Title

> **ttest** — t tests (mean-comparison tests)

Description

ttest performs t tests on the equality of means. The test can be performed for one sample against a hypothesized population mean. Two-sample tests can be conducted for paired and unpaired data. The assumption of equal variances can be optionally relaxed in the unpaired two-sample case.

ttesti is the immediate form of ttest; see [U] **19 Immediate commands**.

Quick start

Test that the mean of v1 is equal between two groups defined by catvar

        ttest v1, by(catvar)

Same as above, but assume unequal variances

        ttest v1, by(catvar) unequal

Paired t test of v2 and v3

        ttest v2 == v3

Same as above, but with unpaired data and conduct test separately for each level of catvar

        by catvar: ttest v2 == v3, unpaired

Test that the mean of v4 is 3 at the 90% confidence level

        ttest v4 == 3, level(90)

Test $\mu_1 = \mu_2$ if $\overline{x}_1 = 3.2$, $sd_1 = 0.1$, $\overline{x}_2 = 3.4$, and $sd_2 = 0.15$ with $n_1 = n_2 = 12$

        ttesti 12 3.2 .1 12 3.4 .15

Menu

ttest

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > t test (mean-comparison test)

ttesti

Statistics > Summaries, tables, and tests > Classical tests of hypotheses > t test calculator

Syntax

One-sample t test

> ttest *varname* == # $\left[\,\textit{if}\,\right]$ $\left[\,\textit{in}\,\right]$ $\left[\,,\ \underline{\texttt{l}}\texttt{evel}(\#)\,\right]$

Two-sample t test using groups

> ttest *varname* $\left[\,\textit{if}\,\right]$ $\left[\,\textit{in}\,\right]$, by(*groupvar*) $\left[\,\textit{options}_1\,\right]$

Two-sample t test using variables

> ttest *varname*$_1$ == *varname*$_2$ $\left[\,\textit{if}\,\right]$ $\left[\,\textit{in}\,\right]$, $\underline{\texttt{unp}}\texttt{aired}$ $\left[\,\underline{\texttt{une}}\texttt{qual}\ \underline{\texttt{w}}\texttt{elch}\ \underline{\texttt{l}}\texttt{evel}(\#)\,\right]$

Paired t test

> ttest *varname*$_1$ == *varname*$_2$ $\left[\,\textit{if}\,\right]$ $\left[\,\textit{in}\,\right]$ $\left[\,,\ \underline{\texttt{l}}\texttt{evel}(\#)\,\right]$

Immediate form of one-sample t test

> ttesti $\#_{\text{obs}}$ $\#_{\text{mean}}$ $\#_{\text{sd}}$ $\#_{\text{val}}$ $\left[\,,\ \underline{\texttt{l}}\texttt{evel}(\#)\,\right]$

Immediate form of two-sample t test

> ttesti $\#_{\text{obs1}}$ $\#_{\text{mean1}}$ $\#_{\text{sd1}}$ $\#_{\text{obs2}}$ $\#_{\text{mean2}}$ $\#_{\text{sd2}}$ $\left[\,,\ \textit{options}_2\,\right]$

options$_1$	Description
Main	
* by(*groupvar*)	variable defining the groups
<u>rev</u>erse	reverse group order for mean difference computation
<u>une</u>qual	unpaired data have unequal variances
<u>w</u>elch	use Welch's approximation
<u>l</u>evel(#)	set confidence level; default is level(95)

*by(*groupvar*) is required.

options$_2$	Description
Main	
<u>une</u>qual	unpaired data have unequal variances
<u>w</u>elch	use Welch's approximation
<u>l</u>evel(#)	set confidence level; default is level(95)

by and collect are allowed with ttest and ttesti; see [U] **11.1.10 Prefix commands**.

