record-per-subject format.

# Testing the proportional-hazards assumption using option tvc()

One way of testing the proportional-hazards assumption for a covariate is to test whether the coefficient associated with that covariate is time invariant. This can be accomplished by including an interaction between that covariate and a function of time in the model and testing whether the corresponding coefficient equals zero. For instance, consider a model with one baseline covariate  $x_1$ . We wish to include an interaction between the covariate and a function of time g(t):

$$h(t) = h_0(t) \exp{\{\beta_1 x_1 + g(t)\gamma_1 x_1\}}$$

Rearranging terms results in

$$h(t) = h_0(t) \exp [\{\beta_1 + \gamma_1 g(t)\} x_1]$$

Given this new arrangement, we consider that  $\beta_1 + \gamma_1 g(t)$  is a (possibly) time-varying coefficient on the covariate  $x_1$  for some specified function of time g(t). The coefficient has a time-invariant component,  $\beta_1$ , with  $\gamma_1$  determining the magnitude of the time-varying deviations from  $\beta_1$ . Thus, a test of  $\gamma_1 = 0$  is a test of time invariance for the coefficient on  $x_1$ . Proportional hazards imply that the relative hazard is fixed over time, and this assumption would be violated if a time interaction proved significant.

The above can be easily accomplished by using the tvc() option with stintcox, where you specify the covariates suspected to violate the proportional-hazards assumption. By default, g(t)=t is used, but you can use the texp() option to specify other functions of time.

## Example 7: Testing the proportional-hazards assumption

Continuing with example 1, we now include time interaction terms for all covariates to test the proportional-hazards assumption for individual covariates and globally:

```
. stintcox age_mean i.male i.needle i.inject i.jail, interval(ltime rtime)
```

Performing EM optimization (showing every 100 iterations):

Iteration 0: Log likelihood = -1086.2564 Iteration 100: Log likelihood = -590.53655 Iteration 200: Log likelihood = -590.45163 Iteration 300: Log likelihood = -590.43665 Iteration 340: Log likelihood = -590.43386

Baseline hazard: Reduced intervals Uncensored = Left-censored = 41 991 Event-time interval: Right-censored =

Lower endpoint: ltime Upper endpoint: rtime

Interval-censored Cox regression

Wald chi2(10) = 31.99Log likelihood = -590.43386Prob > chi2 = 0.0004

	Coefficient	OPG std. err.	z	P> z	[95% conf.	interval]
main age_mean	0310177	.0233817	-1.33	0.185	076845	.0148097
male Yes	-1.271583	.4604788	-2.76	0.006	-2.174105	3690615
needle Yes	1819587	.3297493	-0.55	0.581	8282554	. 464338
inject Yes	.6852961	.3431924	2.00	0.046	.0126513	1.357941
jail Yes	529615	.4021087	-1.32	0.188	-1.317734	. 2585036
tvc age_mean	000129	.0017099	-0.08	0.940	0034804	.0032224
male Yes	.0884102	.042994	2.06	0.040	.0041434	.1726769
needle Yes	. 0358545	.0238562	1.50	0.133	0109027	.0826118
inject Yes	0361192	.0228754	-1.58	0.114	0809541	.0087157
jail Yes	.0916036	.0348915	2.63	0.009	.0232176	. 1599896

Notes: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations. Variables in tvc equation interacted with \_t.

Wald test that [tvc] = 0: chi2(5) = 13.3282

Prob > chi2 = 0.0205

Number of obs = 1,124

Interval-cens. =

<sup>&</sup>gt; tvc(age\_mean i.male i.needle i.inject i.jail) nohr note: using adaptive step size to compute derivatives.

The first equation, main, reports the coefficients for the covariates that do not vary over time; the second equation, tvc, reports the results for covariates interacted with time. We used the default function of time, q(t) = t, although we could have specified other functions with the texp() option.

