Title

Example 30g — Two-level measurement model (multilevel, generalized response)

Description

Remarks and examples

References

Also see

Description

We demonstrate a multilevel measurement model with the same data used in [SEM] Example 29g:

```
. use https://www.stata-press.com/data/r18/gsem_cfa (Fictional math abilities data)
```

. summarize

Variable	Obs	Mean	Std. dev.	Min	Max
school	500	10.5	5.772056	1	20
id	500	50681.71	29081.41	71	100000
q1	500	.506	.5004647	0	1
q2	500	.394	.4891242	0	1
q3	500	.534	.4993423	0	1
q4	500	.424	.4946852	0	1
q5	500	.49	.5004006	0	1
q6	500	.434	.4961212	0	1
q7	500	.52	.5001002	0	1
q8	500	.494	.5004647	0	1
att1	500	2.946	1.607561	1	5
att2	500	2.948	1.561465	1	5
att3	500	2.84	1.640666	1	5
att4	500	2.91	1.566783	1	5
att5	500	3.086	1.581013	1	5
test1	500	75.548	5.948653	55	93
test2	500	80.556	4.976786	65	94
test3	500	75.572	6.677874	50	94
test4	500	74.078	8.845587	43	96

. notes

_dta:

- 1. Fictional data on math ability and attitudes of 500 students from 20 schools.
- 2. Variables q1-q8 are incorrect/correct (0/1) on individual math questions.
- 3. Variables att1-att5 are items from a Likert scale measuring each
- student's attitude toward math.
- 4. Variables test1-test4 are test scores from tests of four different aspects of mathematical abilities. Range of scores: 0-100.

These data record results from a fictional instrument measuring mathematical ability. Variables q1 through q8 are the items from the instrument.

For discussions of multilevel measurement models, including extensions beyond the example we present here, see Mehta and Neale (2005) and Skrondal and Rabe-Hesketh (2004).

See Single-factor measurement models and Multilevel mixed-effects models in [SEM] Intro 5 for background.

Remarks and examples

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Remarks are presented under the following headings:

Fitting the two-level model Fitting the variance-components model Fitting the model with the Builder

Fitting the two-level model

We extend the measurement model fit in [SEM] **Example 29g** to better account for our (fictional) data. In the data, students are nested within school, but we have ignored that so far. In this example, we include a latent variable at the school level to account for possible school-by-school effects.

The model we wish to fit is



The double-ringed $school_1$ is new. That new component of the path diagram is saying, "I am a latent variable at the school level—meaning I am constant within school and vary across schools—and I correspond to a latent variable named M1"; see *Specifying generalized SEMs: Multilevel mixed effects (2 levels)* in [SEM] Intro 2. This new variable will account for the effect, if any, of the identity of the school.

To fit this model without this new, school-level component in it, we would type

. gsem (MathAb -> q1-q8), logit

To include the new school-level component, we add M1[school] to the exogenous variables:

```
. gsem (MathAb M1[school] -> q1-q8), logit
Fitting fixed-effects model:
Iteration 0: Log likelihood = -2750.3114
Iteration 1: Log likelihood = -2749.3709
Iteration 2: Log likelihood = -2749.3708
Refining starting values:
Grid node 0: Log likelihood = -2649.0033
Fitting full model:
Iteration 0:
             Log likelihood = -2649.0033
                                           (not concave)
Iteration 1: Log likelihood = -2645.0613
                                           (not concave)
Iteration 2: Log likelihood = -2641.9755
                                           (not concave)
Iteration 3: Log likelihood = -2634.3857
Iteration 4: Log likelihood = -2631.1111
Iteration 5: Log likelihood = -2630.7898
Iteration 6: Log likelihood = -2630.2477
Iteration 7: Log likelihood = -2630.2402
Iteration 8: Log likelihood = -2630.2074
Iteration 9: Log likelihood = -2630.2063
Iteration 10: Log likelihood = -2630.2063
Generalized structural equation model
                                                           Number of obs = 500
Response: q1
Family:
          Bernoulli
Link:
          Logit
Response: q2
Family:
          Bernoulli
Link:
          Logit
Response: q3
Family:
          Bernoulli
Link:
          Logit
Response: q4
Family:
          Bernoulli
Link:
          Logit
Response: q5
Family:
          Bernoulli
Link:
          Logit
Response: q6
Family:
          Bernoulli
Link:
          Logit
Response: q7
Family:
          Bernoulli
Link:
          Logit
Response: q8
Family:
          Bernoulli
Link:
          Logit
Log likelihood = -2630.2063
 (1)
       [q1]M1[school] = 1
 (2) [q2]MathAb = 1
```