Options

    ⌐ Main ⌐

by(*groupvar*) specifies the *groupvar* that defines the two groups that ttest will use to test the hypothesis that their means are equal. Specifying by(*groupvar*) implies an unpaired (two sample) *t* test. Do not confuse the by() option with the by prefix; you can specify both.

reverse reverses the order of the mean difference between groups defined in by(). By default, the mean of the group corresponding to the largest value in the variable in by() is subtracted from the mean of the group with the smallest value in by(). reverse reverses this behavior and the order in which variables appear on the table.

unpaired specifies that the data be treated as unpaired. The unpaired option is used when the two sets of values to be compared are in different variables.

unequal specifies that the unpaired data not be assumed to have equal variances.

welch specifies that the approximate degrees of freedom for the test be obtained from Welch's formula (1947) rather than from Satterthwaite's approximation formula (1946), which is the default when unequal is specified. Specifying welch implies unequal.

level(*#*) specifies the confidence level, as a percentage, for confidence intervals. The default is level(95) or as set by set level; see [U] **20.8 Specifying the width of confidence intervals**.

Remarks and examples

Remarks are presented under the following headings:

One-sample t test

▷ Example 1

In the first form, ttest tests whether the mean of the sample is equal to a known constant under the assumption of unknown variance. Assume that we have a sample of 74 automobiles. We know each automobile's average mileage rating and wish to test whether the overall average for the sample is 20 miles per gallon.

```
. use https://www.stata-press.com/data/r18/auto
(1978 automobile data)

. ttest mpg==20

One-sample t test
```

Variable	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]
mpg	74	21.2973	.6725511	5.785503	19.9569 22.63769

```
    mean = mean(mpg)                                          t =    1.9289
H0: mean = 20                              Degrees of freedom =        73

    Ha: mean < 20              Ha: mean != 20                Ha: mean > 20
 Pr(T < t) = 0.9712       Pr(|T| > |t|) = 0.0576          Pr(T > t) = 0.0288
```

The test indicates that the underlying mean is not 20 with a significance level of 5.8%.

◁

Two-sample t test

▷ Example 2: Two-sample t test using groups

We are testing the effectiveness of a new fuel additive. We run an experiment in which 12 cars are given the fuel treatment and 12 cars are not. The results of the experiment are as follows:

treated	mpg
0	20
0	23
0	21
0	25
0	18
0	17
0	18
0	24
0	20
0	24
0	23
0	19
1	24
1	25
1	21
1	22
1	23
1	18
1	17
1	28
1	24
1	27
1	21
1	23

The treated variable is coded as 1 if the car received the fuel treatment and 0 otherwise.

We can test the equality of means of the treated and untreated group by typing

```
. use https://www.stata-press.com/data/r18/fuel3
. ttest mpg, by(treated)
```

Two-sample t test with equal variances

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]
0	12	21	.7881701	2.730301	19.26525 22.73475
1	12	22.75	.9384465	3.250874	20.68449 24.81551
Combined	24	21.875	.6264476	3.068954	20.57909 23.17091
diff		-1.75	1.225518		-4.291568 .7915684

```
    diff = mean(0) - mean(1)                                      t =  -1.4280
HO: diff = 0                                    Degrees of freedom =       22

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0837      Pr(|T| > |t|) = 0.1673        Pr(T > t) = 0.9163
```

We do not find a statistically significant difference in the means.

If we were not willing to assume that the variances were equal and wanted to use Welch's formula, we could type

```
. ttest mpg, by(treated) welch
```

Two-sample t test with unequal variances

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]
0	12	21	.7881701	2.730301	19.26525 22.73475
1	12	22.75	.9384465	3.250874	20.68449 24.81551
Combined	24	21.875	.6264476	3.068954	20.57909 23.17091
diff		-1.75	1.225518		-4.28369 .7836902

```
    diff = mean(0) - mean(1)                                      t =  -1.4280
HO: diff = 0                            Welch's degrees of freedom =  23.2465

    Ha: diff < 0                 Ha: diff != 0                 Ha: diff > 0
 Pr(T < t) = 0.0833      Pr(|T| > |t|) = 0.1666        Pr(T > t) = 0.9167
```

◁

❑ Technical note

In two-sample randomized designs, subjects will sometimes refuse the assigned treatment but still be measured for an outcome. In this case, take care to specify the group properly. You might be tempted to let *varname* contain missing where the subject refused and thus let `ttest` drop such observations from the analysis. Zelen (1979) argues that it would be better to specify that the subject belongs to the group in which he or she was randomized, even though such inclusion will dilute the measured effect.