Without the nohr option, the table would present results as hazard ratios for the two equations; exponentiated coefficients can be interpreted as hazard ratios when we are actually modeling timevarying covariates. However, in this case, we are using the tvc() option to incorporate time-varying coefficients; in our model, the coefficient for covariate k is expressed as  $b_k + \gamma_k \times t$ , where  $b_k$ corresponds to the parameter in the main equation and  $\gamma_k$  corresponds to the parameter in the tvc equation. The proportional-hazards assumption does not hold unless  $\gamma_k = 0$ .

There is little evidence to dispute the proportionality of hazards for the covariates age\_mean, needle, and inject with respect to this specific form of misspecification. But the proportional-hazards assumption appears to be violated for covariates male and jail.

At the bottom of the table, stintcox displays results for a Wald test for the global proportionalhazards assumption (that is, the null hypothesis is that all the coefficients in the tyc equation are equal to zero). Alternatively, you can specify the 1rphtest option to perform the likelihood-ratio test between the full model and the model without the time interaction terms:

```
. stintcox age_mean i.male i.needle i.inject i.jail, interval(ltime rtime)
> tvc(age_mean i.male i.needle i.inject i.jail) lrphtest nohr
note: using adaptive step size to compute derivatives.
Performing EM optimization (showing every 100 iterations):
Fitting main model:
Iteration 0: Log likelihood = -1086.2564
Iteration 100: Log likelihood = -597.65634
Iteration 200: Log likelihood = -597.57555
Iteration 295: Log likelihood = -597.56443
Fitting full model:
Iteration 0: Log likelihood = -1086.2564
Iteration 100: Log likelihood = -590.53655
Iteration 200: Log likelihood = -590.45163
Iteration 300: Log likelihood = -590.43665
Iteration 340: Log likelihood = -590.43386
Computing standard errors: .....
```

Log likelihood = -590.43386

Interval-censored Cox regression

Baseline hazard: Reduced intervals

Uncensored = 0

Left-censored = 41

Event-time interval:

Lower endpoint: ltime
Upper endpoint: rtime

Wald chi2(10) = 31.99

Prob > chi2

= 0.0004

	Coefficient	OPG std. err.	z	P> z	[95% conf.	interval]
main age_mean	0310177	.0233817	-1.33	0.185	076845	.0148097
male Yes	-1.271583	.4604788	-2.76	0.006	-2.174105	3690615
needle Yes	1819587	.3297493	-0.55	0.581	8282554	.464338
inject Yes	. 6852961	.3431924	2.00	0.046	.0126513	1.357941
jail Yes	529615	.4021087	-1.32	0.188	-1.317734	. 2585036
tvc age_mean	000129	.0017099	-0.08	0.940	0034804	.0032224
male Yes	.0884102	.042994	2.06	0.040	.0041434	.1726769
needle Yes	. 0358545	.0238562	1.50	0.133	0109027	.0826118
inject Yes	0361192	.0228754	-1.58	0.114	0809541	.0087157
jail Yes	.0916036	.0348915	2.63	0.009	.0232176	.1599896

Notes: Standard error estimates may be more variable for small datasets and datasets with low proportions of interval-censored observations.

Variables in tvc equation interacted with \_t.

LR test that [tvc] = 0: chi2(5) = 14.2611 Prob > chi2 = 0.0140

The results of the likelihood-ratio test agree with the previous Wald test—the proportional-hazards assumption has been violated globally. The likelihood-ratio test may be more stable for sample sizes and when the sampling distribution of regression coefficients is not symmetric.

Specifying the lrphtest option can substantially increase computational time because both the full model and the reduced main model must be fit.