	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
q1 M1[school]	1	(constraine	d)			
III [Belloo1]	-	(constraine)	u)			
MathAb	2.807515	.9468682	2.97	0.003	.9516878	4.663343
_cons	.0388021	.1608489	0.24	0.809	276456	.3540602
q2 M1[school]	.6673925	.3058328	2.18	0.029	.0679712	1.266814
Math∆b	1	(constraine	d)			
_cons	4631159	.1201227	-3.86	0.000	698552	2276798
q3						
M1[school]	.3555867	.3043548	1.17	0.243	2409377	.9521111
MathAb	1 /55520	5197796	2 91	0 005	1397/16	0 170316
_cons	.1537831	.1070288	1.44	0.151	0559894	.3635556
q4 M1[school]	.7073241	.3419273	2.07	0.039	.037159	1.377489
MathAb	.8420897	.3528195	2.39	0.017	.1505762	1.533603
_cons	3252735	.1202088	-2.71	0.007	5608784	0896686
q5 M1[school]	7295553	3330652	2 19	0 028	0767595	1 382351
[001001]			2.10	0.020		1.002001
MathAb	2.399529	.8110973	2.96	0.003	.8098079	3.989251
_cons	0488674	.1378015	-0.35	0.723	3189533	.2212185
q6						
M1[school]	.484903	.2844447	1.70	0.088	0725983	1.042404
MathAb	1.840627	.5934017	3.10	0.002	.6775813	3.003673
_cons	3139302	.1186624	-2.65	0.008	5465042	0813563
M1[school]	.3677241	.2735779	1.34	0.179	1684787	.903927
MathAb	0 444002	0016070	2 05	0 000	8707440	4 015201
_cons	.1062164	.1220796	0.87	0.384	1330552	.3454881
q8 M1[school]	.5851299	.3449508	1.70	0.090	0909612	1.261221
MathAb	1.606287	.536/614	2.99	0.003	.5542541 2593995	2.65832
var(0101016	1510020			050550	9561101
var(MathAb)	.2461246	.1372513			.0825055	.7342217

Notes:

- 1. The variance of M1[school] is estimated to be 0.21.
- 2. So how important is M1[school]? The variance of MathAb is estimated to be 0.25, so math ability and school have roughly the same variance, and both of course have mean 0. The math ability coefficients, meanwhile, are larger—often much larger—than the school coefficients in every case, so math ability is certainly more important than school in explaining whether questions were answered correctly. At this point, we are merely exploring the magnitude of effect.
- 3. You could also include a school-level latent variable for each question. For instance, you could type
 - . gsem (MathAb M1[school] N1[school] -> q1) ///
 (MathAb M1[school] N2[school] -> q2) ///
 (MathAb M1[school] N3[school] -> q3) ///
 (MathAb M1[school] N4[school] -> q4) ///
 (MathAb M1[school] N5[school] -> q5) ///
 (MathAb M1[school] N6[school] -> q6) ///
 (MathAb M1[school] N7[school] -> q7) ///
 (MathAb M1[school] N8[school] -> q8), logit

You will sometimes see such effects included in multilevel measurement models in theoretical discussions of models. Be aware that estimation of models with many latent variables is problematic, requiring both time and luck.

Fitting the variance-components model

In a variance-components model, school would affect math ability which would affect correctness of answers to questions. The model might be drawn like this:



The above is a great way to draw the model. Sadly, gsem cannot understand it. The problem from gsem's perspective is that one latent variable is affecting another and the two latent variables are at different levels.

So we have to draw the model differently:



The models may look different, but constraining the coefficients along the paths from math ability and from school to each question is identical in effect to the model above.