❑

▷ Example 3: Two-sample *t* test using variables

There is a second, inferior way to organize the data in the preceding example. We ran a test on 24 cars, 12 without the additive and 12 with. We now create two new variables, mpg1 and mpg2.

mpg1	mpg2
20	24
23	25
21	21
25	22
18	23
17	18
18	17
24	28
20	24
24	27
23	21
19	23

This method is inferior because it suggests a connection that is not there. There is no link between the car with 20 mpg and the car with 24 mpg in the first row of the data. Each column of data could be arranged in any order. Nevertheless, if our data are organized like this, ttest can accommodate us.

```
. use https://www.stata-press.com/data/r18/fuel
. ttest mpg1==mpg2, unpaired
Two-sample t test with equal variances
```

Variable	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]	
mpg1	12	21	.7881701	2.730301	19.26525	22.73475
mpg2	12	22.75	.9384465	3.250874	20.68449	24.81551
Combined	24	21.875	.6264476	3.068954	20.57909	23.17091
diff		-1.75	1.225518		-4.291568	.7915684

```
    diff = mean(mpg1) - mean(mpg2)                                  t =  -1.4280
HO: diff = 0                                       Degrees of freedom =       22

   Ha: diff < 0                  Ha: diff != 0                  Ha: diff > 0
Pr(T < t) = 0.0837        Pr(|T| > |t|) = 0.1673          Pr(T > t) = 0.9163
```
◁

Paired t test

▷ Example 4

Suppose that the preceding data were actually collected by running a test on 12 cars. Each car was run once with the fuel additive and once without. Our data are stored in the same manner as in example 3, but this time, there is most certainly a connection between the mpg values that appear in the same row. These come from the same car. The variables mpg1 and mpg2 represent mileage without and with the treatment, respectively.

```
. use https://www.stata-press.com/data/r18/fuel
. ttest mpg1==mpg2
Paired t test
```

Variable	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]	
mpg1	12	21	.7881701	2.730301	19.26525	22.73475
mpg2	12	22.75	.9384465	3.250874	20.68449	24.81551
diff	12	-1.75	.7797144	2.70101	-3.46614	-.0338602

```
       mean(diff) = mean(mpg1 - mpg2)                                t =  -2.2444
  H0: mean(diff) = 0                                Degrees of freedom =       11

   Ha: mean(diff) < 0            Ha: mean(diff) != 0            Ha: mean(diff) > 0
  Pr(T < t) = 0.0232         Pr(|T| > |t|) = 0.0463         Pr(T > t) = 0.9768
```

We find that the means are statistically different from each other at any level greater than 4.6%.

◁

Two-sample t test compared with one-way ANOVA

▷ Example 5

In example 2, we saw that ttest can be used to test the equality of a pair of means; see [R] **oneway** for an extension that allows testing the equality of more than two means.

Suppose that we have data on the 50 states. The dataset contains the median age of the population (medage) and the region of the country (region) for each state. Region 1 refers to the Northeast, region 2 to the North Central, region 3 to the South, and region 4 to the West. Using oneway, we can test the equality of all four means.

```
. use https://www.stata-press.com/data/r18/census
(1980 Census data by state)
. oneway medage region
                        Analysis of variance
     Source              SS         df      MS            F      Prob > F
```

Source	SS	df	MS	F	Prob > F
Between groups	46.3961903	3	15.4653968	7.56	0.0003
Within groups	94.1237947	46	2.04616945		
Total	140.519985	49	2.8677548		

```
Bartlett's equal-variances test: chi2(3) =  10.5757    Prob>chi2 = 0.014
```

We find that the means are different, but we are interested only in testing whether the means for the Northeast (region==1) and West (region==4) are different. We could use oneway:

```
. oneway medage region if region==1 | region==4
                        Analysis of variance
     Source              SS         df      MS            F      Prob > F
```