1

### Stored results

stintcox stores the following in e():

```
Scalars
    e(N)
                              number of observations
                              number of subjects
    e(N_sub)
                              number of uncensored subjects
    e(N_unc)
    e(N_lc)
                              number of left-censored subjects
    e(N_rc)
                              number of right-censored subjects
                              number of interval-censored subjects
    e(N_int)
                              number of parameters
    e(k)
                              number of equations in e(b)
    e(k_eq)
                              number of equations in overall model test
    e(k_eq_model)
    e(k_dv)
                              number of dependent variables
                              model degrees of freedom
    e(df_m)
    e(df_tvc)
                              degrees of freedom for proportional-hazards test
                              log likelihood
    e(11)
                              log likelihood, constant-only model
    e(11_0)
    e(11_c)
                              log likelihood, main model
                              number of clusters
    e(N_clust)
                              \chi^2 \over \chi^2 for proportional-hazards test
    e(chi2)
    e(chi2_tvc)
                              p-value for model test
    e(p)
                              p-value for proportional-hazards test
    e(p_tvc)
                              rank of e(V)
    e(rank)
    e(ic)
                              number of iterations
    e(converged)
                              1 if converged, 0 otherwise
    e(delta)
                              multiplier used with fixed step size
    e(delta_n)
                              fixed step size
                              maximum EM iterations
    e(emiterate)
                              EM coefficient tolerance
    e(emtolerance)
    e(emltolerance)
                              EM log-likelihood tolerance
    e(emhsgtolerance)
                              EM scaled-gradient tolerance
                              1 if noemhsgtolerance, 0 otherwise
    e(noemhsgtolerance)
                              maximum VCE iterations
    e(vceiterate)
    e(vcetolerance)
                              VCE log-likelihood tolerance
Macros
    e(cmd)
                           stintcox
    e(cmdline)
                           command as typed
    e(depvar)
                           names of time interval variables specified in interval() or name
                           of examination time variable specified in time()
    e(id)
                           id variable specified in id()
    e(status)
                           name of status variable specified in status()
    e(strata)
                           strata variables
    e(clustvar)
                           name of cluster variable
                           title in estimation output
    e(title)
    e(title2)
                           secondary title in estimation output
                           Wald or LR; type of model \chi^2 test
    e(chi2type)
                           vcetype specified in vce()
    e(vce)
    e(vcetype)
                           title used to label Std. err.
    e(properties)
    e(estat_cmd)
                           program used to implement estat
    e(predict)
                           program used to implement predict
    e(marginsok)
                           predictions allowed by margins
    e(marginsnotok)
                           predictions disallowed by margins
    e(tvc)
                           covariates interacted with time from option tvc()
    e(texp)
                           function of time used for covariates from tvc()
    e(phtest)
                           Wald or LR; type of proportional-hazards assumption test
    e(tvvariables)
                           time-varying variables detected in the data
    e(tvcovimpute)
                           imputation method used for variables in e(tvvariables)
    e(intervals)
                           reduced or full
```

e(filename) name of the file with estimated baseline hazard contributions
e(stepsize) adaptive or fixed

Matrices
e(b) coefficient vector
e(V) variance—covariance matrix of the estimators

Functions
e(sample) marks estimation sample

In addition to the above, the following is stored in r():

Matrices
r(table) matrix containing the coefficients with their standard errors, test statistics, p-values,
and confidence intervals

Note that results stored in r() are updated when the command is replayed and will be replaced when any r-class command is run after the estimation command.

If the vce() option is specified on replay, the following estimation results will be updated only if vce()'s post suboption is specified: e(vce), e(vcetype), e(stepsize), e(delta) and e(delta\_n) (with fixed step size) and e(chi2\_tvc) and e(p\_tvc) (with the tvc() option).

### Methods and formulas

Methods and formulas are presented under the following headings:

Data and model EM algorithm for computing parameter estimates Variance estimation using the profile log-likelihood function Stratified estimation Option tvc()

#### Data and model

For a comprehensive review of the methods in this entry, see Zeng, Mao, and Lin (2016).