The result of fitting the model is

. gsem (MathAb > q101 c) M1[school] - 12@c2 q3@c3	·> q4@c4 q5@c5	q6@c6	q7@c7 q8	30c8), logit	
Fitting fixed-	-effects model	.:				
Iteration 0: Iteration 1: Iteration 2:	Log likelihoo Log likelihoo Log likelihoo	d = -2750.311 d = -2749.370 d = -2749.370	14 09 08			
Refining start	ing values:					
Grid node 0:	Log likelihoo	d = -2642.824	48			
Fitting full m	nodel:					
Iteration 0: Iteration 1: Iteration 2: Iteration 3: Iteration 4: Iteration 5:	Log likelihoo Log likelihoo Log likelihoo Log likelihoo Log likelihoo Log likelihoo	d = -2651.723 $d = -2644.493$ $d = -2634.5$ $d = -2633.933$ $d = -2633.592$ $d = -2633.592$	39 (not 37 92 36 24 22	concave))	
Generalized st	cructural equa	ation model			Number of	obs = 500
(output omitted)					
Log likelihood	1 = -2633.5922	2				
(1) [q1]M1 (2) [q1]Mat (3) [q2]M1 (4) [q3]M1 (5) [q4]M1 (6) [q5]M1 (7) [q6]M1 (8) [q7]M1 (9) [q8]M1	[school] = 1 :hAb = 1 [school] - [q2 [school] - [q3 [school] - [q4 [school] - [q5 [school] - [q6 [school] - [q7 [school] - [q8	<pre>2] MathAb = 0 3] MathAb = 0 4] MathAb = 0 5] MathAb = 0 5] MathAb = 0 7] MathAb = 0 7] MathAb = 0 8] MathAb = 0</pre>				
	Coefficient	Std. err.	z	P> z	[95% conf.	interval]
q1 M1[school]	1					
	-	(constrained))			
MathAb _cons	1	(constrained) (constrained) .1556214) 0.25	0.804	2664601	.3435646
MathAb _cons q2 M1[school]	1 .0385522 .3876281	(constrained) (constrained) .1556214 .1156823) 0.25 3.35	0.804	2664601	. 3435646
MathAb _cons q2 M1[school] MathAb _cons	1 .0385522 .3876281 .3876281 4633143	(constrained) (constrained) .1556214 .1156823 .1156823 .1055062) 0.25 3.35 3.35 -4.39	0.804 0.001 0.001 0.000	2664601 .1608951 .1608951 6701028	.3435646 .6143612 .6143612 2565259
MathAb _cons q2 M1[school] MathAb _cons q3 M1[school]	1 .0385522 .3876281 .3876281 4633143 .4871164	(constrained) (constrained) .1556214 .1156823 .1156823 .1055062 .1295515) 0.25 3.35 3.35 -4.39 3.76	0.804 0.001 0.001 0.000 0.000	2664601 .1608951 .1608951 6701028 .2332001	.3435646 .6143612 .6143612 2565259 .7410328
MathAb _cons q2 M1[school] MathAb _cons q3 M1[school] MathAb _cons	1 .0385522 .3876281 .3876281 4633143 .4871164 .4871164 .1533212	(constrained) (constrained) .1556214 .1156823 .1156823 .1055062 .1295515 .1295515 .1098068	0.25 3.35 3.35 -4.39 3.76 3.76 1.40	0.804 0.001 0.001 0.000 0.000 0.000 0.163	2664601 .1608951 .1608951 6701028 .2332001 .2332001 0618962	.3435646 .6143612 .6143612 2565259 .7410328 .3685386
MathAb _cons q2 M1[school] MathAb _cons q3 M1[school] MathAb _cons q4 M1[school]	1 .0385522 .3876281 .3876281 4633143 .4871164 .4871164 .1533212 .3407151	(constrained) .1556214 .1156823 .1156823 .1055062 .1295515 .1295515 .1098068 .1058542) 0.25 3.35 3.35 -4.39 3.76 3.76 1.40 3.22	0.804 0.001 0.001 0.000 0.000 0.163 0.001	2664601 .1608951 .1608951 6701028 .2332001 .2332001 0618962 .1332446	.3435646 .6143612 2565259 .7410328 .3685386 .5481856

q5 M1[school]	.8327426	.1950955	4.27	0.000	.4503624	1.215123
MathAb _cons	.8327426 0490579	.1950955 .1391324	4.27 -0.35	0.000 0.724	.4503624 3217524	1.215123 .2236365
q6 M1[school]	.6267415	.1572247	3.99	0.000	.3185868	.9348962
MathAb	.6267415	.1572247	3.99	0.000	.3185868	.9348962
_cons	3135398	.1220389	-2.57	0.010	5527317	074348
 q7						
M1[school]	.7660343	.187918	4.08	0.000	.3977219	1.134347
MathAb	.7660343	.187918	4.08	0.000	.3977219	1.134347
_cons	.1039102	.1330652	0.78	0.435	1568927	.3647131
a8						
M1[school]	.5600833	.1416542	3.95	0.000	.2824462	.8377203
MathAb	.5600833	.1416542	3.95	0.000	.2824462	.8377203
_cons	0264193	.1150408	-0.23	0.818	2518951	.1990565
var(
M1[school])	.1719347	.1150138			.0463406	.6379187
var(MathAb)	2.062489	.6900045			1.070589	3.973385

- 1. Note that for each question, the coefficient on MathAb is identical to the coefficient on M1[school].
- 2. We estimate separate variances for M1[school] and MathAb. They are 0.17 and 2.06. Now that the coefficients are the same on school and ability, we can directly compare these variances. We see that math ability has a much larger affect than does school.