Source	SS	df	MS	F	Prob > F
Between groups	46.241247	1	46.241247	20.02	0.0002
Within groups	46.1969169	20	2.30984584		
Total	92.4381638	21	4.40181733		

```
Bartlett's equal-variances test: chi2(1) =   2.4679    Prob>chi2 = 0.116
```

We could also use ttest:

```
. ttest medage if region==1 | region==4, by(region)
```

Two-sample t test with equal variances

Group	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]
NE	9	31.23333	.3411581	1.023474	30.44662	32.02005
West	13	28.28462	.4923577	1.775221	27.21186	29.35737
Combined	22	29.49091	.4473059	2.098051	28.56069	30.42113
diff		2.948718	.6590372		1.57399	4.323445

```
        diff = mean(NE) - mean(West)                            t =    4.4743
HO: diff = 0                                  Degrees of freedom =       20

    Ha: diff < 0                  Ha: diff != 0                  Ha: diff > 0
 Pr(T < t) = 0.9999       Pr(|T| > |t|) = 0.0002       Pr(T > t) = 0.0001
```

The significance levels of both tests are the same.

◁

Immediate form

▷ Example 6

ttesti is like ttest, except that we specify summary statistics rather than variables as arguments. For instance, we are reading an article that reports the mean number of sunspots per month as 62.6 with a standard deviation of 15.8. There are 24 months of data. We wish to test whether the mean is 75:

```
. ttesti 24 62.6 15.8 75
```

One-sample t test

	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]
x	24	62.6	3.225161	15.8	55.92825	69.27175

```
        mean = mean(x)                                          t =   -3.8448
HO: mean = 75                                 Degrees of freedom =       23

    Ha: mean < 75                  Ha: mean != 75                Ha: mean > 75
 Pr(T < t) = 0.0004       Pr(|T| > |t|) = 0.0008       Pr(T > t) = 0.9996
```

◁

▷ Example 7

There is no immediate form of `ttest` with paired data because the test is also a function of the covariance, a number unlikely to be reported in any published source. For unpaired data, however, we might type

```
. ttesti 20 20 5  32 15 4
Two-sample t test with equal variances
```

	Obs	Mean	Std. err.	Std. dev.	[95% conf. interval]	
x	20	20	1.118034	5	17.65993	22.34007
y	32	15	.7071068	4	13.55785	16.44215
Combined	52	16.92308	.6943785	5.007235	15.52905	18.3171
diff		5	1.256135		2.476979	7.523021

```
    diff = mean(x) - mean(y)                                      t =   3.9805
H0: diff = 0                                   Degrees of freedom =        50

    Ha: diff < 0                Ha: diff != 0               Ha: diff > 0
 Pr(T < t) = 0.9999      Pr(|T| > |t|) = 0.0002        Pr(T > t) = 0.0001
```

If we had typed `ttesti 20 20 5  32 15 4, unequal`, the test would have assumed unequal variances.

◁

Video examples

One-sample t test in Stata

t test for two independent samples in Stata

t test for two paired samples in Stata

One-sample t-test calculator

Two-sample t-test calculator

Stored results

`ttest` and `ttesti` store the following in `r()`:

Scalars

r(N_1)	sample size n_1	r(sd_1)	standard deviation for first variable
r(N_2)	sample size n_2	r(sd_2)	standard deviation for second variable
r(p_l)	lower one-sided p-value	r(sd)	combined standard deviation
r(p_u)	upper one-sided p-value	r(mu_1)	$\overline{x}_1$ mean for population 1
r(p)	two-sided p-value	r(mu_2)	$\overline{x}_2$ mean for population 2
r(se)	estimate of standard error	r(df_t)	degrees of freedom
r(t)	t statistic	r(level)	confidence level

Methods and formulas

See, for instance, Hoel (1984, 140–161) or Dixon and Massey (1983, 121–130) for an introduction and explanation of the calculation of these tests. Acock (2023, 165–179) and Hamilton (2013, 145–150) describe t tests using applications in Stata.