Let T denote the event time, and let  $\mathbf{x}(\cdot)$  denote a  $1 \times p$  vector of covariates that can potentially depend on time. Under the Cox proportional hazards model, the hazard function of T conditional on  $\mathbf{x}(\cdot)$  is

$$h(t; \mathbf{x}) = h_0(t) \exp\{\mathbf{x}(t)\boldsymbol{\beta}\}$$

where  $\beta$  is a  $p \times 1$  vector of unknown regression parameters and  $h_0(t)$  is an arbitrary baseline hazard function. Let  $H_0(t) = \int_0^t h_0(s) ds$ , which is the baseline cumulative hazard function.

The occurrence of an asymptomatic event can be detected only through periodic examinations. Let  $(T_l, T_u]$  denote the shortest time interval that brackets T, with  $T_l < T_u$ . Left-censoring is indicated by  $T_l = 0$ , and right-censoring by  $T_u = \infty$ .

Consider a study with n subjects and N observations. In a single-record-per-subject format, n = N and the data consist of  $(t_{li}, t_{ui}, \mathbf{x}_i)$  for  $i = 1, \ldots, n$ , where  $\mathbf{x}_i$  records covariate values for subject i and  $t_{li}$  and  $t_{ui}$  define the observed time interval.

In a multiple-record-per-subject format, n < N (typically) and the data consist of  $(t_{ij}, \delta_{ij}, \mathbf{x}_{ij})$  for  $i = 1, 2, \ldots, n$  and  $j = 1, 2, \ldots, n_i$ , where  $N = \sum_{i=1}^n n_i$ ,  $t_{ij}$  is the examination time j for subject i,  $\delta_{ij}$  denotes whether the event of interest occurs between  $t_{i(j-1)}$  and  $t_{ij}$  ( $t_{i(j-1)} = 0$  if j = 1), and  $\mathbf{x}_{ij}$  records covariate values at  $t_{ij}$ . Here  $\mathbf{x}_{ij}$  includes both baseline and time-varying covariates, where for baseline covariates  $\mathbf{x}_{ij} = \mathbf{x}_i$ . Pairs  $(t_{ij}, \delta_{ij})$ 's are used to form the event-time

interval endpoints  $t_{li}$ 's and  $t_{ui}$ 's as follows. If an event occurs before the first examination time  $t_{i1}$  ( $\delta_{i1}=1$ ), then  $t_{li}=0$ ,  $t_{ui}=t_{i1}$ , and subject i is left-censored. If an event does not occur by the last examination time  $t_{in_i}$  ( $\delta_{in_i}=0$ ), then  $t_{li}=t_{in_i}$ ,  $t_{ui}=+\infty$ , and subject i is right-censored. If, for  $1< j< n_i$ , an event occurs between two examination times  $t_{i(j-1)}$  and  $t_{ij}$  ( $\delta_{ij}=1$ ), then  $t_{li}=t_{i(j-1)}$ ,  $t_{ui}=t_{ij}$ , and subject i is interval-censored.

The observed-data likelihood function for  $\beta$  and  $H_0(t)$  is

$$L_n(\boldsymbol{\beta}, H_0) = \prod_{i=1}^n \left( \exp\left[-\int_0^{t_{li}} \exp\{\mathbf{x}_i(s)\boldsymbol{\beta}\}dH_0(s)\right] - \exp\left[-\int_0^{t_{ui}} \exp\{\mathbf{x}_i(s)\boldsymbol{\beta}\}dH_0(s)\right] \right)$$

where the integral is  $\infty$  if  $t_{ui} = \infty$  and  $\mathbf{x}_i(\cdot)$  is a  $1 \times p$  vector of potentially time-varying covariates for subject i. Under the NPMLE approach,  $H_0$  is regarded as a step function with nonnegative jumps  $h_1, \ldots, h_m$  at  $t_1, \ldots, t_m$ , respectively, where  $t_1 < \cdots < t_m$  are the distinct time points for all  $t_{li} > 0$  and  $t_{ui} < \infty$  for  $i = 1, \ldots, n$ . Thus, we need to maximize the function