Fitting the model with the Builder

Use the diagram in Fitting the two-level model above for reference.

1. Open the dataset.

In the Command window, type

- . use $https://www.stata-press.com/data/r18/gsem_cfa$
- 2. Open a new Builder diagram.

Select menu item Statistics > SEM (structural equation modeling) > Model building and estimation.

- 3. Put the Builder in gsem mode by clicking on the G_{EM}^{G} button.
- 4. Create the measurement component for MathAb.

Select the Add measurement component tool, $^{\heartsuit}$, and then click in the diagram about one-fourth of the way down from the top and slightly left of the center.

In the resulting dialog box,

- a. change the Latent variable name to MathAb;
- b. select q1, q2, q3, q4, q5, q6, q7, and q8 by using the Measurement variables control;

- c. check Make measurements generalized;
- d. select Bernoulli, Logit in the Family/Link control;
- e. select Down in the Measurement direction control;
- f. click on OK.

If you wish, move the component by clicking on any variable and dragging it.

- 5. Create the school-level latent variable.
 - a. Select the Add multilevel latent variable tool, $^{\odot}$, and click about one-fourth of the way up from the bottom and slightly left of the center.
 - b. In the Contextual Toolbar, click on the " button.
 - c. Select the nesting level and nesting variable by selecting 2 from the *Nesting depth* control and selecting school > *Observations* in the next control.
 - d. Specify M1 as the Base name.
 - e. Click on OK.
- 6. Create the factor-loading paths for the multilevel latent variable.
 - a. Select the Add path tool, -.
 - b. Click in the top-left quadrant of the double oval for school₁ (it will highlight when you hover over it), and drag a path to the bottom of the q1 rectangle (it will highlight when you can release to connect the path).
 - c. Continuing with the tool, draw paths from school₁ to each of the remaining rectangles.
- 7. Clean up paths.

If you do not like where a path has been connected to its variables, use the Select tool, \triangleright , to click on the path, and then simply click on where it connects to a rectangle or oval and drag the endpoint.

8. Estimate.

Click on the **Estimate** button, \mathbb{P} , in the Standard Toolbar, and then click on **OK** in the resulting *GSEM estimation options* dialog box.

- 9. To fit the model in *Fitting the variance-components model*, add constraints to the diagram created above.
 - a. From the SEM Builder menu, select **Estimation > Clear estimates** to clear results from the previous model.
 - b. Choose the Select tool, **b**.
 - c. Click on the path from MathAb to q1. In the Contextual Toolbar, type 1 in the β^{β} box and press *Enter*.
 - d. Click on the path from school₁ to q1. In the Contextual Toolbar, type 1 in the ${}^{\alpha\beta}$ box and press *Enter*.
 - c. Click on the path from MathAb to q2. In the Contextual Toolbar, type c2 in the ${}^{\alpha\beta}$ box and press *Enter*.
 - d. Click on the path from school₁ to q2. In the Contextual Toolbar, type c2 in the ${}^{\alpha\beta}$ box and press *Enter*.

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- e. Repeat this process to add the c3 constraint on both paths to q3, the c4 constraint on both paths to q4, ..., and the c8 constraint on both paths to q8.
- 10. Estimate again.

Click on the **Estimate** button, \mathbb{P} , in the Standard Toolbar, and then click on **OK** in the resulting *GSEM estimation options* dialog box.

You can open a completed diagram in the Builder for the first model by typing

. webgetsem gsem_mlcfa1

You can open a completed diagram in the Builder for the second model by typing

. webgetsem gsem_mlcfa2

References

Mehta, P. D., and M. C. Neale. 2005. People are variables too: Multilevel structural equations modeling. Psychological Methods 10: 259–284. https://doi.org/10.1037/1082-989X.10.3.259.

Skrondal, A., and S. Rabe-Hesketh. 2004. Generalized Latent Variable Modeling: Multilevel, Longitudinal, and Structural Equation Models. Boca Raton, FL: Chapman and Hall/CRC.

Also see

- [SEM] Example 27g Single-factor measurement model (generalized response)
- [SEM] Example 29g Two-parameter logistic IRT model
- [SEM] Intro 5 Tour of models
- [SEM] gsem Generalized structural equation model estimation command

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