The test for $\mu = \mu_0$ for unknown σ is given by

$$t = \frac{(\overline{x} - \mu_0)\sqrt{n}}{s}$$

The statistic is distributed as Student's t with $n-1$ degrees of freedom (Gosset [Student, pseud.] 1908).

The test for $\mu_x = \mu_y$ when σ_x and σ_y are unknown but $\sigma_x = \sigma_y$ is given by

$$t = \frac{\overline{x} - \overline{y}}{\left\{ \frac{(n_x-1)s_x^2 + (n_y-1)s_y^2}{n_x + n_y - 2} \right\}^{1/2} \left(\frac{1}{n_x} + \frac{1}{n_y} \right)^{1/2}}$$

The result is distributed as Student's t with $n_x + n_y - 2$ degrees of freedom.

You could perform `ttest` (without the `unequal` option) in a regression setting given that regression assumes a homoskedastic error model. To compare with the `ttest` command, denote the underlying observations on x and y by x_j, $j = 1, \ldots, n_x$, and y_j, $j = 1, \ldots, n_y$. In a regression framework, typing `ttest` without the `unequal` option is equivalent to

1. creating a new variable z_j that represents the stacked observations on x and y (so that $z_j = x_j$ for $j = 1, \ldots, n_x$ and $z_{n_x+j} = y_j$ for $j = 1, \ldots, n_y$)
2. and then estimating the equation $z_j = \beta_0 + \beta_1 d_j + \epsilon_j$, where $d_j = 0$ for $j = 1, \ldots, n_x$ and $d_j = 1$ for $j = n_x + 1, \ldots, n_x + n_y$ (that is, $d_j = 0$ when the z observations represent x, and $d_j = 1$ when the z observations represent y).

The estimated value of β_1, b_1, will equal $\overline{y} - \overline{x}$, and the reported t statistic will be the same t statistic as given by the formula above.

The test for $\mu_x = \mu_y$ when σ_x and σ_y are unknown and $\sigma_x \neq \sigma_y$ is given by

$$t = \frac{\overline{x} - \overline{y}}{\left(s_x^2/n_x + s_y^2/n_y \right)^{1/2}}$$

The result is distributed as Student's t with ν degrees of freedom, where ν is given by (with Satterthwaite's [1946] formula)

$$\frac{\left(s_x^2/n_x + s_y^2/n_y \right)^2}{\dfrac{\left(s_x^2/n_x \right)^2}{n_x - 1} + \dfrac{\left(s_y^2/n_y \right)^2}{n_y - 1}}$$

With Welch's formula (1947), the number of degrees of freedom is given by

$$-2 + \frac{\left(s_x^2/n_x + s_y^2/n_y \right)^2}{\dfrac{\left(s_x^2/n_x \right)^2}{n_x + 1} + \dfrac{\left(s_y^2/n_y \right)^2}{n_y + 1}}$$

The test for $\mu_x = \mu_y$ for matched observations (also known as paired observations, correlated pairs, or permanent components) is given by

$$t = \frac{\overline{d}\sqrt{n}}{s_d}$$

where $\overline{d}$ represents the mean of $x_i - y_i$ and s_d represents the standard deviation. The test statistic t is distributed as Student's t with $n - 1$ degrees of freedom.

You can also use `ttest` without the `unpaired` option in a regression setting because a paired comparison includes the assumption of constant variance. The `ttest` with an unequal variance assumption does not lend itself to an easy representation in regression settings and is not discussed here. $(x_j - y_j) = \beta_0 + \epsilon_j$.

William Sealy Gosset (1876–1937) was born in Canterbury, England. He studied chemistry and mathematics at Oxford and worked as a chemist with the brewers Guinness in Dublin. Gosset became interested in statistical problems, which he discussed with Karl Pearson and later with Fisher and Neyman. He published several important papers under the pseudonym "Student", and he lent that name to the t test he invented.

Stella Cunliffe (1917–2012) was an advocate for increased understanding of the role of human nature in experiments and methodological rigor in social statistics. She was born in Battersea, England. She was the first person from her local public girls' school to attend college, obtaining a bachelor of science from the London School of Economics. Her first job was with the Danish Bacon Company during World War II, where she was in charge of bacon rations for London. After the war, she moved to Germany and again helped to ration food, this time for refugees.