$$L_n(\boldsymbol{\beta}, \{h_k\}) = \prod_{i=1}^n \exp\left\{-\sum_{t_k \le t_{li}} h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta})\right\} \left[1 - \exp\left\{-\sum_{t_{li} < t_k \le t_{ui}} h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta})\right\}\right]^{I(t_{ui} < \infty)}$$
(1)

where  $\mathbf{x}_{ik}^* = \mathbf{x}_i(t_k)$  are the covariate values for subject i at time  $t_k$ . For baseline covariates,  $\mathbf{x}_{ik}^* = \mathbf{x}_i$ . For time-varying covariates, if  $t_k = t_{ij}$  is one of the examination times for subject i, then  $\mathbf{x}_{ik}^* = \mathbf{x}_{ij}$ . Otherwise,  $\mathbf{x}_{ik}^*$  is imputed as described in the tvcovimpute() option in *Options*.

Direct maximization of (1) is difficult because of the lack of an analytic expression for the parameters  $h_k$  (k = 1, ..., m). And an even greater challenge is that not all  $t_{li}$  and  $t_{ui}$  are informative about the event times, so many  $h_k$ 's are zeros and thus lie on the boundary of the parameter space.

To address these challenges, Zeng, Mao, and Lin (2016) construct some latent Poisson variables that yield the same observed-data likelihood as (1). They propose the EM algorithm, in which the E-step involves simple calculations and the M-step amounts to the maximization of a weighted sum of Poisson log-likelihood functions that is strictly concave and has a closed-form solution for  $h_k$ 's.

Turnbull (1976) showed that the NPMLE of the survival distribution is unique only up to a set of intervals, which is called Turnbull's intervals (the innermost intervals or the regions of the maximal cliques). Therefore, for computational reasons, by default or if the reduced option is specified, we estimate the baseline cumulative hazard function at the endpoints of Turnbull's reduced set of intervals. Alternatively, you can specify the full option to estimate the baseline cumulative hazard function at the endpoints of all observed time intervals, which corresponds to the approach of Zeng, Mao, and Lin (2016).

## EM algorithm for computing parameter estimates

Let  $W_{ik}$   $(i=1,\ldots,n;k=1,\ldots,m)$  be independent latent Poisson random variables with means  $h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta})$ . Define  $A_i = \sum_{t_k \leq t_{li}} W_{ik}$  and  $B_i = I(t_{ui} < \infty) \sum_{t_{li} < t_k \leq t_{ui}} W_{ik}$ . The likelihood for the observed data  $\{t_{li}, t_{ui}, \mathbf{x}_i(\cdot), A_i = 0, B_i > 0\}$   $(i=1,\ldots,n)$  is

$$\prod_{i=1}^{n} \prod_{t_k < t_{li}} \Pr(W_{ik} = 0) \left\{ 1 - \Pr\left( \sum_{t_{li} < t_k < t_{ni}} W_{ik} = 0 \right) \right\}^{I(t_{ni} < \infty)}$$
(2)

which is exactly equal to (1). Thus, we can maximize (2) through an EM algorithm treating  $W_{ik}$  as missing data.

The complete-data log likelihood is

$$\sum_{i=1}^{n} \sum_{k=1}^{m} I(t_k \leq t_{ui}^*) \Big[ W_{ik} \log \big\{ h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta}) \big\} - h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta}) - \log W_{ik}! \Big]$$

where  $t_{ui}^* = I(t_{ui} < \infty)t_{ui} + I(t_{ui} = \infty)t_{li}$ . In the E-step, we evaluate the posterior means of  $W_{ik}$  as

$$w_{ik} = \begin{cases} \frac{h_k \exp\left(\mathbf{x}_{ik}^*\boldsymbol{\beta}\right)}{1 - \exp\left\{-\sum_{t_{li} < t_j \le t_{ui}} h_j \exp(\mathbf{x}_{ij}^*\boldsymbol{\beta})\right\}} & \text{if } t_{li} < t_k \le t_{ui} < \infty \\ 0 & \text{otherwise} \end{cases}$$