She then spent a long career in quality control at the Guinness Brewing Company. Cunliffe observed that the weights of rejected casks skewed lighter. Noting that workers had to roll casks that were too light or too heavy uphill to be remade, she had the scales moved to the top of the hill. With workers able to roll rejected casks downhill, the weight of these casks began to follow a normal distribution.

After 25 years at Guinness, Cunliffe joined the British Home Office, where she would go on to become the first woman to serve as director of statistics. During her tenure at the Home Office, she emphasized applying principles of experimental design she had learned at Guinness to the study of such topics as birthrates, recidivism, and criminology. In 1975, she became the first woman to serve as president of the Royal Statistical Society.

References

Acock, A. C. 2023. *A Gentle Introduction to Stata*. Rev. 6th ed. College Station, TX: Stata Press.

Boland, P. J. 2000. William Sealy Gosset—alias 'Student' 1876–1937. In *Creators of Mathematics: The Irish Connection*, ed. K. Houston, 105–112. Dublin: University College Dublin Press.

Dixon, W. J., and F. J. Massey, Jr. 1983. *Introduction to Statistical Analysis*. 4th ed. New York: McGraw–Hill.

Earnest, A. 2017. *Essentials of a Successful Biostatistical Collaboration*. Boca Raton, FL: CRC Press.

Gosset, W. S. 1943. *"Student's" Collected Papers*. London: Biometrika Office, University College.

Gosset [Student, pseud.], W. S. 1908. The probable error of a mean. *Biometrika* 6: 1–25. https://doi.org/10.2307/2331554.

Hamilton, L. C. 2013. *Statistics with Stata: Updated for Version 12*. 8th ed. Boston: Brooks/Cole.

Hoel, P. G. 1984. *Introduction to Mathematical Statistics*. 5th ed. New York: Wiley.

Huber, C. 2013. Measures of effect size in Stata 13. *The Stata Blog: Not Elsewhere Classified*. http://blog.stata.com/2013/09/05/measures-of-effect-size-in-stata-13/.

Kaplan, D. M. 2019. distcomp: Comparing distributions. *Stata Journal* 19: 832–848.

Mehmetoglu, M., and T. G. Jakobsen. 2022. *Applied Statistics Using Stata: A Guide for the Social Sciences*. 2nd ed. Thousand Oaks, CA: Sage.

Pearson, E. S., R. L. Plackett, and G. A. Barnard. 1990. *'Student': A Statistical Biography of William Sealy Gosset*. Oxford: Oxford University Press.

Preece, D. A. 1982. t is for trouble (and textbooks): A critique of some examples of the paired-samples t-test. *Statistician* 31: 169–195. https://doi.org/10.2307/2987888.

Satterthwaite, F. E. 1946. An approximate distribution of estimates of variance components. *Biometrics Bulletin* 2: 110–114. https://doi.org/10.2307/3002019.

Senn, S. J., and W. Richardson. 1994. The first t-test. *Statistics in Medicine* 13: 785–803. https://doi.org/10.1002/sim.4780130802.

Welch, B. L. 1947. The generalization of 'student's' problem when several different population variances are involved. *Biometrika* 34: 28–35. https://doi.org/10.2307/2332510.

Zelen, M. 1979. A new design for randomized clinical trials. *New England Journal of Medicine* 300: 1242–1245. https://doi.org/10.1056/NEJM197905313002203.

Also see

[R] **bitest** — Binomial probability test

[R] **ci** — Confidence intervals for means, proportions, and variances

[R] **esize** — Effect size based on mean comparison

[R] **mean** — Estimate means

[R] **oneway** — One-way analysis of variance

[R] **prtest** — Tests of proportions

[R] **sdtest** — Variance-comparison tests

[R] **ztest** — z tests (mean-comparison tests, known variance)

[MV] **hotelling** — Hotelling's T^2 generalized means test