In the M-step, we update  $\beta$  by solving the following equation via the one-step Newton-Raphson method,

$$\sum_{i=1}^{n} \sum_{k=1}^{m} I(t_k \leq t_{ui}^*) \, w_{ik} \Big\{ \mathbf{x}_{ik}^* - \frac{\sum_{j=1}^{n} I(t_k \leq t_{uj}^*) \exp(\mathbf{x}_{jk}^* \boldsymbol{\beta}) \mathbf{x}_{jk}^*}{\sum_{j=1}^{n} I(t_k \leq t_{uj}^*) \exp(\mathbf{x}_{jk}^* \boldsymbol{\beta})} \Big\} = 0$$

and then update  $h_k$  (k = 1, ..., m) using

$$h_k = \frac{\sum_{i=1}^n I(t_k \le t_{ui}^*) w_{ik}}{\sum_{i=1}^n I(t_k \le t_{ui}^*) \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta})}$$
(3)

Setting the initial values of  $\beta$  to zeros and the initial values of  $h_k$ 's to 1/m, we iterate between the E- and M-steps until the desired convergence criteria are achieved. The convergence tolerances are described in detail in [R] **Maximize**, where emtolerance() is analogous to tolerance(), emltolerance() to ltolerance(), hsgtolerance() to nrtolerance(), and nonrtolerance to nohsgtolerance.

The observed-data log-likelihood function is calculated as

$$\log L = \sum_{i=1}^{n} \log \left\{ I(t_{li} < t_{ui})(S_{li} - S_{ui}) + I(t_{li} = t_{ui}) \sum_{k=1}^{m} I(t_{li} = t_k) h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta}) S_{li} \right\}$$

where 
$$S_{li} = \exp\left\{-\sum_{t_k \leq t_{li}} h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta})\right\}$$
 and  $S_{ui} = \exp\left\{-\sum_{t_k \leq t_{ui}} h_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta})\right\} I(t_{ui} < \infty).$ 

When there are no covariates, the above algorithm becomes iteratively updating  $h_k$  as

$$h_k = \frac{\sum_{i=1}^{n} \left\{ h_k I(t_{li} < t_k \le t_{ui} < \infty) S_{li} / (S_{li} - S_{ui}) + I(t_{li} = t_{ui} = t_k) \right\}}{\sum_{i=1}^{n} I(t_k \le t_{ui}^*)}$$

where 
$$S_{li} = \exp\{-\sum_{t_k < t_{li}} h_k\}$$
 and  $S_{ui} = \exp\{-\sum_{t_k < t_{ui}} h_k\} I(t_{ui} < \infty)$ .

With multiple-record interval-censored data, because uncensored observations are not supported, the terms with  $t_{li} = t_{ui}$  are ignored in the formulas above.

## Variance estimation using the profile log-likelihood function

Denote the estimators of  $\beta$  and  $h_k$ 's by  $\widehat{\beta}$  and  $\widehat{h}_k$ 's, respectively, for  $k = 1, \ldots, m$ . The NPMLES  $\widehat{\beta}$  and  $\widehat{H}_0(t) = \sum_{t_k \leq t} \widehat{h}_k$  are strongly consistent, and  $\widehat{\beta}$  is asymptotically normal and asymptotically efficient (Zeng, Mao, and Lin 2016). The covariance matrix of  $\widehat{\beta}$  can be estimated using profile likelihood (Murphy and van der Vaart 2000; Zeng, Mao, and Lin 2016; Zeng, Gao, and Lin 2017).

The profile log-likelihood function for  $\beta$  takes the form

$$pl(\boldsymbol{\beta}) = \sum_{i=1}^{n} \log \left[ \exp \left\{ -\sum_{t_k \leq t_{ii}} \widetilde{h}_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta}) \right\} - I(t_{iu} < \infty) \exp \left\{ -\sum_{t_k \leq t_{ui}} \widetilde{h}_k \exp(\mathbf{x}_{ik}^* \boldsymbol{\beta}) \right\} \right]$$

where  $\widetilde{h}_k$   $(k=1,\ldots,m)$  are the maximizers of (1) for the given  $\boldsymbol{\beta}$ . These maximizers are obtained from the EM algorithm described in the previous section with  $\widehat{h}_k$   $(k=1,\ldots,m)$  as the initial values and with  $\boldsymbol{\beta}$  fixed over the iterations. Specifically, we apply the same EM algorithm but hold  $\boldsymbol{\beta}$  fixed during the iterations. Thus, the only steps in the EM algorithm are to explicitly evaluate  $w_{ik}$  and to update  $h_k$  using (3).

Two likelihood-based methods, vce(opg) and vce(oim), are available to estimate the covariance matrix of  $\widehat{\beta}$ .

The oim method estimates the covariance matrix of  $\widehat{\beta}$  by the negative inverse of the Hessian matrix,  $-\{D^2pl(\widehat{\beta})\}^{-1}$ , where  $D^2$  is the second-order numerical derivative and its (j,k)th element is

$$\left[D^2 pl(\widehat{\boldsymbol{\beta}})\right]_{j,k} = \frac{pl(\widehat{\boldsymbol{\beta}}) - pl(\widehat{\boldsymbol{\beta}} + \delta_n \mathbf{e}_k) - pl(\widehat{\boldsymbol{\beta}} + \delta_n \mathbf{e}_j) + pl(\widehat{\boldsymbol{\beta}} + \delta_n \mathbf{e}_k + \delta_n \mathbf{e}_j)}{\delta_n^2}$$

where  $\mathbf{e}_j$  is the jth canonical vector in the space of  $\boldsymbol{\beta}$  and  $\delta_n$  is the step size chosen for numerical difference. Zeng, Gao, and Lin (2017) found that the above estimated matrix may be negative definite, especially in small samples. Therefore, they proposed the opg method, which estimates the covariance matrix using  $\left[\sum_{i=1}^n \{Dpl_i(\widehat{\boldsymbol{\beta}})\}^{\otimes 2}\right]^{-1}$ , where D is the first-order numerical derivative and  $\mathbf{a}^{\otimes 2} = \mathbf{a}\mathbf{a}'$ . Specifically, the jth element of D is

$$\left[Dpl_i(\widehat{\boldsymbol{\beta}})\right]_j = \frac{pl_i(\widehat{\boldsymbol{\beta}} + \delta_n \mathbf{e}_j) - pl_i(\widehat{\boldsymbol{\beta}})}{\delta_n}$$

where  $pl_i(\beta)$  is the contribution of subject i to  $pl(\beta)$ . The resulting covariance matrix estimator is guaranteed to be positive semidefinite and more robust with respect to the choice of the step size than the estimator based on the second-order numerical difference.

The stepsize() suboption of the vce() option offers different ways for you to choose the step size with the opg and oim methods when computing numerical derivatives. stepsize(adaptive), the default, uses an adaptive step size; see [M-5] **deriv**(). stepsize(fixed) uses a fixed step size  $\delta_n = 5n^{-1/2}$  (Zeng, Mao, and Lin 2016; Zeng, Gao, and Lin 2017). And stepsize(fixed #) uses a fixed step size equal to  $\# \times n^{-1/2}$ .

This command also supports the Huber/White/sandwich estimator of the variance and its clustered version using vce(robust) and vce(cluster clustvar), respectively. The sandwich estimator is calculated as  $\{D^2pl(\widehat{\boldsymbol{\beta}})\}^{-1}\sum_{m=1}^M Dpl_m(\widehat{\boldsymbol{\beta}})^{\otimes 2}\{D^2pl(\widehat{\boldsymbol{\beta}})\}^{-1}$ , where  $pl_m(\boldsymbol{\beta})$  is the contribution of the mth cluster to  $pl(\boldsymbol{\beta})$  and M is the number of clusters. For datasets where option id(idvar) has been specified, option vce(robust) has been implemented as equivalent to vce(cluster idvar).

#### Stratified estimation

stintcox with the strata() option will produce a stratified proportional hazards model for interval-censored data. Assume that within stratum q (q = 1, ..., Q), the hazard function of T is

$$h_q(t; \mathbf{x}) = h_{q0}(t) \exp{\{\mathbf{x}(t)\boldsymbol{\beta}\}}$$

where  $h_{q0}(t)$  is an unknown baseline hazard function for stratum q.

Let  $S_i$  be the stratum for subject i, which can take value from  $\{1, \ldots, Q\}$ . Analogously to (1), we maximize the function

$$\begin{split} L_n(\boldsymbol{\beta}, \{h_{qk}\}) = & \prod_{i=1}^n \prod_{q=1}^Q \exp \Big\{ - \sum_{t_{qk} \le t_{li}} h_{qk} \exp(\mathbf{x}_{iqk}^* \boldsymbol{\beta}) \Big\} \\ & \Big[ 1 - \exp \Big\{ - \sum_{t_{li} < t_{nk} \le t_{ni}} h_{qk} \exp(\mathbf{x}_{iqk}^* \boldsymbol{\beta}) \Big\} \Big]^{I(t_{ui} < \infty)} \end{split}$$

where  $\mathbf{x}_{iqk}^* = \mathbf{x}_i(t_{qk})$ . Within the NPMLE framework, a stratum-specific baseline cumulative hazard function  $H_{q0}(t) = \int_0^t h_{q0}(s) \, ds$  for  $q = 1, \dots, Q$  is regarded as a step function,  $\sum_{t_{qk} \leq t} h_{qk}$ , with nonnegative jumps  $h_{qk}$  at the distinct values  $t_{qk}$ , ordered from smallest to largest, of the observed interval endpoints from all subjects in stratum q.

## Option tvc()

Let  $\mathbf{z}(\cdot)$  be a  $1 \times q$  vector of potentially time-varying covariates specified in the tvc() option. Let  $g(\cdot)$  be the function of time specified in the texp() option. By default, g(t) = t. Let  $\mathbf{z}_i$ 's and  $\mathbf{z}_{ij}$ 's be the observed covariate values for  $\mathbf{z}(\cdot)$  in the respective single-record-per-subject and multiple-record-per-subject formats.

When the tvc() option is specified, we replace  $\mathbf{x}_{ik}^*oldsymbol{eta}$  in all formulas above with

$$\mathbf{x}_{ik}^*\boldsymbol{\beta} + g(t_k)\mathbf{z}_{ik}^*\boldsymbol{\gamma}$$

where  $\gamma$  is a  $q \times 1$  vector of unknown regression parameters for  $\mathbf{z}(\cdot)$  and  $\mathbf{z}_{ik}^* = \mathbf{z}_i(t_k)$  are the covariate values of the tvc() variables for subject i at time  $t_k$ . For baseline covariates,  $\mathbf{z}_{ik}^* = \mathbf{z}_i$ . For time-varying covariates, if  $t_k = t_{ij}$  is one of the examination times for subject i, then  $\mathbf{z}_{ik}^* = \mathbf{z}_{ij}$ . Otherwise,  $\mathbf{z}_{ik}^*$  is imputed as described in the tvcovimpute() option in *Options*.

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### Also see

- [ST] **stintcox postestimation** Postestimation tools for stintcox
- [ST] stintcox PH-assumption plots Plots of proportional-hazards assumption after stintcox
- [ST] stcurve Plot the survivor or related function after streg, stcox, and more
- [ST] **stcox** Cox proportional hazards model
- [ST] stintreg Parametric models for interval-censored survival-time data
- [U] 20 Estimation and postestimation commands

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